Mustafa Sadiq (NetID: ms3035)

Course: 01:198:206 (04:18156) Section 4 Spring 2020

Homework 6

Answer to question 1

Part (a)

Length	**Possible records**
3	HHH
4	THHH
5	HTHHH, TTHHH
6	TTTHHH, THTHHH, HHTHHH, HTTHHH

Part (b)

P[A1] = 1/8

P[A2] = 4/8

P[A3] = 2/8

P[A4] = 1/8

Part (c)

Get heads on 3 tosses

A1: E[X] ← probability 1/8, total tosses 3

Get tails on first toss which means we need X+1 tosses

A2: E[X+1] ← probability 4/8, total tosses X+1

Get heads then tails on first two tosses which means we need X+2 tosses

A3: E[X+2] ← probability 2/8, total tosses X+2

Get two heads and then tails on first three tosses which means we need X+3 tosses

A4: E[X+3] ← probability 1/8, total tosses X+3

Then we get the equation,

X = 1/8*3 + 4/8*(X+1) + 2/8*(X+2) + 1/8*(X+3)

X = 14

So, expected number of tosses till 3 consecutive heads = **E[X] = 14**

Answer to question 2

Part (a)

We have four multiplication in x*y,

So,

$$\boldsymbol{T(n) = 4T\left(\frac{n}{2}\right) + O(n)}$$

Using Master's theorem,

a = 4, b = 2, d = 1

Log base 2 of 4 = 2

d is less than 2

So, running time = **O(n^2)**

Part (b)

$$\boldsymbol{x * y = 2^{n} * (x_L * y_L) + 2^{\frac{n}{2}} * \big((x_L * x_R) * (y_L * y_R) - (x_L * y_L) - (x_R * Y_R)\big) + (x_R * y_R)}$$

We now have three multiplications,

$$\boldsymbol{(x_L * y_L)}$$

$$\boldsymbol{(x_R * y_R)}$$

$$\boldsymbol{(x_L * x_R) * (y_L * y_R)}$$

So,

$$\boldsymbol{T(n) = 3T\left(\frac{n}{2}\right) + O(n)}$$

Using Master's theorem,

a = 3, b = 2, d = 1

Log base 2 of 3 = 1.5849625007

d is less than 2

So, running time = **O(n^ 1.5849625007)**

Answer to question 3

n	**B(n)**
0	0
1	1
2	1
3	2
4	3
5	5

This squence grows exponentially so we can say b(n) = x^n for some x

Then,

B(n) = b(n-1) + b(n-2)

Where b(0) = 0 and b(1) = 1

X^n = x^(n-1) + x^(x-2)

Divide it by x^(n-2) ,

X^2 = x + 1

Then,

X = $\frac{1\pm\sqrt{5}}{2}$

So,

B(n) = $\left(\frac{1+\sqrt{5}}{2}\right)^n$ or $\left(\frac{1-\sqrt{5}}{2}\right)^n$

Using the theorem of sum of homogenoes linear recurrence, this is also a solution to b(n):

$$f(n) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^n + \beta\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$f(0) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^0 + \beta\left(\frac{1-\sqrt{5}}{2}\right)^0 = 0$$

$$f(0) = \alpha + \beta = 0$$

$$f(1) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^1 + \beta\left(\frac{1-\sqrt{5}}{2}\right)^1 = 1$$

$$solving\ these\ two\ equations\ simultaneously,$$

$$\alpha = \frac{1}{\sqrt{5}}$$

$$\beta = -\frac{\sqrt{5}}{5}$$

$$so,$$

$$\boldsymbol{B(n) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \frac{\sqrt{5}}{5}\left(\frac{1-\sqrt{5}}{2}\right)^{n}}$$

$$where\ b(1) = 1\ and\ b(2) = 1$$