VICSEK MODEL

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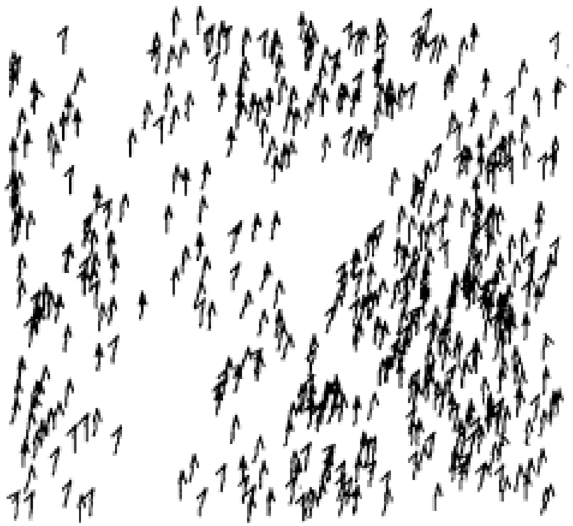
Critical and Cooperative Phenomena

- *N* self-propelled particles inside a box of size *L.*
- Constant velocity but different direction at each time depending on the neighbouring particles.
- Local interactions up to radius R_o.

Small density and small noise



Higher density and small noise

Classical Kinematics

$$\mathbf{r}_j^{t+\Delta t} = \mathbf{r}_j^t + \Delta t \mathbf{v}_j^{t+\Delta t} \qquad v_{xj}^{t+\Delta t} = v_0 \cos \theta_j^{t+\Delta t}$$

$$\mathbf{v}_j^{t+\Delta t} = (v_{xj}^{t+\Delta t}, v_{yj}^{t+\Delta t}) \qquad v_{yj}^{t+\Delta t} = v_0 \sin \theta_j^{t+\Delta t}$$

Alignment and noise addition

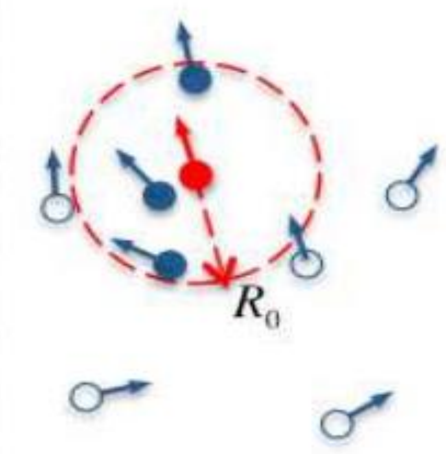
Angular noise

$$\theta_i^{t+1} = \mathrm{Arg}\left[\sum_{\langle i,j \rangle} \mathrm{e}^{\mathrm{i}\theta_j^t}\right] + \eta \xi_i^t$$

Vectorial noise

$$\theta_j^{t+1} = arg\left[\sum_{j\sim k} e^{\iota\theta_k^t} + \eta n_j^t e^{\iota \xi_j^t}\right]$$

$$n_{jk}^t = \begin{cases} 1 & \text{si } \|r_j^t - r_k^t\| < R_0 \\ 0 & \text{si } \|r_j^t - r_k^t\| > R_0 \end{cases}$$



Order parameter $$v_a = \frac{1}{N v_0} \left| \sum_i \mathbf{v}_i \right|$$

Algorithm

- Randomly initiate the *N* positions and the angles.
- Define functions for the *distance* and *neighbours velocity.*
- Each time compute the deviation angle from the neighbours velocity, add the noise to the velocities (angular or vectorial) and update the positions.
- For each noise save the magnitudes

Magnitudes used in the simulations

$\rho = N/L^2$ (density)

$v_0 = 1$

$\eta \in [0.1, 0.9]$

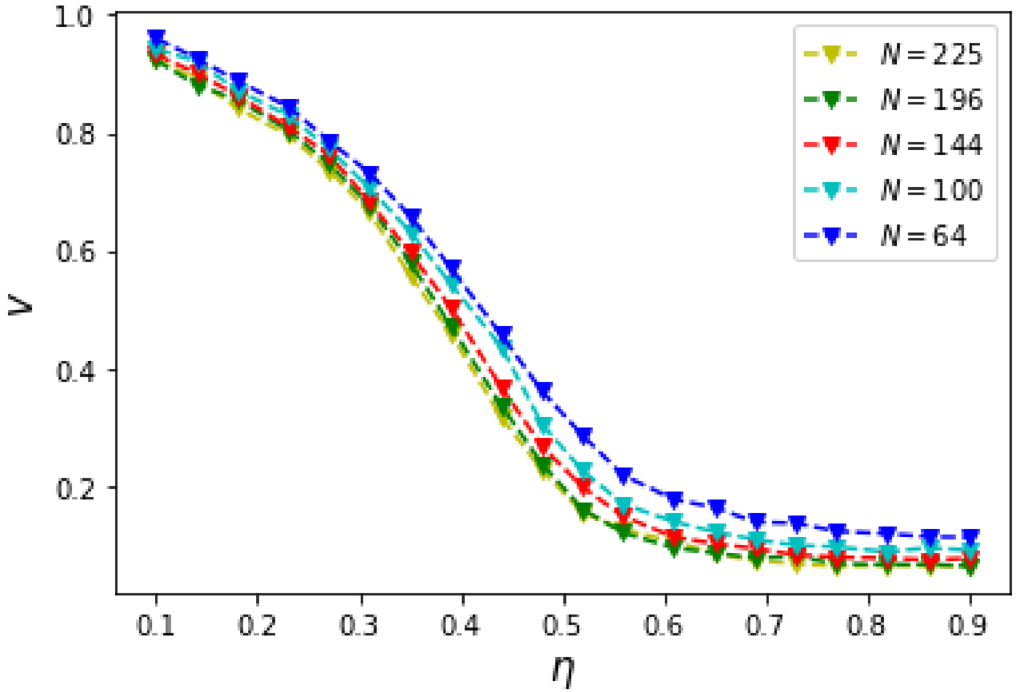
$\rho = 1$

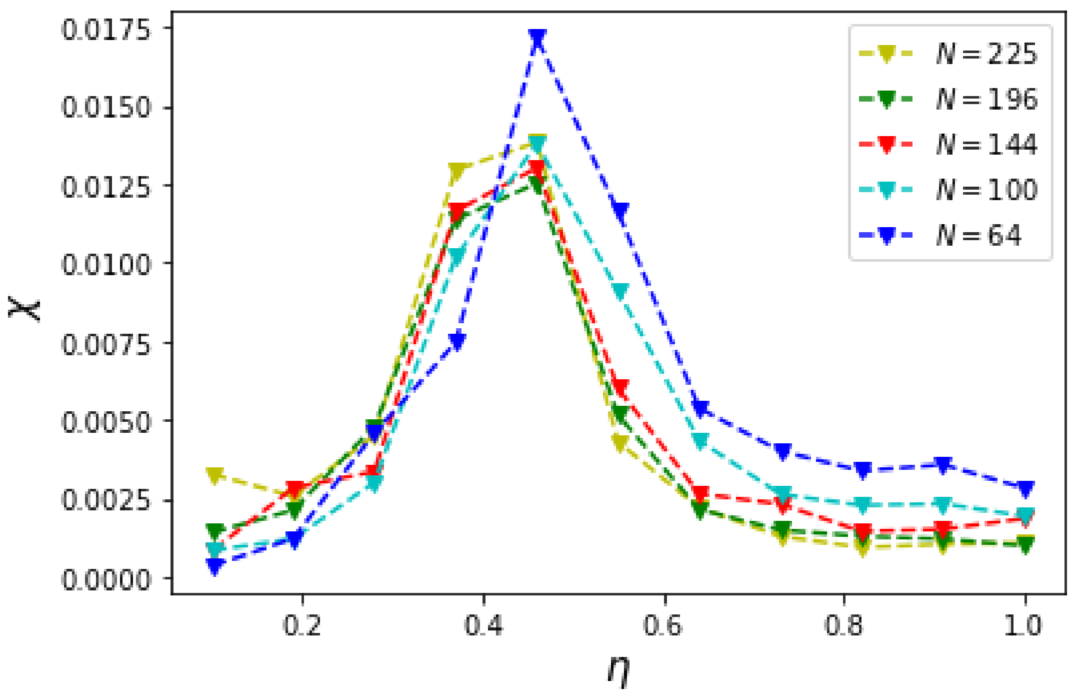
$\Delta t = 100$ (time between measures)

$t_f = 20000$

$N = [225, 196, 144, 100, 64]$

Order parameter and Susceptibility for the Angular noise

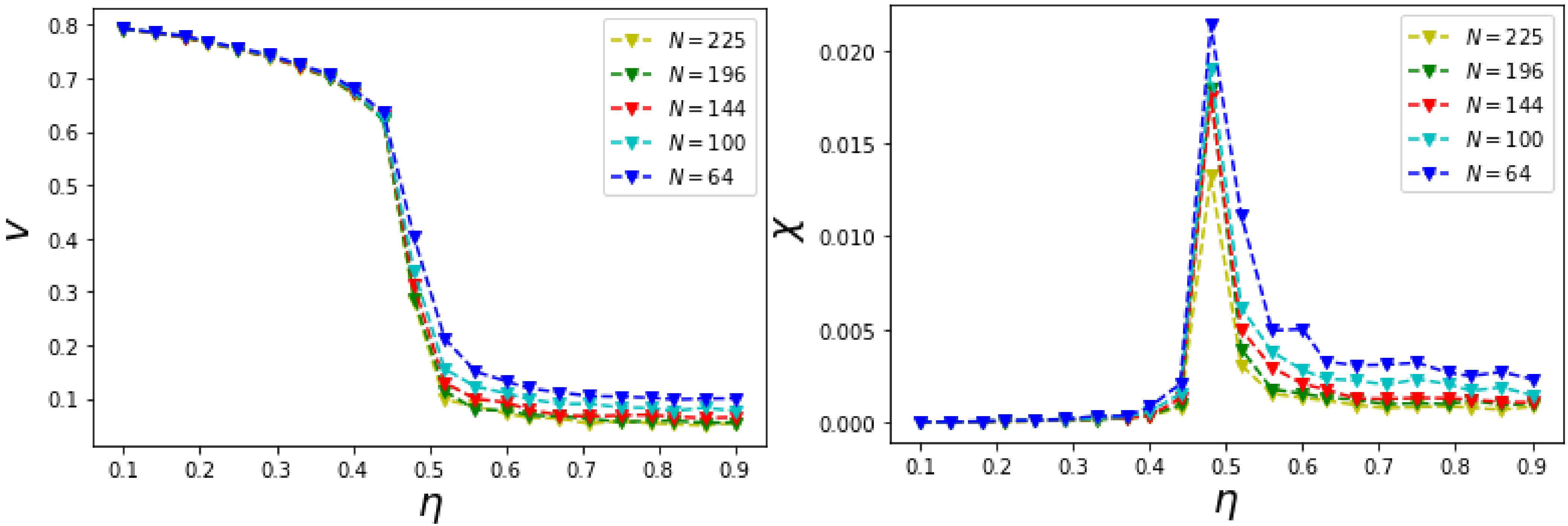




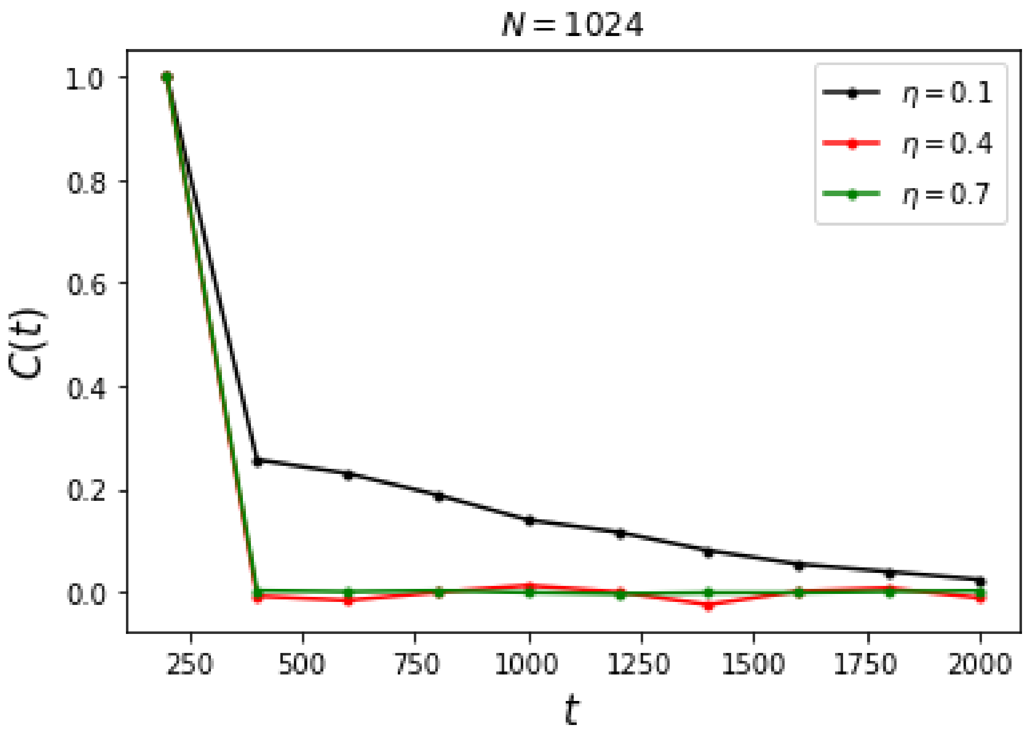
$$\chi = \sigma^2[v]$$

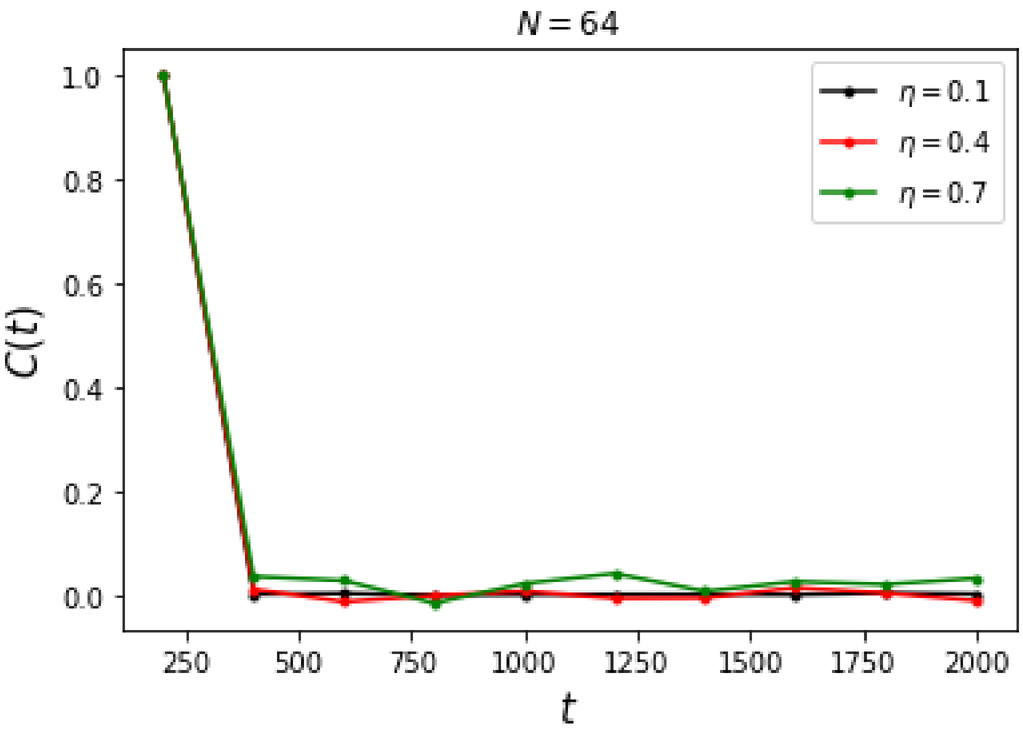
- Ordered collective movement for small noise.
- The phase transition is continuous (2nd order transition)
- Critical noise η_C=0.46
- No collapse for the order parameter
- The peaks heights have no clear dependency with *N.*

Order parameter and Susceptibility for the Vectorial noise



- The phase transition is discontinuous (1st order transition)
- Critical noise η_C=0.48
- Collapse of the order parameter for different *N.*
- The peaks are higher the smaller *N.*
- Ising resemblance → convenient scaling, but not enough points!

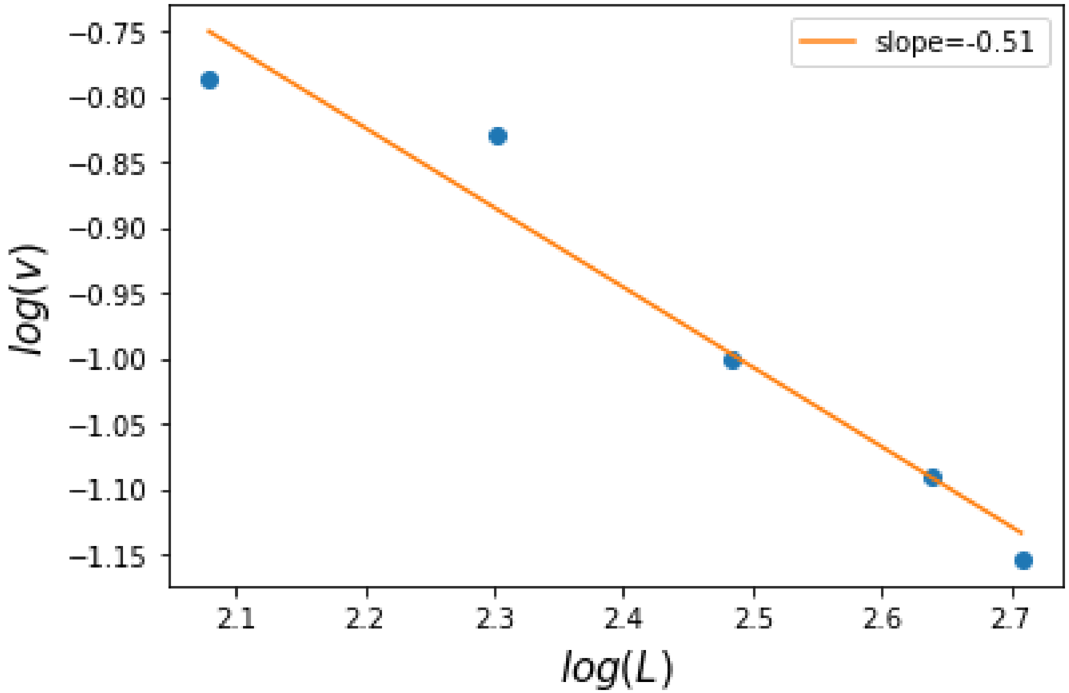




$$C(\tau) = \frac{< (v(t) - \bar{v})(v(t+\tau) - \bar{v}) >_t}{\sigma^2[v]}$$

- Normalized autocorrelation function
- The equilibrium time is small for small systems (around 500 *MCS*).
- No need of many equilibrium steps like in the Ising Model.
- Large systems need large with small noise need many equilibrium steps that should not be included in the averages.

Critical exponents in the Angular case



$$v \sim |\eta_c - \eta|^{\beta} \sim L^{-\beta/\nu}$$

- The value I get is $\beta/\nu = 0.51$
- In the paper [1] $\beta/\nu = 0.45$
- To get a proper scaling more points are needed and higher systems need to be analysed.
- Computationally heavy to consider very large systems.

Conclusions

- The angular and the vectorial noise do not give the same type of phase transition, but they both predict an ordered phase for small noise.
- The figures of the order parameter are slightly different from the ones in [1].
- Vectorial case resembles the most the Ising model.
- Higher system sizes need to be explored and more points are needed for a scaling function procedure.
- The critical exponents cannot be estimated correctly with the obtained data.

References

[1] Vicsek et al. Phys. Rev. Lett. 75, 1226 (1995)

[2] Francesco Ginelli. *The physics of the vicsek model*. The European Physical Journal Special Topics, 225(11-12):2099–2117, 2016.

[3] Hugues Chat´e, Francesco Ginelli, Guillaume Gr´egoire, Fernando Peruani, and Franck Raynaud. *Modeling collective motion: variations on the vicsek model*. The European Physical Journal B, 64(3-4):451– 456, 2008.

THANK YOU

for your attention