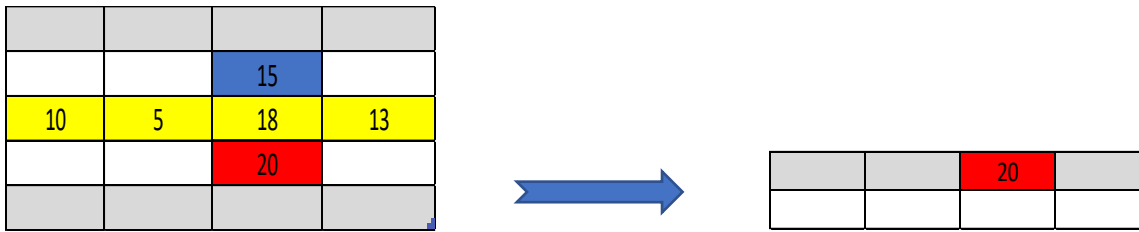


PEAK FINDER 2D

- 1- Find middle row/column, then pick a number in middle row/column ==> $\theta(1)$
- 2- Check if the number is peak number, if it is a peak number then return it ==> $\theta(1)$
- 3- Find max number in row/column ==> $\theta(m)$ or $\theta(n)$
- 4- Check if the number is peak number, if it is a peak number then return it ==> $\theta(1)$
- 5- Compare max number with its neighbors ==> $\theta(1)$
 - If the max's neighbor is bigger than max, then maintain search on neighbor's part
 $\text{arr}[\text{midRow}+1][\text{max}] > \text{arr}[\text{midRow}][\text{max}]$
 $\text{arr}[\text{max}][\text{midCol}+1] > \text{arr}[\text{max}][\text{midCol}]$
 - If the max's other neighbor is bigger, then maintain search on this neighbor's part
 $\text{arr}[\text{midRow}-1][\text{max}] > \text{arr}[\text{midRow}][\text{max}]$
 $\text{arr}[\text{max}][\text{midCol}-1] > \text{arr}[\text{max}][\text{midCol}]$

n (number of rows)	m (number of columns)	rowOrCol	Time(ns)
10	100	0	13800
10	100	1	12500
10	100	2	3900
100	10	0	11800
100	10	1	15100
100	10	2	9100
50	200	0	197599
50	200	1	151499
50	200	2	85700
200	50	0	245500
200	50	1	37600
200	50	2	474699
50	1000	0	219700
50	1000	1	158200
50	1000	2	449800
1000	50	0	241600
1000	50	1	135099
1000	50	2	78700
50	10000	0	1661400
50	10000	1	126201
50	10000	2	680400
10000	50	0	230900
10000	50	1	143999
10000	50	2	1238900

Condition 1: rowOrCol = 0 ---> array is split by row



To find the peak number in an nXm-sized array;

n: number of rows

m: number of columns

$$T(n, m) = \begin{cases} m, & n = 1 \\ T\left(\frac{n}{2}, m\right) + m, & n > 1 \end{cases}$$

$$T(n, m) = m + T\left(\frac{n}{2}, m\right) \quad \text{1.Equation}$$

In 1.Equation, we write $T\left(\frac{n}{2}, m\right) = m + T\left(\frac{n}{4}, m\right)$ instead of $T\left(\frac{n}{2}, m\right)$

$$T(n, m) = 2m + T\left(\frac{n}{4}, m\right) \quad \text{2.Equation}$$

$$T(n, m) = km + T\left(\frac{n}{2^k}, m\right) \quad \text{General Formula}$$

$$T(1, m) = m$$

$$\frac{n}{2^k} = 1, \quad k = \log n$$

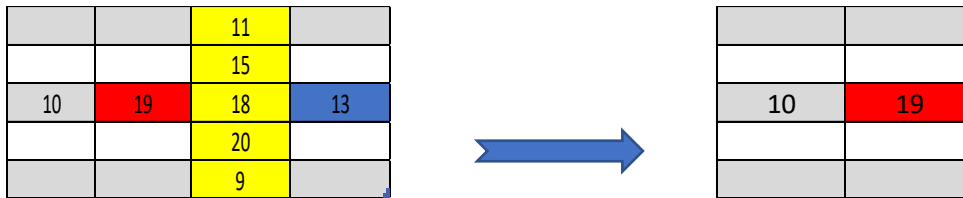
Substitute k value in the general formula

$$T(n, m) = \log n \cdot m + T(1, m)$$

$$T(n, m) = \log n \cdot m + m$$

$$O(n) = n \cdot \log m$$

Condition 2: rowOrCol = 1 ---> array is split by column



To find the peak number in an $n \times m$ -sized array;

n : number of rows

m : number of columns

$$T(n, m) = \begin{cases} n, & m = 1 \\ T(n, \frac{m}{2}) + m, & m > 1 \end{cases}$$

$$T(n, m) = n + T(n, \frac{m}{2}) \quad \text{1.Equation}$$

In 1.Equation, we write $T(n, \frac{m}{2}) = n + T(n, \frac{m}{4})$ instead of $T(n, \frac{m}{2})$

$$T(n, m) = 2n + T(n, \frac{m}{4}) \quad \text{2.Equation}$$

$$T(n, m) = kn + T(n, \frac{m}{2^k}) \quad \text{General Formula}$$

$$T(n, 1) = n$$

$$\frac{n}{2^k} = 1 \quad k = \log n$$

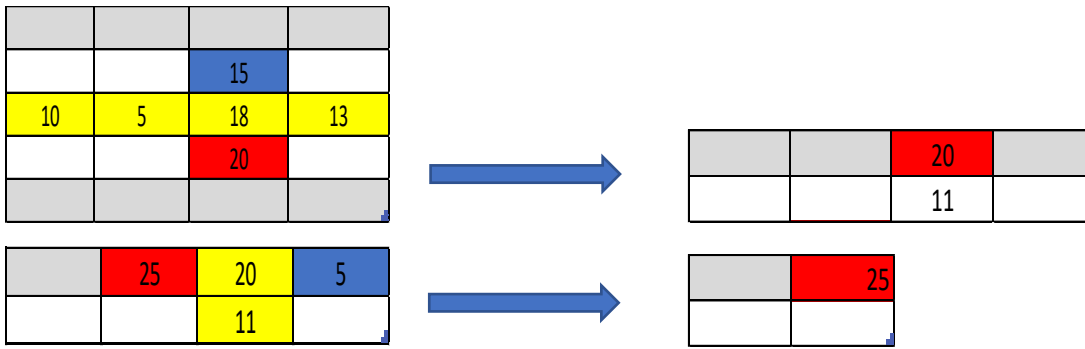
Substitute k value in the general formula

$$T(n, m) = \log n \cdot m + T(n, 1)$$

$$T(n, m) = \log n \cdot m + n$$

$$O(n) = m \cdot \log n$$

Condition 3: rowOrCol = 2 ---> first, array is split by row then column



To find the peak number in an $n \times m$ -sized array;

n : number of rows

m : number of columns

$$T(n, m) = \begin{cases} n, m = 1 \\ m, n = 1 \\ T\left(\frac{n}{2}, m\right) + m, n > 1 \\ T\left(n, \frac{m}{2}\right) + n, m > 1 \end{cases}$$

$$T(n, m) = m + T\left(\frac{n}{2}, m\right) \text{ 1. Equation}$$

$$T\left(\frac{n}{2}, m\right) = m + T\left(\frac{n}{2}, \frac{m}{2}\right) \text{ 2. Equation}$$

$$T\left(\frac{n}{2}, \frac{m}{2}\right) = \frac{m}{2} + T\left(\frac{n}{4}, \frac{m}{2}\right) \text{ 3. Equation}$$

$$T\left(\frac{n}{4}, \frac{m}{2}\right) = \frac{n}{4} + T\left(\frac{n}{4}, \frac{m}{4}\right) \text{ 4. Equation}$$

$$T\left(\frac{n}{4}, \frac{m}{4}\right) = \frac{m}{4} + T\left(\frac{n}{8}, \frac{m}{4}\right) \text{ 5. Equation}$$

We write the above equations, starting from the 5th equation, backwards to the 1st equation.

$$T(n, m) = m + n + \frac{m}{2} + \frac{n}{2} + \frac{m}{4} + \frac{n}{4} + T\left(\frac{n}{8}, \frac{m}{4}\right)$$

$$T(n, m) = m + n + \frac{m}{2^k} + \frac{n}{2^k} + T\left(\frac{n}{2^k}, \frac{m}{2^k}\right) \text{ General Formula}$$

$$n \sum_{k=1}^{\infty} \frac{1}{2^k} = n$$

$$m \sum_{k=1}^{\infty} \frac{1}{2^k} = m$$

$$T(n, m) = m + n + m + n = 2m + 2n$$

$$O(n, m) = m + n$$

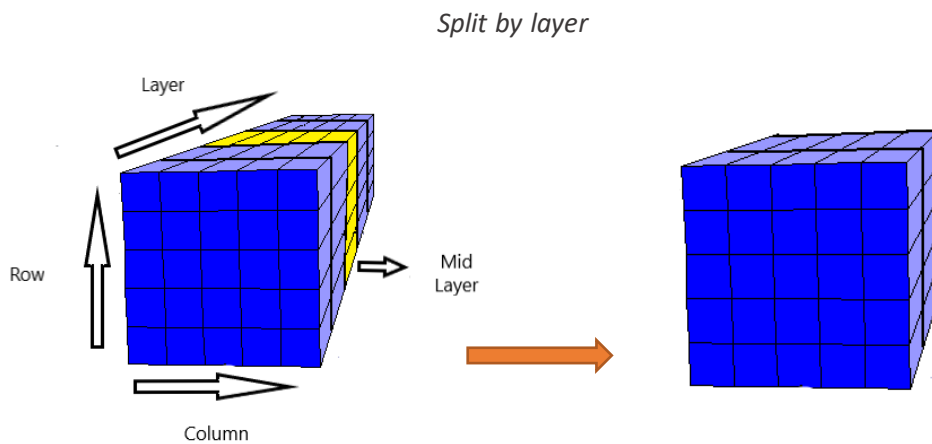
PEAK FINDER 3D

- 1- Find middle layer, then pick a number in midLayer $\Rightarrow \theta(1)$
- 2- Check if the number is peak number, if it is a peak number then return it $\Rightarrow \theta(1)$
- 3- Find max number in midLayer $\Rightarrow \theta(mn)$
- 4- Check if the number is peak number, if it is a peak number then return it $\Rightarrow \theta(1)$
- 5- Compare max number with its neighbors $\Rightarrow \theta(1)$
 - If the max's neighbor is bigger than max, then maintain search on neighbor's part $\Rightarrow \theta(1)$

$\text{arr}[\text{max}][\text{max}][\text{midLayer}] > \text{arr}[\text{max}][\text{max}][\text{midLayer}+1]$

- If the max's other neighbor is bigger, then maintain search on this neighbor's part $\Rightarrow \theta(1)$

$\text{arr}[\text{max}][\text{max}][\text{midLayer}] > \text{arr}[\text{max}][\text{max}][\text{midLayer}-1]$



To find the peak number in an $n \times m \times d$ -sized array;

n : number of rows

m : number of columns

d : number of layers

$$T(n, m, d) = \begin{cases} n.m, & d = 1 \\ T\left(n, m, \frac{d}{2}\right) + m.n, & d > 1 \end{cases}$$

$$T(n, m, d) = n.m + T\left(n, m, \frac{d}{2}\right) \quad \text{1.Equation}$$

In 1.Equation, we write $T\left(n, m, \frac{d}{2}\right) = n.m + T\left(n, m, \frac{d}{4}\right)$ instead of $T\left(n, m, \frac{d}{2}\right)$

$$T(n, m, d) = 2.n.m + T\left(n, m, \frac{d}{4}\right) \quad \text{2.Equation}$$

$$T(n, m, d) = k.n.m + T\left(n, m, \frac{d}{2^k}\right) \quad \text{General Formula}$$

$$T(n, m, 1) = m \cdot n, \quad \frac{d}{2^k} = 1$$

$$k = \log d$$

Substitute k value in the general formula

$$T(n, m, d) = \log d \cdot m \cdot n + T(n, m, 1)$$

$$T(n, m, d) = \log d \cdot m \cdot n + n \cdot m$$

$$O(n, m, d) = m \cdot n \cdot \log d$$