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Homework 1

Answer to Question 1

1. First character must be a letter (upper or lowercase) gives us 26+26 = 52 choices.
2. Followed by either 2, 3, 4, 5 characters. Can be either letter (upper or lowercase), numbers or underscores. Total choices = 52 + 10 + 1 = 63.

3 chars	52 * 63 * 63 choices
4 chars	52 * 63 * 63 * 63 choices
5 chars	52 * 63 * 63 * 63 * 63 choices
6 chars	52 * 63 * 63 * 63 * 63 * 63 choices

Total variable names possible = 52*(63^2) + 52*(63^3) + 52*(63^4) + 52*(63^5)

= 52,439,063,040 ways

Answer to Question 2

Which policy is more secure can be determined with number of possible combinations. The greater number of combinations the more secure the policy is.

Policy 1:

- Letter (upper or lowercase), numbers, 5 special characters.
- Length exactly 16 characters.

Total choices for each character = 26 + 26 + 10 + 5 = 67

Total combinations = 67^16 = 1.6489096e+29

Policy 2:

- Letter (lowercase)
- Length exactly 24 characters.

Total choice for each character = 26

Total combinations = 26^24 = 9.1066858e+33

Policy 2 is more secure as it has more combinations.

Answer to Question 3

- **From 1 to 1,000,000**
- **Contains either a 1 or a 5 (or both)**

Total #digits 0-9 = 10

#Digits 0, 2-4, 6-9 = 8

Numbers that don't contain either a 1 or a 5 in a XXXXXX number

(8 choose 1) ^6 = 8^6 = 262,144

Total numbers in XXXXXX number

(10 choose 1) ^6 = 1,000,000

Total numbers – numbers that don't contain 1/5 = 1,000,000 – 262,144 = 737,856 = numbers that contain either a 1 or a 5 (or both) in a XXXXXX number

Including 1,000,000 which contains a 1, increment count by 1

737,856 + 1 = 737,857

Total numbers that contain either a 1 or a 5 (or both) = 737,857

Answer to Question 4

One bijection from X to X if X is of size 3:

A	→	A
B	→	B
C	→	C

One other can be:

A	→	C
B	→	B
C	→	A

How many bijections from X to X exist?: n!

n choices for element 1, n-1 for element 2, n-2 for element 3 so n! bijections. This is 1-1 and onto functions.

Bx is the set of all bijections from X to X so Bx is of size n!

Set {1, 2, 3, ..., n} has n! permutations.

Find a bijection between Bx and permutations of the set {1, 2, 3, …, n}.

Bijection 1	→	2
Bijection 2	→	48
Bijection 3	→	69
Bijection 4	→	8

Or

Bijection 2	→	1
Bijection 48	→	2
Bijection 69	→	3
Bijection 8	→	4

Since both have the size n!, there are (n!)! bijection from Bx to permutation of set {1, 2, 3, …, n}.

Answer to Question 5

Part a)

No restriction so chooses from 8 possible chairs, but seating is same if table is rotated so divide by 8.

So total arrangements = 8! /8 = (8-1)! = **5,040**

Part b)

Consider 7 groups and Anna and Brian need to choose one. Then they must choose either the left or right chair depending on where either of them sit so 2! choices. Divide by 7 to negate double counting table rotations.

So total arrangement = 2 * 7! /7 = **1,440**

Part c)

Consider 6 groups as Anna and Brian need to choose one. Then they must choose between the three chairs in this group giving 2! choices as Anna/Carol can sit at either end of Brian. Divide by 6 to negate double counting table rotations.

So total arrangements = 2! * 6! /6= **240**

Part d)

Brian sitting next to Anna = 2*7! /7 = 1440

Brian sitting next to Carol = 2*7! /7 = 1440

So total arrangement = 1440 + 1440 – when Brian is next to both (double counted) = 2880 – 140 = **2640**

Answer to Question 6

Ways we can select at least 1 woman = total ways – ways there are no women

Total faculty = 6 women + 9 men = 15 totals

Total ways regardless of gender = 15 choose 5 = 3003

Total ways if no women = 9 men choose 5 = 126

Ways we can select at least 1 woman = total ways – ways there are no women

= 3003 – 126

= **2,877**

Answer to Question 7

1 to 15 list = {1 2 3 4 5 6 7 8 9 10 11 12 13 14 15}

1 to 15 Modulus 3 list = {1 2 0 1 2 0 1 2 0 1 2 0 1 2 0}

5 2's

5 1's

5 0's

For the sum of 3 numbers to be divisible by 3 that is congruent to 0 mod 3 we can have the following combinations:

- 1 + 1 + 1 = 3 → 5 choose 3 = 10
- 2 + 2 + 2 = 6 → 5 choose 3 = 10
- 0 + 0 + 0 = 0 → 5 choose 3 = 10
- 0 + 1 + 2 = 3 → (5 choose 1) ^3 = 125

Total combinations = 10 + 10 + 10 + 25 = **155 ways**