Mustafa Sadiq (ms3035)

HW5 for Statistics 1 Fall 2020

Answer to 9.16 part (a)

$$null\ hypothesis: \mu = 23$$

Answer to 9.16 part (b)

$$alternative\ hypothesis: \mu \neq 23$$

Answer to 9.16 part (c)

$$two\ tailed$$

Answer to 9.63, 9.64 and 9.66

9.63a

$$p - value\ is\ P(z \geq 2.03) = 1 - 0.9788 = 0.0212$$

$$(p - value = 0.0212) \leq (\alpha = 0.05)$$

$$reject\ the\ null\ hypothesis$$

9.63b

$$p - value\ is\ P(z \geq -0.31) = 1 - 0.3783 = 0.6217$$

$$(p - value = 0.6217) \geq (\alpha = 0.05)$$

$$do\ not\ reject\ null\ hypothesis$$

9.64a

$$p - value\ is\ P(z \leq -1.84) = 0.0329$$

$$(p - value = 0.0329) \leq (\alpha = 0.05)$$

$$reject\ the\ null\ hypothesis$$

9.64b

$$p - value\ is\ P(z \leq 1.25) = 0.8944$$

$$(p - value = 0.8944) \geq (\alpha = 0.05)$$

$$do\ not\ reject\ null\ hypothesis$$

9.66a

$$p - value\ is\ P(z \leq -3.08\ or\ z \geq 3.08) = 2 * 0.001$$

$$(p - value = 0.002) \leq (\alpha = 0.05)$$

$$reject\ the\ null\ hypothesis$$

9.66b

$$p - value\ is\ P(z \leq -2.42\ or\ z \geq 2.42) = 2 * 0.0078$$

$$(p - value = 0.0156) \leq (\alpha = 0.05)$$

$$reject\ the\ null\ hypothesis$$

Answer to 9.83

$$H_0: \mu = 0.5ppm$$

$$H_a: \mu > 0.5ppm$$

$$\alpha = 0.05$$

$$\sigma = 0.37$$

$$sample\ mean = \frac{6.31}{12} = 0.5258$$

$$z = \frac{0.5258 - 0.5}{\frac{0.37}{\sqrt{12}}} = 0.2416$$

$$critical\ value = 1.645$$

$$do\ not\ reject\ null\ hypothesis$$

Answer to 9.113

$$H_0: \mu = 4.55$$

$$H_a: \mu \neq 4.55$$

$$\alpha = 0.10$$

$$t = \frac{4.760 - 4.55}{\frac{2.297}{\sqrt{20}}} = 0.41$$

$$critical\ value = 1.729$$

$$do\ not\ reject\ null\ hypothesis$$

Answer to 10.51

$$s_p = \sqrt{\frac{(10-1)(4.90)^2 + (10-1)(4.64)^2}{10+10-2}} = 4.772$$

$$t_{0.05} = 1.734$$

$$(10.12 - 18.78\ ) \pm 1.734 * 4.772\left(\sqrt{\frac{1}{10} + \frac{1}{10}}\right)$$

$$-12.36\ \ to\ \ -4.96$$

Answer to 10.85

$$t_{0.05} = 1.676$$

$$(25.8 - 22.1\ ) \pm 1.676\left(\sqrt{\frac{9.2^2}{32} + \frac{5.7^2}{20}}\right)$$

$$0.2\ to\ 7.2$$

Answer to 10.160

$$H_0 = u_{water} \geq u_{slurry}$$

$$H_a = u_{water} < u_{slurry}$$

$$d.f = 10 - 1 = 9$$

$$t_{\alpha/2} = 3.250$$

$$t = \frac{\bar{d} - D_0}{\frac{s_d}{\sqrt{n_d}}} = \frac{-5.9}{\frac{1.6}{\sqrt{10}}} = -11.66$$

$$reject\ the\ null\ hypothesis$$

$$enough\ evidence\ that\ cold\ water\ is\ less\ effective\ than\ ice\ slurry$$

Answer to 10.166

$$d.f = 9$$

$$t_{\alpha/2} = 2.821$$

$$\bar{d} \pm t_{\frac{\alpha}{2}} * \frac{s_d}{\sqrt{n_d}} = -5.9 \pm 2.821 * \frac{1.6}{\sqrt{10}}$$

$$-7.3\ to\ -4.5$$

Answer to 11.27

$$s = 2.501$$

$$s^2 = 6.255$$

$$For\ 13\ degrees\ of\ freedom, and\ a\ 90\%\ CI$$

$$\chi^2_{0.05} = 22.362\ \ and\ \chi^2_{0.95} = 5.892$$

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}$$

$$\sqrt{\frac{(n-1)S^2}{\chi^2_{\alpha/2}}} \leq s \leq \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}}$$

$$\sqrt{\frac{(14-1)}{22.362}} \leq s \leq \sqrt{\frac{(14-1)}{5.892}} * 2.501 * 2.501$$

$$1.906\ to\ 3.715$$

Answer to 12.51

$$n = 500$$

$$\alpha = 0.05$$

$$\hat{p} = \frac{38}{500} = 0.076$$

$$Z_{\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}} \leq p \leq \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}$$

$$0.076 - 1.96 * \sqrt{\frac{0.076(0.924)}{500}} \leq p \leq 0.076 + 1.96 * \sqrt{\frac{0.076(0.924)}{500}}$$

$$0.0528\ to\ 0.0992$$

Answer to 12.59 part (a)

$$E = z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} = 1.96 * \sqrt{\frac{0.076(0.924)}{500}} = 0.0232$$

Answer to 12.59 part (b)

$$n = \frac{\left(z_{\frac{\alpha}{2}}\right)^2 p(1-p)}{E^2}$$

$$p = 0.5$$

$$n = \frac{(1.96)^2 0.25}{0.01^2} = 9604$$

Answer to 12.59 part (c)

$$p = 0.071$$

$$n = 9604$$

$$0.071 - 1.96 * \sqrt{\frac{0.071(0.929)}{9604}} \leq p \leq 0.071 + 1.96 * \sqrt{\frac{0.071(0.929)}{9604}}$$

$$0.0659 \; to \; 0.0761$$

Answer to 12.59 part (d)

$$1.96 * \sqrt{\frac{0.071(0.929)}{9604}} = 0.0051$$

$$less \; than \; 0.01$$

Answer to 12.59 part (e)

$$p = 0.1$$

$$n = \frac{(1.96)^2\big(0.1(0.9)\big)}{0.01^2} = 3458$$

$$0.071 - 1.96 * \sqrt{\frac{0.071(0.929)}{3458}} \leq p \leq 0.071 + 1.96 * \sqrt{\frac{0.071(0.929)}{3458}}$$

$$0.0624 \; to \; 0.0796$$

$$1.96 * \sqrt{\frac{0.071(0.929)}{3458}} = 0.0086$$

$$less \; than \; 0.01$$

Answer to 12.59 part (f)

required sample size is reduced by 6146 and only 0.35% of accuracy is lost while margin of error rises from 0.0051 to 0.0086.

Answer to 12.85 part (a)

$$n = 1008$$

$$x = 534$$

$$p = \frac{534}{1008} = 0.53$$

$$53\%\ of\ populaton\ disapprove$$

Answer to 12.85 part (b)

$$\alpha = 0.05$$

$$critical\ value = z_{0.05} = 1.645$$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

$$z = \frac{0.53 - 0.5}{\sqrt{\frac{0.5(0.5)}{1008}}} = 1.9$$

$$reject\ null\ hypothesis$$

Answer to 12.118 part (b)

$$\hat{p}_1 = \frac{40}{2229} = 0.0179$$

$$\hat{p}_2 = \frac{21}{2303} = 0.0091$$

$$z_{\alpha/2} = 1.645$$

$$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}} * \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$0.0179 - 0.0091 \pm 1.645 * \sqrt{\frac{0.0179(1-0.0179)}{2229} + \frac{0.0091(1-0.0091)}{2303}}$$

$$0.0031\ to\ 0.0145$$