

The Mathematics Behind Machine Learning and Artificial Intelligence

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Abstract

The development of artificial intelligence is a long journey that demonstrates how a technology, initially thought to be impossible, was propelled forward by mathematical foundations. This paper explains the mathematical concepts that enable the functioning of artificial intelligence and machine learning systems. It emphasizes how fundamental disciplines such as linear algebra, calculus, probability and statistics, discrete mathematics, and optimization form the cornerstone of AI algorithms.

The linear algebra section explains how matrix operations and vector spaces serve as the building blocks of neural networks. The calculus section explains the impact of optimization techniques such as gradients, backpropagation, and gradient descent on model training. In the probability and statistics section, methods such as uncertainty modeling, cross-entropy, and the Kullback-Leibler divergence are explained clearly in the context of enhancing model accuracy. Discrete mathematics lays the mathematical foundation for graph theory and combinatorial analysis, emphasizing their applications in network structures and feature selection within AI models. Finally, the optimization section explains the minimization of loss functions.

This study highlights how mathematics transitions from abstract theories to practical applications in data processing, learning, and decision-making within AI systems. It underscores the critical role that mathematical methods play in the development of this transformative technology.

Keywords

Artificial Intelligence, Linear Algebra, Calculus,
Probability, Statistics, Discrete Mathematics, Optimization

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