Numerical work II

Mustapha Bousakla

Study the dynamics of two coupled Hodgkin-Huxley neurons described by the equations:

$$
\begin{aligned}
C_m\frac{dV}{dt} &= \overline{G}_{Na}m^3h(E_{Na}-V)+\overline{G}_K n^4(E_K-V) \\
&\quad +G_m(V_{rest}-V)+I+\sum I_{syn} \\
\frac{dx}{dt} &= \alpha_x(V)(1-x)-\beta_x(V)x\,,
\end{aligned}
\qquad
\begin{aligned}
\alpha_n(V) &= \frac{10-V}{100(e^{(10-V)/10}-1)}, \\
\beta_n(V) &= 0.125e^{-V/80}, \\
\alpha_m(V) &= \frac{25-V}{10(e^{(25-V)/10}-1)}, \\
\beta_m(V) &= 4e^{-V/18}, \\
\alpha_h(V) &= 0.07e^{-V/20}, \\
\beta_h(V) &= \frac{1}{(e^{(30-V)/10}+1)}.
\end{aligned}
$$

With $x = n, m, h$ and the parameters:
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$$
\begin{aligned}
C_m &= 9\pi \\
E_{Na} &= 115\ \text{mV} \\
E_K &= -12\ \text{mV} \\
V_{rest} &= 10.6\ \text{mV} \\
G_{Na} &= 1080\pi \\
G_K &= 324\pi \\
G_m &= 2.7\pi \\
I &= 280\ \text{pA}
\end{aligned}
$$

The synaptic currents are mediated by AMPA and GABAA and are given by:

$$I_i^{syn} = g_i r_i (E_i - V), \qquad i = A, G$$

g_i (=10 nS for AMPA and GABAA) is the maximum synaptic conductance, $E_A = 60$ mV and $E_G = -20$ mV are the reversal potentials for AMPA and GABA, respectively, and r_i is the fraction of bounded (i.e. open) synaptic receptors whose dynamics is given by:

$$\frac{dr_i}{dt} = \alpha_i f(V_{pre})(1 - r_i) - \beta_i r_i$$

where α_i and β_i are the rate constants ($\alpha_A = 1.1\ \text{mV}^{-1}$,$\alpha_G = 5\ \text{mV}^{-1}$, $\beta_A = 0.19\ \text{mV}^{-1}$, $\beta_G = 0.3\ \text{mV}^{-1}$) and $f(V_{pre})$ is the neurotransmitter concentration in the presynpatic cleft. For simplicity, we assume $f(V_{pre})$ to be an instantaneous function of the presynaptic potential given by:

$$f(V_{pre}) = \frac{T_{max}}{1 + e^{-\frac{V_{pre}-V_p}{K_p}}}$$

where $T_{max} = 1\,\mathrm{mM}^{-1}$ is the maximal value of f $K_p = 5\,\mathrm{mV}$ gives the steepness of the sigmoid and $V_p = 62\,\mathrm{mV}$ sets the value at which the function is half-activated.

Analyze the case where the two neurons are unidirectionally coupled (excitatory and inhibitory) and the situation in which they are mutually coupled and the two couplings are excitatory, the two are inhibitory and one is excitatory and the other inhibitory. Represent the results by plotting the time evolution of the membrane potential

Results and Discussion

Figure 1 depicts the dynamics of two unidirectionally coupled neurons for the same external current $I_{ext} = 280\,\mathrm{pA}$. Both the presynaptic and the postsynaptic neurons spike with the same frequency but difference phase. The spiking of the postsynaptic neuron happens after the presynaptic one as usual (delayed synchronization) and in the inhibitory case the phase difference is larger. The zoomed figures on the right may be misleading since they may wrongly indicate an anticipated synchronization depending on the zoomed interval (like the figure on the bottom right), but the whole plots on the left correctly show a normal delayed synchronization.

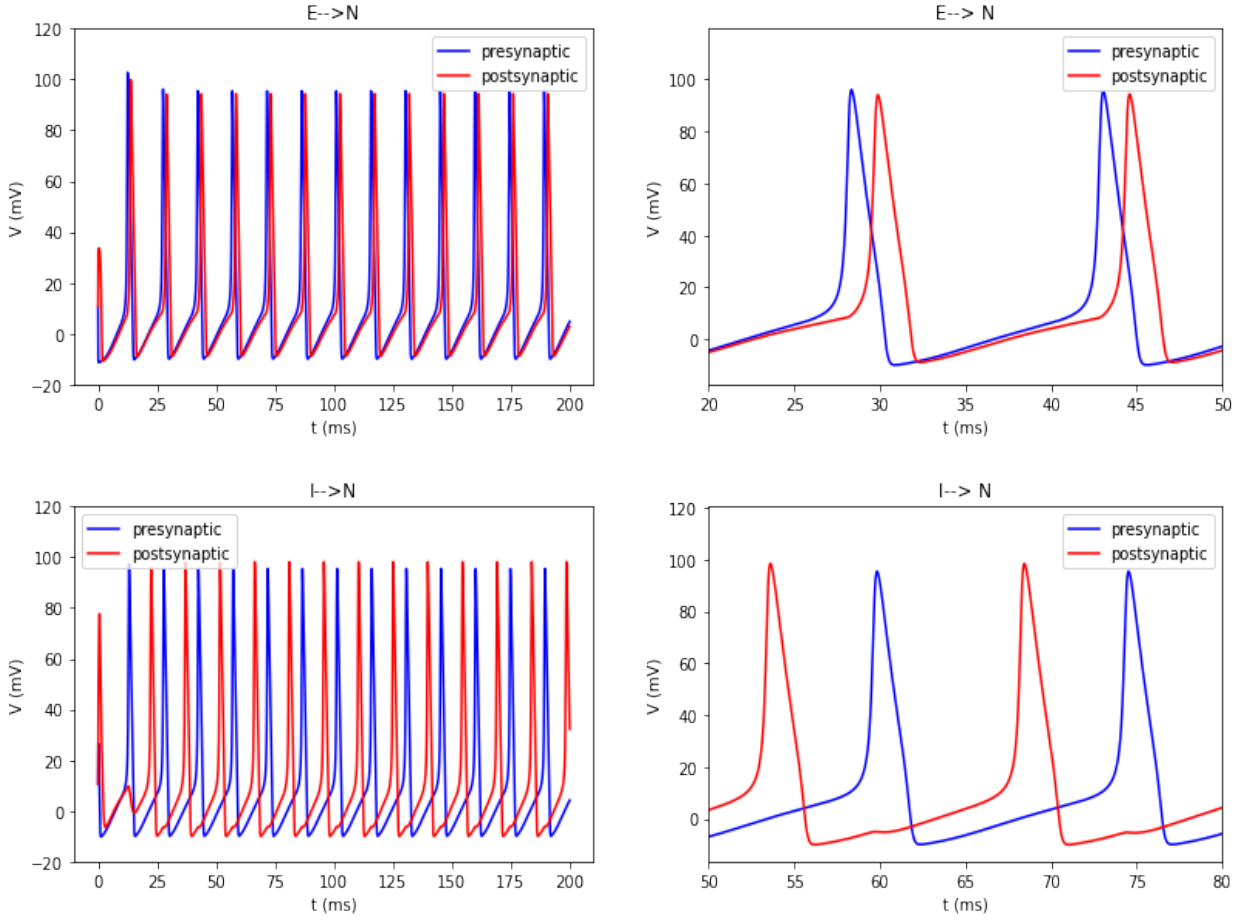

Figure 1: V of the presynaptic and postsynaptic neurons for unidirectional coupling. On the right there are the respective zooms of the figures on the left. The external current is 280 pA.

Figure 2 clearly shows how different couplings affect the relative phase of the spikings: excitatory connections promote synchronous firing and inhibitory ones trigger alternate spikings (delayed synchronization as usual). If we combine two neurons of each type as represented in the bottom figures, the inhibitory neuron prevents a perfect synchronization that would be expected if they were both excitatory (upper figures).

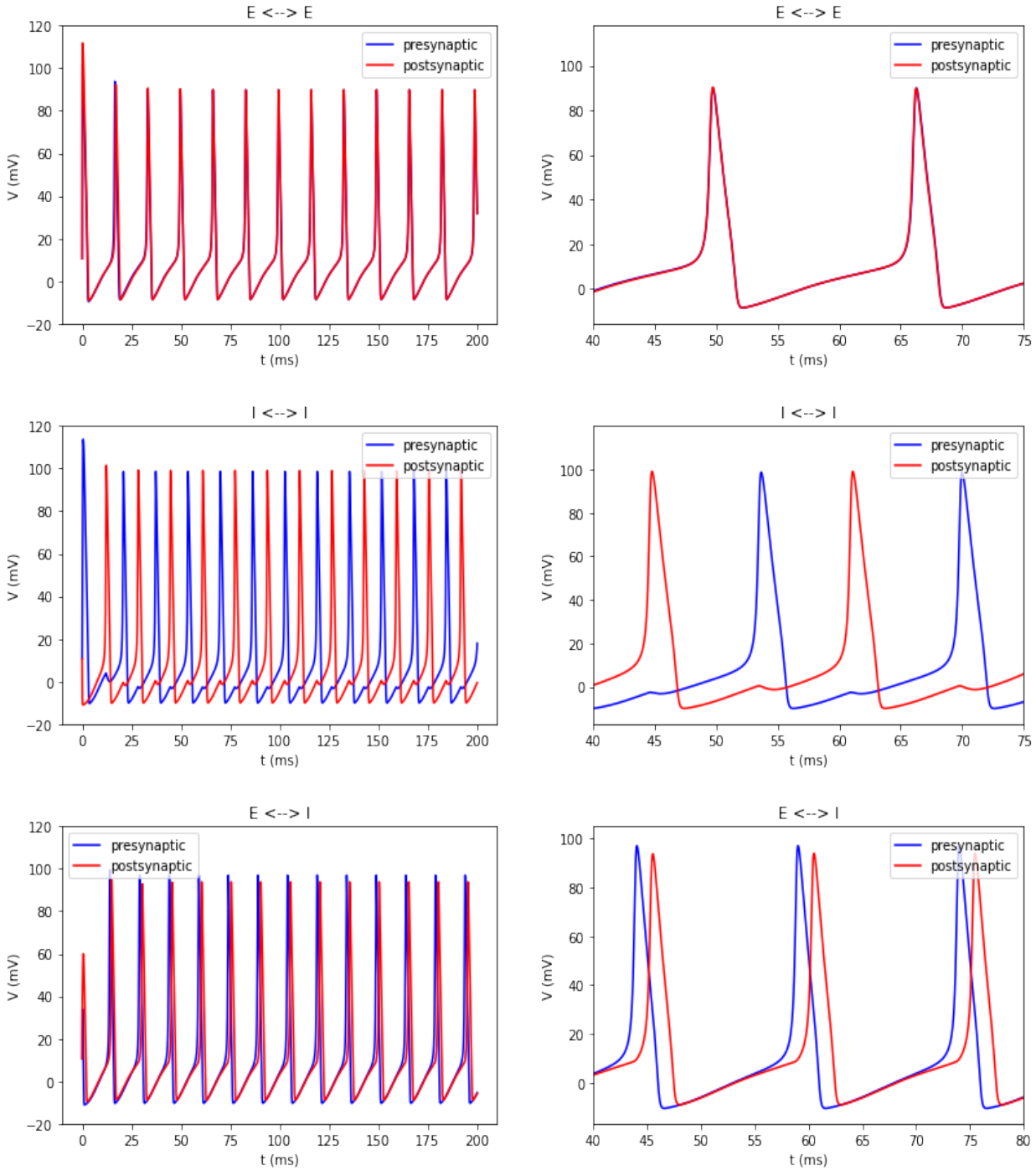

Figure 2: V of the presynaptic and postsynaptic neurons for bidirectional coupling. Three different types of couplings are analyzed as indicated by the titles of each plot (excitatory-excitatory, inhibitory-inhibitory and excitatory-inhibitory). On the right there are the respective zooms of the figures on the left.

Numerical work I

Mustapha Bousakla

We are firstly asked to numerically integrate the Hodgkin-Huxley model:

$$
\begin{aligned}
C_m\frac{dV}{dt} &= \overline{G}_{Na}m^3h(E_{Na}-V)+\overline{G}_K n^4(E_K-V) \\
&\quad +G_m(V_{rest}-V)+I \\
\frac{dx}{dt} &= \alpha_x(V)(1-x)-\beta_x(V)x\,,
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1) Plot the time evolution for V, I_{Na}, I_K, I_L for I=280 pA and I=350 pA.

2) Compute the pulsating frequency f vs. I curve for $I = I_0$, increasing I_0 from 150 pA to 350 pA in steps of 1 pA.

3) Repeat point 2) going from 350 pA to 150 pA.

Hint: in 2) and 3) for the first value of I (I=150 pA in 2)) start the simulation with different arbitrary initial conditions for V, n, m and h. For the following values of I use the values of V, n, m and h obtained at the end of the previous simulation with the previous current value.

Results and Discussion

Figures 1 and 2 depict the membrane potential and the ionic currents for two external currents. The numerical details of the simulation can be found in the appended code. The integration method is the 4th Runge-Kutta and it is efficiently implemented using the scipy package. The initial condition for V is the resting voltage V_{rest} and the initial gate probabilities m, n, h are taking randomly in the interval $(0, 1)$.

The Hodgkin-Huxley model for a constant external current reproduces a spiking model in which the train of pulses have a constant frequency that depends on the external current as seen in Figure 4. The ionic currents correctly simulate the opening and closing of the ionic channels and their relation with the membrane potential.

Figure 3 shows more details about how two different external currents affect the frequency of the spikes. The difference in these currents is not enough to visualize a difference in the period of the spikes but it is clearly observed a phase difference between the potentials.

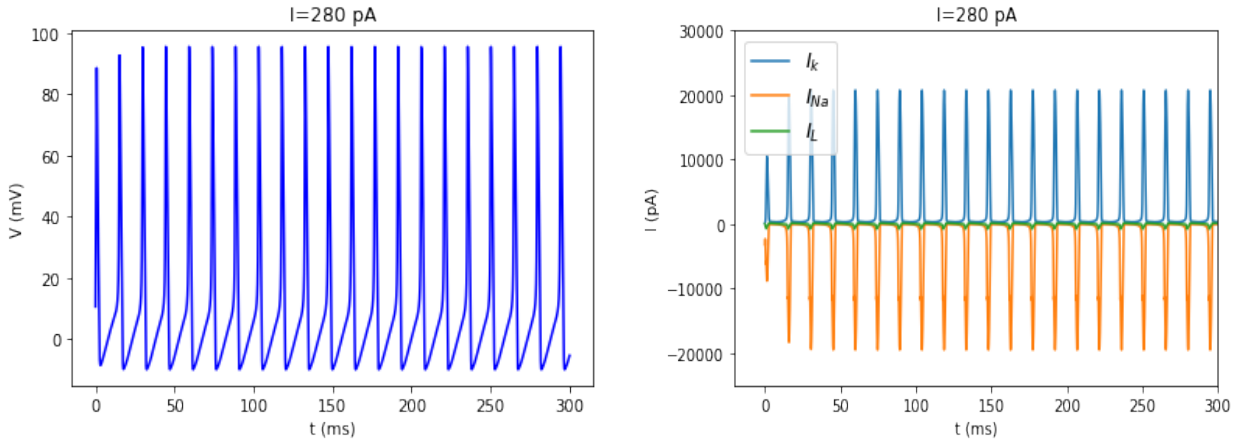

Figure 1: Membrane potential V and ionic currents for external current $I_{ext} = 280$ pA.

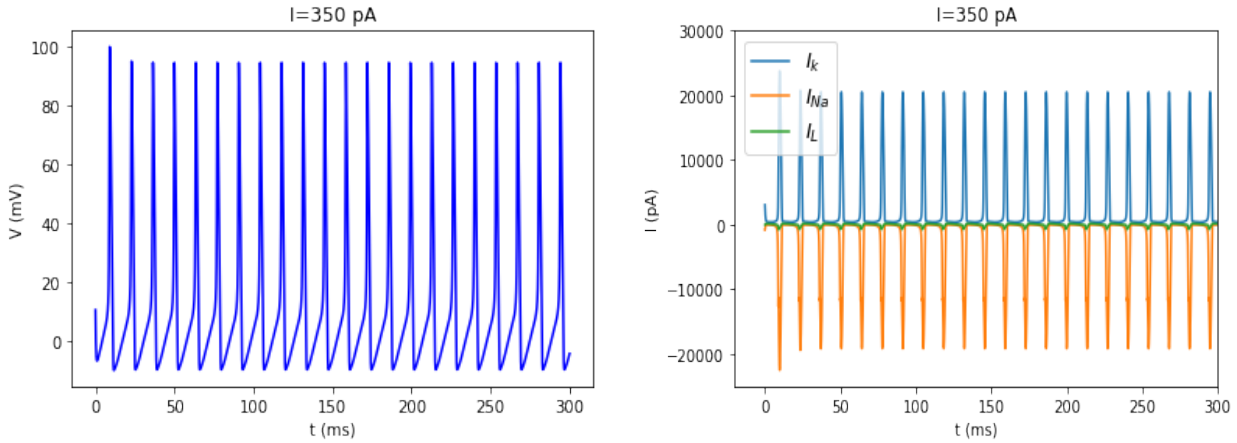

Figure 2: Membrane potential V and ionic currents for external current $I_{ext} = 350$ pA

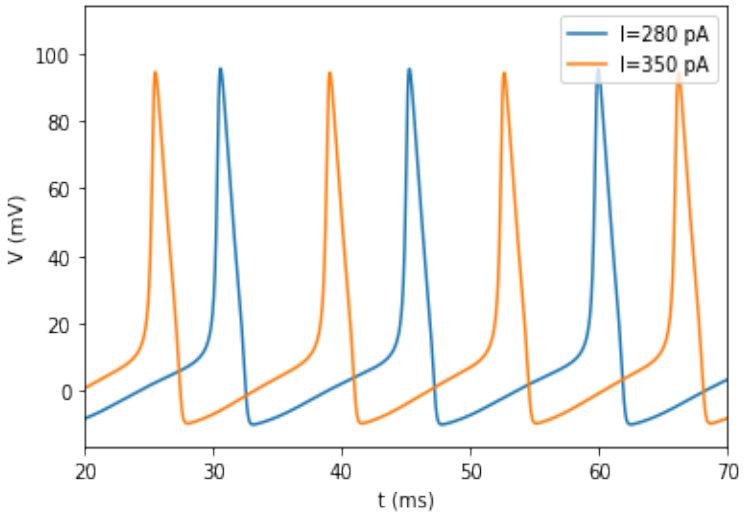

Figure 3: Comparison between V for two different external currents. The difference in frequencies is not apparent for a small difference in the intensities.

Figure 4 not only proves that the spiking frequency of the membrane potential depends on the external current but it also illustrates how it depends on the way this current is changed. For increasing currents in the interval $(150, 350)$ (orange curve) we do not observe any spiking until $I = 280$ pA approximately, but if start from $I = 350$ pA and we constantly decrease the external current (blue curve), the neuron shows a spiking behaviour even for intensities smaller than $I = 280$ pA. In other words, the bifurcation points are different in both experiments, the processes are not reversible.

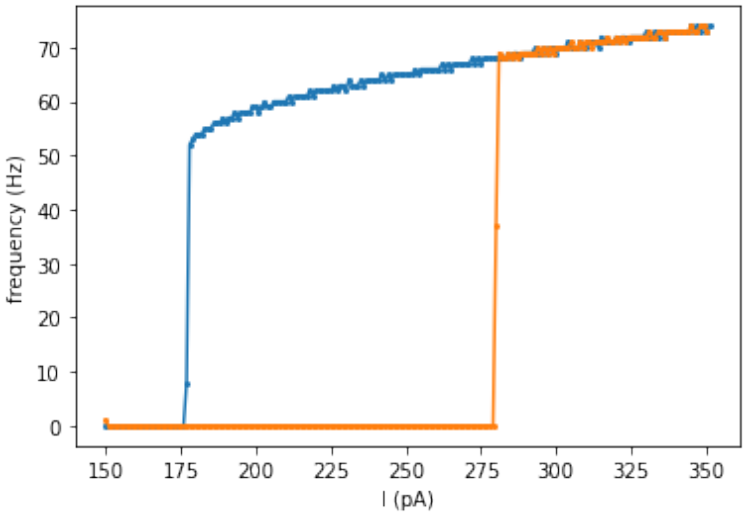

Figure 4: Frequency of the spikes in two different experiments: when the external frequency is increased (orange) and when the external frequency is decreased (blue). The plot clearly shows a different response for the respective reversal experiment.