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Take-home assignment/ midterm 1

Answer to question 1

E = {100, 010, 001, 111}

|E| = 4

F = {100, 110, 101, 111}

|F| = 4

E intersection F = {100, 111}

| E intersection F | = 2

Possible strings with length 3 = 2^3 = 8

P(E) = 4/8

P(F) = 4/8

P(E intersection F) = 2/8

P(E intersection F) = P(E) * P(F) ?

2/8 = 4/8 * 4/8

1/4 = 1/4

Thus, **E and F are independent events.**

Answer to question 2

Part (a)

Total combination of length 10 strings = 2^10 = 1024

Strings that are missing one alphabet = 1^10 * 2 = 2

A a a a a a a a a a (no b)

B b b b b b b b b b (no a)

Rest all have at least one of each letter

String of length 10 that have at least one of each letter = 1024 – 2 = **1022**

Part (b)

Total combination of length 10 string = 3^10 = 59,049

Possibilities string is missing one of the alphabets = 2^10 * 3 = 3,072

Now we need to account for the overlap where we took away more than once

Possibilities string is missing two of the alphabets = 3

Aaaaaaaaaa (no b or c)

Bbbbbbbbb (no a or c)

Cccccccccc (no a or b)

String of length 10 that have at least one of each letter =

59049 – 3072 + 3 = **55,980**

Answer to question 3

One-to-one functions from A to B where |A| = n and |B| = m (assuming m >= n)

m ways to map first element in A and m-1 ways to map the second one, etc.

if m = n then total ways = m!

if m >= n then account for extra choices,

Total ways = **m! / ((m-n)!)**

Where if m = n then total ways = m!

Answer to question 4

A 2 3 4 5 6 7 8 9 10 J Q K (Clubs)

A 2 3 4 5 6 7 8 9 10 J Q K (Diamonds)

A 2 3 4 5 6 7 8 9 10 J Q K (Hearts)

A 2 3 4 5 6 7 8 9 10 J Q K (Spades)

Total cards: 52

Total combinations for a 5-card poker hand: 52 choose 5 = 2598960

Total combinations for a 5-card poker hand with no A = 48 choose 5 = 1712304

Total combinations for a 5-card poker hand with at least one A = (52 choose 5) – (48 choose 5) = 886656

Probability that a 5-card poker hand has at least one A = (52 choose 5 – 48 choose 5)/52 choose 5 =

0.34115

Answer to question 5

X4 can be:

Case0: 0

Case1: 1

Case2: 2

Case0

X1+x2+x3 = 50

As x2 >= 2 and x3 >= 5 we can give them their minimum values

Remaining numbers which we can distribute = 50 – 5 – 2 = 43

Total ways = $\binom{43+3-1}{3-1} = \binom{45}{2} = 990$

Case1

X1+x2+x3 = 49

As x2 >= 2 and x3 >= 5 we can give them their minimum values

Remaining numbers which we can distribute = 49 – 5 – 2 = 42

Total ways = $\binom{42+3-1}{3-1} = \binom{44}{2} = 946$

Case0

X1+x2+x3 = 28

As x2 >= 2 and x3 >= 5 we can give them their minimum values

Remaining numbers which we can distribute = 48 – 5 – 2 = 41

Total ways = $\binom{41+3-1}{3-1} = \binom{43}{2} = 903$

Total ways = 990 + 946 + 903 = **2839**

Answer to question 6

Part (a)

To go from 0,0 to 4,4 we need 4 downs and 4 rights.

Examples include:

rrrrdddd, rdrdrdrd, ddddrrrr etc.

Total ways = 8!/(4! * 4!) = **70**

Part (b)

To go from 0,0 to 4,4 through 3,2 we divide journey into two parts

0,0 to 3,2 then 3,2 to 4,4

We need 2 downs and 3 rights for 0,0 to 3,2

We need 2 downs and 1 right for 3,2 to 4,4

Total ways =

(5!/(2! * 3!)) * (3!/(2! * 1!) = 10 * 3 = **30**

Answer to question 7

Given 1^{st} flip is a head we need at least 2 heads to appear in next 4 flips.

Probability 0 or 1 head in 4 flips = Probability all 4 tails + Probability 1 head 3 tails

(4 places for head)

$= (\frac{1}{2})^4 + (4 * (\frac{1}{2})^1 (\frac{1}{2})^3)$

= 5/16 = 0.3125

Probability at least 2 heads in 4 flips = 1 - Probability 0 or 1 head in 4 flips

= 1 – 5/16

= 11/16

= 0.6875

Probability at least 2 heads in 4 flips given 1^{st} is head = Probability at least 2 heads in 4 flips

= 11/16

= **0.6875**

Answer to question 8

Probability of any side: 1/6

Probability of getting a 3 or 4 on single roll = 2/6 = 1/3

Probability of not getting a 3 or 4 on a single roll = 1 – 1/3 = 2/3

Probability of not getting a 3 or 4 is same on any roll:

Probability of not getting a 3 or 4 on 6 rolls = (2/3) ^6 = **0.08779**