Problem Set 2, M366

Problem 1 Calculate $3^{162} \mod 17$

Problem 2 Let $n \geq 0$ be an integer. Show that $(2 * 4^{3n+1} + 5^{2n+1}) \equiv 0 \pmod{13}$.

Problem 3 Let $\phi(n)$ denote the Euler Totient function applied to n.

a) Let $n = 7^3 * 5^2$. Find $\phi(n)$.

b) Let $n = 7^3 * 5^2 * 11^5$. Find $\phi(n)$.

c) Find $\phi(10000)$.

Problem 4 Use the Euclidean algorithm to:

a) Show that 100 and 31 are coprime.

b) Find an integer k with $0 < k < 100$ such that $31k \equiv 1 \mod 100$.

c) Find an integer m with $0 < m < 31$ such that $100m \equiv 1 \mod 31$.

Problem 5 a) Find the equivalence classes of integers that are units under multiplication mod 15.

b) Write out the multiplication table for the equivalence classes of integers that are units under multiplication $\mod 14$.

c) Write out the addition table for the equivalence classes of integers $\mod 5$.

Problem 6 Let G be a group.

a) Show that G has exactly one identity element.

b) Show that if $a \in G$ then a has exactly one inverse.

c) Show that if $a, b \in G$ then there is exactly one element $c \in G$ such that $ac = b$.