

## Problem Set 3 - Cosets, Normal Subgroups, the Symmetric Group

**Problem 1** Write the following product as a product of disjoint cycles

- a)  $(1\ 2\ 3\ 4)(2\ 3\ 4\ 5)(2\ 4\ 3)(1\ 2)(1\ 4)$
- b)  $(1\ 2)(1\ 3)(1\ 4)(1\ 5)$
- c)  $(1\ 2)(1\ 3)(1\ 2)$
- d)  $(1\ 2)(1\ 2\ 3)(1\ 2)$
- e)  $(1\ 2)(1\ 2\ 3\ 4)(1\ 2)$

**Problem 2** What are the powers of  $(1\ 2\ 3\ 4\ 5\ 6)$ ?

**Problem 3** Find the order of

- a)  $(1\ 2)(3\ 4)(5\ 6)$
- b)  $(1\ 2)(3\ 4\ 5\ 6)$
- c)  $(1\ 2)(3\ 4\ 5)$

**Problem 3** Let  $G = S_3$ . Let  $H = \{(), (1\ 2)\}$ .

- a) Show that  $H < G$ .
- b) What groups are conjugate to  $H$  in  $G$ ?
- c) What are the cosets of  $H$  in  $G$ ?
- d) Is  $H \triangleleft G$ ?

- Problem 4**
- a) What do the conjugacy classes of elements in  $S_5$  look like?
  - b) How many conjugacy classes are there in  $S_5$ ?
  - c) How many elements are in each conjugacy class in  $S_5$ ?