Problem Set 3

P1. write product as a product of disjoint cycles

a) (1 4 2 3 5)

c) (2 3)

e) (1 3 4 2)

P2. Powers of (1 2 3 4 5 6)

This cycle has order 6, so power 1: ↑ Power 2: (1 3 5)(2 4 6)

3: (1 4)(2 5)(3 6)

4: (1 5 3)(2 6 4)

5: (1 6 5 4 3 2)

P3. Find Order

a) (1 2)(3 4)(5 6) has order of $LCM(2,2,2) = 2$

b) (1 2)(3 4 5 6) $LCM(2,4) = 4$

c) (1 2)(3 4 5) $LCM(2,3) = 6$

P4. $G = S_3$ $H = \{(),(1\ 2)\}$

a) show $H < G$ subgroups must hold (1) closer under operation (2) associative property (3) existence of identity and (4) inverse for every element

() is the identity (3) ✓

all possible operations maintain a result within the group (1) ✓

()(1 2) = (1 2)() associativity ✓

(1 2)(1 2) = () inverse ✓

b) what groups are conjugate to H in G

(1 2)H(1 2) find a group in G that is conjugate to H : gHg^{-1}

(2 3)H(2 3) → {(), (1 3)}

(1 3)H(1 3) → {(), (2 3)} and {(),(1 2)} | () is always ()

The cycles of order 3 won't make a ~~group~~ conjugate to H

c) what are the cosets of H in G

find sets s.t. $gH = \{gh \mid h \in H\}$

(1 2 3)H = {(1 2 3), (2 3)}

(1 3 2)H = {(1 3 2), (1 3)}

()H = {(), (1 2)} These add up to make the whole group G

P5.

a) Conjugacy classes of S_5

$()()()()()$ | $(1\,2)()()()$ | $(1\,2)(3\,4)()$ | 3,1,1 | 3,2 | 4,1 | 5

1,1,1,1,1 size: 1 order 1 — 2,1,1,1 size: 10 order 2 — 2,2,1 size 15 order 2 ...

b) How many conjugacy classes in S_5 7

c) How many elements in each conjugacy class of S_5 $\frac{5!}{\prod_{k=1}^{n} k^{\alpha_k} \cdot \alpha_k!}$

Problem Set ~~3~~ 4

P1. $\sigma = (1\,3\,5\,7)(2\,4\,6)$ $\tau = (5\,3\,1\,2)$

a) $\sigma^{-1} = (1\,7\,5\,3)(2\,6\,4)$

b) $\sigma^{-1}\tau\sigma = (3\,4\,7\,5)$

c) $(\sigma^{-1}\tau\sigma)^2 = (3\,7)(4\,5)$

d) order of ~~$\sigma^{-1}\tau\sigma$~~ τ 4

e) order of $\sigma^{-1}\tau\sigma$ 4

f) Find α s.t. $\alpha^{-1}\tau\alpha = (1\,2\,3\,4)$

5 3 1 2
↓ ↓ ↓ ↓
1 2 3 4 $(1\,3\,2\,4\,5)$

$(1\,5\,4\,2\,3)(5\,3\,1\,2)(1\,3\,2\,4\,5)$ ✓

P2. Find the order

a) $(1\,2)(3\,4\,5)$ LCM(2, 3) = 6

c) $(1\,2)(3\,4\,5)(6\,7\,8\,9)$ LCM(2, 3, 4) = 12.

e) $(1\,2\,3\,4)\ldots$ LCM(4, 4, 3, 2, 2) = 12

☆ P3.

a) Possible orders of elements in S_5 1, 2, 3, 6, 4, 5

b) " " in S_6 1, 2, 3, 4, 5, 6,

c) largest order an element can have in S_n when n = 6, 7, 8, 9, 10

6 : 6 7 : 4,3 → 12 8 : 5,3 → 15

9 : 5,4 → 20 10 : 5,3,2 → 30

P4.

a) How many elements in S_{10} have cycle type 3, 2, 2, 2, 1

$$\frac{10!}{3^1 2^3 1^1 \cdot 1! 3! 1!} \qquad \frac{10!}{8 \cdot 6} = \frac{10!}{48}$$

b) Give an example for ↑ (1 2 3)(4 5)(6 7)(8 9)

c) How many conjugacy classes in S_4, S_5, and S_6

S_4 : 5 $\quad S_5$: 7 $\quad S_6$: 11

d) How many elements are in each conjugacy class of S_5

$$\frac{5!}{2^1 1^3 \cdot 1! 3!} = \frac{120}{12} = 10 \qquad \frac{5!}{2^2 1 \cdot 2!} = \frac{120}{8} = 15 \qquad \frac{5!}{3 1^2 \cdot 2!} = \frac{120}{6} = 20$$

$$\frac{5!}{3 \cdot 2 \cdot 1! 1!} = \frac{120}{6} = 20 \qquad \frac{5!}{4 1 \cdot 1! 1!} = \frac{120}{4} = 30 \qquad \frac{5!}{5} = 24$$

2,2,2
2,2,1,1
2,1,1,1,1
1,1,1,1,1,1
3,1,1,1
3,3
3,2,1
4,1,1
4,2

e) if $H < S_4$ what are the possibilities for number of elements in H

by Lagrange's Theorem $|H|$ must divide $|S_4| = 4! = 24$

so 1, 2, 3, 4, 6, 8, 12, 24

f) if $H \trianglelefteq S_5$ what are the possibilities for number of elements in H

$|S_5| = 120 \qquad$ 1, 60, 120

P5.

a) write (1 2 3 4 5) as a product of transpositions (1 2)(1 3)(1 4)(1 5)

b) (1 3 2)(1 2 3 4)(1 2 3) as a product of transpositions in two ways

(1 3)(1 2)(1 2)(1 3)(1 4)(1 2)(1 3) $\quad$ 1 by term

(1 4 2 3) → (1 4)(1 2)(1 3) $\quad$ 2 multiply out

c) The group A_5 has 5 conjugacy classes. The number of elements in each are 1, 12, 12, 15, 20. Show A_5 has no trivial normal subgroups

for a normal subgroup we need the identity element

12 + 12 + 1 = 25 $\quad$ there is no combination of the classes that would result a size that divides 60

Problem Set 5

$(1\ 4\ 6\ 2\ 5\ 7)$

P1. $a = (1\ 2\ 5\ 7)(2\ 4\ 6) \qquad b = (5\ 3\ 1\ 2) \qquad c = (5\ 4\ 1\ 2\ 3)$

a) Order of ab $\quad (1\ 4\ 6\ 5\ 7\ 2\ 3) \quad 7$

b) Order of $(ac)^{-1} b (ac)$ $\quad$ conjugation on b, order maintains $\rightarrow 4$

c) Order of $(a^{-1}ba)^2$ and $(a^{-1}ba)^4$

$a^{-1}ba\,a^{-1}ba = a^{-1}b^2a \longrightarrow b^2 : (5\ 1)(3\ 2) \quad \underline{2} \qquad 4/\gcd(4,p)$

$(a^{-1}ba)^4 = a^{-1}b^4a \rightarrow b^4 : (2)(1)(3)(5) \quad \underline{1}$

d) Order of $(a^{-1}ca)^3$ and $(a^{-1}ca)^5$

$c^3 : \quad 5/\gcd(5,3) = 5/1 = 5$

$c^5 : \quad 5/\gcd(5,5) = 5/5 = 1$

e) Determine whether the element is odd or even for a, b, c, ab, ac

a has order 6 odd $\qquad b$ has order 4 odd

c has order 5 even $\qquad ab \quad 7 \quad$ even

$ac \quad$ odd + even : odd

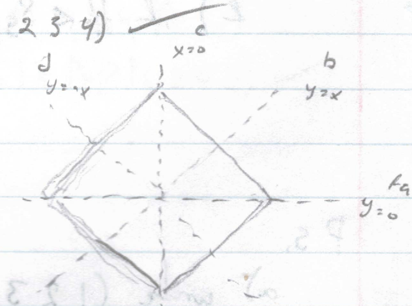
f) Find an element d such that $d^{-1}bd = (1\ 2\ 3\ 4)$

$d^{-1}(1\ 2\ 5\ 3)d$

$(1\ 2\ 3\ 4) \Rightarrow (3\ 4\ 5)$

test out $(3\ 5\ 4)(1\ 2\ 5\ 3)(3\ 4\ 5)$

$(1\ 2\ 3\ 4)$ ✓



P2. D_4 Counterclock-wise rotations : $R_0, R_{90}, R_{270}, R_{180}$

a) compute $F_a R_{90} F_b$ $\quad (2\ 4)(1\ 2\ 3\ 4)(1\ 2)(3\ 4) = () \quad R_0$

b) What is the order of each element in D_4 →

c) Order of $F_a R_{270} F_b F_c$ $\quad (2\ 4)(1\ 4\ 3\ 2)(1\ 2)(3\ 4)(1\ 3) = (2\ 4)$ order 2

d) Find the elements in S_4 corresponding to each of the elements in D_4 →

1 $R_0 : ()$

4 $R_{90} : (1\ 2\ 3\ 4)$

2 $R_{180} : (1\ 3)(2\ 4)$

4 $R_{270} : (1\ 4\ 3\ 2)$

2 $F_A : (2\ 4)$

2 $F_B : (1\ 2)(3\ 4)$

2 $F_C : (1\ 3)$

2 $F_D : (1\ 4)(2\ 3)$

e) Illustrate orbit-stabilizer theorem by considering number of elements in D_4 $\quad 4! = 24$

F_A and F_C fix a vertex and R_0 $\quad 2$

orbit of x is all the possible vertices : 4

$|orb(x)| \cdot |stab(x)| = 8$

$4 \cdot 2 = 8$