

STAT212 Assignment 5

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Answer to Question 1

$$\begin{aligned}\hat{p} &= \frac{\text{wins by sven}}{\text{total games}} \\ &= \frac{56}{100} = 0.56\end{aligned}$$

Answer to Question 2

The confidence interval for p is from:

$$\hat{p} - z_{\alpha_0/2} \times \sqrt{\frac{\hat{p} \times (1 - \hat{p})}{n}} \text{ to } \hat{p} + z_{\alpha_0/2} \times \sqrt{\frac{\hat{p} \times (1 - \hat{p})}{n}}$$

We know:

$$n = 100$$

$$\hat{p} = 0.56$$

$$\alpha_0 = 1 - 0.8 = 0.2$$

$$z_{\alpha_0/2} = 1.282$$

Plugging these in, 80% confidence interval for p :

$$0.496 \text{ to } 0.624$$

Answer to Question 3

The margin of error for the estimate of p is:

$$E = z_{\alpha_0/2} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

We know:

$$n = 100$$

$$\hat{p} = 0.56$$

$$z_{\alpha_0/2} = 1.282$$

Plugging these in:

$$E = 0.064$$

Answer to Question 4

Let p denote the proportion of all games won by Sven. Then the null and alternative hypothesis are, respectively:

$$H_0 : p = 0.50 \text{ (evenly matched)}$$

$$H_a : p \neq 0.50 \text{ (one is better than the other)}$$

Answer to Question 5

The sample size needed to obtain a desired power of $(1 - \beta)$ when $p = p_a$ is given by:

$$n = \lceil (\sqrt{p_A(1 - p_A)} \times \frac{z_\beta + z_{\alpha_0/2}}{p_0 - p_A})^2 \rceil$$

We know:

$$p_0 = 0.5$$

$$p_A = 0.51$$

$$\beta = 1 - [\text{power}] = 1 - 0.99 = 0.01$$

$$\alpha_0 = 1 - 0.99 = 0.01$$

$$z_\beta = 2.326$$

$$z_{\alpha_0/2} = 2.576$$

Plugging these in:

$$n = 60055$$

100 games is not enough and they should play 60055 games.