

Problem Set 4 - More on Cosets, Normal Subgroups, the Symmetric Group

Problem 1 Let $\sigma = (1\ 3\ 5\ 7)(2\ 4\ 6)$. Let $\tau = (5\ 3\ 1\ 2)$. Write the following product as a product of disjoint cycles

- a) Find σ^{-1}
- b) Find $\sigma^{-1}\tau\sigma$.
- c) Find $(\sigma^{-1}\tau\sigma)^2$
- d) What is the order of τ ?
- e) What is the order of $\sigma^{-1}\tau\sigma$?
- f) Find an element a such that $a^{-1}\tau a = (1\ 2\ 3\ 4)$.

Problem 2 Find the order of each of the following:

- a) $(1\ 2)(3\ 4\ 5)$
- b) $(1\ 2)(3\ 4\ 5)(6\ 7\ 8)$
- c) $(1\ 2)(3\ 4\ 5)(6\ 7\ 8\ 9)$
- d) $(1\ 2)(3\ 4\ 5)(6\ 7\ 8\ 9\ 10)$
- e) $(1\ 2\ 3\ 4)(2\ 3\ 4\ 5)(2\ 4\ 3)(1\ 2)(1\ 4)$

Problem 3 a) What are the possible orders of elements in S_5 ?

b) What are the possible orders of elements in S_6 ?

c) What is the largest order an element can have in S_n when $n = 6, 7, 8, 9, 10$?

Problem 4 Recall that if an element of S_n has cycle type $1^{\alpha_1} 2^{\alpha_2} 3^{\alpha_3} \dots k^{\alpha_k}$, then the number of elements in its conjugacy class in S_n is the same as the number of elements with the same cycle type. This number is

$$\frac{n!}{1^{\alpha_1} 2^{\alpha_2} 3^{\alpha_3} \dots k^{\alpha_k} \alpha_1! \alpha_2! \alpha_3! \dots \alpha_k!}$$

- a) How many elements in S_{10} have cycle type $3, 2, 2, 2, 1$?
- b) Give an example of an element in S_{10} with cycle type $3, 2, 2, 2, 1$.
- c) How many conjugacy classes are in S_4, S_5 and S_6 ?
- d) How many elements are in each conjugacy class of S_5 ?
- e) If $H < S_4$ then what are the possibilities for the number of elements in H ?
- f) If $H < S_5$ then what are the possibilities for the number of elements in H ?

Problem 5 a) Write $(1\ 2\ 3\ 4\ 5)$ as a product of transpositions.

b) Write $(1\ 3\ 2)(1\ 2\ 3\ 4)(1\ 2\ 3)$ as a product of transpositions in two ways, first by writing each term in the product as a product of transpositions, second by multiplying the expression out then writing the result as a product of transpositions.

c) The group A_5 has 5 conjugacy classes. The number of elements in the conjugacy classes are 1, 12, 12, 15, 20. Show that A_5 has no nontrivial normal subgroups.