Sol:

① $\lim_{x \to a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \times \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\cancel{4}^{2}\cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\cancel{a}}$

$= \frac{2}{3\sqrt{3}}$

Limits & Continuity



* PRACTICAL - 01 *

2. $\lim_{y \to 0} \left[ \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left[ \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\lim_{y \to 0} \left[ \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})} \right]$

$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$= \frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}(2\sqrt{a})}$

$= \frac{1}{2a}$

3. $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

→ By substituting $x - \pi/6 = h$

$x = h + \pi/6$, where $h \to 0$

65

$$= \lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\,\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

; using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \lim_{x\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\,\sin h \cdot \cos\frac{\pi}{6} + \cos h \cdot \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$$

$\cos\frac{\pi}{6} = \cos 30^\circ = \sqrt{3}/2$

$\sin\frac{\pi}{6} = \sin 30^\circ = 1/\sqrt{2}$

$$= \lim_{h\to 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$= \lim_{h\to 0} \frac{\cancel{\cos\frac{\sqrt{3}h}{2}} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}h}{2}}}{-6h}$$

$$= \lim_{h\to 0} \frac{\cancel{-}\sin 4h/2}{\cancel{-}6h}$$

$$= \lim_{h\to 0} \frac{\sin 4h}{\sin 12h} = \frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

30

4] $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

By rationalizing numerator & Denominator both.

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{(\cancel{x^2}+5-\cancel{x^2}+3)\,(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\cancel{x^2}+3-\cancel{x^2}-1)\,(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{4\cancel{8}\,(\sqrt{x^2+3} + \sqrt{x^2+1})}{\cancel{2}\,(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,

$= \underline{4}$

5] $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$
$= \frac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$
at $x = \pi/2$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore$ for $(\pi/2) = 0$

$f$ at $x = \pi/2$ define.

b) $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ Using, $\sin 2x = 2\sin x \cdot \cos x$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x \to \frac{\pi}{2}^-} \frac{2\cancel{\sin x} \cdot \cos x}{\sqrt{2}\,\cancel{\sin x}}$

$\lim_{x \to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\therefore$ LHL $\neq$ RHL

$\therefore$ $f$ is not continuous at $x = \pi/2$.


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18,

i) $f(3) = \frac{x^2-9}{x-3} = 0$

$f$ at $x=3$ define

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

$f$ is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

$f$ is continuous at $x=3$

2) $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$.

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$= \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continuous



6 i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$ ; $= K$ $x=0$ } at $x=0$

$\rightarrow$ $f$ is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$

$2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = K$

$2 \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$\therefore \underline{\underline{K = 8}}$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ ; $= K$ $x=0$ } at $x=0$

$\rightarrow$ $f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

Using,

$\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1+\tan^2 x$

&

$\cot^2 x = \frac{1}{\tan^2 x}$

we know that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$\therefore$ $\underline{k = e}$

iii) $f(x) = \begin{cases} \dfrac{\sqrt{3}-\tan x}{\pi - 3x} & x \neq \pi/3 \\ = k & x = \pi/3 \end{cases}$ at $x = \pi/3$

$x - \frac{\pi}{3} = h$

$x = h + \pi/3$ , where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

using,
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan\pi/3 + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1 - \tan\pi/3 \cdot \tan h) - (\tan\pi/3 + \tan h)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

using $\tan\pi/3 = \tan$
$= \sqrt{3}$



$$\lim_{h\to 0} \frac{\left(\dfrac{\sqrt{3} - 3\cdot\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\cdot\tan h}\right)}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3} \cdot \frac{1}{(1 - \sqrt{3}(0))} = \frac{4}{3}(1/1) = 4/3$$

7) $f(x) = \begin{cases} \dfrac{1 - \cos^3 x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1 - \cos^3 x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{\dfrac{2\sin^2 3x/2}{x^2} \times x^2}{\dfrac{x\cdot\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x \to 0} \frac{(3/2)^2}{1} = 7 \times \frac{9}{4^2} = 9/2$$

LL

$$\lim_{h \to 0} f(x) = 9/2 \quad 9 = f(0)$$

$\therefore$ f is not continuous at $x = 0$

Rectifine function.

$$f(x) = \begin{cases} \frac{1 - \cos 3x}{x \tan x} & x \neq 0 \\ = 9/2 & x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$.

7)(ii) $f(x) = \begin{cases} \frac{(e^{3x} - 1) \sin x^\circ}{x^2} & x \neq 0 \\ & x = 0 \end{cases}$ at $x = 0$

$$\lim_{x \to 0} \frac{(e^{3x} - 1) \sin(\pi x/180)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{3x} - 1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x \to 0} 3 \cdot \frac{e^{3x} - 1}{3x} \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right) \frac{1}{x}$$

$$3 \lim_{h \to 0} \frac{e^{3h} - 1}{3h} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$



$$3 \log e \frac{\pi}{\cancel{180}\,60} = \frac{\pi}{60} = f(0)$$

f is continuous at $x = 0$

Given

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x = 0$

is continuous at $x = 0$

$\therefore$ Given,

f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\therefore \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$$

1.E.

$\log e + 2 \lim_{x \to 0} \left( \frac{\sin x/2}{x} \right)^2$

Multiply with 2 in Num & Den.

$= 1 + \cancel{2} \times \frac{1}{\cancel{4}2} = \frac{3}{2} = f(0)$

$f(x) = \frac{\sqrt{2} - \sqrt{1-\sin x}}{\cos^2 x} \quad x \neq \pi/2$

$f$ is continuous at $x = \pi/2$

9] $\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$= f(\pi/2) = \frac{1}{4\sqrt{2}}$

5/12/17

# PRACTICAL - 2



TOPIC :- Derivative

Q.1] Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable :

i) $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$

$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(a) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$

FORMULA :- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$

$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) cosec $x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$

$x = a+h$

as $x \to a,\ h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$



$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$\left[\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{\cancel{2}} \times \cancel{2}\frac{\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) sec $x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a+h$

as $x \to a,\ h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$


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Formula :- $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\cos a \cos(a+h) \times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a$$

Q.2] If $f(x) = 4x+1$ , $x \le 2$
$= x^2+5$ , $x>0$ , at $x=2$ , then find function is differential or not.

Solution :-

LHD :-

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2} = 4$$

$Df(2^-) = 4$



RHD :-

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$

$\therefore$ RHD = LHD

f is differentiable at $x=2$.

Q.3] If $f(x) = 4x+7$ , $x<3$
$= x^2+3x+1$ , $x \ge 3$ at $x=3$ , then find f is differentiable or not?

Solution :- RHD :-

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$$



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<u>LHD</u> :- $Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$

$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 8$

$Df(2^-) = 8$

LHD = RHD

$\therefore$ f is differentiable at $x = 3$



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Now f is decreasing iff

$f'(x) < 0$

$\therefore\ 3x^2 - 5 < 0$

$\therefore\ x = \pm\sqrt{5/3}$

$\therefore\ x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

→ f is increasing iff

$f'(x) > 0$

$\because\ f(x) = x^2 - 4x$

$\therefore\ f'(x) = 2x - 4$

$\therefore\ 2x - 4 > 0$

$2(x-2) > 0$

$\therefore\ x = 2$

$\therefore\ x \in (2, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$\because\ f(x) = x^2 - 4x$

$\therefore\ 2x - 4 < 0$

$\therefore\ 2(x-2) < 0$

$\therefore\ x - 2 < 0$

$\therefore\ x = 2$

$\therefore\ x \in (-\infty, 2)$



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Now $f$ is increasing iff

$f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$

$\therefore x \in (-3; 3)$

5) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f(x) = 2x^3 - 9x^2 - 24x + 69$

$f$ is increasing iff

$f'(x) > 0$

$\because f(x) = 2x^3 - 9x^2 - 24x + 69$

$\therefore f'(x) = 6x^2 - 18x - 24 > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6x^2 - 24x + 6x - 24 > 0$

$\therefore 6x(x-4) + 6(x-4) > 0$

$\therefore (x-4)(6x+6) > 0$

$\therefore x = +4, -1$

$\therefore x \in (-\infty, 4) \cup (-1, \infty)$

Now $f$ is decreasing iff

$f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore (x-4)(6x+6) < 0$



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Q.2) 1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$\therefore x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (-1/2, \infty)$



2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\rightarrow \therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore$ f is concave upwards iff

$f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$\therefore x = 2, 1$

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore$ f is concave downwards iff

$f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

$\therefore x = 2, 1$

3) $y = x^3 - 27x + 5$

$y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27x$

$\therefore f''(x) = 6x - 27$



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$f$ is concave upwards iff

iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$x = 0$

$\therefore x \in (0, \infty)$

$\therefore f$ is concave downwards iff

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$

$\therefore x \in (-\infty, 0)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$\rightarrow y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$\therefore f$ is concave upwards iff

$f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 6(2x - 3) > 0$

$\therefore 2x - 3 > 0$

$\therefore x = 3/2$

$\therefore x \in (3/2, \infty)$



$\therefore f$ is concave downwards iff

$f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 12x - 18 < 0$

$\therefore 6(2x - 3) < 0$

$\therefore (2x - 3) < 0$

$\therefore x = 3/2$

$\therefore x \in \left(-\infty, \frac{3}{2}\right)$

5) $y = 2x^3 + x^2 - 20x + 4$

$\rightarrow y = f(x)$

$f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

$f$ is concave upwards iff

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x + 1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$ $\quad\therefore x \in (-1/6, \infty)$

$f$ is concave downwards iff

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 12) < 0$

$\therefore 6x + 21 < 0$

$\therefore x = -1/6$

$\therefore x \in (-\infty, -1/6)$

## * PRACTICAL - 04 *

Topic :- Application of Derivative & Newtons Method

Q.1] Find maximum & minimum value of following functions.

1) $f(x) = x^2 + \frac{16}{x^2}$

2) $f(x) = 3 - 5x^3 + 3x^5$

3) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2] Find the root of following equation by Newton's method. (Take 4 iteration only) correct upto 4 decimal.

1) $f(x) = x^3 - 2x^2 - 55x + 95$ (take $x_0 = 0$)

2) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$



Q.1]

1] $f(x) = x^2 + \frac{16}{x^2}$

$\rightarrow f'(x) = 2x - \frac{32}{x^3}$

For maxima / minima

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = 32/x^3$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$\therefore f''(2) = f''(-2) = \frac{2+96}{(\pm 2)^4} = \frac{2+96}{16} = 8 > 0$

$\therefore f(x)$ is minimum at $x = \pm 2$

$\therefore f(2) = 8$ is minimum value.

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= \frac{2+96}{16}$

$= 8 > 0$

f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2, x = -2$.

3) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x = 2$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$.

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

f has maximum value 1 at $x = 0$ & minimum value at $x = 2$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 - x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x+1)(x-2) = 0$

$\therefore x = -1$ & $x = 2$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -18 < 0$

$\therefore$ f has min. value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has max. value 8 at $x = -1$ & min. value 9 at $x = 2$



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Q2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 0.1727 - 0.0829 / 55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f''(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$

$= 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712.



2) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - (4 \cdot 2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - \dfrac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$




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$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$
$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$
$= 2.7065$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1,2]
$f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= 1 - 1.8 - 10 + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$





Let $x_0 = 2$ be initial approximation
By Newton's Method.
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.06755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4603 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$
$= 1.6592$

$f(x_2) = (1.6592)^3 + 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 + 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$



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31

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 + \frac{0.0204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.86618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= 1.6618$

The root of equation is 1.6618

PRACTICAL - 5.

INTEGRATION

47

Solve the following integration.

1) $\int \frac{1}{\sqrt{x^2+2x-3}}\, dx$

$I = \int \frac{1}{\sqrt{x^2+2x-3}}\, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\, dx = \int \frac{1}{\sqrt{(x+1)^2-(2)^2}}\, dx$

$\because a^2+2ab+b^2 = (a+b)^2$

Substitute

$x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t=1$, $t = x+1$

$\int \frac{1}{t^2-4}\, dt$

$= \log(|t + \sqrt{t^2-4}|)$

$\left[\because \int \frac{1}{\sqrt{x^2-a^2}}\, dx = \log|x+\sqrt{x^2-a^2}|\right]$

$= \log(|x+1 + \sqrt{\cancel{(x+1)^2}} - 4|)$

$= \log(|x+1 + \sqrt{x^2-2x-3}| + c$





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2) $\int (4e^{3x}+1)\,dx$

$I = \int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}dx + \int 1\,dx$

$= 4\int e^{3x}dx + \int 1\,dx \quad [\because \int e^{ax}dx = 1/a\, e^{ax}]$

$= \frac{4e^{3x}}{3} + x$

$= 4e^{3x}/3 + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$I = \int 2x^2 - 3\sin x + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}dx$

$= \int 2x^2 dx - 3\int \sin x\,dx + 5\int x^{1/2}dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt[3]{x}}{3} + C \quad [\because \int x^n dx = \frac{x^{n+1}}{n+1} + C]$

$= \frac{2x^3 + 10\sqrt[3]{3}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx$

$= \int \frac{x^3}{x^{1/2}}dx + 3\int x^{1/2}dx + 4\int x^{-1/2}dx$

$= \int x^{5/2}dx + 3\int x^{1/2}dx + 4\int x^{-1/2}dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3\cdot\frac{x^{1/2+1}}{1/2+1} + 4\,\frac{x^{-1/2+1}}{}$



$= \frac{x^{7/2}}{7/2} + 3\cdot\frac{x^{3/2}}{3/2} + 4\cdot\frac{x^{1/2}}{1/2}$

$= \frac{2x^{7/2}}{7} + 2\cdot x^{3/2} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

Substitute $t^4$ with $u/2$

$= \int \frac{u/2 \times \sin(2u/2)}{8}\,du$

$= \int \frac{\frac{u/2 \times \sin(u)}{2}}{8}\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16}(u \times (-\cos(u)) - \int -\cos(u)\,du$

$[\because \int u\,dv = uv - \int v\,du$ where $u = v$

$dv = \sin(u) \times du$

$du = 1\,du \quad v = -\cos(u)]$

$= \frac{1}{16} \times [u \times (-\cos u)) + (\cos(u)\,du$



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$= \frac{1}{16} \times (4 \times (-\cos(u)) + \sin(u))$ [$\because \int \cos x\, dx$ -

Resubstituting $u = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\cdot dx$

$= \int x^{1/2}(x^2-1)\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1}$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2}$

$= \frac{2x^{7/2}}{7} - \frac{2x^3}{3} + C$

7) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^3}}\,dx$

$I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \frac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin x$

$\therefore dt = \cos x\,dx$

$\frac{1}{(t)^{2/3}}\,dt$

$\therefore (t)^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/3}}{1/3}$

$= 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$



9) $\int e^{\cos^2 x}\cdot \sin 2x\,dx$

$I = \int e^{\cos^2 x}\cdot \sin 2x\,dx$

put,

$\cos^2 x = t$

$2\cos x\,(-\sin x)\,dx = dt$

$-\sin 2x\,dx = dt$

$\sin x\,dx = -dt$

$\therefore \int e^t\cdot(-dt)$

$= -\int e^t\cdot dt$ [$\because \int e^x dx = e^x + c$]

$= -e^t + C$

Resubstituting $\cos^2 x = t$

$= -e^{\cos^2 x} + C$

10) $\int \frac{x^2-2x}{x^3-3x+1}\,dx$

$\rightarrow$ put $x^3-3x^2+1=t$

$\therefore (3x^2-6x)\,dx = dt$

$3(x^2-2x)\,dx = dt$

$(x^2-2x)\,dx = dt/3$

$\therefore \int (1/t)\,dt/3$

$\therefore 1/3 \int (1/t)\,dt$

$= 1/3 \log |t| + c \quad [\because \int (1/x)\,dx = \log|x| + c]$

Resubstituting $x^3-3x^2+1=t$

$\therefore 1/3 \cdot \log |x^3-3x^2+1| + c$





Application of integration & Numeric integration.

Q.1] Find length of the following :-

1) $x = 1-\sin t$ ; $y = 1-\cos t$ $\quad \in [0, 2\pi]$

$\rightarrow$ length $= \int_{2}^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\therefore dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\,dt$

$= \int_0^{2\pi} \sqrt{1-2\cos t + \cos^2 t + \sin^2 t + dt}$

$= \sqrt{1-2\cos t + 1 dt}$

$= \int_0^{2\pi} \sqrt{2-2\cos t}\,dt$

$= \int_0^{2\pi} \sqrt{2-2\cos t}\,dt$

$= \int_0^{2\pi} 2\sin \frac{1}{2}\,dt$

$= [-9 \cos 1/2]_0^{2\pi}$

$= (-4\cos\pi) + 4\cos 0$

$= 8$ units

2) $y = \sqrt{4-x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$

$= \frac{-x}{\sqrt{4-x^2}}$



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$$L = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{(2)^2-(x)}}\, dx$$

$$= 2\left[\sin^{-1}(x/2)\right]^{2}$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

3) $y = x^{3/2}$ in $[0,4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{3/2-1}$$

$$l = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Shot on OnePlus

$$= \int_0^4 \sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2}\, dx$$



$$= \int_0^4 \sqrt{\left(1+\left(\frac{9x}{4}\right)\right)}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= 1/2 \int_0^4 \sqrt{4+9x}\, dx$$

$$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$$

$$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{3/2} \times 1/9\right]_0^4$$

$$= 1/27 \left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= 1/27\, (4)^{3/2} - (40)^{3/2}$$

4) $x = 3\sin t$ ; $y = 3\cos t$

$dx/dy = 3\cos t$ , $dy/dx = -3\sin t$

$$= \int_0^{2\pi} \sqrt{(dy/dt)^2 + (dy/dt)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[t]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

5) $\frac{dx}{dy} = \frac{1}{6}\frac{d}{dy}(y^3) + 1/2 \frac{dx}{dy}(1/y)$

18.

$= 1/6\ 3y^2 + 1/2\,[-1/y^2]$

$= y^2/2 - 1/2y^2$

$= \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}$

$= \int_1^2 \sqrt{1+\left(\frac{y^4-1}{2y^2}\right)^2}\,dy$

$= \int_1^2 \sqrt{1+\left(\frac{y^4-1}{4y^4}\right)^2}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{4y^4}}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{4y^4}}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\,dy$

$= \cancel{\int_1^2 \frac{y^4+1}{2y^2}\,dy}$

$= \int_1^2 \frac{y^4}{2y^2}\,dy + \int_1^2 1/2y^2\,dy$

$= 1/2 \int_1^2 y\,dy + 1/2 \int 1/y^2\,dy$

$= 1/2\,[y^3/3 - y^{-2}]_1^2$





$= 1/2\,[8/3 - 1/2 - 1/3 + 1]$

$= 1/2\,[7/3 + 1/6]$

$= 1/2\,[17/6]$

$= \frac{17}{12}$ units.

II) 1) $\int_0^4 x^2\,dx$

$h = \frac{4-0}{4} = 1$

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 0 | 1 | 4 | 9 | 16 |

$\int_0^4 x^2\,dx = 1/3\,[(y_0+y_4)+4(y_1+y_3)2(y_2)]$

$= 1/3\,[16+4(10)+8]$

$= \frac{64}{3}$

$\int_0^4 x^2\,dx = \cancel{21.533}$



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**Q.3** $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n=6$

$L = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 0 | $\pi/18$ | $2\pi/18$ | $3\pi/18$ | $4\pi/18$ | $6\pi/18$ |
| y | 0 | 0.4166 | 0.58 | 0.70 | 0.80087 | 0.8727 |

$\int_0^{\pi/3} \sqrt{\sin x} = 1/9 [y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_0+y_4)$

$= \pi/54 \times 12.1163$

$= \int_0^{\pi/3} \sqrt{\sin x\, dx}$

$= 0.7049$ //



## Differential Equation.

**Q.1]** Solve the following Equation:-

1) $x \frac{dy}{dx} + y = e^x$

Solⁿ:- Dividing by x

$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

By comparing with

$\frac{dy}{dx} + P(x) y = Q(x)$

$If = e^{\int p \cdot dx}$

$= e^{\int 1/x\, dx}$

$= \int e^{\log x}\, dx = x$

$y(IF) = \int Q (I.F) \times dx + C$

$y[x] = \int \frac{e^x}{\cancel{x}} \cdot \cancel{x}\, dx + C$

$y[x] = \int e^x + C$

$y[x] = e^x + C$

2) $e^x \frac{dy}{dx} + 2y = 1$

Dividing by $e^x$

$\frac{dy}{dx} + 2 \cdot y = 1/e^x$

By comparing with

$$\frac{dy}{dx} + p(x)y = g(x)$$

$$I.f = e^{-\int p\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(I.f) = \int g\,(I.f)\times dx + C$$

$$y(e^{2x}) = \int \frac{1}{e^{x}} \cdot e^{2x} \times dx + C$$

$$= \int e^{2x} \cdot e^{x}\,dx + C$$

$$= \int e^{x}\,dx + C$$

$$y\,e^{2x} = e^{x} + C$$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

comparing with

$$\frac{dy}{dx} + p(x)y = g(x)$$

$$I.f = e^{\int p\,dx}$$

$$= e^{\int 2/x\,dx}$$

$$= e^{2/x}x = \ln x^2 = x^2$$

$$y(I.f) = \int g(x)\,(I.f)\,dx + C$$

$$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + C$$

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$$= \int \cos x + C$$

$$x^2 y = \sin x + C$$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$ (÷ by x on both side)

$p(x) = 3/x \quad g(x) = \sin x / x^3$

$$I.f = e^{\int p\,dx}$$

$$= e^{\int 3/x\,dx}$$

$$= e^{3/x}\,x$$

$$= e^{\ln x^3}$$

$$I.f = x^3$$

$$y(I.f) = \int g(x)\,(I.f)\,dx + C$$

$$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + C$$

$$= \int \sin x\,dx + C$$

$$x^3 y = -\cos x + C$$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$p(x) = 2 \quad g(x) = 2x / e^{2x} = 2xe^{-2x}$

$$I.f = e^{\int p(x)\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(g) = \int g(x)\,(g.f)\,dx + C$$
$$= \int 2xe^{-2x}\,e^{2x}\,dx + C$$
$$= \int 2x\,dx + C$$

$$ye^{2x} = x^2 + C$$

6) $\sec^2 x \tan y\,dx + \sec^2 y \cdot \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$$\frac{\sec^2 x\,dx}{\tan x} = \frac{-\sec^2 y\,dy}{\tan y}$$

$$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$$

$\therefore \log|\tan x| = -\log|\tan y| + C$

$\log|\tan x - \tan y| = C$

$\tan x \cdot \tan y = e^C$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both side

$x-y+1 = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$



$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$\int \sec^2 v\,dv = \int dx$

$\tan v = x + C$

$\tan(x+y-1) = x + C$

8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$$

$v + \log(x) = 3x + C$

$2x + 3y + \log|2x+3y+1| = 3x + C$

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$y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ , $y(0) = 1$ , $h = 0.2$ ; find $y(1)$

$y(0) = 1$ , $x_0 = 0$ , $h = 0.2$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0.2 | 1 | 0.4472 | 1.0894 |
| 2 | 0.4 | 1.6894 | 0.6059 | 1.2105 |
| 3 | 0.6 | 1.2105 | 0.7040 | 1.3813 |
| 4 | 0.8 | 1.3813 | 0.7694 | 1.5051 |
| 5 | 1 | 1.5051 | | |

$y(1) = 1.5051$

$\frac{dy}{dx} = 3x^2+1$ , $y(1) = 2$ find $y(\overset{2}{\cancel{x}})$

For $h = 0.5$ & $h = 0.25$

$h = 0.5$ & $h = 0.25$ $y_0 = 02$ $x_0 = 1$


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| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 3 |
| 1 | 1.25 | 3 | 5.8875 | 4.4218 |
| 2 | 1.5 | 4.4218 | 59.6569 | 19.3360 |
| 3 | 1.75 | 19.3360 | 112.6482 | 299.9966 |
| 4 | 2 | 299.9966 | | |

$y(1) = 299.9966$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$ , $y(1) = 1$ find $y(1.2)$ with

$h = 0.2$

$y(0) = 1$ $x(0) = 1$ $h = 0.2$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 3.6 |
| 1 | 1.2 | 3.6 | | |

$y(1) = 3.6$

# * PRACTICAL - 09 *



TOPIC :- Limits & Partial order derivatives.

Q.1] Evaluate the following limits.

1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

$= \lim_{(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

$= \frac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1) + 5}$

$= -\frac{52}{9}$

2) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$= \lim_{(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$= \frac{(0+1)(2^2 + 0^2 + 4(2))}{2 + 3(0)}$

$= \frac{4-8}{2}$

$= -2$

3) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$

$= \frac{(1)^2 - (1)^2 (1)^2}{(1)^3 - (1)^2 (1)^2} = \frac{1-1}{1-1} = 0/0$ $\therefore$ Limit does not exists.

$\therefore\ f_x = 2,\ f_y = 0$

$$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$$

$$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

$\therefore\ f_x = 2,\ f_y = 0$

Find all second order partial derivatives of $f$. Also verify whether $f_{xy} = f_{yx}$.

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$\therefore\ f_{x\cancel{y}x} = \frac{d^2y}{dx^2} \qquad f_{yy} = \frac{d^2f}{dy^2}$$

$$f_x = \frac{x^2(0-y)-(y^2-xy)2x}{x^4}$$

$$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$$

$$\therefore f_x = \frac{x^2y - 2xy^2 + 2x^2y}{x^4}$$

$$f_{xx} = \frac{x^4(2xy-2y^2)-(x^2y-2xy^2)(4x^3)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$$

$$= \frac{-2x^5y + 6x^4y^2}{x^8}$$

$$= \frac{6x^4y^2 - 2x^5y}{x^8}$$

$f_y = \frac{1}{x^2}(2y-x) \quad \therefore f_y = \frac{2x}{x^2}$

$\therefore f_{yy} = \frac{1}{x^2} \cdot 2 = \frac{2}{x^2}$

$$\therefore f_{xy} = \frac{2y-x}{x^2}$$

$$= \frac{x^2(-1)-(2y-x)(2x)}{x^4}$$

$$= \frac{-x^2-4xy+2x^2}{x^4}$$

$$= \frac{x^2-4xy}{x^4}$$

$$= \frac{x-4y}{x^3}$$

$$\therefore f_{xy} = \frac{x-4y}{x^3}$$

$$\therefore f_{yx} = \frac{x^2y-2xy}{x^4}$$

$$= \frac{x-4y}{x^3}$$

$$= f_{xy}$$

$$\therefore f_{yx} = f_{xy}$$

Hence verified.

Q.5] Find the linearization of $f(x,y)$ at given point.

ii] $f(x,y) = 1-x+y\sin x$ at $\pi/2$

$f(\pi/2, 0) = 1-\pi/2 + 0 + \sin \pi/2$

$f(\pi/2, 0) = \frac{2-\pi}{2}$

$f_x = -1+y\cos x \qquad f_y = \sin x$

$f_x(\pi/2, 0) = -1 + 0\cdot\cos\pi/2$

$= -1$

$f_y(\pi/2, 0) = \sin \pi/2$

$= 1$

$2(x,y) = f(\pi/2,0) + f_x(\pi/2,0)(x-\pi/2) + f_y(\pi/2,0)(y-0)$

$= 2-\pi/2 + (-1)(x-\pi/2) + 1(y)$

$= 1 \cdot \pi/2 - x + \pi/2 + y$

$$= 0 + 6yx^2 - 0$$

$$f_y = 6x^2y$$

$$f_{xx} = \frac{2}{2x} f_x$$

$$= \frac{2}{2x}\left(3x^2 + 6x^2y - \frac{2x}{x^2+1}\right)$$

$$\therefore f_{xx} = 6x + 6y^2 \frac{-4x - 2x^2 + 2}{(x^2+1)^2}$$

$$\therefore f_{yy} = \frac{2}{2y} f_y$$


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$\therefore\ f_{yx} = 12xy$

$\therefore\ f_{xy} = f_{yx}$

3] $f(x,y) = \sin(x,y) + e^{x+y}$

$f(x,y) = \sin(xy) + e^x e^y$

$f_x = \frac{\partial}{\partial x}\ (\sin(xy) + e^x e^y)$

$f_x = y\cos(xy) + e^x e^y$

$f_y = \frac{\partial}{\partial y}\ (\sin(xy) + e^x e^y$

$= x\cos(xy) + e^x \cdot e^y$

$f_{xx} = \frac{\partial}{\partial x}\ f_x$

$= \frac{\partial}{\partial x}\ y\cos(xy) + e^x \cdot e^y$

$f_{xx} = -y^2 \sin(xy) + e^{x+y}$

$f_{yy} = \frac{\partial}{\partial y}\ f_y$

$= \frac{\partial}{\partial y}\ (x\cos(xy) + e^x \cdot e^y)$

$f_{yy} = -x^2 \sin^2(xy) + e^x \cdot e^y$

$f_{xy} = \frac{\partial}{\partial y}\ \cdot y \cdot \cos(xy) + e^x \cdot e^y$

$f_{yx} = \frac{\partial}{\partial x}\ f_y$

$= \frac{\partial}{\partial x}\ x \cdot \cos(xy) + e^x \cdot e^y$

$f_{xy} = -xy^2 \sin(xy) + \cos(xy) + e^x \cdot e^y$

$f_{yx} = \frac{\partial}{\partial x}\ f_y$

$f_{yx} = \frac{\partial}{\partial x}\ x \cdot \cos(xy) + e^x \cdot e^y$

$\therefore\ f_{yx} = -xy\sin(xy) + \cos(xy) + e^x \cdot e^y$

$\therefore\ f_{xy} = f_{yx}$

PRACTICAL - 10

TOPIC :- Directional Derivatives, Gradient vector, maxima, minima & Tangent & Normal vectors

Q.1] Find the directional derivative of the following functions at given points & in the direction of given vector.

1] $f(x,y) = x+2y-3$ , $a = (1,-1)$ , $u = 3i-j$

$\rightarrow$ $u = 3\hat{i}-\hat{j}$

$\therefore \hat{u} = \frac{\bar{u}}{|u|} = \frac{1}{\sqrt{3^2+(-1)^2}}(3\hat{i}-\hat{j})$

$\therefore \hat{u} = \frac{1}{\sqrt{10}}(3\hat{i}-\hat{j})$

$\hat{u} = (3/\sqrt{10}, \frac{-1}{\sqrt{10}})$

$a = (1,-1)$

$\therefore f(a) = 1+2(-1)-3$

$= 1+(-2)-3$

$= -4$

$f(a+hu) = f((1,-1)+h(\frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{10}}))$

$= f((1+\frac{3}{\sqrt{10}}h), (-1-\frac{h}{\sqrt{10}}))$

$= 1+\frac{3}{\sqrt{10}}h+2(-1-\frac{h}{\sqrt{10}})-3$

$= 1+\frac{3}{\sqrt{10}}h-2-\frac{2h}{\sqrt{10}}-3$

$\therefore f(a+hu) = \frac{h}{\sqrt{10}}-4$

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61

$\therefore D_u f = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$= \lim_{h\to 0} \frac{h/\sqrt{10}-4-(-4)}{h}$

$= \frac{1}{\sqrt{10}} \lim_{h\to 0} \frac{h}{h}$

$= \frac{1}{\sqrt{10}}$

2] $f(x,y) = y^2-4x+1$ , $a = (3,4)$ , $u = \hat{i}+5\hat{j}$

$\rightarrow$ $f(x,y) = y^2-4x+1$

$\bar{u} = \hat{i}+5\hat{j}$

$\therefore \hat{u} = \frac{\bar{u}}{|u|} = \frac{\hat{i}+5\hat{j}}{\sqrt{1^2+5^2}} = \frac{1}{\sqrt{26}}(\hat{i}+5\hat{j})$

$\therefore \hat{u} = (\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}})$

$f(a) = (4)^2-4(3)+1$

$= 16-12+1$

$= 5$

$f(a+hu) = f((3,4)+h(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}))$

$= f((3+\frac{h}{\sqrt{26}}), (4+\frac{5h}{\sqrt{26}}))$

$= (4+\frac{5h}{\sqrt{26}})^2-4(3+\frac{h}{\sqrt{26}})+1$

$= 16+\frac{40h}{\sqrt{26}}+\frac{25h}{26}-12-\frac{4h}{\sqrt{26}}$

$= \frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5$

$= 18h/5 + 8$

62

$Df(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$\therefore Df(a) = \lim_{h \to 0} \frac{18h/5 + 8 - 8}{h}$

$= \lim_{h \to 0} \frac{18}{5}h/h$

$\therefore Df(a) = 18/5$

Q.2] Find gradient vector for the following function at given point.

① $f(x,y) = x^y + y^x$, $a = (1,1)$

$\rightarrow f(x,y) = x^y + y^x$

$f_x = \frac{\partial}{\partial x}(x^y + y^x)$

$f_x = yx^{y-1} + y^x \cdot \log y$

$f_y = \frac{\partial}{\partial x} x^y + y^x$

$\therefore f_y = xy^{x-1} + x^y \cdot \log x$

$\nabla f(x,y) = (f_x, f_y)$

$\nabla f(x,y) = (yx^{y-1} + y^x \cdot \log y,\ xy^{x-1} + x^y \log x)$



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$(2x\cos y + ye^{xy})(x-1) + (-x^2\sin y + xe^{xy})(y-0) = 0$

$2x^2\cos y + xye^{xy} - 2x\cos y - ye^{xy} - (x^2\sin y - xe^{xy})y = 0$

(2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2 = 0$

$f_x = 2x - 2 \qquad f_x(2,-2) = 2$

$f_y = 2y + 3 \qquad f_y(2,-2) = 1$

**Tangent** :- $f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(x-x_0) = 0$

$2(x-2) + 1(y+2) = 0$

$2x - 4 + y + 2 = 0$

$2x + y - 2 = 0$

**Normal** :- $x - 2y + d = 0$

$2 - 2(-2) + d = 0$

$\therefore d = 2$

$\therefore x - 2y + 2 = 0$

5.8

∴ **Normal** :-

$$\frac{x-x_0}{f_x(x_0,y_0,z_0)} = \frac{y-y_0}{f_y(x_0,y_0,z_0)} = \frac{z-z_0}{f_z(x_0,y_0,z_0)}$$

$$\frac{x-2}{4} = \frac{y-1}{1} = \frac{z-0}{0}$$

(2) $3xyz - x - y + z = -4$ at $(1,-1,2)$

→ $f(x,y,z) = 3xyz - x - y + z + 4 = 0$

$f_x = 3yz - 1$ $\quad f_x(x_0,y_0,z_0) = -7$

$f_y = 3xz - 1$ $\quad f_y(x_0,y_0,z_0) = 5$

$f_z = 3xy + 1$ $\quad f_z(x_0,y_0,z_0) = -2$

**Tangent** :-

$f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0)$

∴ $-7(x-1) + 5(y+1) - 2(z-2) = 0$

∴ $-7x + 7 + 5y + 5 - 2z + 4 = 0$

∴ $7x - 5y + 2z + 6 = 0$

**Normal** :-

$$\frac{x-x_0}{f_x(x_0,y_0,z_0)} = \frac{y-y_0}{f_y(x_0,y_0,z_0)} = \frac{z-z_0}{f_z(x_0,y_0,z_0)}$$

∴ $$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

**Q5** Find the local maxima & minima for the following functions.

61

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

→ $f_x = 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0$ $\qquad f_y = 0$

$6x - 3y = -6$ $\qquad 3x - 2y = -4$

$\qquad 6x - 4y = -8$

∴ $y = 2$

∴ $x = 0$

∴ $(x,y) = (0,2)$

$r = f_{xx} = 6$

$s = f_{xy} = -3$

$t = f_{yy} = 2$

$rt - s^2 = 6(2) - (-3)^2 = 12 - 9 = 3 > 0$

$r > 0$

∴ $f$ is minimum at $(0,2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= -4$

(2) $f(x,y) = 2x^4 + 3x^2y - y^2$

→ $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = -2y + 3x^2$

$f_x = 0$ $\qquad f_y = 0$

∴ $8x^3 + 6xy = 0$ $\qquad -2y + 3(0) = 0$

∴ $x(8x^2 + 6y) = 0$

$x = 0$ $\qquad$ ∴ $y = 0$


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$(x,y) = (0,0)$ is a root

1.2 $8x^2 + 6y = 0$

$x^2 = -2/3\, y$

$\therefore f y = -2y - 2/3 \times 3y$

$\therefore -4y = 0$

$\therefore y = 0$

$\therefore x^2 = 0$

$\therefore x = 0$

$\therefore (x,y) = (0,0)$ is the only root.

$r = f_{xx} = 24x^2 + 6y = 0$

$s = f_{xy} = 6x = 0$

$t = f_{yy} = -2 + 0 = -2$

$r = 0$

$rt - s^2 = (0)(0)(-2)^2$

$\therefore rt - s^2 = -4 < 0$

$\therefore (0,0)$ is the saddlepoint)

③ $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$ $\therefore 2x + 2 = 0$

$\therefore x = -2/2 = -1$

$f_y = 0$ $-2y + 8 = 0$

$y = -8/-2 = 4$

Critical point is $(-1,4)$

~~$r = f_{xx} =$~~