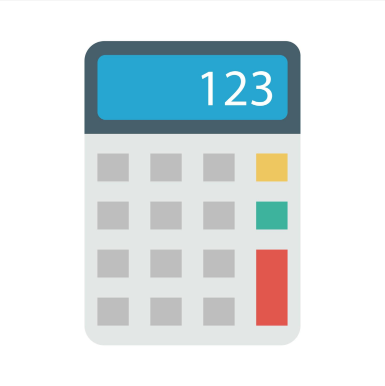
Department of Computer Science and Engineering, BRAC University

Lecture 1

Number System & Calculations



CSE260 : Digital Logic Design

Objective

- Distinguish between analog and digital system
- Understand the advantage and limitation of digital system
- Understand the meaning of digital logic

Analog vs. Digital

- Analog data can vary over a continuous range of values.
  Example: speedometer
- Digital quantities can take on only discrete values (0 and 1, high and low).
  Example: Digital Computer, Decimal Digits, Alphabets

Digital System

- A digital system is a combination of devices designed to manipulate physical quantities or information that are represented in digital form.

- “A discreet information processing system”
- Signals: Discreet information

Digital System Advantages

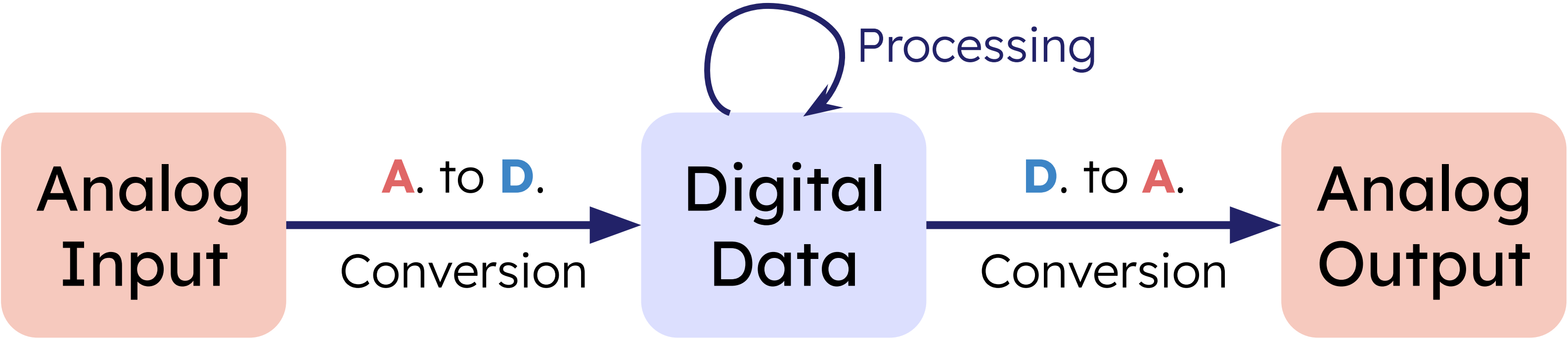
- Greater accuracy or precision
- Easier to design (generality)
- Easier information storage
- Programmability (instructions)
- Speed
- Economical

Limitation

- The real world is mainly analog

How to Overcome the Limitations

- Convert the real world analog input data into digital
- Process this digital data
- Then again convert into analog form



Digital Logic

- Design logic is a term used to denote the design and analysis of digital system
- Digital logic is concerned with the interconnection among digital components and modules
- Digital logic design is engineering and engineering means problem solving

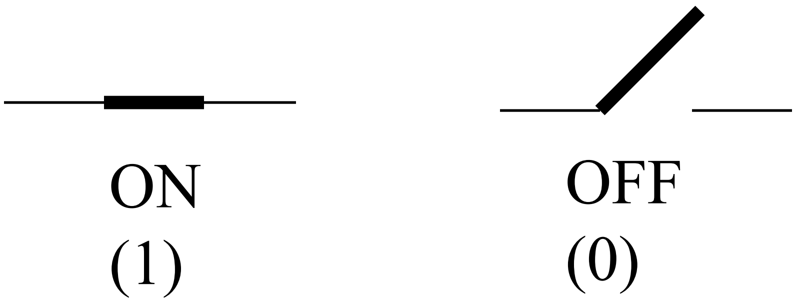
Number systems and codes

- Digital Systems are built from circuits that process binary digits. BUT very few real-life problems are based on binary numbers

- So, a digital system designer must establish some correspondence between the binary digits processed by digital circuits and real-life numbers, events and conditions

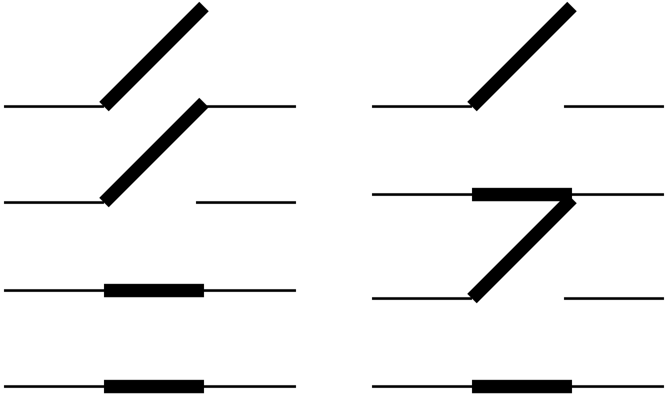
Information representation

- Human decisions tends to be binary i.e. **Yes or No**
- Elementary storage units inside computer are *electronic switches*. Each switch holds one of two states: **on (1) or off (0).**



Information representation [cont.]

- We use a **bit** (**bi**nary digi**t**), **0** or **1**, to represent the state
- Storage units can be grouped together to cater for larger range of numbers. Example: 2 switches to represent 4 values



0 (00)

1 (01)

2 (10)

3 (11)

Information representation [cont.]

- In general, N bits can represent 2^N different values.

  1 bit $\rightarrow$ represents up to 2 values (0 or 1)
  2 bits $\rightarrow$ rep. up to 4 values (00, 01, 10 or 11)
  3 bits $\rightarrow$ rep. up to 8 values (000, 001, 010. ..., 110, 111)
  4 bits $\rightarrow$ rep. up to 16 values (0000, 0001, 0010, ..., 1111)

- For M values, $\lceil \log_2 M \rceil$ bits are needed.

  32 values $\rightarrow$ requires 5 bits
  64 values $\rightarrow$ requires 6 bits
  1024 values $\rightarrow$ requires 10 bits
  40 values $\rightarrow$ requires 6 bits
  100 values $\rightarrow$ requires 7 bits

Positional Notations

- Decimal number system, symbols = { 0, 1, 2, 3, ..., 9 }
- Position is important

Example:

$$(7594)_{10} = (7\text{x}10^3) + (5\text{x}10^2) + (9\text{x}10^1) + (4\text{x}10^0)$$

- In general,

$$(a_n a_{n-1} \dots a_0)_{10} = (a_n \text{ x } 10^n) + (a_{n-1} \text{ x } 10^{n-1}) + \dots + (a_0 \text{ x } 10^0)$$

$$(2.75)_{10} = (2 \text{ x } 10^0) + (7 \text{ x } 10^{-1}) + (5 \text{ x } 10^{-2})$$

- In general,
- $(a_n a_{n-1} \dots a_0 \,.\, f_1 f_2 \dots f_m)_{10} = (a_n \text{ x } 10^n) + (a_{n-1}\text{x}10^{n-1}) + \dots + (a_0 \text{ x } 10^0) + (f_1 \text{ x } 10^{-1}) + (f_2 \text{ x } 10^{-2}) + \dots + (f_m \text{ x } 10^{-m})$

Other Number Systems

- Binary (base 2): weights in powers-of-2. Binary digits (bits): ***0,1***
- Octal (base 8): weights in powers-of-8. Octal digits: ***0,1,2,3,4,5,6,7***
- Hexadecimal (base 16): weights in powers-of-16. Hexadecimal digits: ***0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F***

Note: when base is r, coefficient values range from 0 to r-1.

Other Number Systems [cont]

- Binary (base 2): weights in powers-of-2. Binary digits (bits): ***0,1***
- Octal (base 8): weights in powers-of-8. Octal digits: ***0,1,2,3,4,5,6,7***
- Hexadecimal (base 16): weights in powers-of-16. Hexadecimal digits: ***0,1,2,3,4,5,6,7,8,9,A,B,C, D,E,F***

Binary	Octal	Decimal	Hexadecimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	10	A
1011	13	11	B
1100	14	12	C
1101	15	13	D
1110	16	14	E
1111	17	15	F

Base-R to Decimal Conversion

***Formula= Σ digit * source_base ^{position}

$(1101.101)_2$

$= 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-3}$

$= 8 + 4 + 1 + 0.5 + 0.125$

$= (13.625)_{10}$

$(2A.8)_{16}$

$= 2\times16^1 + 10\times16^0 + 8\times16^{-1}$

$= 32 + 10 + 0.5$

$= (42.5)_{10}$

$(572.6)_8$

$= 5\times8^2 + 7\times8^1 + 2\times8^0 + 6\times8^{-1}$

$= 320 + 56 + 2 + 0.75$

$= (378.75)_{10}$

$(341.24)_5$

$= 3\times5^2 + 4\times5^1 + 1\times5^0 + 2\times5^{-1} + 4\times5^{-2}$

$= 75 + 20 + 1 + 0.4 + 0.16$

$= (96.56)_{10}$

Decimal to Base-R Conversion

- Whole numbers: repeated **division**-by-R
- Fractions: repeated **multiplication**-by-R

Repeated Division-by-2 Method

- To convert a whole number to **binary**, use successive division by 2 until the quotient is 0. The remainders form the answer, with the first remainder as the *least significant bit (LSB)* and the last as the *most significant bit (MSB)*.

$$(43)_{10} = (101011)_2$$

2	43		
2	21	rem 1	← LSB
2	10	rem 1	
2	5	rem 0	
2	2	rem 1	
2	1	rem 0	
	0	rem 1	← MSB

Repeated Multiplication-by-2 Method

- To convert decimal fractions to binary, repeated multiplication by 2 is used, until the fractional product is 0 (or until the desired number of decimal places). The carried digits, or *carries*, produce the answer, with the first carry as the MSB, and the last as the LSB.

	Carry	
$0.3125 \times 2 = 0.625$	0	←MSB
$0.625 \times 2 = 1.25$	1	
$0.25 \times 2 = 0.50$	0	
$0.5 \times 2 = 1.00$	1	←LSB

$(0.3125)_{10} = (.0101)_2$

Repeated Multiplication-by-2 Method

Integer

- Division by base of target no. system
- Remainders are accumulated
- By division we obtain LSB to MSB

Fraction

- Multiplication by base of target no. system
- Integers are accumulated
- By multiplication we obtain MSB to LSB

Binary-Octal/Hexadecimal Conversion

- **Binary → Octal**: Partition in groups of 3

  $(\underbrace{10}\ \underbrace{111}\ \underbrace{011}\ \underbrace{001}\ .\ \underbrace{101}\ \underbrace{110})_2 = (2731.56)_8$

- **Octal → Binary**: reverse

  $(2731.56)_8 = (10\ 111\ 011\ 001\ .\ 101\ 110)_2$

- **Binary → Hexadecimal**: Partition in groups of 4

  $(\underbrace{101}\ \underbrace{1101}\ \underbrace{1001}\ .\ \underbrace{1011}\ \underbrace{1000})_2 = (5D9.B8)_{16}$

- **Hexadecimal → Binary**: reverse

  $(5D9.B8)_{16} = (101\ 1101\ 1001\ .\ 1011\ 1000)_2$

Exercise

(1) Try converting this to

$(10110001101011.111100000110)_2$

a) octal

b) hexadecimal

(2) Try converting these to binary

a) $(673.124)_8$

b) $(306.D)_{16}$

Exercise Answers

1. a) $(26153.7406)_8$
   b) $(2C6B.F06)_{16}$

2. a) $(110\ 111\ 011\ .\ 001\ 010\ 100)_2$
   b) $(0011\ 0000\ 0110\ .\ 1101)_2$

Exercise

$(1054)_6 = (?)_{15}$

Solution:

Step 1: $(1054)_6 = (?)_{10}$

$=> 1 \times 6^3 + 0 \times 6^2 + 5 \times 6^1 + 4 \times 6^0$

$=> 250$

So, $(1054)_6 = (250)_{10}$

Exercise

Step 2: $(250)_{10} = (?)_{15}$

15	250	
15	16 rem A	← LSB
15	1 rem 1	
	0 rem 1	← MSB

So, $(250)_{10} = (11A)_{15}$

$(1054)_6 = (11A)_{15}$

Exercise

$(10A1B)_{13} = (?)_{18}$

Solution:

Step 1: $(10A1B)_{13} = (?)_{10}$

$\Rightarrow 1 \times 13^4 + 0 \times 13^3 + A \times 13^2 + A \times 13^1 + B \times 13^0$

$\Rightarrow 30275$

So, $(10A1B)_{13} = (30275)_{10}$

Exercise

Step 2: $(30275)_{10} = (?)_{18}$

18	30275
18	1681 rem H ⬅ LSB
18	93 rem 7
18	5 rem 3
	0 rem 5 ⬅ MSB

10	A
11	B
12	C
13	D
14	E
15	F
16	G
17	H

So, $(30275)_{10} = (537H)_{18}$

$(10A1B)_6 = (537H)_{18}$

Binary Addition: Addition Rules w/Carries

For 2 bit

- 0 + 0 = 0 0
  - (0 with a 0 carry)
- 0 + 1 = 0 1
  - (1 with a 0 carry)
- 1 + 0 = 0 1
  - (1 with a 0 carry)
- 1 + 1 = 1 0
  - (0 with a 1 carry)

For 3 bit

- 0+0+0 = 0 0
  - (0 with 0 carry)
- 0+0+1 = 0 1
  - (1 with 0 carry)
- 0+1+1 = 1 0
  - (0 with 1 carry)
- 1+1+1 = 1 1
  - (1 with 1 carry)

Adding Binary Numbers

$$\begin{array}{r} 28 \\ +\ 43 \\ \hline 71 \end{array} \quad \longrightarrow \quad \begin{array}{r} \color{red}{0\ 1\ 1\ 1\ 0\ 0\ 0\ \ } \\ 00011100 \\ +\ 00101011 \\ \hline 01000111 \end{array}$$

Exercise

(3) Add $(101101)_2$ with $(100111)_2$

Exercise Solution

(3) $(1010100)_2$

Working:

Augend:	101101
Addend:	+100111
Sum:	1010100

Addition of base-r

Example:

$(34)_5 + (41)_5 + (24)_5$

$$
\begin{array}{rccl}
 & 2 & 1 & \\
 & (3 & 4)_5 & \\
 & (4 & 1)_5 & \\
+ & (2 & 4)_5 & \\
\hline
 & (2\ 0 & 4)_5 & \text{(Ans)}
\end{array}
$$

$4+1+4= 9$

$9\%5=4$ (sum)

$9/5=1$ (carry)

$1+3+4+2= 10$

$10\%5=0$ (sum)

$10/5=2$ (carry)

Exercise

(4) (a) $(FF)_{16}+(F1)_{16}$
(b) $(66)_7 + (55)_7$

Solution

(a) $(1F0)_{16}$

(b) $(154)_7$

Binary Multiplication

- The multiplication of two binary numbers can be carried out in the same manner as the decimal multiplication.
- Unlike decimal multiplication, only two values are generated as the outcome of multiplying the multiplication bit by 0 or 1 in the binary multiplication. These values are either 0 or 1.
- The binary multiplication can also be considered as repeated binary addition. Therefore, the binary multiplication is performed in conjunction with the binary addition operation.

Exercise

(5) Multiply 1011 with 101

Multiplicand	1011
Multiplier	x 101
Partial Product	1011
	0000X
	1011XX
Product	110111

Multiplication with base-r

```
  2A3C
   xB7
------
 127A4
1D094X
------
1E30E4
```

Working

7*C = 7*12 = 84 =(write 4, carry 5)
7*3+5 = 26 = 1A (write A, carry 1)
7*A+1 = 71 = 0x47 (write 7, carry 4)
7*2+4 = 18 = 0x12
This completes the 7*2A3C = 127A4 partial product.

B*C = 11*12 = 132 = (write 4, carry 8)
B*3+8 = 11*3+8 = 41 = (write 9, carry 2)
B*A+2 = 11*10+2 = 112 = (write 0, carry 7)
B*2+7 = 11*2+7 = 29 = 1D
This completes the B[0]*2A3C = 1D094[0] partial product, where I'm noting the [0] digits to remind us this is in the 16s column.

Adding the partial products: 127A4 + 1D0940
4+0 = 4
A+4 = E
7+9 = 16 = 0x10 (write 0, carry 1)
2+0+1 = 3
1+D = E
1 = 1

Exercise

(6) (a) Multiply $(34)_5$ with $(42)_5$
(b) Multiply $(25)_9$ with $(36)_9$

Solution

(a) $(3133)_5$

(b) $(1033)_9$

Binary Subtraction: Rules w/Carries

For 2 bit

- 0 - 0 = 0 0 (0 with a 0 carry)
- 1 - 1 = 0 0 (0 with a 0 carry)
- 1 - 0 = 0 1 (1 with a 0 carry)
- 0 - 1 = ? (???)

Binary Subtraction: Rules w/Carries

$$\begin{array}{r} 2 \\ -\ 1 \\ \hline 1 \end{array} \quad \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \quad \begin{array}{r} \overset{0}{\cancel{1}}\overset{2}{\cancel{0}} \\ -\ 01 \\ \hline 01 \end{array}$$

$$\begin{array}{r} 4 \\ -\ 1 \\ \hline 3 \end{array} \quad \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \quad \begin{array}{r} 100 \\ -\ 001 \\ \hline 011 \end{array}$$

Exercise

(7) (b) Subtract $(100111)_2$ from $(101101)_2$

Solution

$(000110)_2$

Working

Minuend:	101101
Subtrahend:	-100111
Difference:	000110

Binary Subtraction: Rules w/Carries

$$
\begin{array}{rccc}
 & \scriptstyle 3 & \scriptstyle 16 & \\
 & (\cancel{4} & A & 6)_{16} \\
- & (1 & \cancel{B} & 3)_{16} \\
\hline
 & (2 & F & 3)_{16}
\end{array}
$$

$$
\begin{array}{rcc}
 & \scriptstyle 4 & \scriptstyle 6 \\
 & (\cancel{5} & 4)_{6} \\
- & (3 & \cancel{5})_{6} \\
\hline
 & (1 & 5)_{6}
\end{array}
$$

Exercise

(8) (a) $(71)_8-(56)_8$ (b) $(21)_3-(12)_3$

Solution

a) $(13)_8$

b) $(2)_3$

Division of base-r

Dividend: $x_1x_2x_3...x_m$; Divisor: $y_1y_2...y_n = y$;
Perform $101_2 / 10_2$; Dividend = 101 and Divisor = 10

Step1: Start with the first digit of the dividend. (x_1)
Step2: Compare it with the divisor;
If (it is smaller, than the divisor): ($x_1 < y$)
2.a append 0 to the quotient.
else: ($x_1 >= y$)
proceed with division.
2.a Find the largest multiple of the divisor that fits.
2.b Append the multiplier to the quotient.
Step3: Subtract and bring down the next digit.
Step4: From now on compare the newly formed number in step3 with the divisor while repeating step2.
Step5. Repeat this step 2, 3 & 4 until all digits of the dividend ($x_1x_2x_3...x_m$) are processed.

Iteration 1

```
10| 101 |0
  | 0   |
  -------
    10
```

Iteration 2

```
10| 101 |01
  | 0   |
  -------
    10
   -10
  -------
     0
```

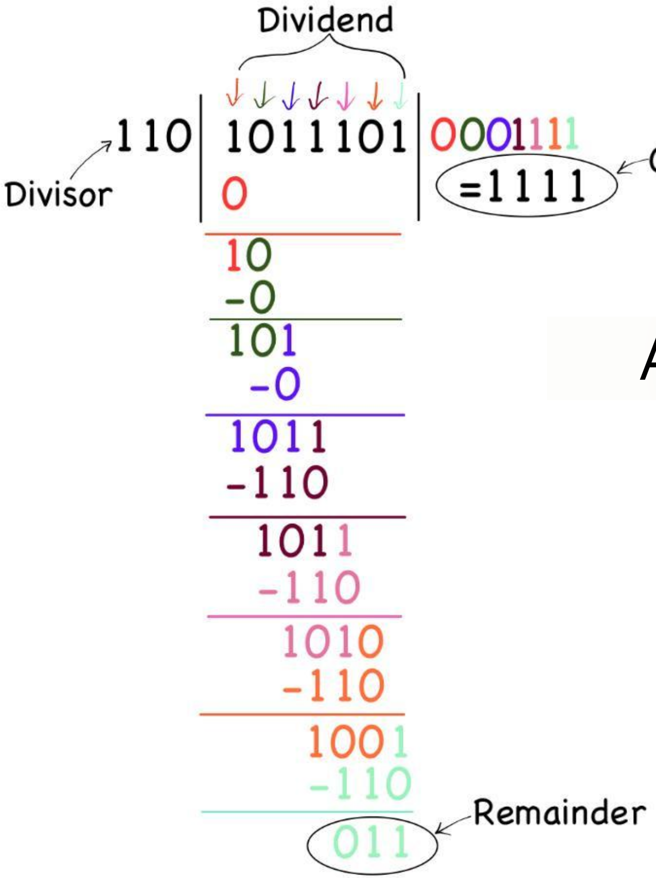
Iteration 3

```
10| 101 |010
  | 0   |
  -------
    10
   -10
  -------
     01
   -  0
  -------
      1
```

Exercise

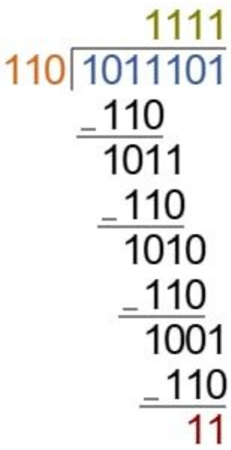
(9) Perform 1011101_2 / 110_2; Find the quotient and remainder

Solution



110 x 0 = 0
110 x 1 = 110

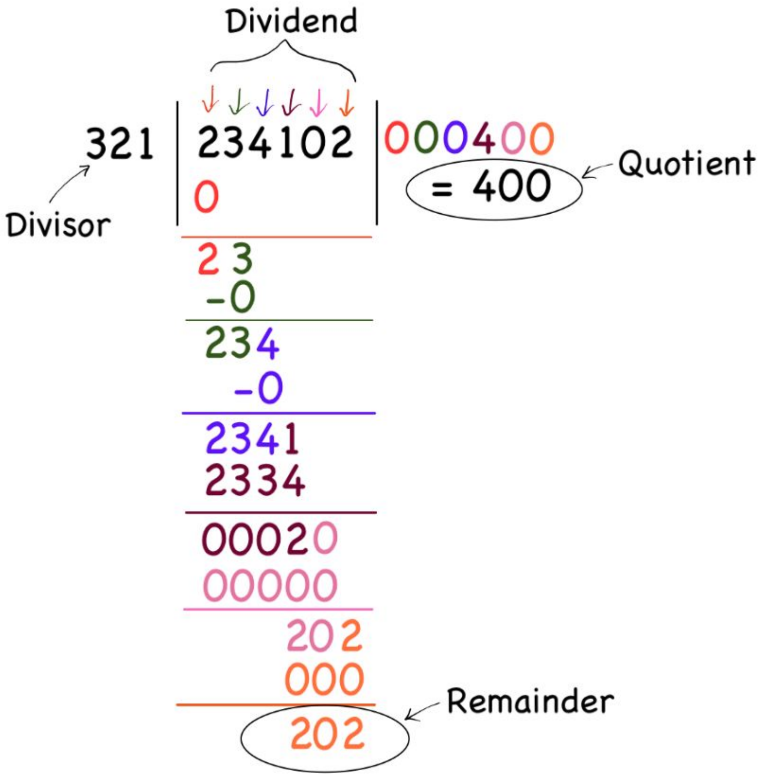
Alt. Solution



Exercise

(10) Perform 234102_5 / 321_5; Find the quotient and remainder.

Solution



$321 \times 0 = 0$

$321 \times 1 = 321$

$321 \times 2 = 1142$

$321 \times 3 = 2013$

$321 \times 4 = 2334$

Exercise

(11) (a) $(71)_8 / (56)_8$ (b) $(21)_3 / (12)_3$

Find the quotient and remainder.

Solution

Do it yourself & Compare the answer with your peers.