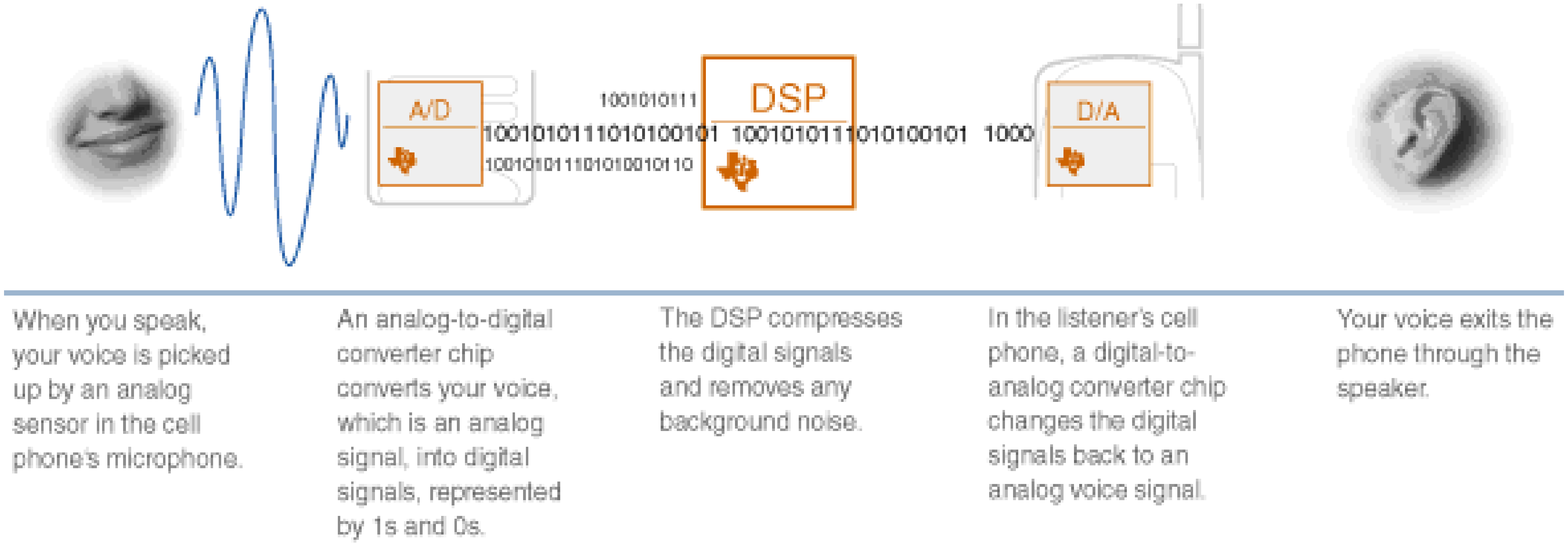


EHM 310
SAYISAL İŞARET İŞLEME
MATLAB UYGULAMALAR (2)

An LTI system is specified by the difference equation

$$y(n) = 0.8y(n-1) + x(n)$$

a. Determine $H(e^{j\omega})$.
b. Calculate and plot the steady-state response $y_{ss}(n)$ to

$$x(n) = \cos(0.05\pi n)u(n)$$

Rewrite the difference equation as $y(n) - 0.8y(n-1) = x(n)$.

a. Using (3.21), we obtain

$$H(e^{j\omega}) = \frac{1}{1 - 0.8e^{-j\omega}}$$

b. In the steady state the input is $x(n) = \cos(0.05\pi n)$ with frequency $\omega_0 = 0.05\pi$ and $\theta_0 = 0°$. The response of the system is

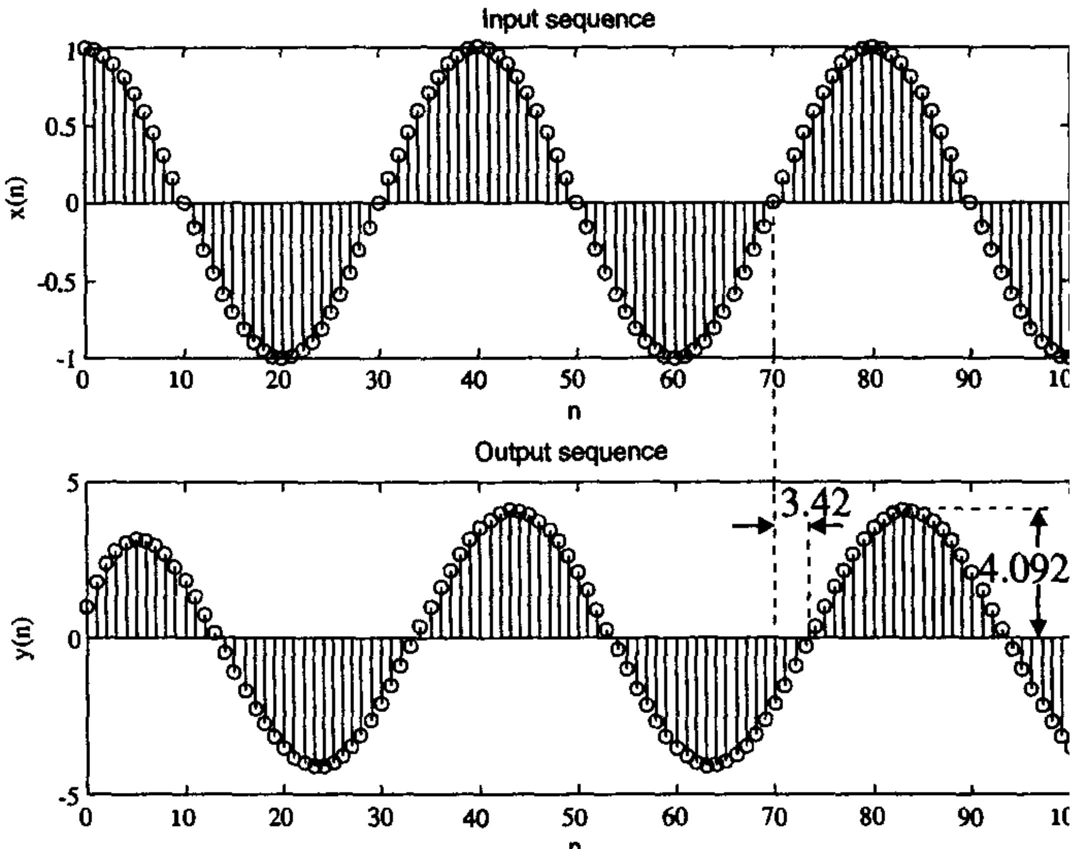
$$H(e^{j0.05\pi}) = \frac{1}{1 - 0.8e^{j0.05\pi}} = 4.0928e^{-j0.5377}$$

Therefore

$$y_{ss}(n) = 4.0928\cos(0.05\pi n - 0.5377) = 4.0928\cos\left[0.05\pi(n - 3.42)\right]$$

This means that at the output the sinusoid is scaled by 4.0928 and shifted by 3.42 samples. This can be verified using MATLAB.

```
subplot(1,1,1)
b = 1; a = [1,-0.8];
n=[0:100];x = cos(0.05*pi*n);
y = filter(b,a,x);
subplot(2,1,1); stem(n,x);
xlabel('n'); ylabel('x(n)'); title('Input sequence')
subplot(2,1,2); stem(n,y);
xlabel('n'); ylabel('y(n)'); title('Output sequence')
```



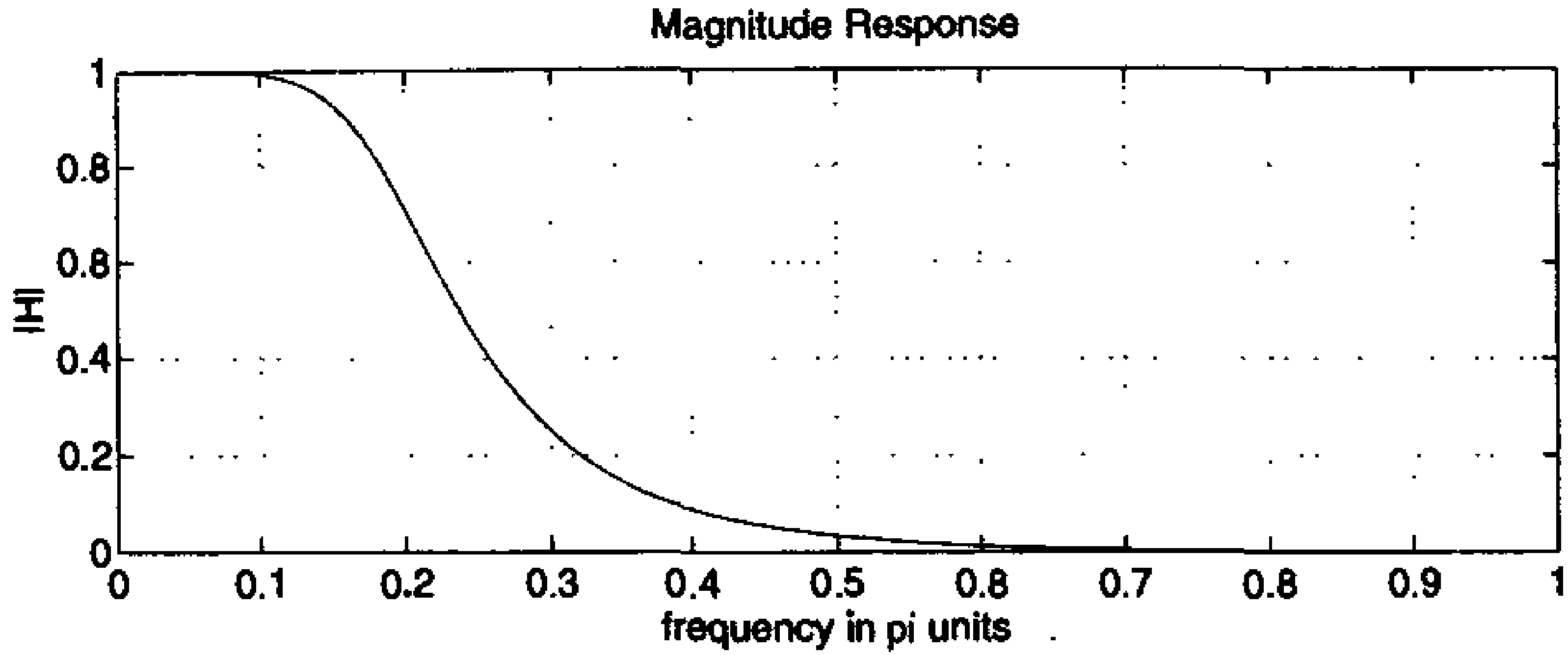
A 3rd-order lowpass filter is described by the difference equation

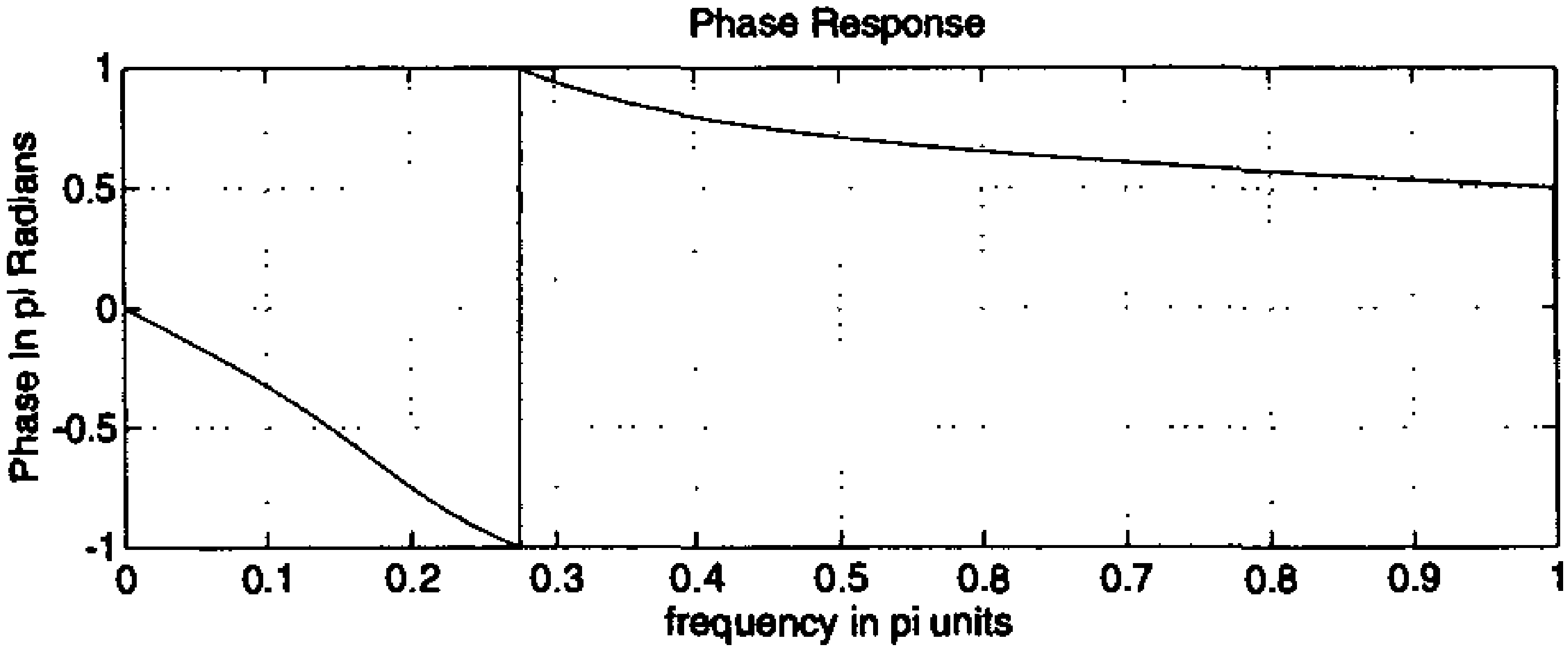
$$y(n) = 0.0181x(n) + 0.0543x(n-1) + 0.0543x(n-2) + 0.0181x(n-3)$$
$$+ 1.76y(n-1) - 1.1829y(n-2) + 0.2781y(n-3)$$

Plot the magnitude and the phase response of this filter and verify that it is a lowpass filter.

```
b = [0.0181,  0.0543, 0.0543,  0.0181]; % filter coefficient array b
a = [1.0000, -1.7600, 1.1829, -0.2781]; % filter coefficient array a
m = 0:length(b)-1; l = 0:length(a)-1;   % index arrays m and l
K = 500; k = 0:1:K;                     % index array k for frequencies
w = pi*k/K;                             % [0, pi] axis divided into 501 points.
num = b * exp(-j*m'*w);                 % Numerator calculations

den = a * exp(-j*l'*w);                 % Denominator calculations
H = num ./ den;                         % Frequency response
magH = abs(H); angH = angle(H);         % mag and phase responses
subplot(1,1,1);
subplot(2,1,1); plot(w/pi,magH); grid; axis([0,1,0,1])
xlabel('frequency in pi units'); ylabel('|H|');
title('Magnitude Response');
subplot(2,1,2); plot(w/pi,angH/pi); grid
xlabel('frequency in pi units'); ylabel('Phase in pi Radians');
title('Phase Response');
```


Magnitude Response
|H|
1
0.8
0.8
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.8
0.7
0.8
0.9
1
frequency in pi units

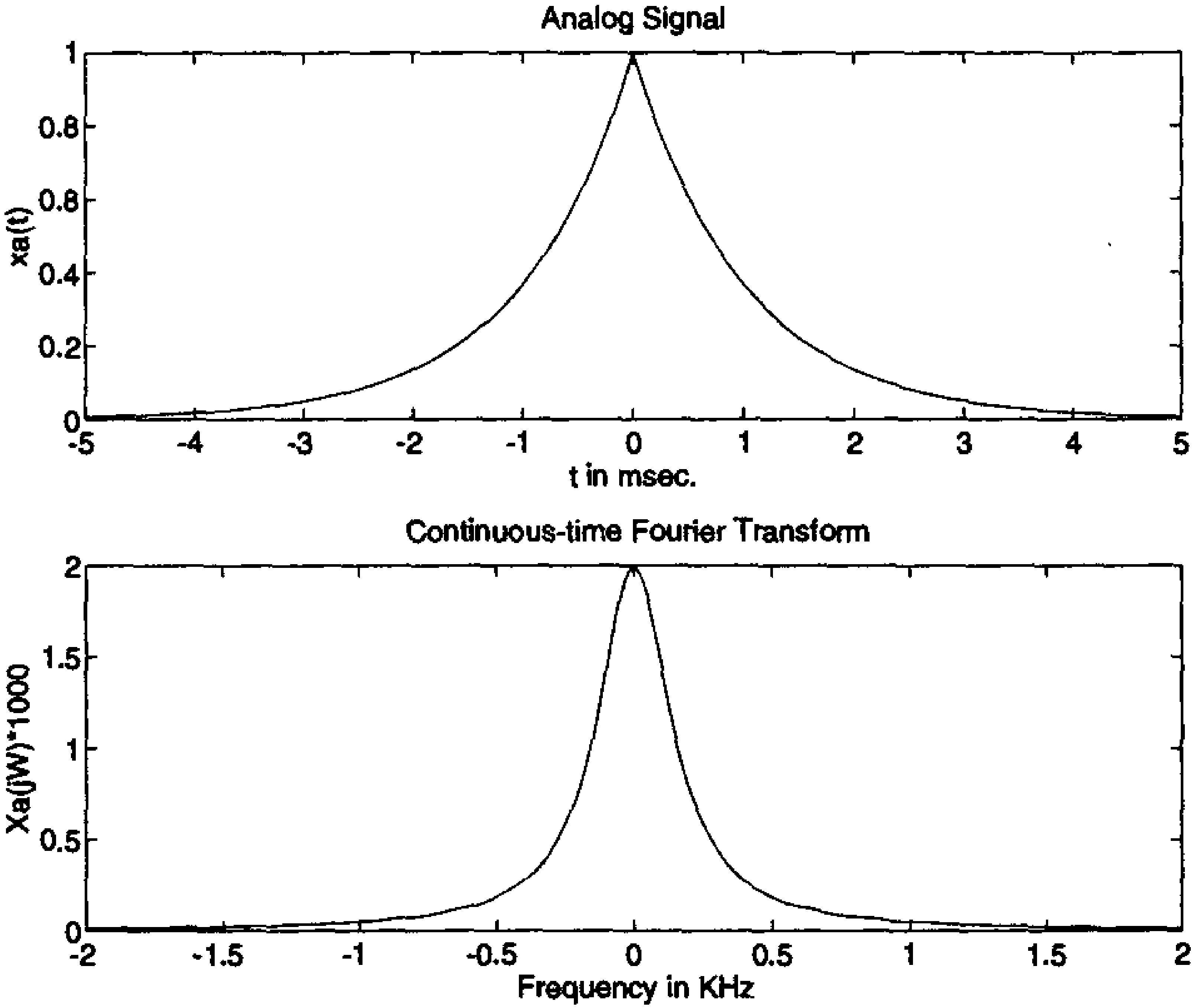

Phase Response
Phase In pi Radians
1
0.5
0
-0.5
-1
0
0.1
0.2
0.3
0.4
0.5
0.8
0.7
0.8
0.9
1
frequency in pi units

$x_a(t) = e^{-1000|t|}$. Determine and plot its Fourier transform.

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt = \int_{-\infty}^{0} e^{1000t}e^{-j\Omega t}dt + \int_{0}^{\infty} e^{-1000t}e^{-j\Omega t}dt$$

$$= \frac{0.002}{1 + \left(\frac{\Omega}{1000}\right)^2}$$

```
% Analog Signal
>> Dt = 0.00005; t = -0.005:Dt:0.005; xa = exp(-1000*abs(t));
% Continuous-time Fourier Transform
>>Wmax = 2*pi*2000; K = 500; k = 0:1:K; W = k*Wmax/K;
>>Xa = xa * exp(-j*t'*W) * Dt; Xa = real(Xa);
>>W = [-fliplr(W), W(2:501)]; % Omega from -Wmax to Wmax
>>Xa = [fliplr(Xa), Xa(2:501)]; % Xa over -Wmax to Wmax interval
>>subplot(1,1,1)
>>subplot(2,1,1);plot(t*1000,xa);
>>xlabel('t in msec.'); ylabel('xa(t)')
>>title('Analog Signal')
>>subplot(2,1,2);plot(W/(2*pi*1000),Xa*1000);
>>xlabel('Frequency in KHz'); ylabel('Xa(jW)*1000')
>>title('Continuous-time Fourier Transform')
```



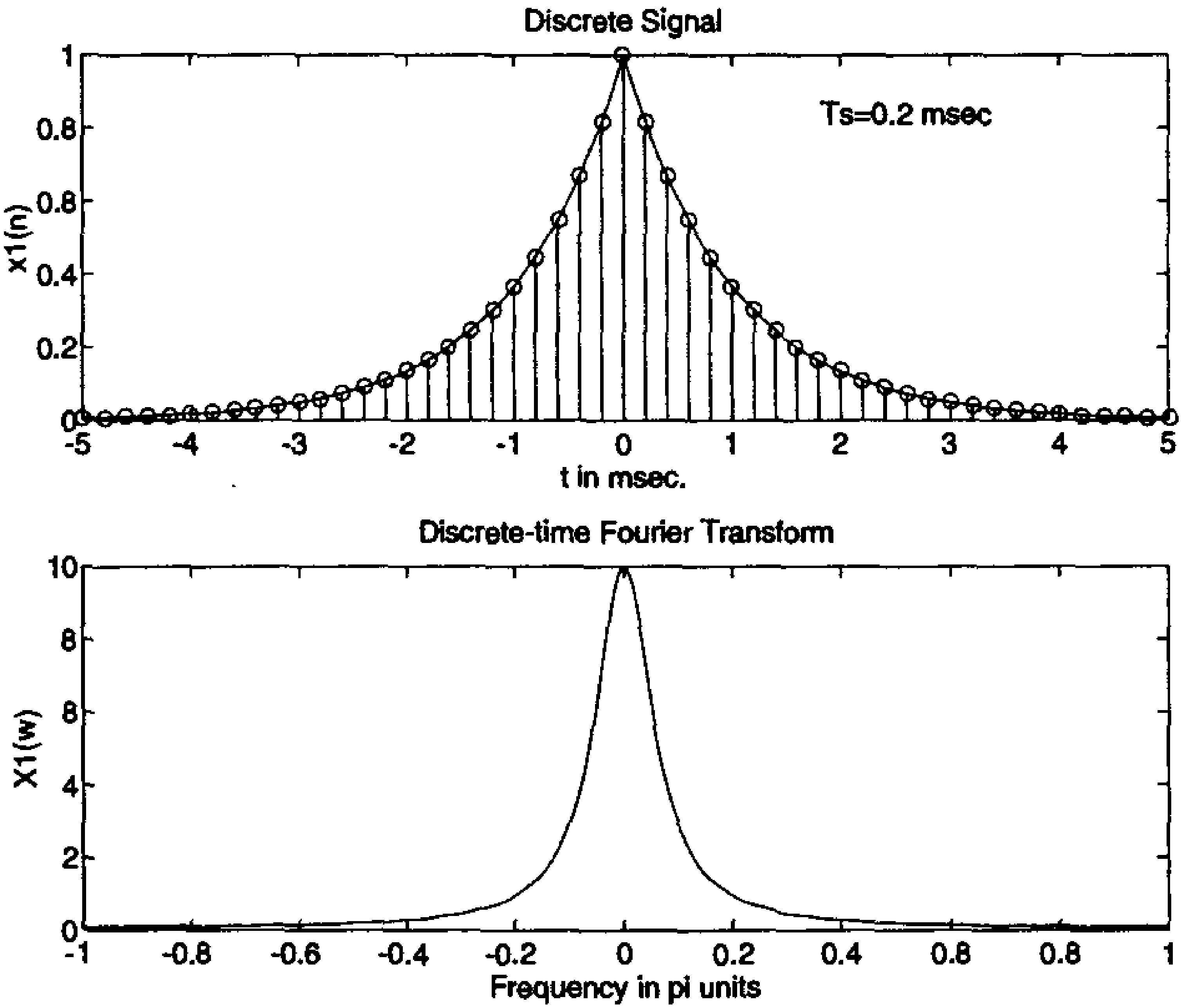
Analog Signal
xa(t)
1
0.8
0.8
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
t in msec.
Continuous-time Fourier Transform
Xa(jW)*1000
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Frequency in KHz

a. Sample $x_a(t)$ at $F_s = 5000$ sam/sec to obtain $x_1(n)$. Determine and plot $X_1(e^{j\omega})$.

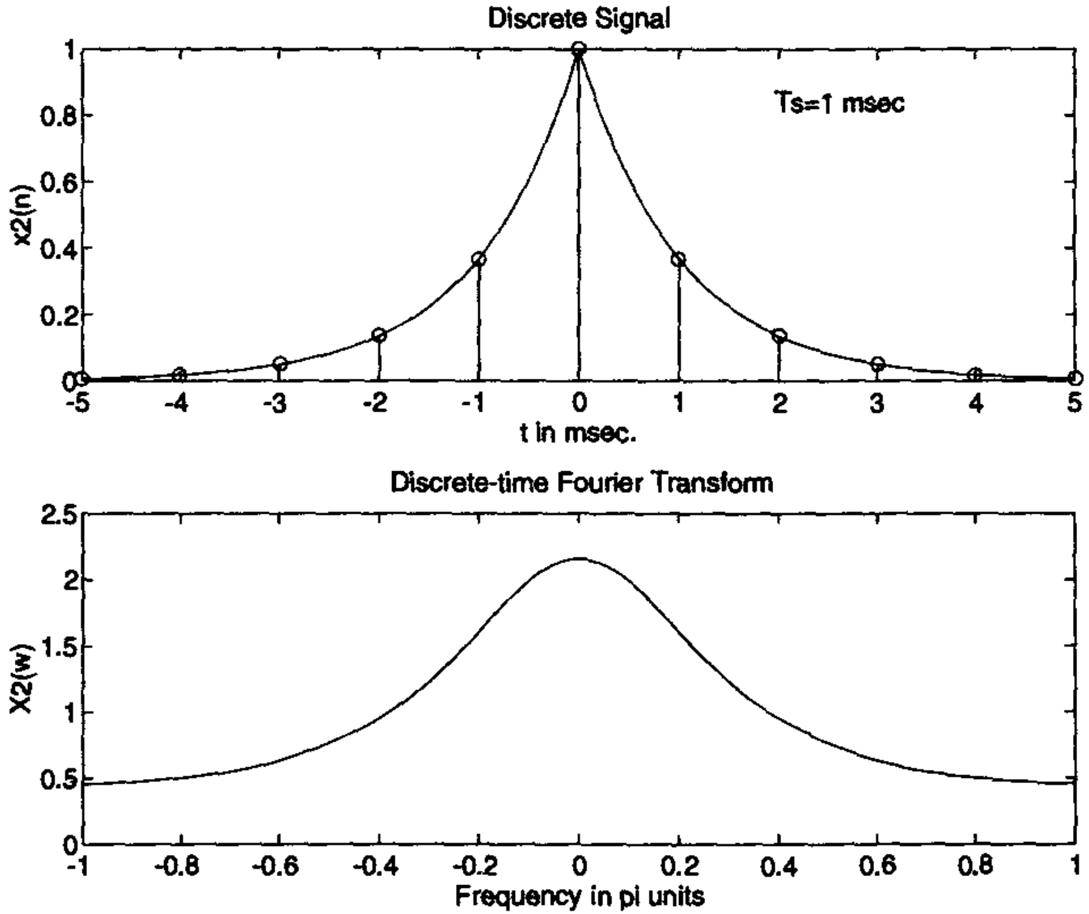
b. Sample $x_a(t)$ at $F_s = 1000$ sam/sec to obtain $x_2(n)$. Determine and plot $X_2(e^{j\omega})$.

a. Since the bandwidth of $x_a(t)$ is 2KHz, the Nyquist rate is 4000 sam/sec, which is less than the given F_s. Therefore aliasing will be (almost) nonexistent.

```
% Analog Signal
>> Dt = 0.00005; t = -0.005:Dt:0.005; xa = exp(-1000*abs(t));
% Discrete-time Signal
>> Ts = 0.0002; n = -25:1:25; x = exp(-1000*abs(n*Ts));
% Discrete-time Fourier transform
>> K = 500; k = 0:1:K; w = pi*k/K;
>> X = x * exp(-j*n'*w); X = real(X);
>> w = [-fliplr(w), w(2:K+1)];
>> X = [fliplr(X), X(2:K+1)];
>> subplot(1,1,1)
>> subplot(2,1,1);plot(t*1000,xa);
>> xlabel('t in msec.'); ylabel('x1(n)')
>> title('Discrete Signal'); hold on
>> stem(n*Ts*1000,x); gtext('Ts=0.2 msec'); hold off
>> subplot(2,1,2);plot(w/pi,X);
>> xlabel('Frequency in pi units'); ylabel('X1(w)')
>> title('Discrete-time Fourier Transform')
```


Discrete Signal
Ts=0.2 msec
x1(n)
1
0.8
0.8
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
t in msec.
Discrete-time Fourier Transform
10
8
8
4
2
0
X1(w)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.8
0.8
1
Frequency in pi units

b. Here $F_s = 1000 < 4000$. Hence there will be a considerable amount of aliasing. This is evident from Figure 3.13, in which the shape of $X(e^{j\omega})$ is different from that of $X_a(j\Omega)$ and can be seen to be a result of adding overlapping replicas of $X_a(j\Omega)$. □



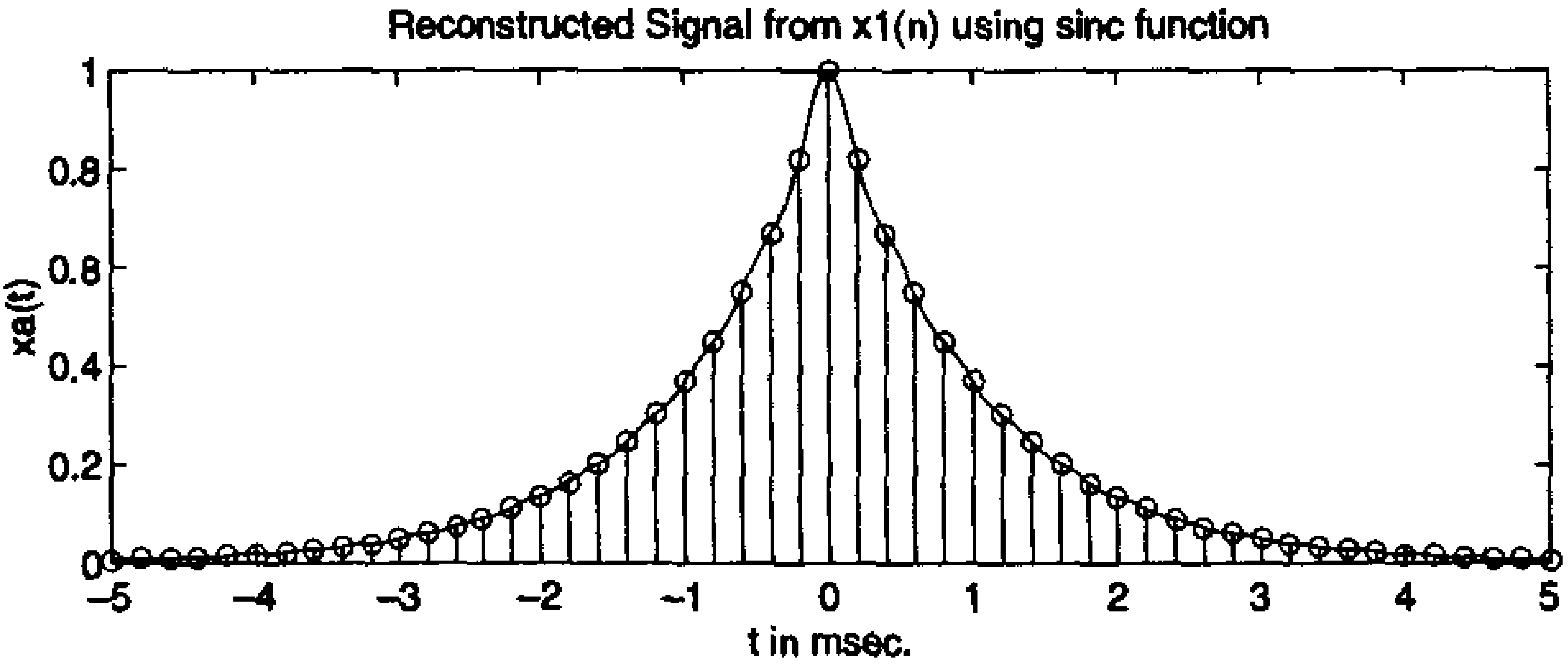
$$x_a(t) = e^{-1000|t|}.$$

Reconstruct (Fs=5000 samp/sec)

Note that $x_1(n)$ was obtained by sampling $x_a(t)$ at $T_s = 1/F_s = 0.0002$ sec. W will use the grid spacing of 0.00005 sec over $-0.005 \le t \le 0.005$, which giv $x(n)$ over $-25 \le n \le 25$.

```
% Discrete-time Signal x1(n)
>> Ts = 0.0002; n = -25:1:25; nTs = n*Ts;
>> x = exp(-1000*abs(nTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = x * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
% check
>> error = max(abs(xa - exp(-1000*abs(t))))
error =
     0.0363
```

The maximum error between the reconstructed and the actual analog signal 0.0363, which is due to the fact that $x_a(t)$ is not strictly band-limited (and al we have a finite number of samples). From Figure 3.15 we note that visuall the reconstruction is excellent.

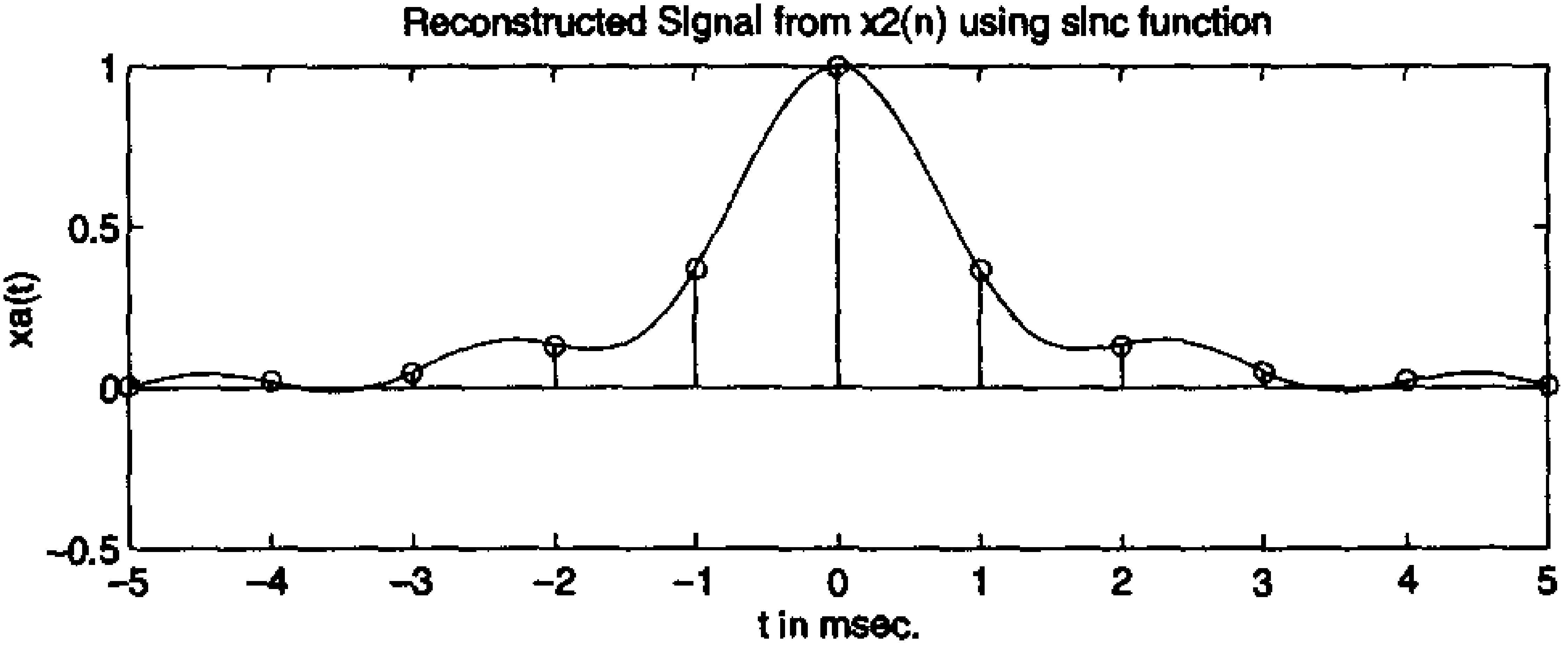

Reconstructed Signal from x1(n) using sinc function
xa(t)
1
0.8
0.8
0.4
0.2
0
−5
−4
−3
−2
−1
0
1
2
3
4
5
t in msec.

Reconstruct (Fs=1000 samp/sec) $x_a(t) = e^{-1000|t|}$.

In this case $x_2(n)$ was obtained by sampling $x_a(t)$ at $T_s = 1/F_s = 0.001$ sec. We will again use the grid spacing of 0.00005 sec over $-0.005 \le t \le 0.005$, which gives $x(n)$ over $-5 \le n \le 5$.

```
% Discrete-time Signal x2(n)
>> Ts = 0.001; n = -5:1:5; nTs = n*Ts;
>> x = exp(-1000*abs(nTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = x * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
% check
>> error = max(abs(xa - exp(-1000*abs(t))))
error =
     0.1852
```

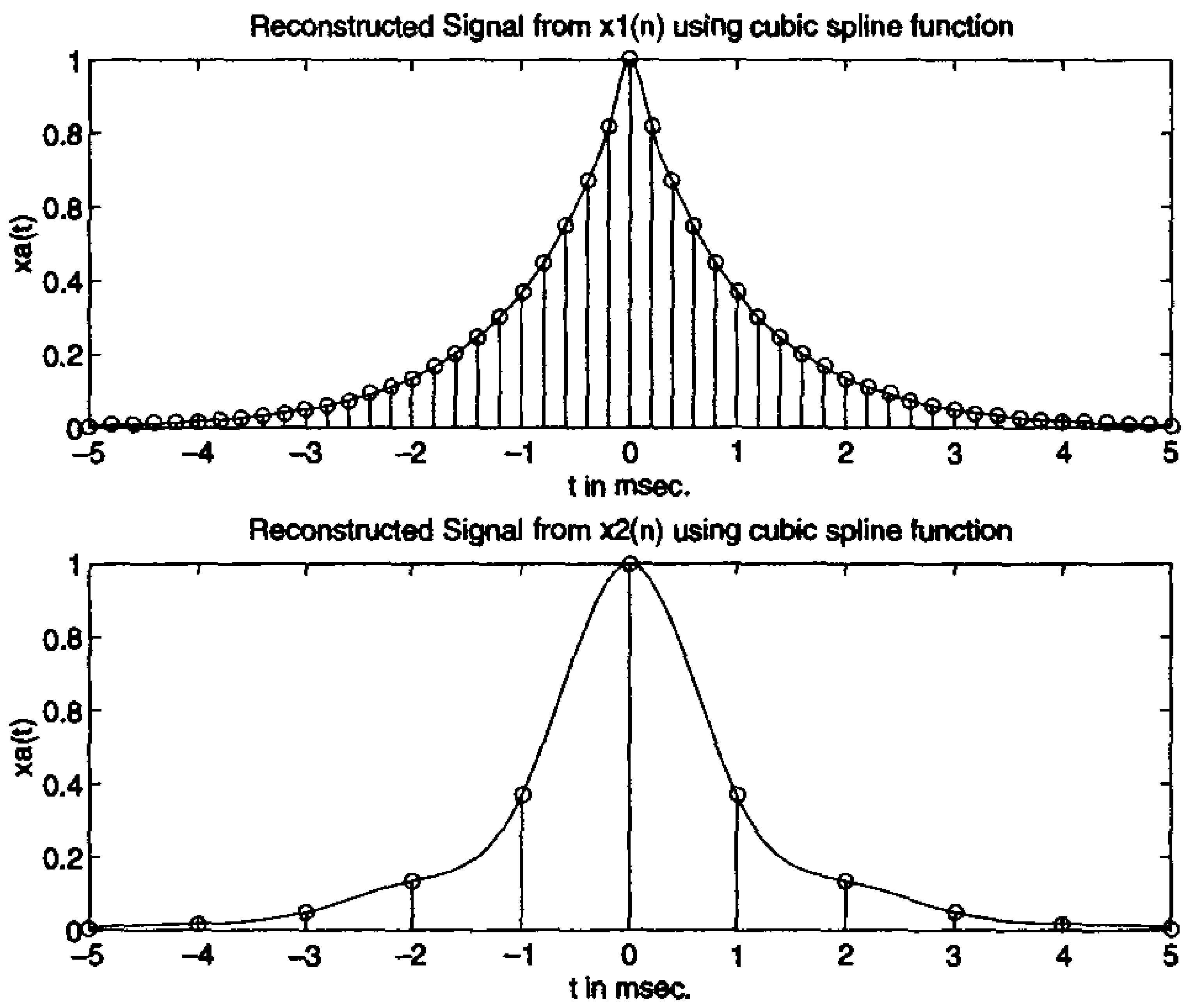
The maximum error between the reconstructed and the actual analog signal is 0.1852, which is significant and cannot be attributed to the nonband-limitedness of $x_a(t)$ alone. From Figure 3.16 observe that the reconstructed signal differs from the actual one in many places over the interpolated regions. This is the visual demonstration of aliasing in the time domain. □



Reconstructed Signal from x2(n) using sinc function
1
0.5
0
-0.5
xa(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5
t in msec.

```
% a) Discrete-time Signal x1(n): Ts = 0.0002
>> Ts = 0.0002; n = -25:1:25; nTs = n*Ts;
>> x = exp(-1000*abs(nTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = spline(nTs,x,t);
% check
>> error = max(abs(xa - exp(-1000*abs(t))))
error = 0.0317
```

```
% Discrete-time Signal x2(n): Ts = 0.001
>> Ts = 0.001; n = -5:1:5; nTs = n*Ts;
>> x = exp(-1000*abs(nTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = spline(nTs,x,t);
% check
>> error = max(abs(xa - exp(-1000*abs(t))))
error = 0.1679
```


Reconstructed Signal from x1(n) using cubic spline function
xa(t)
1
0.8
0.8
0.4
0.2
0
–5
–4
–3
–2
–1
0
1
2
3
4
5
t in msec.
Reconstructed Signal from x2(n) using cubic spline function
xa(t)
1
0.8
0.8
0.4
0.2
0
–5
–4
–3
–2
–1
0
1
2
3
4
5
t in msec.

Z Dönüşümü

$$X_1(z) = 2 + 3z^{-1} + 4z^{-2} \text{ and } X_2(z) = 3 + 4z^{-1} + 5z^{-2} + 6z^{-3}.$$

$$X_3(z) = X_1(z)X_2(z).$$

$$x_1(n) = \{\underset{\uparrow}{2}, 3, 4\} \qquad \text{and} \qquad x_2(n) = \{\underset{\uparrow}{3}, 4, 5, 6\}$$

```
>> x1 = [2,3,4]; x2 = [3,4,5,6];
>> x3 = conv(x1,x2)
x3 =          6     17     34     43     38     24
```

$$X_3(z) = 6 + 17z^{-1} + 34z^{-2} + 43z^{-3} + 38z^{-4} + 24z^{-5}$$

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

$$X(z) = \frac{z^{-1}}{3 - 4z^{-1} + z^{-2}} = \frac{0 + z^{-1}}{3 - 4z^{-1} + z^{-2}}$$

```
>> b = [0,1]; a = [3,-4,1];
>> [R,p,C] = residuez(b,a)
R =
    0.5000
   -0.5000
p =
    1.0000
    0.3333
c =
     []
```

$$X(z) = \frac{\frac{1}{2}}{1 - z^{-1}} - \frac{\frac{1}{2}}{1 - \frac{1}{3}z^{-1}}$$

```
>> [b,a] = residuez(R,p,C)
b =
    0.0000
    0.3333

 a =
     1.0000
    -1.3333
     0.3333
```

$$X(z) = \frac{0 + \frac{1}{3}z^{-1}}{1 - \frac{4}{3}z^{-1} + \frac{1}{3}z^{-2}} = \frac{z^{-1}}{3 - 4z^{-1} + z^{-2}} = \frac{z}{3z^2 - 4z + 1}$$

$$X(z) = \frac{1}{\left(1-0.9z^{-1}\right)^2\left(1+0.9z^{-1}\right)}, \quad |z| > 0.9$$

```
>> b = 1; a = poly([0.9,0.9,-0.9])
a =
     1.0000    -0.9000    -0.8100     0.7290
>> [R,p,C]=residuez(b,a)
R =
    0.2500
    0.5000
    0.2500
p =
    0.9000
    0.9000
   -0.9000
c =
     []
```

$$X(z) = \frac{0.25}{1-0.9z^{-1}} + \frac{0.5}{\left(1-0.9z^{-1}\right)^2} + \frac{0.25}{1+0.9z^{-1}}, \qquad |z| > 0.9$$

$$= \frac{0.25}{1-0.9z^{-1}} + \frac{0.5}{0.9} z \frac{\left(0.9z^{-1}\right)}{\left(1-0.9z^{-1}\right)^2} + \frac{0.25}{1+0.9z^{-1}}, \quad |z| > 0.9$$

$$x(n) = 0.25\,(0.9)^n\,u(n) + \frac{5}{9}\,(n+1)\,(0.9)^{n+1}\,u(n+1) + 0.25\,(-0.9)^n\,u(n)$$

$$x(n) = 0.75\,(0.9)^n\,u(n) + 0.5n\,(0.9)^n\,u(n) + 0.25\,(-0.9)^n\,u(n)$$

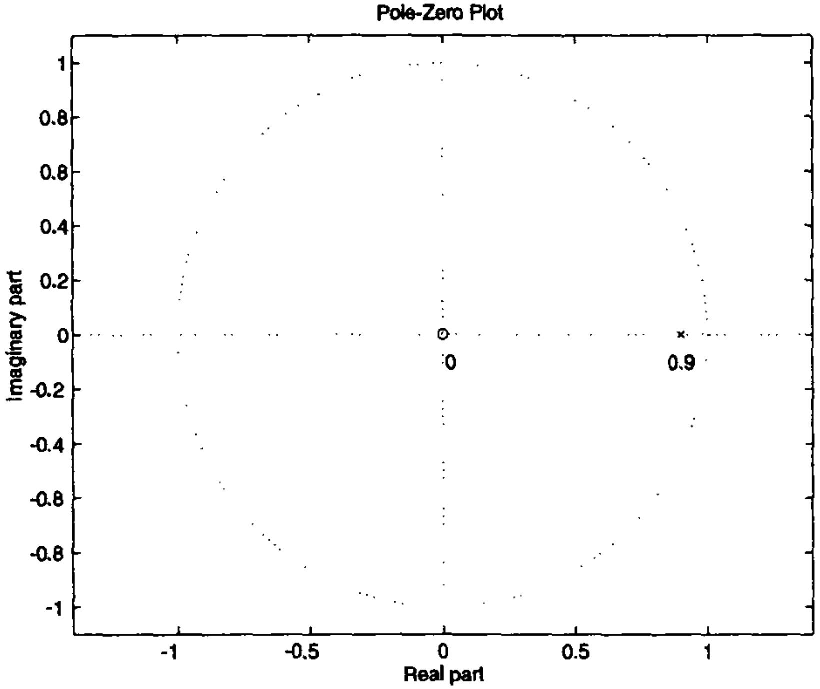
Given a causal system

$$y(n) = 0.9y(n-1) + x(n)$$

a. Find $H(z)$ and sketch its pole-zero plot.
b. Plot $\left|H(e^{j\omega})\right|$ and $\angle H(e^{j\omega})$.
c. Determine the impulse response $h(n)$.

$$y(n) - 0.9y(n-1) = x(n) \qquad H(z) = \frac{1}{1-0.9z^{-1}}; \quad |z| > 0.9$$

```
b = [1, 0]; a = [1, -0.9];
zplane(b,a)
```



```
[H,w] = freqz(b,a,100);
magH = abs(H); phaH = angle(H);
subplot(2,1,1);plot(w/pi,magH);grid
xlabel('frequency in pi units'); ylabel('Magnitude');
title('Magnitude Response')
subplot(2,1,2);plot(w/pi,phaH/pi);grid
xlabel('frequency in pi units'); ylabel('Phase in pi units');
title('Phase Response')
```

$$h(n) = \mathcal{Z}^{-1}\left[\frac{1}{1-0.9z^{-1}},\ |z| > 0.9\right] = (0.9)^n u(n)$$