Stochastic Resonance in a Noise-Driven Excitable System

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$$\varepsilon\frac{dx}{dt} = x - \frac{x^3}{3} - y$$
$$\frac{dy}{dt} = x + a + D\xi(t)$$

Non-driven

Fitz Hugh-Nagumo system

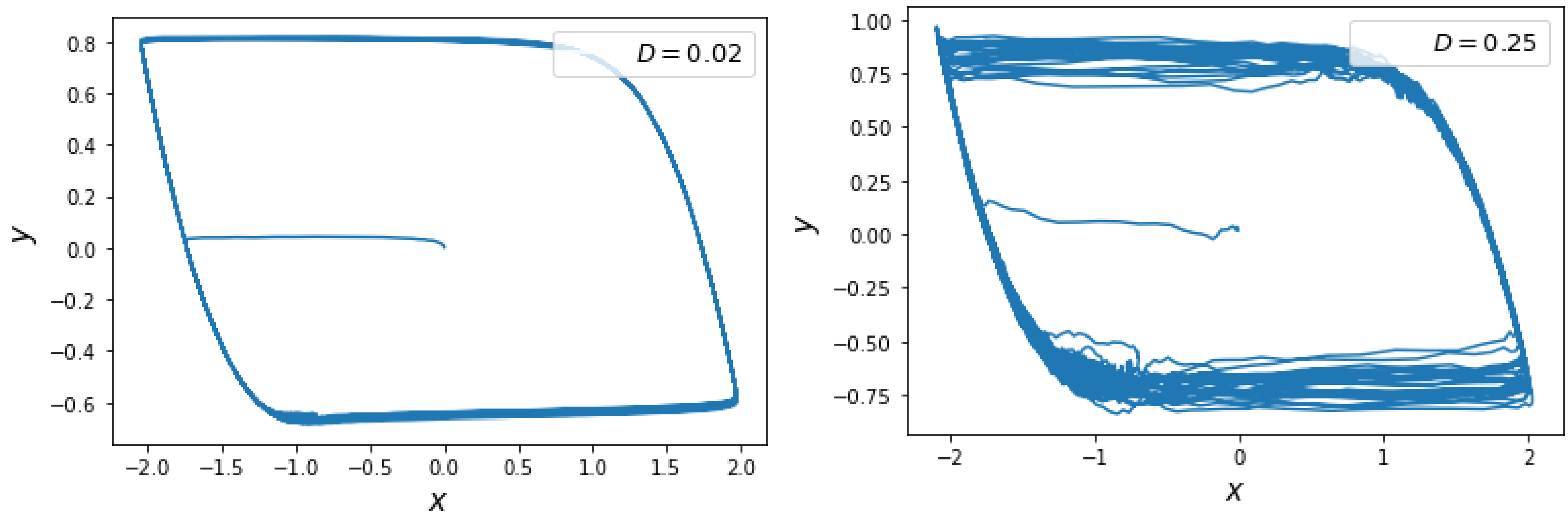
$$\varepsilon\frac{dx}{dt} = x - \frac{x^3}{3} - y$$
$$\frac{dy}{dt} = x + a + Acos(\Omega t) + D\xi(t)$$

Driven

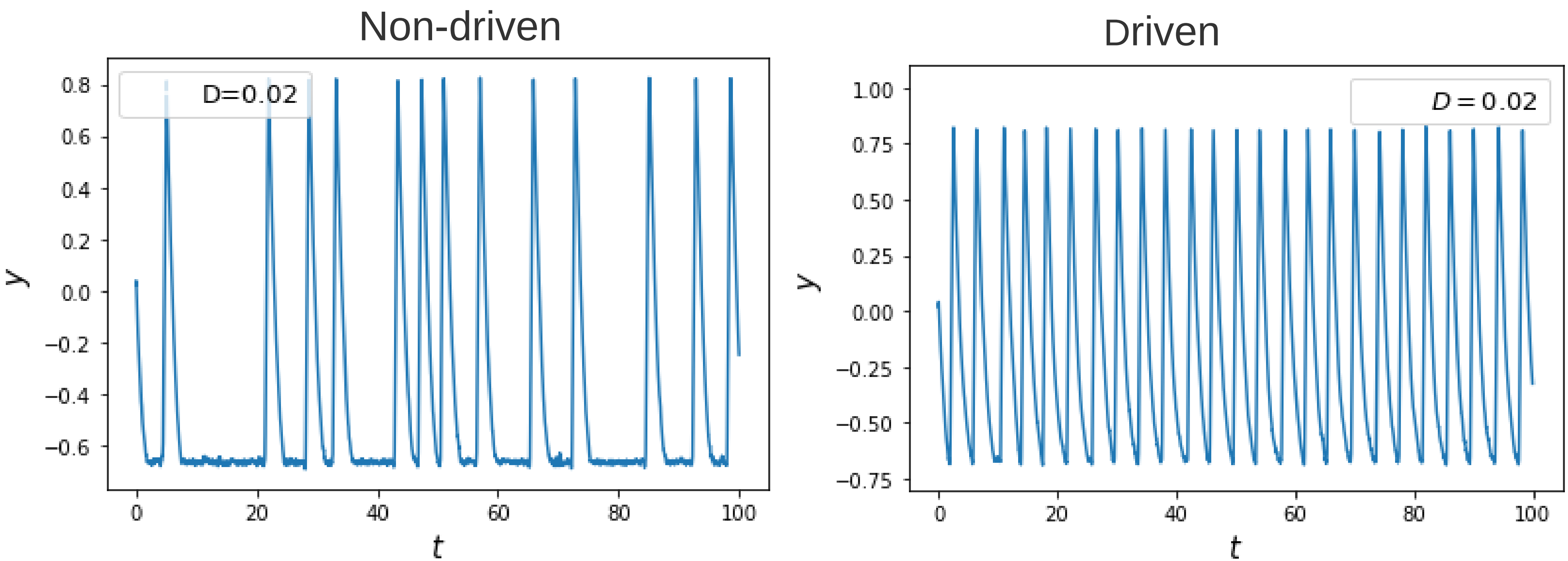
Fitz Hugh-Nagumo system

Milstein Algorithm Ito SDE

$$x(t_{i+1}) = x(t_i) + \frac{h}{\varepsilon}q_1(x(t_i), y(t_i)) \qquad q_1(x,y) = x - \frac{x^3}{3} - y$$
$$y(t_{i+1}) = y(t_i) + hq_2(x(t_i), y(t_i), t) + Dh^{1/2}u_i \qquad q_2(x,y,t) = x + a + Acos(\Omega t)$$



- Noise clearly affects the resemblance of the trajectory with the deterministic trajectory for both forced and non-forced cases.
- Fixed points deterministic system $x^* = -a$ $y^* = a^3/3 - a$
- Trajectory is a periodic limit cycle.
- Parameters: a=1.05, ε=0.01, A=0.1, Δt=0.02

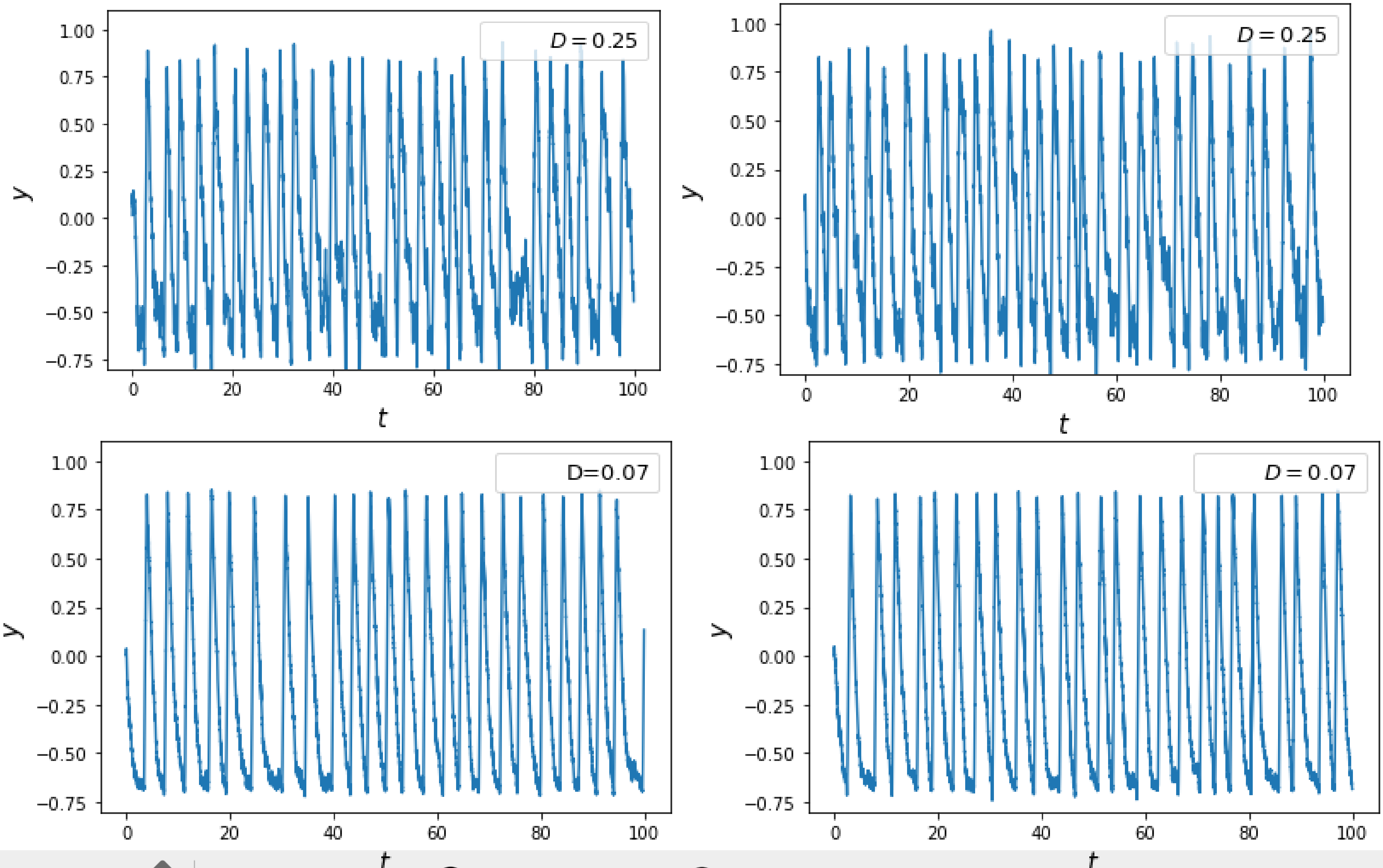


- Non-coherent pulses for non-driven case.
- Non-driven: Irregular peaks, not well defined period for all noises *D.*
- Driven case always shows coherence for well chosen periods.
- Driven: period of the oscillations is the same as the external “force”, that is *stochastic resonance*.

Pulses

Non-driven

Driven

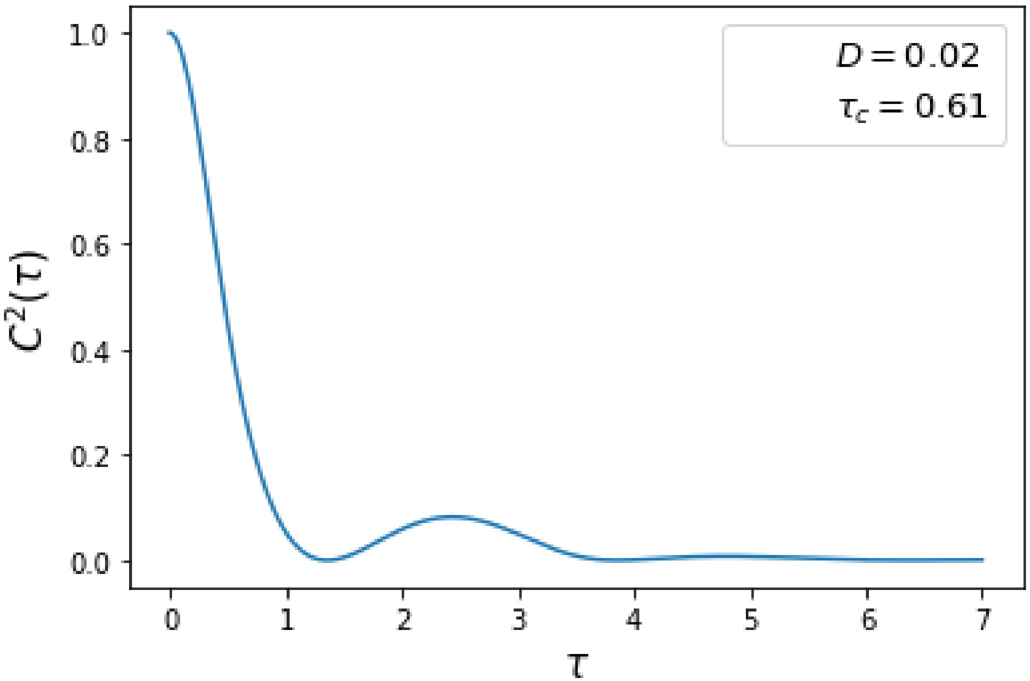


Coherence of the pulses

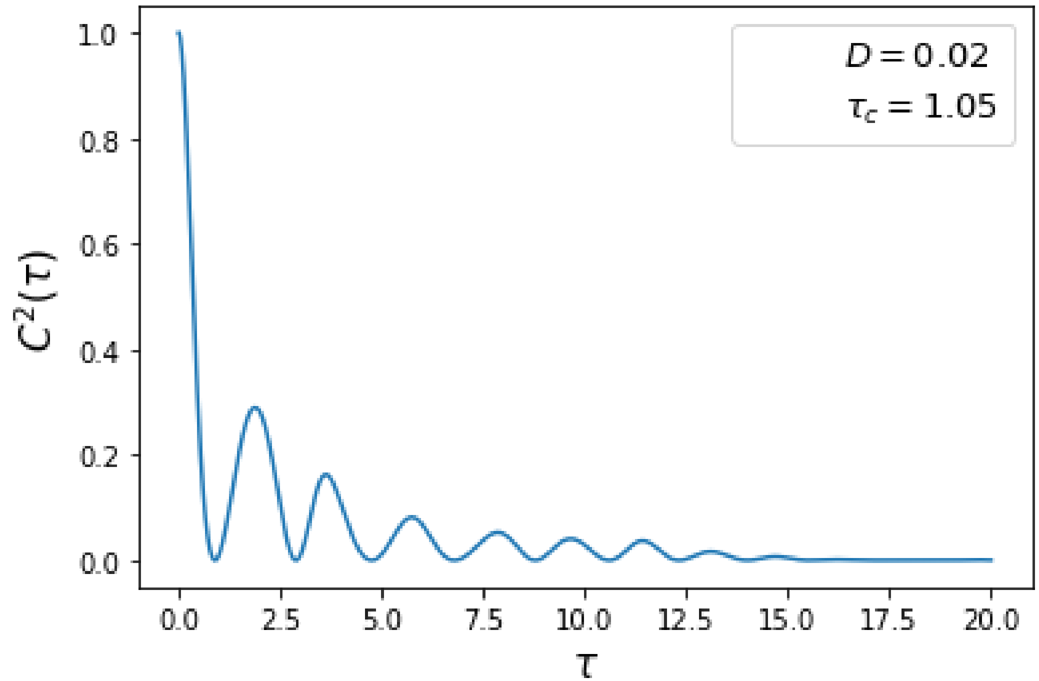
$$C(\tau) = \frac{\langle \tilde{y}(t)\tilde{y}(t+\tau)\rangle_t}{\langle \tilde{y}^2 \rangle} \quad ; \quad \tilde{y} = y - \langle y \rangle$$

- Characteristic correlation time $\tau_c = \int_0^\infty C(\tau)^2 d\tau$
- Correlation time as a measure of *coherence*, similarity.
- The higher the correlation time, the higher the coherence.
- Normalized correlation function.
- We need a long trajectory, like t_{final}=11000 or a set of many trajectories.

Non-driven

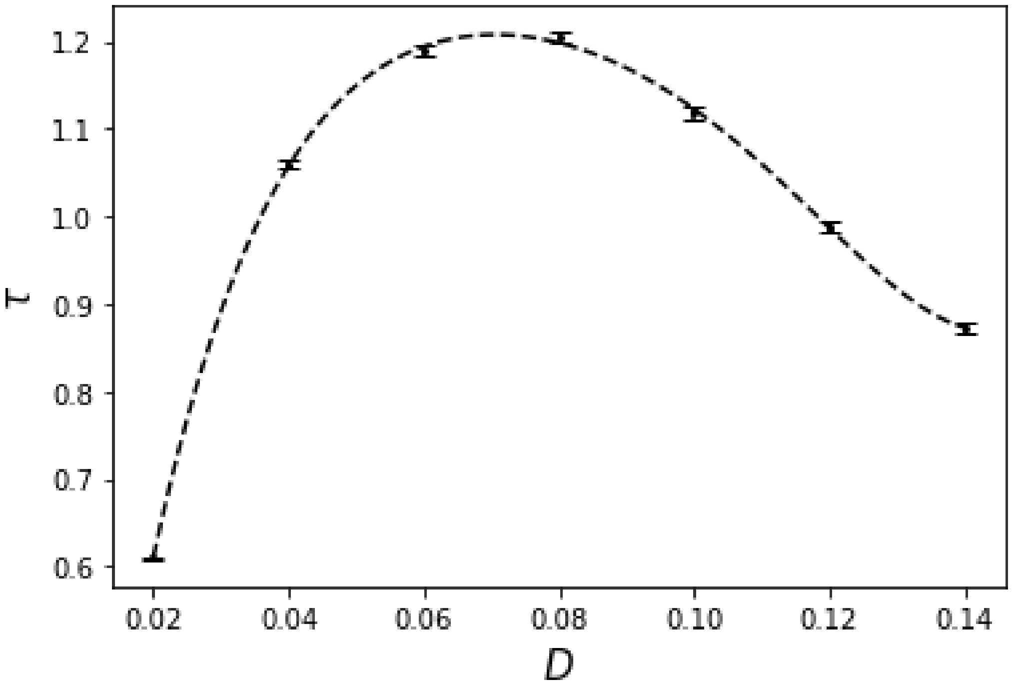


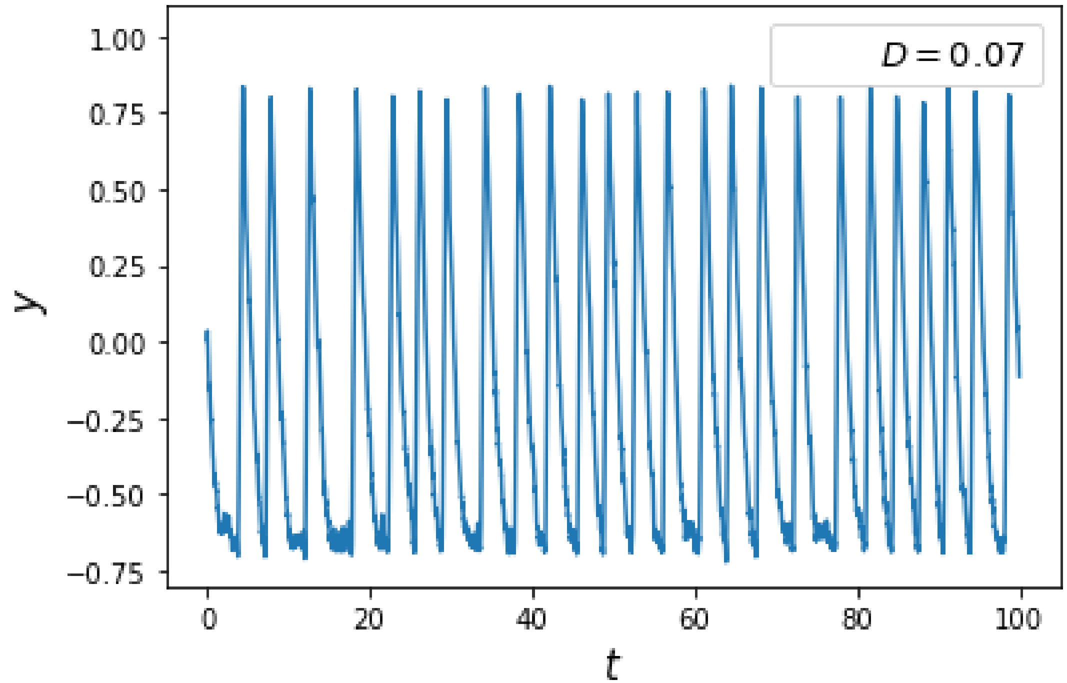
Driven



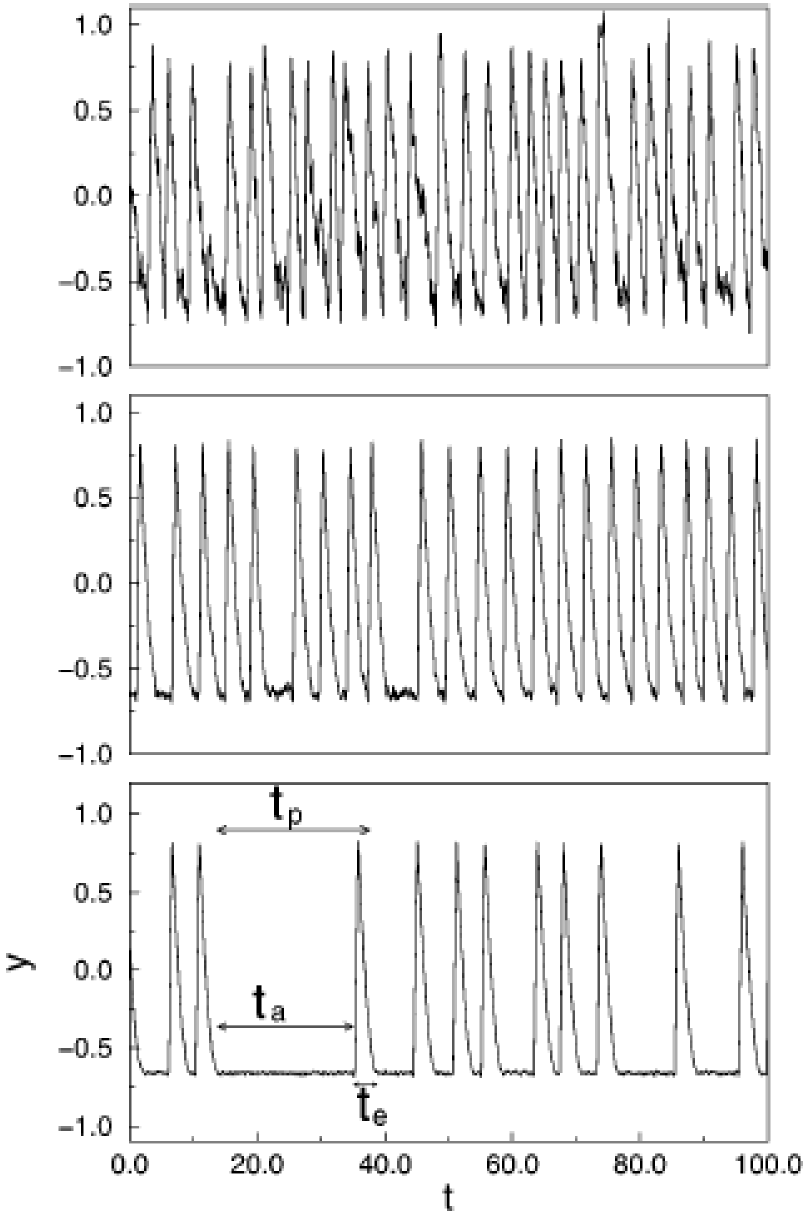
- Data were taken every $\Delta\tau = 0.02$.
- The driving term clearly increases the characteristic correlation time.
- Non-driven case: the correlation time depends on the noise.
- Is there a value of the noise amplitude *D* for which the correlation time is máximum?

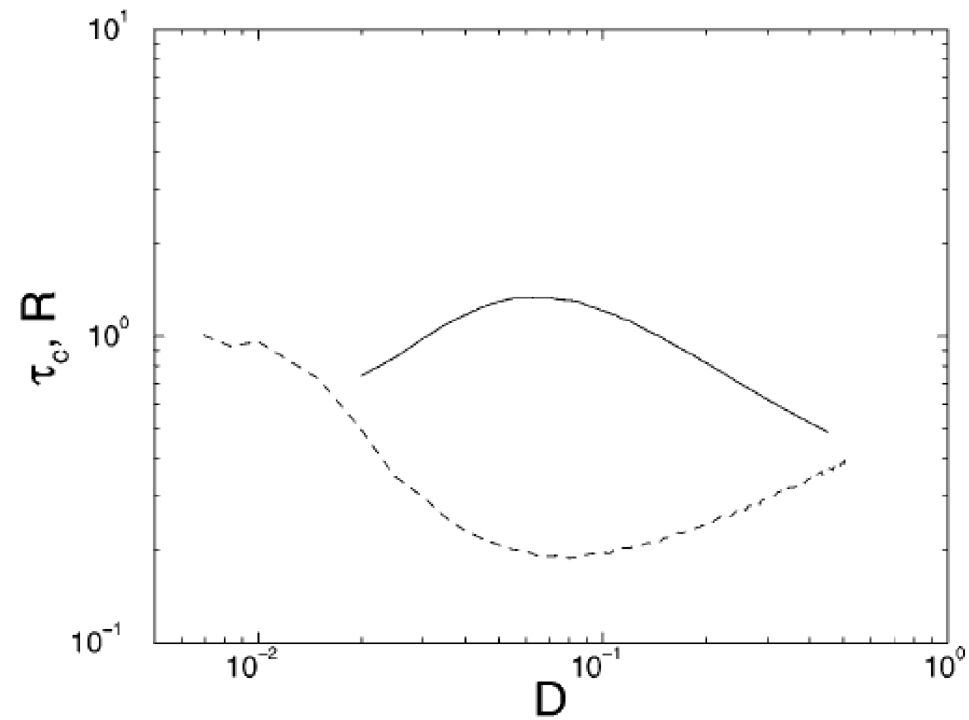
Optimal noise





- 7 noise amplitudes *D* in the interval (0.02, 0.14)
- Trapezoids integration error for array: $f''(\xi)\dfrac{b^3}{12N^2}$ $\quad N = \dfrac{b}{\Delta\tau}$
- 5th order polynomial.
- Maximum coherence for *D*=0.07. $\quad \tau_c = \int_0^\infty C(\tau)^2 d\tau$
- Notorious visual coherence.





A. Pikovsky, J. Kurths, *Coherence resonance in a noise-driven excitable system*. Phys. Rev. Lett. 78 (5) (1997) 775–778

Conclusions

- The driven system always shows periodic coherent pulses with the external term.
- Non-driven systems show irregular peaks for high and small noises.
- In between we have an optimal noise amplitude with maximum coherence and well defined period for the oscillations
- No defined maximum of the characteristic correlation time for driven case.

THANK YOU

for your attention