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**April, 2016**

**JEL : C18;C52;R40**

# Estimating travel mode choice including rail in regional area, based on a new family of regression models

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## Abstract

In this paper, we model mode choice with the new specification of generalized linear models proposed by Peyhardi et al. (2015). In logit models used by economists, the link function can be decomposed into the reference ratio of probabilities and a cumulative distribution function (cdf). Alternative cdfs (Student, Cauchy, Gumbel, Gompertz, Laplace, Normal) can be used. These cdfs differ in their symmetry (symetric or asymmetric distributions) and in their tails (heavy or thin tails), each allowing a different distribution of behaviors. We test the statistic and economic implications of changing the cdf. First, we investigate the goodness-of-fit indicators (AIC, BIC, Mc-Fadden  $R^2$ ). Then, we compare estimated parameters in terms of sign and significativity. And finally, we look at behavioural outputs such as value of time and demand elasticities. We use a recent stated preferences survey conducted by the author in the Rhône-Alpes Région (France). Its specificity is to specifically address the question of mode choice (rail, coach and car) in a regional context. Attributes include travel time, cost and comfort. We also investigate the cross effect of travel time and comfort. Comparisons between cdfs are made on the basis of three models, including only attributes variables or only individual variables or both. Our results show that the different cdfs provide quite similar results. But, in our estimations, the logistic cdf never ranks among the best options. In terms of significance and sign of coefficients, parameters' estimation are globally the same even if some special features can be noticed. Looking at time equivalence of comfort, we notice that in the model without individual variables, the cdf has a major influence on outputs. In particular, the Student cdf provides very consistent results while some other cdfs (e.g. Gompertz, Logistic, Normal) are extreme.

## 1 Introduction

Mode choice modeling is a recurring challenge, broadly discussed in the transportation economics literature. It is the support of the development of discrete choice models: conditional logit (McFadden, 1974), nested logit (Ben-Akiva,

1974), mixed logit (McFadden et al., 2000) were all developed and illustrated with mode choice applications. While developing these models, the authors proved their compatibility with economics theory, namely the random utility models (Thurstone, 1927). Thus, they are used to explore the determinants of mode choice behavior and to derive economic concepts such as willingness-to-pay, value of time, elasticities or market shares. For example, Wardman et al. (2004) uses a multinomial logit to calculate the rail demand elasticities of a general cost function, a clockfaced index and a timetable’s memorability index. Yanez et al. (2010) calculate subjective value of time, wait and walk with an integrated choice latent variables model. They also predict changes in market shares following an artificial increase of individual income.

Multiple variations of the above mentioned models can be found in the literature. In particular, a significant body of literature on discrete choice model deals with heterogeneity in behaviour and preferences (Fiebig et al., 2010; Hensher et al., 2015). They focus on changes in error terms specification. For example, HEV models specify errors which are not independently distributed. In mixed logit, parameters are random since the error terms are divided into two parts with one following any distribution specified by the author. Latent class models authorizes variations of coefficients between groups.

For the multinomial logit, the conditional logit and the mixed logit model, the link function between the probabilities and the predictors is the multinomial logit function (canonical link function). It can be remarked that this function is defined using the logistic cdf. Alternative cdfs were proposed by Peyhardi et al. (2015), e.g. Student, Cauchy or Laplace cdf. These cdf differ in their symmetry (symetric or asymmetric distributions) and in their tails (heavy or thin tails), each allowing a different distribution of behaviors. For example, heavy tails distributions, such as the Cauchy or Student cdf, permit a greater dispersion of the response behavior to changes in predictors. Our aim is to test and compare these new specifications with the logistic cdf and to address the economic implications of such models on a real and recent database collected by the author. Beyond explanatory consequences and potential differences in significance of predictors, we test to which extent these new models affect mode choice and behavioral outputs like value of time, equivalent between time and comfort as well as elasticities.

In this paper, we use a dataset that explores the determinants explaining train as a mode choice. Train is a relevant alternative for some suburban trips but mainly for interurban trips. We also believe that to understand the determinants of rail use, it is important to study the behavior of train users as well as non-train users such as car users and coach users. Nevertheless, mode choice models including rail have previously been studied either on a single specific rail corridor (Kottenhoff, 1995; Ben-Akiva and Morikawa, 2002; Börjesson and Eliasson, 2011) or interviewing only train users (Wardman 2004; Richter and Keuchel, 2012). One exception is Arentze and Molin (2013) who conduct four stated preferences survey (SP) in the Netherlands, interviewing all types of users. In three surveys, rail alternatives are proposed. But there is no car alternative, except for medium distances (20 kms). Yet it seems that, in SP surveys, offering alternatives to the train is a condition for understanding mode choice. We do

not know of any paper that examines regional trips, offering differentiated travel modes to travelers. We intend to fill this gap with the data analyzed in this paper.

Ben-Akiva and Morikawa (2002) provides a comprehensive list of factors which are important in a choice of a mode between train and coach but which are usually not specified in the models. It includes for example, reliability, comfort (e.g. seating availability) or safety from accidents. For these specific characteristics of train, stated preferences data may be preferred over revealed preference data. Indeed, in a real choice world the variables may not vary sufficiently (e.g. frequency) ; there may be strong correlations between variables of interest (e.g. time and cost) ; it is not possible to directly evaluate demand under conditions which do not yet exist (e.g. a new timetable with clockfacedness) ; variables must be expressed in objective units (such as euros for cost) which is difficult for some secondary variables such as seat design. Nevertheless, stated preferences data also have drawbacks (Wardman, 1988). Stated preferences may not correspond to actual preferences due to systematic bias responses or because of difficulties in carrying out the choice task.

In our survey, we study comfort which is solely addressed in the literature. Comfort, and more precisely the possibility of working or resting during the trip, is often seen as an asset of rail trips. But this advantage is being challenged in trains where seating availability is not guaranteed. Richter and Keuchel (2012) find out that seat availability is the most important component of comfort in trains. Most papers in which values of time are differentiated according to the mode show that value of time is greater for car trips than for public transport trips. Fosgerau et al. (2010) investigate the reasons explaining this result: strategic bias responses, self-selection (car users having higher values of time) or linked to comfort (travel time is seen less as a waste of time if mode choice is the train). There may thus be a link between comfort and travel time: the longer the trip, the more important the guarantee of a seating availability. For more literature on travel time use, see (Lyons G, 2013; Joly and Vincent-Geslin, 2016).

The paper is organized as follows. Section 2 presents the new specification in the context of discrete choice models. Section 3 presents the comparison method of the estimations with the different specifications and presents the data. Results are discussed in section 4. Section 5 concludes.

## 2 Generalized linear models for categorical data

### 2.1 Logit models

The family of logit models is a particular subset of generalized linear models (Nelder and Baker, 1972) which are composed of a random response variable associated to a probability distribution, a set of predictors which are deterministic and a link function which describes the relation between the predictors and the expectation of the response variable. Such models are characterized by the distribution of the response variable  $Y$ , the linear predictor  $\eta$  and the link function between the expectation  $E(Y)$  and  $\eta$ .

Let  $Y_i$  denote the response variable corresponding to the choice of individual  $i$  (with alternatives  $j = 1, \dots, J$ ),  $\mathbf{x}_i$  denote the vector of individual variables and  $\boldsymbol{\omega}_{i,j}$  denote the vector of alternative-specific variables. In the following, we will suppress the individual subscript  $i$  without loss of generality. Luce's choice axiom (Luce, 1959) and the principle of random utility maximization lead to the logit defined by

$$\pi_j = \frac{\exp(\eta_j)}{1 + \sum_{k=1}^{J-1} \exp(\eta_k)} \quad (1)$$

where  $\pi_j = P(Y = j)$ . Depending on the form of the linear predictors  $\eta_j$ , we obtain different logit models:

- $\eta_j = \alpha_j + \mathbf{x}^T \delta_j$ . Individual characteristics  $x$  are used with  $J - 1$  different slopes  $\delta_j$ . This is the classical multinomial logit model.
- $\eta_j = \alpha_j + \tilde{\boldsymbol{\omega}}_j^T \gamma$  where  $\tilde{\boldsymbol{\omega}}_j = \boldsymbol{\omega}_j - \boldsymbol{\omega}_J$ . Choice characteristics  $\omega_j$  are used with common slope  $\gamma$ . This is the conditional logit model introduced by McFadden (1974).
- $\eta_j = \alpha_j + \mathbf{x}^T \delta_j + \tilde{\boldsymbol{\omega}}_j^T \gamma$ . This is a combination of the two previous parametrizations.

## 2.2 A new specification

To introduce the new specification, let us remark that the equation (1) is equivalent to:

$$\frac{\pi_j}{\pi_j + \pi_J} = \frac{\exp(\eta_j)}{1 + \exp(\eta_j)}, \quad j = 1, \dots, J - 1. \quad (2)$$

More generally, all the classical regression models for categorical responses (Tutz, 2012; Agresti, 2013) share the generic equation

$$r_j(\boldsymbol{\pi}) = F(\eta_j), \quad j = 1, \dots, J - 1,$$

where  $\mathbf{r}$  is a differentiable and invertible map between the simplex  $\Delta = \{\boldsymbol{\pi} \in (0, 1)^{J-1} \mid \sum_{j=1}^{J-1} \pi_j < 1\}$  and an open subset of the hypercube  $(0, 1)^{J-1}$ ,  $\boldsymbol{\pi}$  is the vector of probabilities  $(\pi_1, \dots, \pi_{J-1})^T$  and  $F$  is a continuous and strictly increasing cdf (Peyhardi et al., 2015).

The ratio of probabilities  $\mathbf{r}$  is chosen according to the ordinal or nominal nature of the response variable. The reference ratio (see the left part of the equation (2)) is appropriate in the context of qualitative choices because there is no natural ordering among the different alternatives. Therefore, this part of the equation is conserved. Now, remark that the right part of the equation (2) is the logistic cdf. The main idea of our methodology is to replace the logistic cdf by other continuous and strictly increasing cdfs. The proposed models are thus described by the equations

$$\frac{\pi_j}{\pi_j + \pi_J} = F(\eta_j), \quad j = 1, \dots, J - 1.$$

where  $F$  is either the logistic, normal, Laplace, Gumbel, Gompertz or the Cauchy cdf. The heavy tails of the Cauchy distribution or the asymmetry of

the Gumbel and Gompertz distributions may markedly improve the model fit.

Since all these cdfs are strictly increasing functions with  $\eta_j$ , the parameters can still be easily interpreted (Equation (3)). For instance, if the slope parameter of the alternatives cost is significantly negative then the proportion of alternative  $j$  is decreasing when its cost is increasing.

$$\frac{\pi_j}{\pi_J} = \frac{F}{1-F}(\eta_j) \quad (3)$$

In the particular case of the logistic cdf, we have :

$$\frac{\pi_j}{\pi_J} = \exp(\eta_j) \quad (4)$$

This family of models stays easily estimated using the Fisher scoring algorithm. The scores and the Fisher's information matrix computation are a few changed by the use of different cdf  $F$ ; see (Peyhardi et al., 2015) for more details. In the same way, the elasticities of a single alternative specific characteristic  $\omega$  are given by

$$\frac{\partial \ln \pi_j}{\partial \ln \omega_k} = \gamma \omega_k d_k \begin{cases} (1 - \pi_k) & , j = k, \\ -\pi_k & , j \neq k, \end{cases} \quad (5)$$

where  $d_k = f(\eta_k)/[F(\eta_k)\{1 - F(\eta_k)\}]$ ,  $f$  denoting the derivative of  $F$ , i.e. the associated probability density function. For the particular case of logit model (i.e.  $F$  is the logistic cdf) we have  $f = F(1-F)$  (i.e.  $d_k = 1$  for all alternatives  $k$ ) and (5) is the classical result of elasticities computation for logit model, given in (Greene and Hensher, 2003) for instance. One can remark that the IIA property is conserved since  $d_k$  is not depending on alternative  $j$ .

## 3 Methods, data and models

### 3.1 Methods for comparing models

Our objective is to compare the different specifications of the model, in terms of explanation and in terms of prediction. We test 7 seven cdfs: Cauchy, Gompertz, Gumbel, Laplace, Logistic, Normal and Student.

Comparison of models is a classical task. For example, Greene and Hensher (2003) compare latent-class model to mixed logit and multinomial logit. To do so, they use Log-Likelihood, pseudo-R<sup>2</sup>, estimates of Value Of Time (VOT) kernel density for choice probabilities (mixed and latent-class model), direct elasticities of time and cost as well as probabilities profiles of mixed and latent-class models. Yanez et al. (2010) uses likelihood ratio tests to compare the fit of a multinomial logit and two hybrid choice models. They also compare predictions of market shares and so-called "subjective value of time". Fiebig et al. (2010) propose a systematic comparison of multinomial logit, scale multinomial logit, mixed logit and generalized multinomial logit with AIC, BIC and CAIC indicators. They also compare the willingness-to-pay between models and face them to the comparison of the percent of individual choosing an alternative versus another facing a given price change.

The methods to be used depend on the objectives (e.g. AIC is preferred to BIC to predict an outcome), on the comparability of the models (LR tests require the models to be nested) and on the nature of the data (comparisons on VOT or elasticities show which estimates are in the range of plausible values but do not point to a "better" model if data are real and not simulated).

The 7 models we propose are comparable in terms of goodness-of-fit: McFadden R<sup>2</sup>, Loglikelihood, AIC and BIC. The three last indicators will offer the same results since all models have the same number of parameters except the one with the Student cdf which requires the estimation of only one additional parameter, the degrees of freedom.

As a second step, we focus on the outputs of the models: significativity of parameters, elasticities, value of time and equivalence between time and comfort. With this approach, we can grasp the scopes of values and estimate the impact of choosing one model over the others. Even if the scales of parameters are partly normalized to provide comparable estimations, a residual scale effect preclude a direct comparison of estimated parameters. We thus focus on a comparison of significativity and on the relation linking coefficients of a model. Indeed, the parameters' estimations should correspond to identical preference structures between models (Viney et al., 2005). This linear structure is frequently illustrated plotting estimates of coefficients from two models (Flynn et al., 2008; Kerr and Sharp, 2009). We also propose a comparison of significativity. Concerning elasticities and equivalences cost-time-comfort, there is no scale effects, but the formulas integrate derivatives of utility with respect to the attributes (see part2.2).

### 3.2 Data

We conducted a choice experiment (web and face-to-face) among 1,774 inhabitants of the Rhône-Alpes region, in February and March 2015. Each respondent had to choose between three travels modes: train, coach and car. In the experiment, alternatives are described in terms of mode (train, coach and car), cost, time, probability and time delay, frequency, clock-face timetable and comfort. To avoid a cognitive burden, attributes describing the journey are split into three exercises. In exercise 3, on which we focus here, modes vary according to time, cost and comfort (Table 1). Respondents had to answer to four choices questions in exercise 3, leading to a database with 6,373 observations since some rare respondents didn't answer all four questions.

Respondents described a reference journey in terms of time, cost, purpose, origin and destination... This reference journey is then used to personalize the choice questions and minimize the well-known hypothetical bias. Levels of attributes are also presented in Table 1. Levels of time and cost attributes are pivoted around the values collected for a reference journey. One of the three alternatives was systematically a status quo alternative with the mode, travel time and travel cost identical to the reference journey.

To improve the efficiency of the design, a Bayesian efficient design was implemented. A priori weights of attributes were taken from the literature and



adjusted during the pilot tests.

Table 1: Attributes by exercise

	Variations and unit	Exercise 1	Exercise 2	Exercise 3
<b>Mode</b>		Train, train, car	Train, train, car	Train, coach, car
<b>Travel time</b>	-30%,0%,+30% (minutes)	X	X	X
<b>Travel cost</b>	-30%,0%,+30% (euros/travel)	X	X	X
<b>Frequency</b>	In number of trains or coach/hour	X	X	-
<b>Time delay</b>	In minutes	X	-	-
<b>Probability of delay</b>	In %	X	-	-
<b>Clockfacedness</b>	Yes (1) or No(0)	-	X	-
<b>Comfort</b>	Guarantee of a seat (1) or not (0)	-	-	X






	Option A	Option B	Option C
Moyen de transport			
Place Assise	 Place assise non garantie	 Place assise garantie	
Temps de déplacement	1H30	1H52	34 minutes
Coût du déplacement	7€ / déplacement	5€ / déplacement	6€ / déplacement
Cochez l'option choisie :	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Figure 1: Example of choice question for exercise 3

### 3.3 Variables selected in the model

We used stepwise selection to select the variables that enter into the models. It defines a full model, S3, that includes alternative-specific and individual variables. To get a deeper understanding of what the new specification provides to the modeling of discrete choices, we divide this full models into two submodels. Model S1 includes only alternative-specific variables. Model S2 includes only individual variables.

Along modal alternative-specific constants (ASC), we include variables describing the alternatives (time, cost, comfort and time \* comfort) as well as individual variables. Individual variables are related to socio-economic characteristics (age, income), spatial characteristics (% of the origin and destination municipalities in high density area), journey characteristics (type of user (car or coach user for the reference trip), frequency of the trip, imperative schedule) and general mobility (access to car, frequency of use of modes alternative to car). Summary statistics for alternative-specific variables are in table 2. For coach and car, comfort is set to 1 since seating position is guaranteed. Summary statistics for individual variables are in table 3. Concerning the choice made by respondents, 29.6 % chose the train alternative, 21% chose the coach alternative and 49,4% chose the car alternative.

Table 2: Descriptive statistics on alternative-specific variables

	Mean	S.D.	Min	Max
CostA	8.88	7.56	0.63	62.00
CostB	8.90	7.80	0.46	78.00
CostC	9.99	8.64	0.38	62.00
TimeA	69.89	51.86	7.00	325.00
TimeB	70.22	53.23	7.00	325.00
TimeC	57.24	36.89	4.00	330.00
ComfortA	0.50	0.50	0.00	1.00
TimeA*ComfortA	37.16	53.04	0.00	325.00

Table 3: Descriptive statistics on individual variables

Label	Definition	Mean	S.D.	Min	Max
age	In years	47.57	15.76	19.00	90.00
size_hh	Size of the household	2.63	1.27	1.00	8.00
nb_car	number of cars available in the household	1.69	0.72	1.00	5.00
access_car	nb_car/size_hh	0.76	0.38	0.12	4.00
income_h	1 : Income above 4 000 euros	0.28	0.45	0.00	1.00
dep_dens	% of the origin municipality in high density area	0.74	0.40	0.00	1.00
arr_dens	% of the destination municipality in high density area	0.86	0.30	0.00	1.00
type_coach	1 : user of coach for the reference trip	0.03	0.16	0.00	1.00
type_car	1 : user of car for the reference trip	0.54	0.50	0.00	1.00
regular	1 : regular trip	0.28	0.45	0.00	1.00
imperative	1 : imperative schedule at destination	0.45	0.50	0.00	1.00
alt_pt	1 : car user who had already used public transports to do the reference trip	0.15	0.35	0.00	1.00
alt_car	1 : train or coach user who had already used car to do the reference trip	0.26	0.44	0.00	1.00
freq_alt	1 : frequency of use of modes other than car	0.41	0.49	0.00	1.00

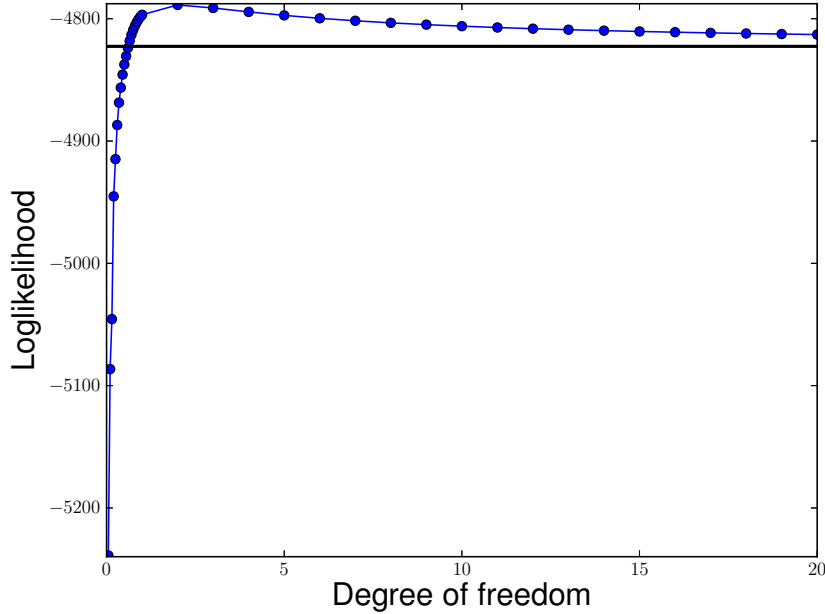


Figure 2: Log-likelihood depending on the degrees of freedom - S3

## 4 Results

Seven specifications are estimated and compared on three models (S1, S2 and S3). Models are estimated using the software "Statiskit" developed by Pierre Fernique and Jean Peyhardi, in C++ with an interface in Python. For the logistic distribution, we compared and validated estimations' results with the one provided by the package "Mlogit" in R. The Student distribution depends on the degrees of freedom (df) chosen. Several Student distributions were tested to select the one that maximizes the log-likelihood which appears to be empirically generally concave. As shown in 2 and the zoom 3, it is the Student distribution with 2 degrees of liberty which meet that criterion for the S3 model (and  $df=0.35$  for the S1 and  $df=1.5$  for S2; 4 to 7 ).

Note that for model S3, the specification with the Gompertz cdf did not converge.

### 4.1 Comparison of models

In this subsection, we compare the models considering the usual goodness of fit indicators (table 4): Log-likelihood, AIC, BIC, and McFadden  $R^2$ .

For all distributions we focus on the rank of the models in terms of AIC, BIC, and McFadden  $R^2$ . In model S1, all indicators converge in a common finding: the Student cdf performs the best. It is followed by the Cauchy cdf. In model S2, the Laplace cdf performs best. Cauchy and Student (1.5) follow depending on the criteria. In model S3, the Gumbel cdf ranks first, followed by Student (2) and Laplace cdfs.

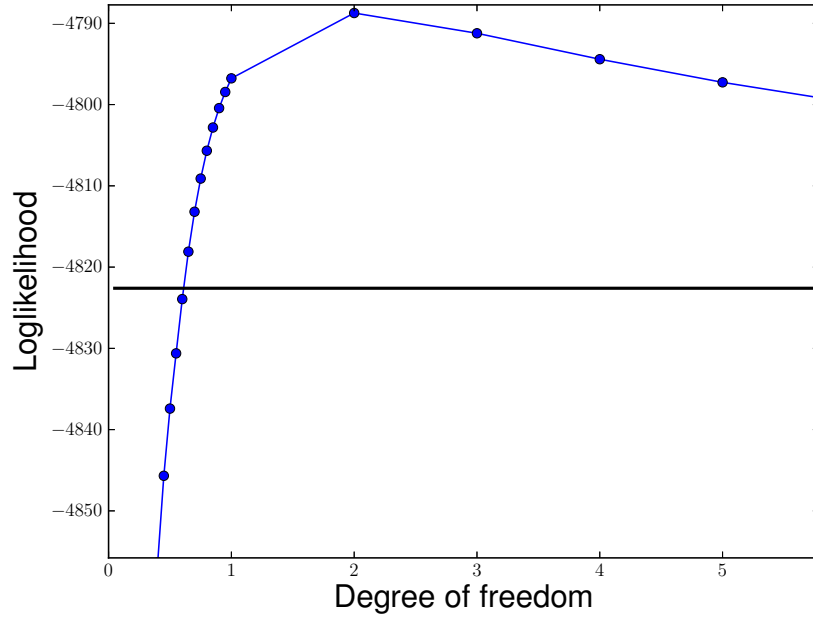


Figure 3: Log-likelihood depending on the degrees of freedom (zoom) - S3

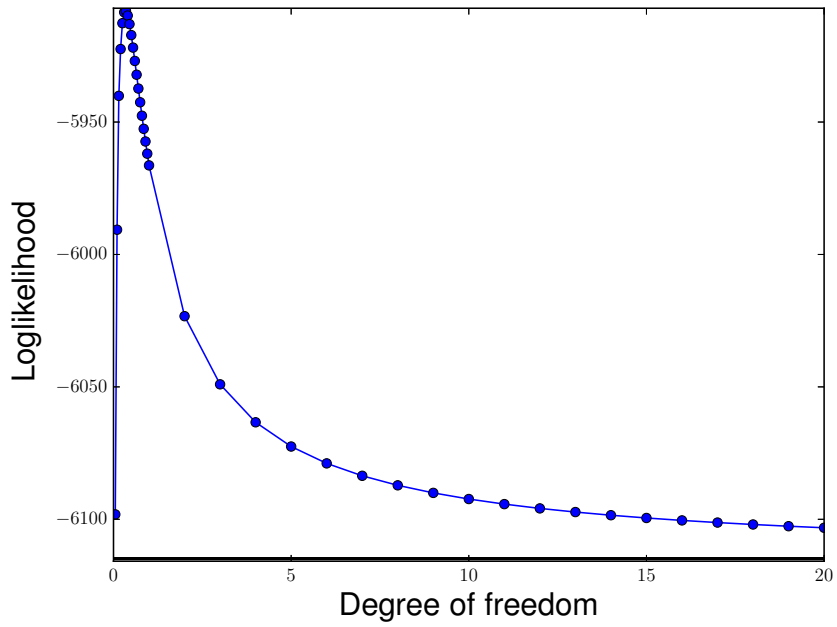


Figure 4: Log-likelihood depending on the degrees of freedom - S1

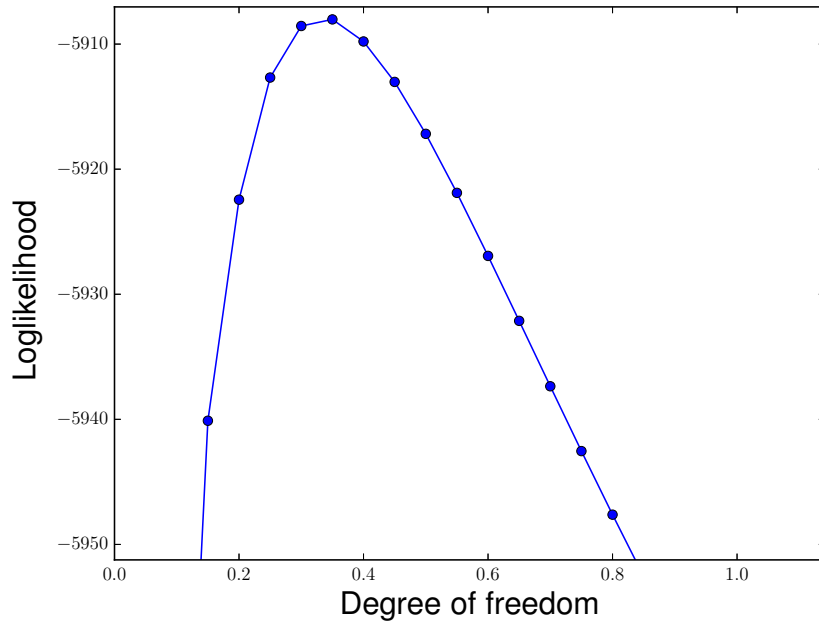


Figure 5: Log-likelihood depending on the degrees of freedom (zoom) - S1

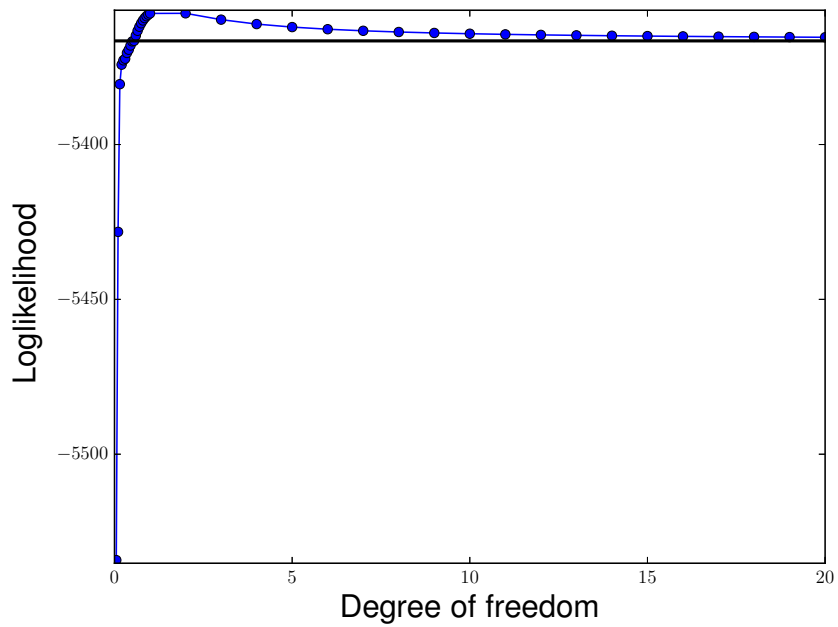


Figure 6: Log-likelihood depending on the degrees of freedom - S2

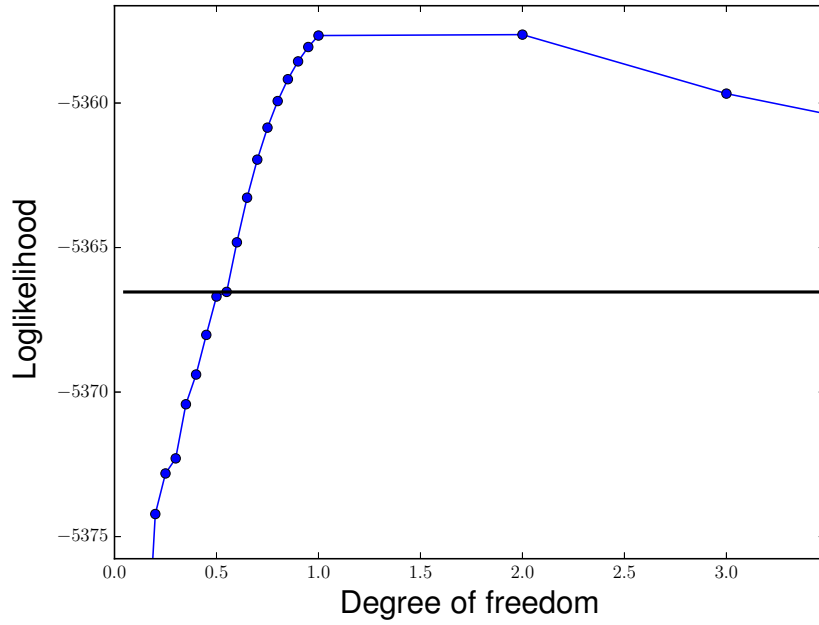


Figure 7: Log-likelihood depending on the degrees of freedom (zoom) - S2

Globally, the Logistic and Normal cdfs performs badly. Student performs well. Its worst rank is three, but the premiums are globally low, except for the McFadden  $R^2$  in model S1.

Table 4: Comparison of fit indicators

		Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Value of indicators (S1)	Log-likelihood	-5,966	-6,194	-6,073	-6,028	-6,083	-6,115	-5,908
	Mc Fadden R2	0.097	0.062	0.081	0.087	0.079	0.074	0.105
	AIC	11,945	12,400	12,157	12,068	12,178	12,242	11,830
	BIC	11,985	12,441	12,198	12,109	12,219	12,282	11,877
Rank (S1)	Mc Fadden R2	2	7	4	3	5	6	1
	AIC	2	7	4	3	5	6	1
	BIC	2	7	4	3	5	6	1
		Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (1.5)
Value of indicators (S2)	Log-likelihood	-5,358	-5,382	-5,366	-5,356	-5,363	-5,367	-5,357
	Mc Fadden R2	0.189	0.185	0.188	0.189	0.188	0.187	0.189
	AIC	10,767	10,816	10,784	10,765	10,778	10,785	10,767
	BIC	10,943	10,992	10,959	10,941	10,954	10,961	10,950
Rank (S2)	Mc Fadden R2	3	7	5	1	4	6	2
	AIC	2	7	5	1	4	6	3
	BIC	2	7	5	1	4	6	3
		Cauchy		Gumbel	Laplace	Logistic	Normal	Student (2)
Value of indicators (S3)	Log-likelihood	-4,797		-4,782	-4,791	-4,803	-4,823	-4,789
	Mc Fadden R2	0.274		0.276	0.275	0.273	0.270	0.275
	AIC	9,654		9,624	9,642	9,665	9,705	9,639
	BIC	9,856		9,826	9,845	9,868	9,908	9,849
Rank (S3)	Mc Fadden R2	4		1	3	5	6	2
	AIC	4		1	3	5	6	2
	BIC	4		1	2	5	6	3

## 4.2 Estimation of models

Before comparing estimations between models, we can draw some conclusions on the effects of the included variables on the probability to choose the coach or the car option relative to the train option. All significant effects are consistent with theory, in the final model S3 including alternative and individual variables. Some exceptions appear in S1 and S2 models.

As expected, time and cost are associated to significant and negative effects. Comfort is significant and positive. In model S3, the crossed effect of time and comfort positive and globally significant, indicating the propensity to accept longer time in comfortable situation. In model S1, it is either non significant or significant and negative which is contrary to intuition. Except with the Cauchy cdf in model 1, car and coach are associated to a significantly smaller baseline utility than train.

Age, high income, using coach and car in the reference journey, having already used a car alternative for the reference journey and level of accessibility to a car all increase the probability to choose the coach or the car options in the SP survey. Population density at destination, regular trip and frequent use of modes other than car, all decrease the probability to choose the alternatives to train. Other variables have adverse effects. For example, density at the origin has no influence on the coach choice's probability but has a positive effect on car choice which is surprising. Indeed, our expectation was a positive effect: high density being a motive to take public transports. On the contrary, having an imperative at destination and having already used a public transport alternative for the reference journey decrease the probability of choosing the car option.

Comparisons of estimations are done on the sign of coefficients, their significance and the structure of preferences (figures 8 to 22).

First, we can notice that the signs of coefficients are the same whatever the chosen cdf and the model investigated. Exception is, as mentioned before, in the model S1, for the car intercept and for the cross-variable (Time\*Comfort). Indeed, the car intercept in model S1 with the Cauchy cdf is positive whereas it is negative in all other situations. The cross-variable is significantly negative in model S1 (Gompertz, Logistic, Normal) and significantly positive in model S3 (all cdfs taken together).

Second, significance of variables can vary greatly between estimations. But we can notice some patterns. In model S1, the only differences in significance concerns the cross variable (time\*comfort) which is either not significant, either highly significant. In model S2, some variables' significance is stable over specifications (e.g., car intercept) and others are highly heterogeneous. Some individual variables have thus higher significance depending on the model. For example, age associated with the car option is significant at the 0.1% level with the Cauchy or Laplace cdf and not significant with the Gompertz cdf. In model S3, the estimations done with the Cauchy cdf seem to be atypical. The car intercept is not significant, the density at destination has a highly significant negative effect. These differences in terms of variables significance show that the choice of the distribution matters if the model is used for explanatory purposes.



The general structure of the preferences seems to be captured almost equivalently by the different model specifications. Figures 8 to 22 illustrate relatively stable patterns of the estimates. The Cauchy and the Student give estimates closer to the logistic ones than the Normal, the Gumbel and Laplace distributions.

Table 5: Estimation of models S1

		Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Intercepts	Coach	-0.608 ***	-1.16 ***	-0.497 ***	-0.403 ***	-0.67 ***	-0.413 ***	-1.223 ***
	Car	0.173 ***	-0.617 ***	0.164 ***	0.108 ***	0.176 ***	0.104 ***	0.338 ***
Alternative-specific variables	Time	-0.021 ***	-0.004 ***	-0.01 ***	-0.01 ***	-0.012 ***	-0.006 ***	-0.061 ***
	Cost	-0.195 ***	-0.072 ***	-0.099 ***	-0.106 ***	-0.147 ***	-0.079 ***	-0.496 ***
	Comfort	0.606 ***	0.598 ***	0.49 ***	0.433 ***	0.796 ***	0.511 ***	1.089 ***
	Time*Comfort	0.0003	-0.004 ***	-0.0001	-0.001	-0.003 **	-0.002 ***	0.003

\*\*\* = significant at the 0.1 % level

\*\* = significant at the 1 % level

\* = significant at the 5 % level

. = significant at the 10 % level

Table 6: Estimation of models S2

		Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (1.5)
Intercepts	Coach	-0.649 **	-1.13 ***	-0.491 **	-0.385 *	-0.635 *	-0.376 *	-0.56 **
	Car	-0.804 **	-1.311 ***	-0.49 **	-0.478 **	-0.765 **	-0.442 **	-0.714 **
Individual variables	Coach	Age	0.076 *	0.009	0.042 .	0.05 *	0.059	0.064 *
		Income_h	0.269 ***	0.209 ***	0.193 ***	0.202 ***	0.302 ***	0.18 ***
		Dep_dens	-0.018	0.041	-0.018	-0.004	0.024	0.015
		Arr_dens	-0.372 ***	-0.232 *	-0.246 **	-0.28 ***	-0.404 **	-0.233 **
		Type_coach	2.1 ***	1.319 ***	1.591 ***	1.358 ***	2.07 ***	1.247 ***
		Type_car	0.888 ***	0.786 ***	0.767 ***	0.603 ***	1.075 ***	0.666 ***
		Regular	-0.12	-0.054	-0.079	-0.087	-0.128	-0.072
		Imperative	0.021	-0.068	-0.033	-0.021	-0.072	-0.051
		Alt_pt	0.107	-0.089	0.112	0.03	0.021	0.005
		Alt_car	0.049	-0.016	0.044	0.009	0.014	0.004
		Freq_alt	-0.095	-0.11 .	-0.048	-0.096 .	-0.129	-0.077
		Access_car	0.205	0.096	0.169 *	0.157 *	0.23 .	0.13 .
	Car	Age	0.134 **	0.01	0.061 *	0.07 **	0.082 *	0.037 .
		Income_h	0.353 **	0.099 *	0.159 **	0.186 **	0.221 **	0.116 *
		Dep_dens	0.148	0.094 .	0.018	0.092	0.145	0.077
		Arr_dens	-0.47 **	-0.101 .	-0.182 *	-0.247 *	-0.286 *	-0.141 *
		Type_coach	1.805 ***	1.229 ***	1.285 ***	1.17 ***	1.789 ***	1.088 ***
		Type_car	3.441 ***	1.806 ***	2.127 ***	1.954 ***	2.796 ***	1.652 ***
		Regular	-1.119 ***	-0.301 ***	-0.459 ***	-0.538 ***	-0.662 ***	-0.361 ***
		Imperative	-0.359 ***	-0.167 ***	-0.215 ***	-0.228 ***	-0.321 ***	-0.182 ***
		Alt_pt	-0.693 ***	-0.389 ***	-0.189 *	-0.404 ***	-0.533 ***	-0.306 ***
		Alt_car	0.305 **	0.167 *	0.087	0.151 *	0.17	0.088
		Freq_alt	-0.629 ***	-0.306 ***	-0.342 ***	-0.408 ***	-0.544 ***	-0.311 ***
		Access_car	0.53 ***	0.126 *	0.358 ***	0.308 ***	0.415 ***	0.216 **

\*\*\* = significant at the 0.1 % level

\*\* = significant at the 1 % level

\* = significant at the 5 % level

. = significant at the 10 % level

Table 7: Estimation of model S3

		Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)	
Intercepts	Coach	-0.414 .	-0.475 **	-0.254	-0.54 *	-0.43 **	-0.332 .	
	Car	-0.348	-0.577 ***	-0.534 **	-1.137 ***	-0.762 ***	-0.625 **	
Alternative-specific variables	Time	-0.041 ***	-0.021 ***	-0.022 ***	-0.03 ***	-0.016 ***	-0.027 ***	
	Cost	-0.197 ***	-0.087 ***	-0.098 ***	-0.127 ***	-0.069 ***	-0.122 ***	
	Comfort	0.493 ***	0.26 ***	0.319 ***	0.46 ***	0.275 ***	0.364 ***	
	Time*Comfort	0.003 **	0.003 ***	0.002 **	0.003 **	0.002 **	0.003 **	
Individual variables	Coach	Age	-0.009 **	-0.005 **	-0.006 ***	-0.009 **	-0.005 **	-0.007 ***
		Income_h	0.381 ***	0.221 ***	0.247 ***	0.371 ***	0.202 ***	0.302 ***
		Dep_dens	-0.067	-0.096	-0.027	-0.035	0.003	-0.051
		Arr_dens	-0.355 **	-0.141	-0.22 *	-0.273 *	-0.101	-0.267 *
		Type_coach	3.51 ***	1.848 ***	1.75 ***	2.38 ***	1.282 ***	2.224 ***
		Type_car	1.062 ***	0.917 ***	0.758 ***	1.289 ***	0.792 ***	0.93 ***
		Regular	-0.079	0.006	-0.042	-0.028	0.001	-0.043
		Imperative	0.024	-0.019	-0.008	-0.04	-0.023	-0.015
		Alt_pt	0.157	0.121 .	0.03	0.026	0.006	0.058
		Alt_car	0.028	0.045	0.004	-0.01	-0.02	0.009
		Freq_alt	-0.131	-0.099	-0.143 *	-0.213 *	-0.128 *	-0.139 .
		Access_car	0.012	0.099	0.056	0.138	0.103	0.064
	Car	Age	-0.034 ***	-0.013 ***	-0.016 ***	-0.018 ***	-0.009 ***	-0.019 ***
		Income_h	0.697 ***	0.227 ***	0.297 ***	0.346 ***	0.173 ***	0.374 ***
		Dep_dens	0.249	-0.046	0.126	0.15	0.079	0.146
		Arr_dens	-0.561 **	0.017	-0.143	-0.057	0.021	-0.205
		Type_coach	3.283 ***	1.563 ***	1.709 ***	2.34 ***	1.357 ***	2.084 ***
		Type_car	4.678 ***	2.626 ***	2.62 ***	3.701 ***	2.13 ***	3.158 ***
		Regular	-1.133 ***	-0.327 ***	-0.445 ***	-0.469 ***	-0.228 ***	-0.556 ***
		Imperative	-0.348 **	-0.153 **	-0.184 **	-0.248 **	-0.135 **	-0.238 **
		Alt_pt	-0.413 *	-0.091	-0.234 **	-0.276 *	-0.147 *	-0.272 *
		Alt_car	0.219	0.125 .	0.149 .	0.231 .	0.142 *	0.167
		Freq_alt	-0.708 ***	-0.375 ***	-0.458 ***	-0.599 ***	-0.338 ***	-0.5 ***
		Access_car	0.601 ***	0.367 ***	0.323 ***	0.442 ***	0.247 ***	0.404 ***

\*\*\* = significant at the 0.1 % level

\*\* = significant at the 1 % level

\* = significant at the 5 % level

. = significant at the 10 % level

### 4.3 Value of Time and equivalence time of comfort

Due to the introduction of the cross variable (Time\*Comfort), values of time (VOT) differ depending on the guarantee to have a seat in the train. VOT should be greater when seat is not guaranteed since travel time can't be used to rest or to work. Values depend also greatly on the model (S1 or S3) and on the chosen cdf (Table 8). For model S1, if seat is not guaranteed, VOT ranges from 4.44 to 7.42 euros per hour. In the same conditions, it ranges from 12.53 to 14.56 euros per hour in model S3. If seating position is guaranteed, then VOT is comprised between 5.95 and 7.2 euros par hour (model S1) or between 11.47 and 12.66 (model S3). Minimum and maximum values are not always observed with the same cdf. But can notice that the smallest ratio between VOT estimated with models S1 and VOT estimated with model S3 are observed with the Student and Cauchy cdfs which are the best performing model regarding likelihood based indicators for model S1.

Despite very heterogeneous across models and cdfs, VOT found in this paper are in line with literature. Literature's VOT also prove to be very heterogeneous depending on mode, travel purpose, type of survey... (Abrantes and Wardman, 2011; Wardman and Wheat, 2013). For example, in a stated preference survey, Arentze and Molin (2013) find values between 14.4 and 17.4 euros per hour for train travels. Even if we do not have a great confidence to the values found in model S1, we can notice that they are not totally inconsistent with the french rail context. A stated preference survey done by the french rail operator (RFF) in 2013 provide values between 3.9 and 7.7 euros per hour.

Concerning comfort, we convert it in minutes to get a time equivalent. First of all, we notice that most values found with models S1 are totally inconsistent. Cauchy and Student cdfs are the only one providing believable values. The values found with the Student distribution in model S1 are even very close to the values found with models S3. This result tends to support that, even if the model is misspecified, the Student distribution captures well the effects of time and comfort.

We now focus on equivalence time of comfort in models S3. For a short travel time of 30 minutes (corresponding to the first quartile of the sample), having a seated position guaranteed is equivalent to 14.51 to 20.07 minutes of travel time. For medium travel time of 60 minutes, it ranges from 17.05 to 23.17 minutes. And for a long travel time of 60 minutes (corresponding to the last quartile of the sample), seated position is equivalent to 19.58 to 26.27 minutes of travel time. In the literature, values of comfort are rare. RFF (2013) finds that users are ready to spend between 13 and 51 more minutes in the train to avoid a standing position in comparison to a seated position.

We also notice that choosing one cdf over the others has strong implications in terms of prediction and thus in terms of transport policy which commonly uses these kinds of values to calculate a generalized travel cost or time function. Indeed, even considering only model S3, VOT may be increased by up to 16% depending on the chosen cdf and equivalent time of comfort by up to 38%. This result has strong implications.

Table 8: Estimated mean value of time and equivalence time of comfort

S1	Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
VOT (comfort = 0)	-6.40	-3.44	-6.26	-5.82	-4.89	-4.56	-7.42
VOT (comfort = 1)	-6.32	7.18	-6.32	-6.16	-5.95	-6.13	-7.10
Eq. Time of comfort (time = 30)	29.45	120.04	47.15	40.51	60.14	75.25	19.04
Eq. Time of comfort (time = 60)	29.82	94.17	46.85	38.76	53.64	64.93	20.32
Eq. Time of comfort (time = 90)	30.19	68.31	46.54	37.00	47.13	54.61	21.61

S3	Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)
VOT (comfort = 0)	-12.53	-14.57	-13.56	-14.10	-14.12	-13.46
VOT (comfort = 1)	-11.47	-12.55	-12.24	-12.57	-12.66	-12.03
Eq. Time of comfort (time = 30)	14.51	16.45	17.37	18.63	20.07	16.51
Eq. Time of comfort (time = 60)	17.05	20.60	20.28	21.87	23.17	19.70
Eq. Time of comfort (time = 90)	19.58	24.74	23.18	25.12	26.27	22.88

#### 4.4 Elasticities

As a first step, elasticities are presented without probabilities' weighting (Table 9 and Table 10). Weighted elasticities are available from authors upon request. Own-elasticities measure how the probability of choosing alternative  $i$  is influenced by increasing time or cost of the same alternative  $i$ . Cross-elasticities measure how the probability of choosing alternative  $j$  is influenced by increasing time or cost of alternative  $i$  ( $\forall j \neq i$ ).

Probability of choosing the train and coach options is much more sensitive to their own cost and time than probability to choose the car option. It is hard to find a specific pattern for cross-elasticities, which may be due to the IIA property.

Comparison between cdfs show that there is a strong heterogeneity, especially for models S1, models S3 being more homogeneous. As for VOT and equivalent time of comfort, we notice that with some distributions, misspecification has more impacts. For example, own-time-elasticities with the Normal or the Gumbel cdf are very different in model S1 and in model S3.

Table 9: Elasticities for models S1

<b>Own-time-elasticities</b>	Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Train	-1.329	-0.227	-0.702	-1.013	-0.583	-0.461	-2.973
Coach	-1.156	-0.179	-0.944	-0.885	-0.665	-0.547	-1.345
Car	-0.583	-0.115	-0.551	-0.551	-0.37381	-0.314	-0.624
<b>Own-cost-elasticities</b>	Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Train	-1.636	-0.677	-0.884	-1.377	-0.946	-0.803	-3.128
Coach	-1.380	-0.520	-1.196	-1.170	-1.057	-0.934	-1.377
Car	-0.953	-0.456	-1.004	-0.877	-0.827	-0.746	-0.842
<b>Cross-time elasticities</b>	Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Train	0.525	0.103	0.298	0.419	0.252	0.205	1.172
Coach	0.340	0.046	0.218	0.251	0.174	0.139	0.447
Car	0.446	0.112	0.340	0.340	0.310	0.266	0.431
<b>Cross-cost elasticities</b>	Cauchy	Gompertz	Gumbel	Laplace	Logistic	Normal	Student (0.35)
Train	0.570	0.274	0.33462	0.500	0.355	0.310	1.131
Coach	0.361	0.128	0.24522	0.296	0.248	0.215	0.417
Car	0.679	0.431	0.55179	0.647	0.637	0.589	0.552

## 5 Conclusion

Based on the work of Peyhardi et al. (2015), we estimate and compare to the multinomial logit six models with different cdfs (Cauchy, Gumbel, Gompertz Laplace, Normal and Student). This work is done on three different types of

Table 10: Elasticities for models S3

<b>Own-time-elasticities</b>	Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)
Train	-2.547	-1.389	-2.12384	-1.42001	-1.21497	-1.86245
Coach	-1.956	-2.177	-1.77218	-1.67352	-1.55391	-1.83435
Car	-0.703	-2.239	-2.23863	-0.96663	-1.01252	-0.84285
<b>Own-cost-elasticities</b>	Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)
Train	-1.679	-0.793	-1.298	-0.837	-0.716	-1.147
Coach	-1.268	-1.046	-1.030	-0.910	-0.835	-1.070
Car	-0.596	-1.605	-0.675	-0.712	-0.739	-0.659
<b>Cross-time-elasticities</b>	Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)
Train	1.114	0.655	0.962	0.671	0.592	0.842
Coach	0.558	0.420	0.474	0.427	0.382	0.497
Car	0.497	0.631	0.631	0.746	0.801	0.620
<b>Cross-cost-elasticities</b>	Cauchy	Gumbel	Laplace	Logistic	Normal	Student (2)
Train	0.549	0.277	0.437	0.293	0.259	0.385
Coach	0.324	0.205	0.257	0.224	0.200	0.271
Car	0.416	0.460	0.517	0.559	0.605	0.483

models. The full model S3 gathers the best adapted variables (individual and alternatives) to the data. S1 and S2 are sub-models that include only alternative-specific variables (model S1) and only individual variables (model S2). Our comparisons are done based on goodness-of-fit indicators signs and significance of variables. We also draw some conclusions based on behavioural outputs (structure of preferences, value of time, equivalence time-comfort and elasticities).

For a same type of model (S1, S2 or S3), our results suggest that changing the cdf does not alter the estimated sign of the covariables effects on the choice probability (apart one exception in model S1), but can lead to variations in the significance level of the coefficients. Whereas the coefficients can not be compared in absolute value between models, we can observe relatively stable structure of the preferences (or weight of covariables in the decision).

Models S1 and S3 are voluntary misspecified since important variables are withdrawn. We show that some cdfs can encounter this misspecification and provide behavioral outputs consistent with literature or expectations. In model S1, the Student (0.35) cdf largely outperforms the other cdfs. Even if the model is misspecified, the Student (0.35) cdf captures the effects of time and comfort. In model S2, it is less clear but we can notice that the Laplace, Cauchy and Student (1.5) cdfs stand out.

Conclusions drawn with model S3 are consistent with expectations. Analysis of mode choice between rail, coach and car indicates usual significant negative effects of time and monetary cost. The comfort attribute is a significant and



positive determinant of mode choice. We also find a crossed effect with travel time. Comfort increases the propensity to accept higher travel time. Estimated value of time are around 12 euros / hour if a seated position is guaranteed, and around 14 euros / hour if it is not. Our results suggest that for example substituting the Logistic cdf for the Cauchy cdf induces a reduction in value of time (with comfort = 1) of approximately 10% or 1.1 euro / hour. Thus, even with a model that we believe to be correctly specified, changing the cdf has operational impacts. Impacts are also in terms of explanation, since the significant explanatory variables differ from one model to the other. In terms of goodness-of-fit indicators, changing the cdf is associated with a low premium. Yet, we notice that, with our data, the logit function is never associated with the best fit indicators.

Globally, testing a variety of cdfs allowed us to point out the one that best fit the data. If behaviour are heterogeneous, it is interesting to select cdfs with heavy tails. Some behaviour patterns, such as high value of time for some types of users (e.g. car users or travelers with a business purpose), may be preferably modeled with asymmetric cdfs. These assumptions still have to be tested on simulated data sets but we can already state that playing on the form of the cdf allows more flexibility and a better fit to the data. Further research on the cdf influence is thus needed to evaluate the potential gains of the different specifications. This first evaluation is conditional to our data and need to be extended to data from different fields, survey types, etc. Evaluations and comparisons have to be done including the well established and well known models in the literature (as for example, the probit model, nested and mixed logit).

Acknowledgments: This work would not have been possible without the funding of the survey by the Regional Rhône-Alpes Board and its partners.

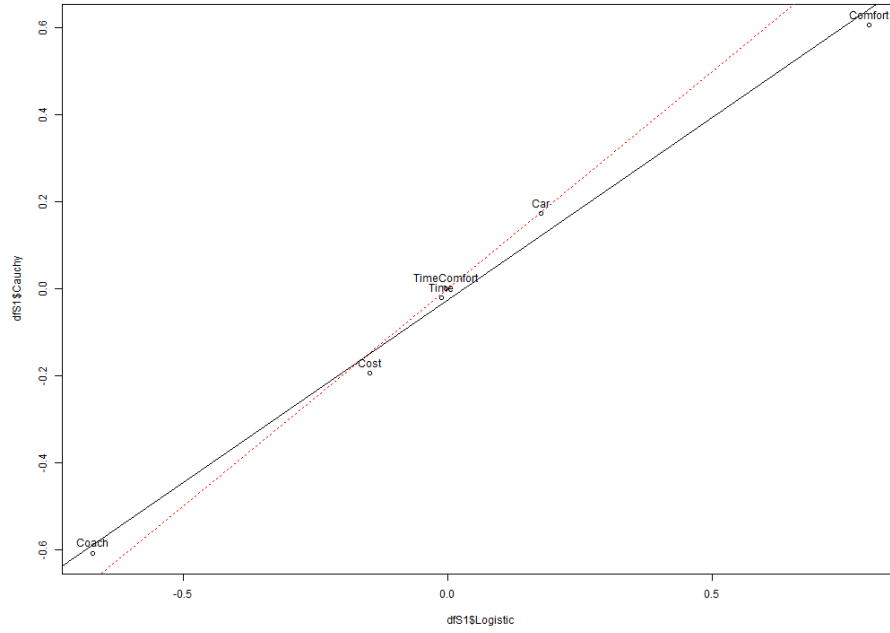


Figure 8: Coefficients Comparison - S1 - Cauchy : Logistic

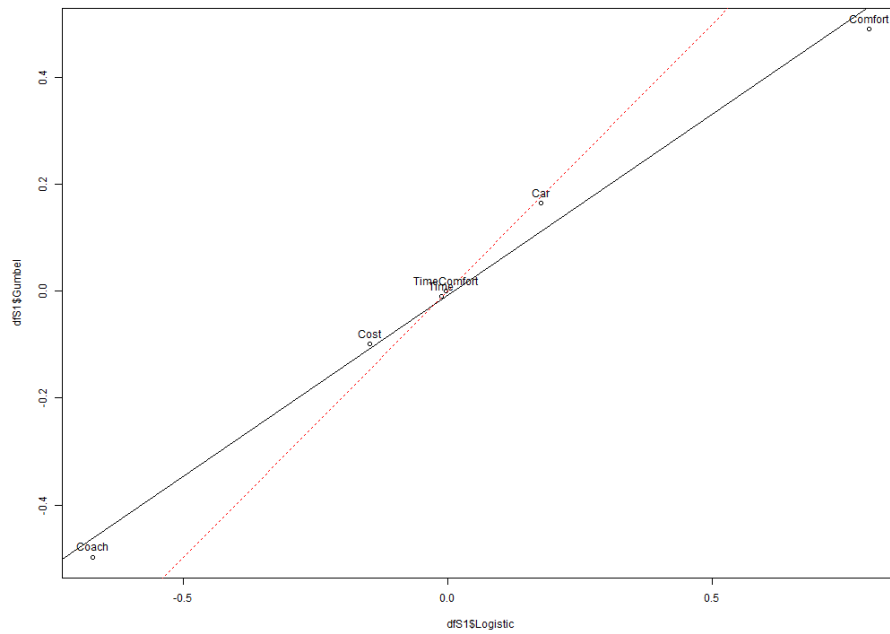


Figure 9: Coefficients Comparison - S1 - Gumbel : Logistic

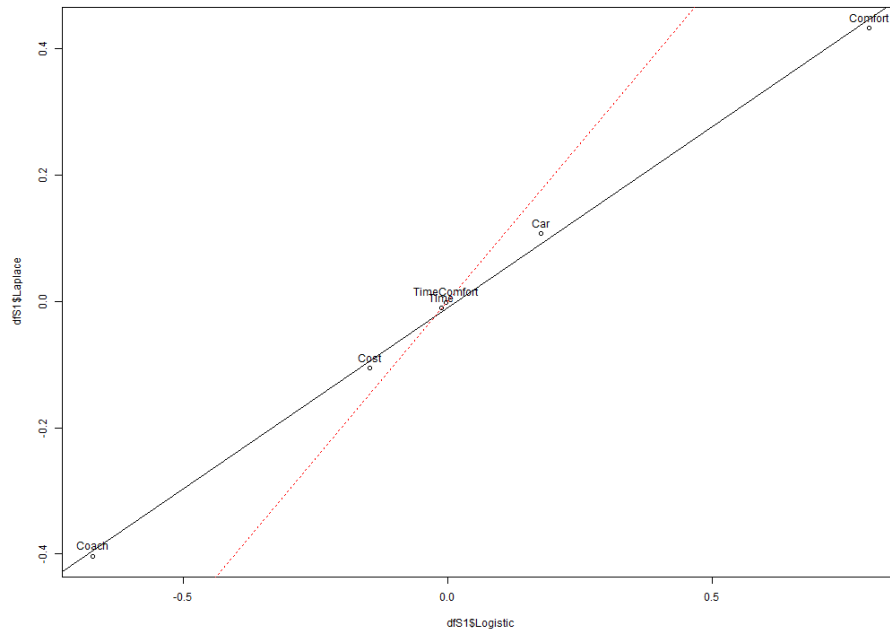


Figure 10: Coefficients Comparison - S1 - Laplace : Logistic

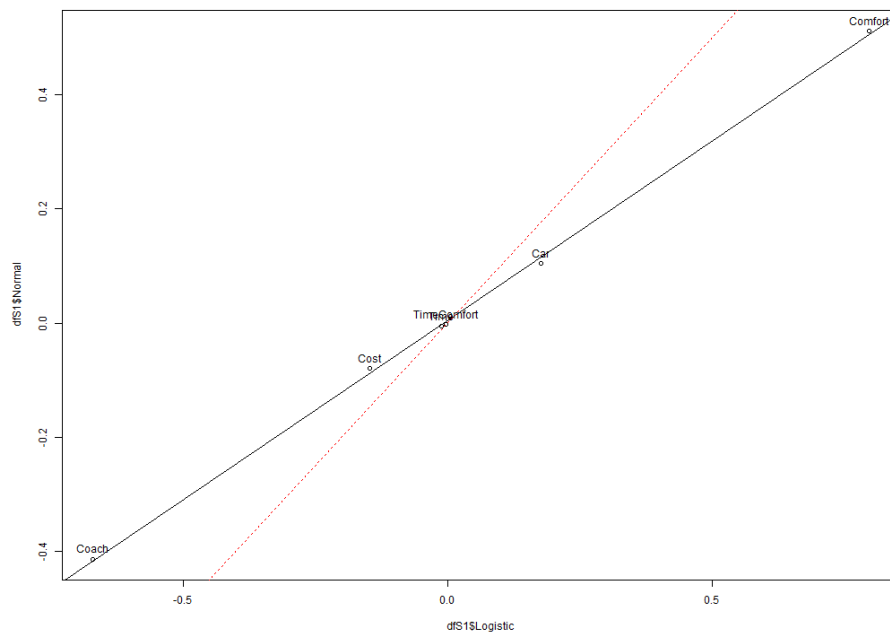


Figure 11: Coefficients Comparison - S1 - Normal : Logistic

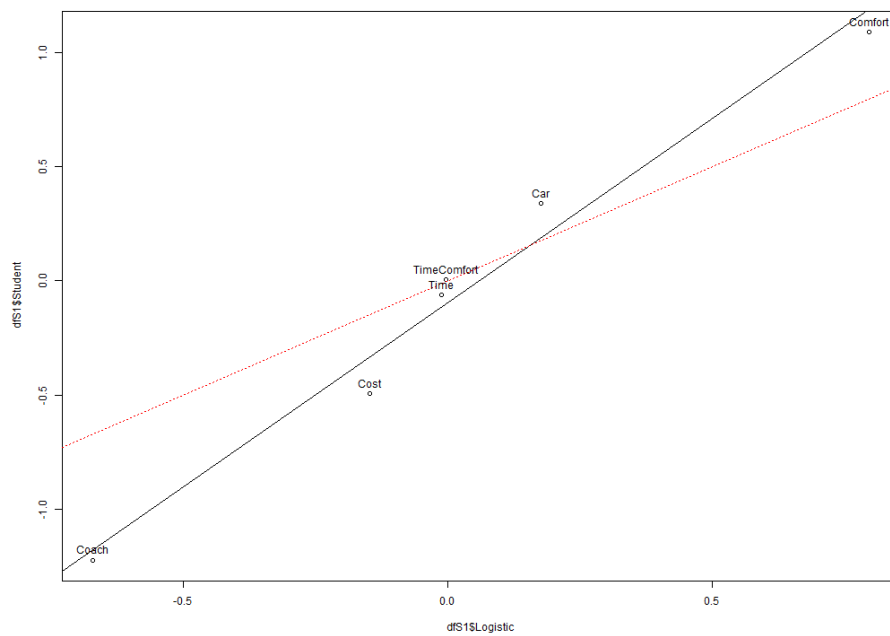


Figure 12: Coefficients Comparison - S1 - Student : Logistic

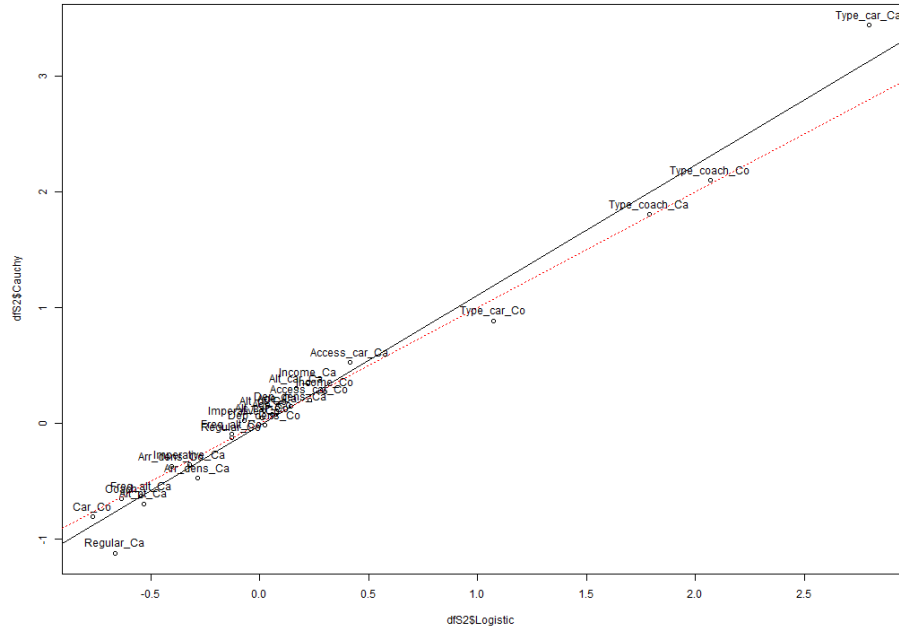


Figure 13: Coefficients Comparison - S2 - Cauchy : Logistic

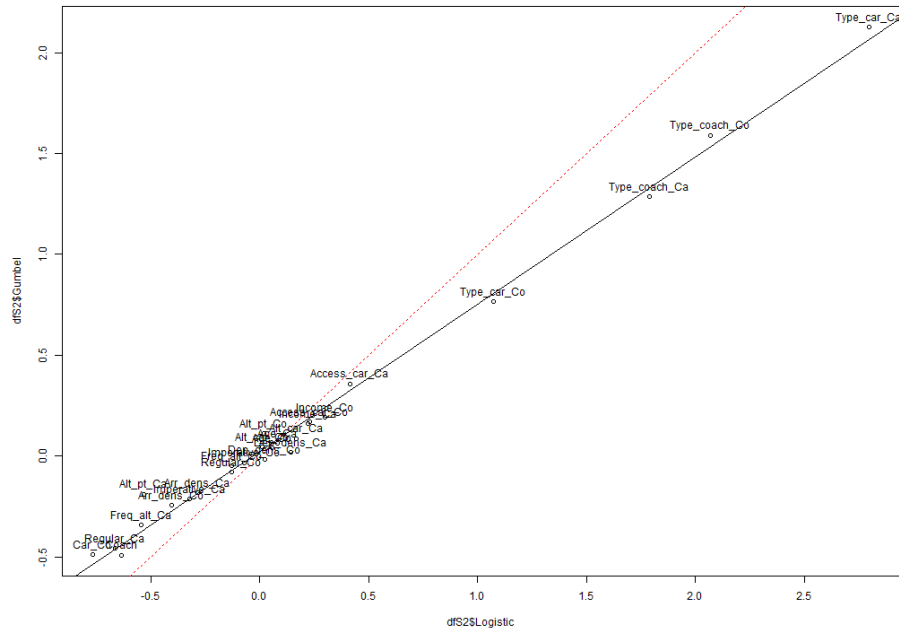


Figure 14: Coefficients Comparison - S2 - Gumbel : Logistic

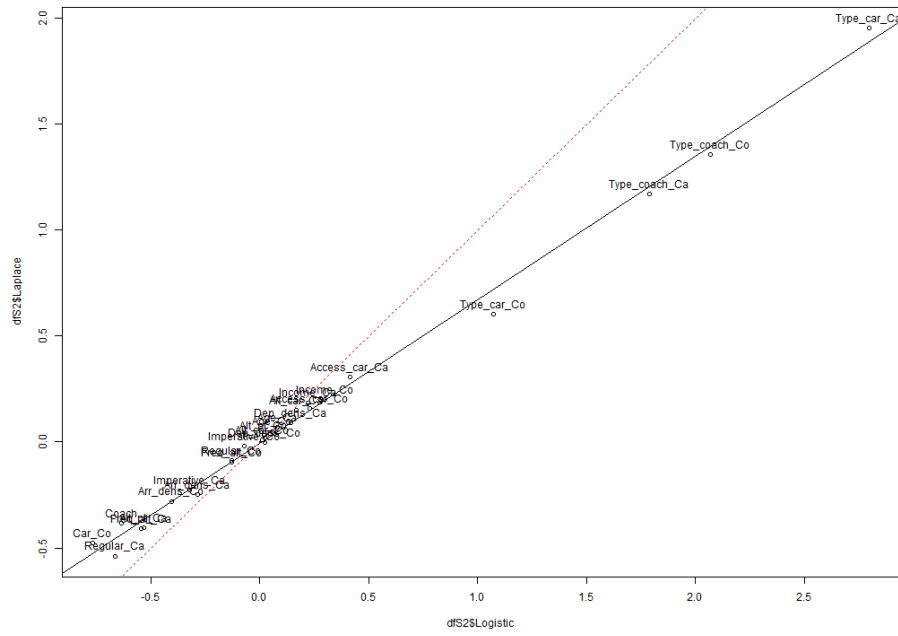


Figure 15: Coefficients Comparison - S2 - Laplace : Logistic

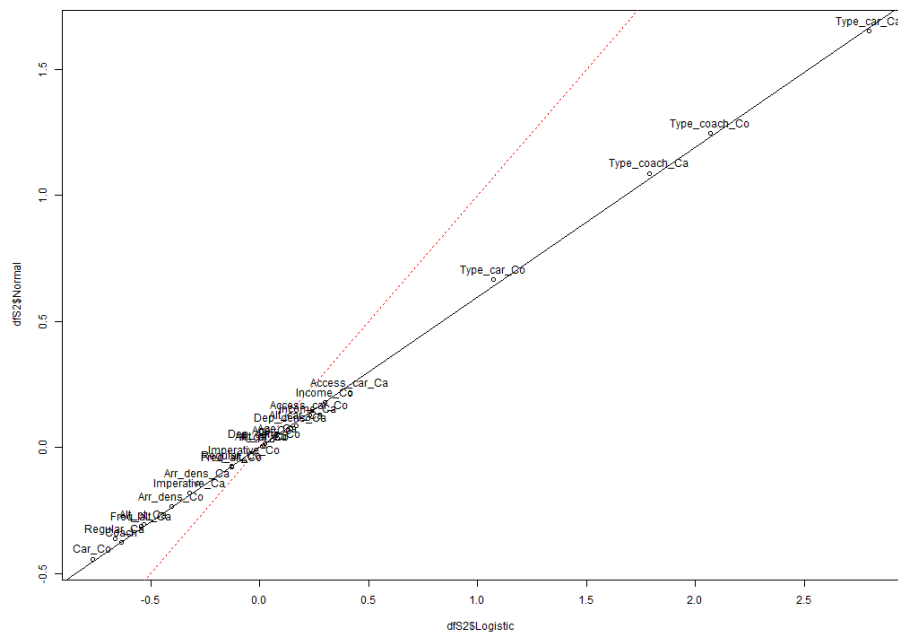


Figure 16: Coefficients Comparison - S2 - Normal : Logistic

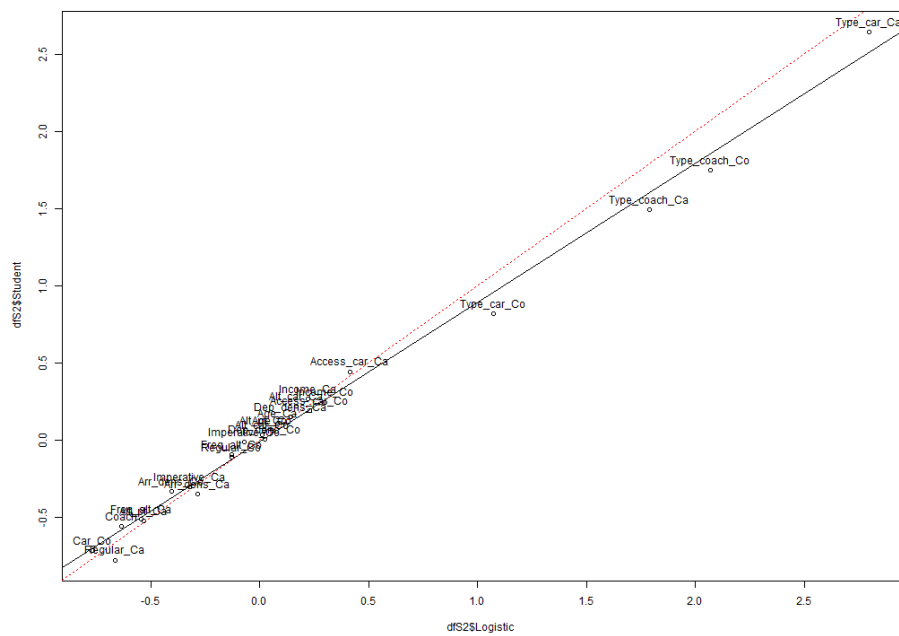


Figure 17: Coefficients Comparison - S2 - Student : Logistic

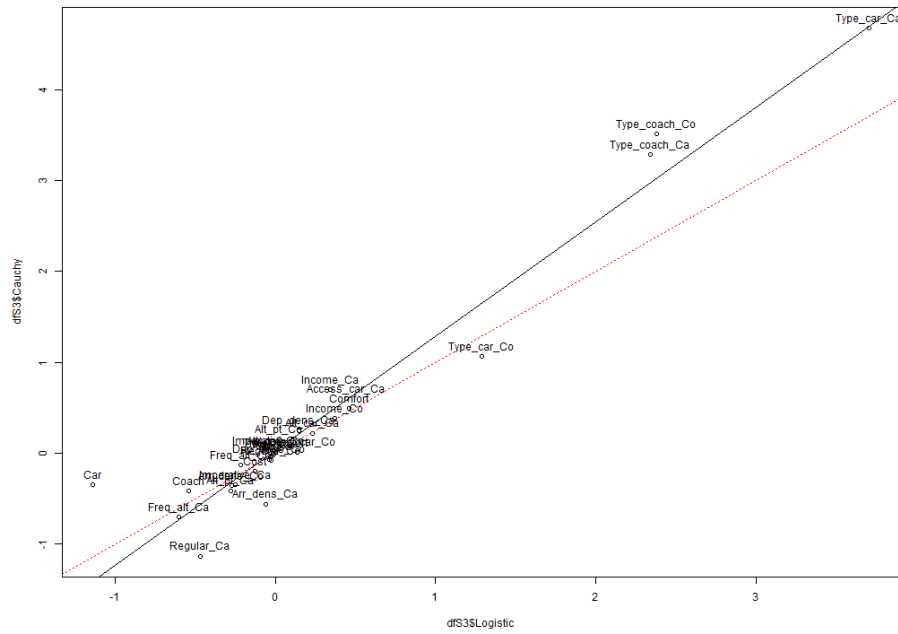


Figure 18: Coefficients Comparison - S3 - Cauchy : Logistic

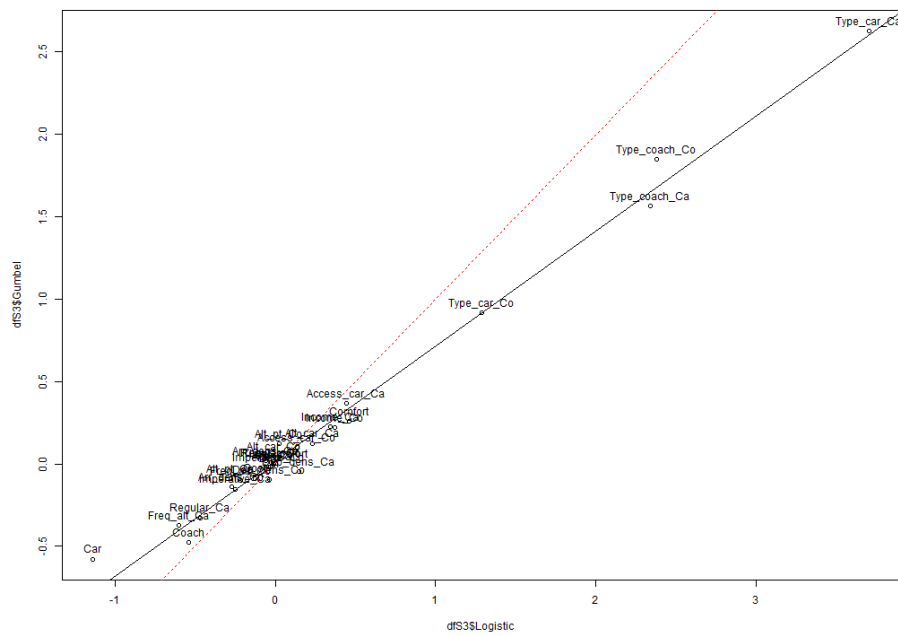


Figure 19: Coefficients Comparison - S3 - Gumbel : Logistic



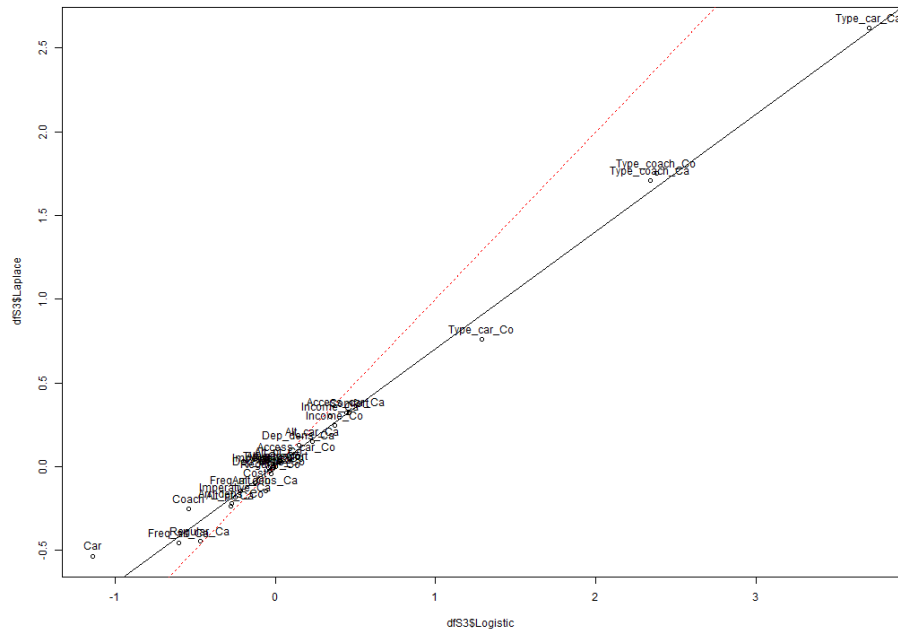


Figure 20: Coefficients Comparison - S3 - Laplace : Logistic

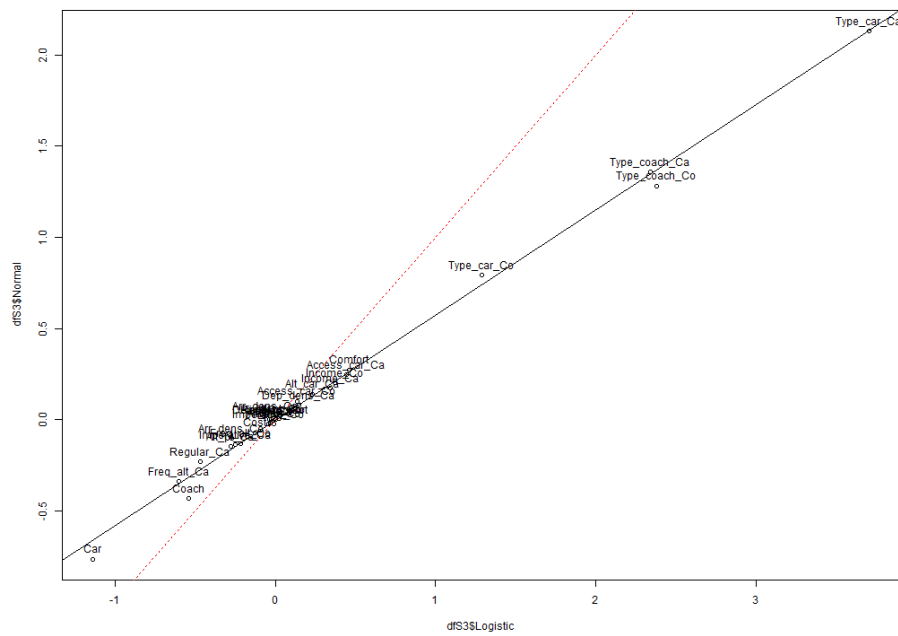


Figure 21: Coefficients Comparison - S3 - Normal : Logistic



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