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Course: 01:198:206 (04:18156) Section 4 Spring 2020

Homework 2

Answer to Question 1

17 + 23 + 12 = 52

Total permutations = 52!

Permutations of 17 twos = 17!

Permutations of 23 fives = 23!

Permutations of 12 nines = 12!

Permutations that contain exactly 17 twos, 23 fives and 12 nines = 52! / (17! * 23! * 12!)

= **1.831253571 * (10^22)**

Answer to Question 2

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \ldots + b^n \text{, where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

a) $\binom{11}{5}1^{11-5}x^5$ = 462x^5

Coefficient = **462**

b) $\binom{17}{9}3x^8 2y^9 = 17\ choose\ 9 * (3x)^8 * (2y)^9$

$= 24310 * 3^8 * 2^9 * x^8 * y^9 = 81662929920x^8y^9$

Coefficient = **81662929920**

c) $\binom{5}{2}a^{2^3}b^{3^2} = 10a^6b^6$

Coefficient = **10**

Answer to Question 3

a)

- 6 choices of vertices
- Choose 3 to form a triangle

  6 choose 3 = **20** triangles

b)

- 6 vertices
- 5 links from each vertex
- Choose 2 to form an incident pair

  5 choose 2 * 6 = 10 * 6 = **60** incident pairs

c)

- {((u, v), (v, w))} → {(u, v), (v, w), (u, w)}
- (u, v) and (v, w) are multicolored
- {((u, v), (v, w))} maps to multicolored triangle {(u, v), (v, w), (u, w)}
- Three i.p form a triangle in which two i.p are of same color.
- Since a triangle is of three i.p and two i.p are same color and third can be any color then two combinations pairs of multicolored i.p will map to the same triangle.
- Hence {(v, w), (u, w)} also maps to the same triangle {(u, v), (v, w), (u, w)}.

d)

- 5 links for each vertex
- Choose 2 to form an incident pair
- For 3 blue 2 red (or 2 red 3 blue) we have 5 choose 2 = 10 choices including non-multicolored
- 3 non-multicolored choices from 3 blue and 1 from 2 red
- Total i.ps that are multicolored with same center = 10 – 3- 1 = 6

e)

- 6 vertices
- Every vertex has 6 multicolored choices when it is a center
- So total multicolored incident pairs = 6*6 = **36**

f)

- From a vertex are five edges
- Suppose each is color red or blue and three must be of the same color; for if less than three are suppose red then there must be at least three that are blue
- Let A, B, C be the other ends of these three edges, all the same color, say blue.
- If any one of AB, BC, CA is blue, then that edge together with the two edges from P to the edge's endpoints forms a blue triangle. If none of AB, BC, CA is blue, then all three edges are red and we have a red triangle, namely, ABC.

Answer to Question 4

a)

- Ranks in sequence: A 2 3 4 5 6 7 8 9 10 J Q K or 2 3 4 5 6 7 8 9 10 J Q K A
- Sequence is hand consisting of five consecutive cards of any suit
- E.g A, 2, 3, 4, 5 or 10, J, Q, K, A
- Since a sequence must have five cards there are 10 choices to start a sequence i.e. A, 2, 3, 4, 5, 6, 7, 8, 9, 10
- For every card in a sequence there are 4 suits so total combinations in a sequence = 4*4*4*4
- Total sequences of hands possible = 10 * (4^5) = **10,240**

b)

- 4 suits total
- For one suit: A 2 3 4 5 6 7 8 9 10 J Q K
- For a hand of same suit total combinations = 13 choose 5 = 1,287
- Total hands that consist of cards that are all the same suit = 1,287 * 4 = **5,148**

c)

- 4 suits total
- For one suit: A 2 3 4 5 6 7 8 9 10 J Q K or 2 3 4 5 6 7 8 9 10 J Q K A
- 10 choices for a sequence of hand from same suit.
- How many hands consist of cards that are all the same suit = 10 * 4 = **40**