Mustafa Sadiq

HW3 for Statistics 1 Fall 2020

Solution to problem 1 part (a) (Textbook Problem 5.32)

$$mean(Y) = 1 * 0.265 + 2 * 0.327 + 3 * 0.161 + 4 * 0.147 + 5 * 0.065 + 6 * 0.022 + 7 * 0.013 = 2.538$$

Solution to problem 1 part (b)

$$s.d(Y) = \sqrt{\sum (y - \mu)^2 * p(y)}$$

$$= 1.409$$

Solution to problem 1 part (c)

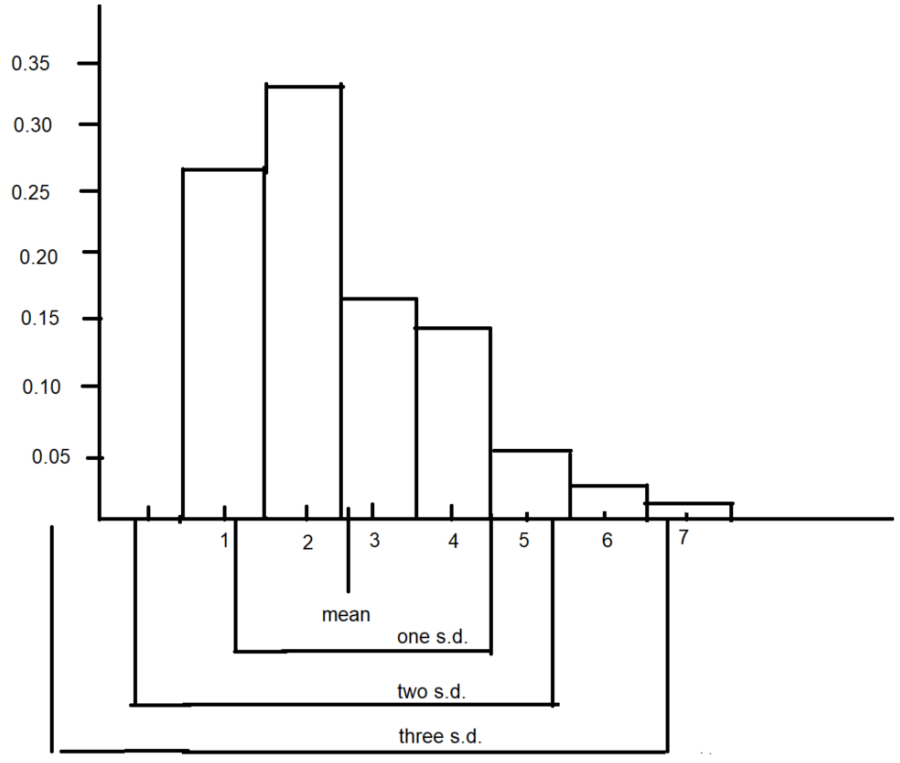
$$mean = 2.538$$

$$standard\ deviation = 1.409$$

$$one - standard\ deviation\ interval: 1.129\ to\ 3.947$$

$$two - standard\ deviation\ interval: -0.280\ to\ 5.356$$

$$three - standard\ deviation\ interval: -1.689\ to\ 6.765$$



Solution to problem 2 part (a) (Textbook Problem 5.42)

Investment A:

$$mean\ of\ Y = \big((0 * 0.1) + (1 * 0.5) + (4 * 0.4)\big) = 2.1$$

Investment B:

$$mean\ of\ Y = \big((0 * 0.5) + (1 * 0.3) + (16 * 0.2)\big) = 3.5$$

Comparison: Investment B has higher expected yield than Investment A.

Solution to problem 2 part (b)

Investment A:

$$mean\ of\ \sqrt{Y} = \big((0 * 0.1) + (1 * 0.5) + (2 * 0.4)\big) = 1.3$$

Investment B:

$$mean\ of\ \sqrt{Y} = \big((0 * 0.5) + (1 * 0.3) + (4 * 0.2)\big) = 1.1$$

Comparison: Investment A has higher expected utility than Investment B.

Solution to problem 2 part (c)

Investment A:

$$mean\ of\ Y^{\frac{3}{2}} = \big((0 * 0.1) + (1 * 0.5) + (8 * 0.4)\big) = 3.7$$

Investment B:

$$mean\ of\ Y^{3/2} = \big((0 * 0.5) + (1 * 0.3) + (64 * 0.2)\big) = 13.1$$

Comparison: Investment B has higher expected utility than Investment A.

Solution to problem 3 part (a) (Textbook Problem 5.79)

Exactly three:

$$\binom{8}{3}(0.4)^3(0.6)^{8-3} = 0.279$$

At least three:

$$1 - Exactly\ zero - Exactly\ one - Exactly\ two - Exactly\ three$$

$$= 1 - \binom{8}{0}(0.40)^0(0.60)^{8-0} - \binom{8}{1}(0.40)^1(0.60)^{8-1} - \binom{8}{2}(0.40)^2(0.60)^{8-2}$$

$$= 0.685$$

At most three:

$$Exactly\ zero + Exactly\ one + Exactly\ two + Exactly\ three$$

$$= \binom{8}{0}(0.40)^0(0.60)^{8-0} + \binom{8}{1}(0.40)^1(0.60)^{8-1} + \binom{8}{2}(0.40)^2(0.60)^{8-2} + \binom{8}{3}(0.40)^3(0.60)^{8-3}$$

$$= 0.594$$

Solution to problem 3 part (b)

$$Exactly\ two + Exactly\ three + Exactly\ four$$

$$= \binom{8}{2}(0.40)^2(0.60)^{8-2} + \binom{8}{3}(0.40)^3(0.60)^{8-3} + \binom{8}{4}(0.40)^4(0.60)^{8-4}$$

$$= 0.720$$

Solution to problem 3 part (c)

$$mean\ of\ random\ variable\ Y = np = 8 * 0.4 = 3.20$$

Solution to problem 3 part (d)

$$standard\ deviation\ of\ Y = \sqrt{np(1-p)}$$

$$= \sqrt{8 * 0.4 * 0.6} = 1.4$$

Solution to problem 4 part (a) (Textbook Problem 5.83)

$$p = 0.09, n = 7$$

$$P(X = x) = \binom{7}{x} (0.09)^x (1 - 0.09)^{7-x}$$

x	P(X=x)
0	0.517
1	0.358
2	0.106
3	0.017
4	0.002
5	0.000
6	0.000
7	0.000

Solution to problem 4 part (b)

$$mean\ of\ X = 0 * 0.517 + 1 * 0.358 + 2 * 0.106 + 3 * 0.017 + 4 * 0.002 = 0.63$$

Solution to problem 4 part (c)

$$P(X \geq 3) = 0.017 + 0.002 = 0.019$$

Yes, as only 1.9% chance that three or more of the seven youths selected would be PVGU.

Solution to problem 4 part (d)

$$P(X \geq 2) = 0.106 + 0.017 + 0.002 = 0.125$$

No, as 12.5% chance that two or more of the seven youths selected would be PVGU.

Solution to problem 5 part (a) (Textbook Problem 5.88, parts a and b)

$$N = 250, n = 4, p = 0.94$$

Solution to problem 5 part (b)

$$P(x) = \frac{\binom{250*0.94}{x}\binom{250*(1-0.94)}{4-x}}{\binom{250}{4}} = \frac{\binom{235}{x}\binom{15}{4-x}}{\binom{250}{4}}$$

$$P(0) = \frac{\binom{235}{0}\binom{15}{4-0}}{\binom{250}{4}} = 0.000009$$

$$P(1) = \frac{\binom{235}{1}\binom{15}{4-1}}{\binom{250}{4}} = 0.000673$$

$$P(2) = \frac{\binom{235}{2}\binom{15}{4-2}}{\binom{250}{4}} = 0.018170$$

$$P(3) = \frac{\binom{235}{3}\binom{15}{4-3}}{\binom{250}{4}} = 0.201606$$

$$P(4) = \frac{\binom{235}{4}\binom{15}{4-4}}{\binom{250}{4}} = 0.779542$$

Solution to problem 6 part (a) (Textbook Problem 5.103)

$$P(X = 0) = e^{-0.7} * \frac{0.7^0}{0!}$$

$$= 0.497$$

Solution to problem 6 part (b)

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= e^{-0.7} * \frac{0.7^0}{0!} + e^{-0.7} * \frac{0.7^1}{1!} + e^{-0.7} * \frac{0.7^2}{2!}$$

$$= 0.966$$

Solution to problem 6 part (c)

$$P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= e^{-0.7} * \frac{0.7^1}{1!} + e^{-0.7} * \frac{0.7^2}{2!} + e^{-0.7} * \frac{0.7^3}{3!}$$

$$0.498$$

Solution to problem 6 part (d)

$$mean = \lambda = 0.7$$

Interpretation: On average, 0.7 wars begin during a calendar year.

Solution to problem 6 part (e)

$$standard\ deviation = \sqrt{\lambda} = \sqrt{0.7} = 0.8$$

Solution to problem 7 part (a) (Textbook Problem 5.111)

1 in every 30 million

3.33 in every 100 million

$$p = \frac{1}{30000000}$$

$$n = 100000000$$

$$np = \frac{10}{3} = 3.33$$

$$P(3 \leq X \leq 5) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= e^{-3.33} * \frac{3.33^3}{3!} + e^{-3.33} * \frac{3.33^4}{4!} + e^{-3.33} * \frac{3.33^5}{5!}$$

$$= 0.523$$

Solution to problem 7 part (b)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) \leq 0.1$$

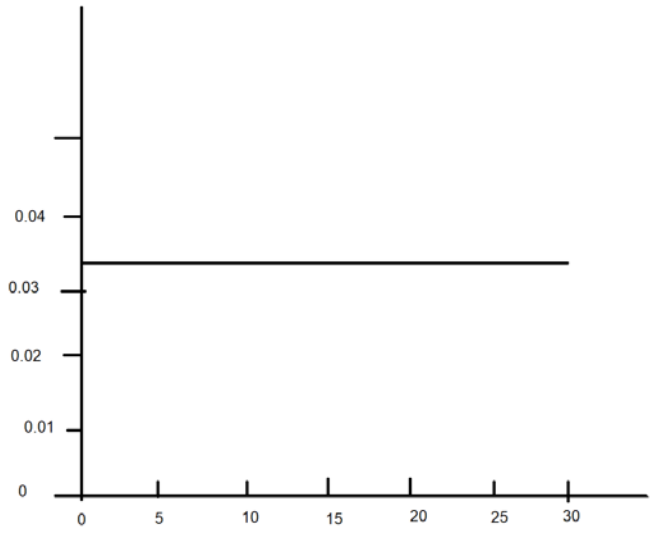
$$n = ?$$

$$p = 1/30000000$$

$$e^{-\frac{n}{30000000}} \leq 0.1$$

$$n \geq 69077553$$

Solution to problem 8 part (a) (Textbook Problem 6.31)



Solution to problem 8 part (b)

Total area from $0\ to\ 30$ is $\frac{1}{30} * 30 = 1$

Area to the left of $10 = 10 * \frac{1}{30}$

Area to the left of $20 = 20 * \frac{1}{30}$ and so on.

Solution to problem 8 part (c)

$$5 * \frac{1}{30} = 0.167 = 16.7\%$$

Solution to problem 8 part (d)

$$|10 - 15| * \frac{1}{30} = 0.167 = 16.7\%$$

Solution to problem 8 part (e)

$$|20 - 30| * \frac{1}{30} = 0.333 = 33.3\%$$

Solution to problem 9 part (a) (Not in Textbook)

Since it is an exponential distribution and probability of a single point is 0:

$$A = P(x \geq a) = e^{-\frac{a}{3}}$$

$$P(X > 5) = e^{-\frac{5}{3}} = 0.188$$

Solution to problem 9 part (b)

$$P(X > 10) = e^{-\frac{10}{3}} = 0.036$$

Solution to problem 10 part (a) (Textbook Problem 6.99 parts a, b, c)

$$mean = 61, s.d. = 9$$

$$P(50 < X < 70)$$

$$P(-1.22 < Z < 1) = 0.8413 - 0.1112 = 0.7301 = 73.01\%$$

Solution to problem 10 part (b)

$$P(X < 75)$$

$$P(Z < 1.56) = 0.9406 = 94.06\%$$

Solution to problem 10 part (c)

$$P(Z < z) = 40\%$$

$$z = -0.23$$

$$x = z * s.d. + mean = -0.23 * 9 + 61 = 58.84$$

Interpretation: 40% of finishers in the New York City 10-km run have times less than 58.8 minutes.