Economic MPC for Room Temperature Control

Muhammad Khan
Matr.: 243214

Syed Umair Ali
Matr.: 257944

Omer Mostafa
Matr.: 242816

Moustafa Elbery
Matr.: 259703

***Abstract*— Ensuring energy efficiency and sustainable building management is an attractive point of interest nowadays. Utilizing Model Predictive Control (MPC) to achieve this is highly efficient. In this project, the use of MPC to softly track room temperatures while capitalizing on optimal energy consumption was explored. Implementing an economic MPC controller to minimize the consumed energy while sustaining a bounded room temperature was also investigated. In addition, adapting weather forecasts into the prediction horizon, and exploiting latent thermal potential within building structures was achieved. The system model was discretized using the implicit Euler scheme. An advanced Kalman Filter state observer was integrated with MPC in order to simulate a real system with minimum number of measurements. The results obtained were optimal and the controller maintained room temperatures while being Eco-efficient.**

I. Introduction [S.U. Ali]

Model Predictive Control (MPC)[1] has become a key innovation in advancing energy efficiency and sustainable building management[2]. Central to its success is incorporating weather forecasts into its predictive framework, allowing for preemptive adjustments to heating and cooling strategies tailored to expected conditions. By leveraging forecasted weather data, MPC maximizes energy efficiency by harnessing the thermal properties of building materials.

For instance, MPC can retain heat accumulated in walls during warmer periods to reduce heating requirements and utilize cooler nighttime temperatures to lessen the need for air conditioning. This approach optimizes energy usage and enhances building sustainability. Additionally, MPC can strategically pre-heat or pre-cool spaces, ensuring occupant comfort while minimizing energy waste. As buildings become smarter, MPC's role in achieving sustainable and energy-efficient environments will continue to grow.

II. Methodology

A. System Description [M. Khan O. Mostafa]

The system model for the building, illustrated in Figures [1] and [2], includes the interaction between room air temperature, wall temperatures, heat sources, and environmental parameters. By accurately modeling these dynamics, the MPC can effectively control the room temperature while optimizing energy use.

The building system under consideration consists of multiple rooms, R_i, each defined by four walls. Walls can either separate two rooms (indicated by dashed lines) or separate the building from the external environment (indicated by solid lines). Figure [1] illustrates the schematic layout of the building and the rooms.

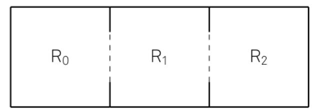

Fig. 1. Schematic illustration of a building with rooms Ri

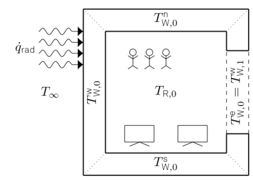

Fig. 2. Schematic illustration of R_0

1) Rooms and Walls Details: Figure [2] provides a detailed overview of the compartments within a room, exemplified by room R_0. The components of a room are as follows:

- Room Temperatures $T_{R,i}$: The temperature of the air within the room.
- Wall Temperatures $T^d_{W,i}$: where superscript d represents the direction (north, east, south, west).
- Other Heat Sources: Humans and PCs each producing a specific heat.

R_0 is connected to adjacent room R_1 through an open inner wall, allowing for convective heat transfer. Outer walls are exposed to environmental temperature T_∞ and can also absorb solar radiation $\dot{q}_{rad}$.

2) System Model Equations: There are three types of ODEs that describe our system:

$$Room\ Temperatures: \frac{\partial T_{R,i}}{\partial t} = f_1(u(t), x(t)) \qquad (1)$$

$$Inner\ Walls\ Temperatures: \frac{\partial T_{W,i}}{\partial t} = f_2(x(t)) \qquad (2)$$

$$Outer\ Walls\ Temperatures: \frac{\partial T_{W,i}}{\partial t} = f_3(x(t), d(t)) \qquad (3)$$

$u(t)$ is the control input vector. These represent the current inputs to the system. It consists of:

- $\dot{Q}_{heat,i}(t)$: Heating duty in room R_i at time t.
- $\dot{Q}_{cool,i}(t)$: Cooling duty in room R_i at time t.

$x(t)$ is the state variables vector. These represent the current state of the system. It consists of:

- $T_{R,i}(t)$: Air temperature in room R_i at time t.
- $T_{W,i}(t)$: Temperature of the wall W_i at time t.

$d(t)$ is the environmental parameters vector. These are external factors that influence the system but are not under our control. It consists of:

- $T_\infty(t)$: Environmental temperature at time at time t.
- $\dot{q}_{rad}(t)$: Solar radiation heat input at time t.

After analysing the system ODEs, it was apparent that the *system is* ***linear***. It can be written in state space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t) + \mathbf{F} \tag{4}$$

A is the system dynamics matrix, **B** is the input matrix, **E** is the parameters matrix, and **F** is a constants matrix.

After formulating the system equations into the state space form, the eigenvalues of matrix **A** were observed to be negative, indicating that the *system is* ***stable***.

B. Discretization using Implicit Euler Scheme [O. Mostafa, M. Elbery]

Disctretizing the model is a crucial step in order to be able to employ MPC. This ensures that we have a tractable optimization problem with a finite amount of optimization variables. Implicit Euler scheme[3] was implemented, which is a direct full discretization approach.

The implicit Euler scheme is a numerical method used to solve ordinary differential equations (ODEs). Unlike the explicit Euler method[3], which uses the current known values to estimate the next step, the implicit Euler method involves solving for the next value implicitly. This means it requires solving an equation that involves the unknown future value. It results in a more stable behaviour.

The model equation using this method is:

$$x_{k+1} = x_k + h * f(x_{k+1}, u_k, d_k) \tag{5}$$

with h being the stepsize of the discretization scheme. Choosing an appropriate h value is sometimes challenging and requires investigation while having a good understanding of the system. After dicretizing the system, the eigenvalues of the discrete A matrix are within the unit circle, which indicates that the system is stable even after discretizing.

C. Kalman Filter State Estimation [M. Elbery O. Mostafa]

In order to simulate a real world scenario, a Kalman Filter [4] state observer was implemented based on the minimum amount of measurements possible. The Kalman filter is an algorithm that provides estimates of the unknown variables of a dynamic system from a series of noisy measurements. It works in a two-step process: prediction and update. The Kalman filter is widely used in applications like navigation, control systems, and signal processing.

For deciding on the minimum number of measured states, a grid search was done trying different combinations of observers distributed over a number of rooms and walls. The result was that at least five observers are needed to achieve full observability of the system. The locations of observers/sensors are described by Fig. 3.

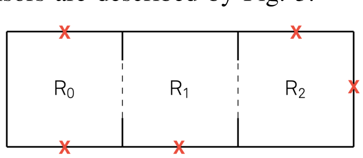

Fig. 3. Locations of sensors in the building structure.

Random process and measurement noises are added to the system to mimic real world conditions. The variance for the process noise added is 10^{-5}, whereas, the variance of the measurement noise added is 10^{-2}. **Q** and **R** matrices of the Kalman filter were tuned accordingly.

D. MPC Design and Implementation [M. Elbery S.U. Ali]

MPC is an advanced control strategy that uses a dynamic model of the system to predict future behavior and optimize control actions. At each time step, MPC solves an optimization problem to find the control inputs that will minimize a cost function over a future time horizon, subject to constraints on the inputs and states. It then applies the first control input and repeats the process at the next step, updating predictions based on new measurements. This approach allows MPC to handle multi-variable control problems and constraints effectively.

The optimization problem for applying an MPC controller to the system under study was constructed using CasAdi library [5], consisting of the following ingredients:

1) Constraints: There are four types of constraints to be considered:

- **State Constraints**: The state constraints are implemented as inequality constraints. The room temperatures are bounded between 19°C and 23°C with soft constraints. Soft constraints were implemented using slack variables. The wall temperatures should be bounded between 5°C and 40°C with hard constraints.
- **Input Constraints**: The maximum heating and cooling duties for the temperature control devices are 3kW, implemented as inequalities.
- Boundary Conditions: The initial temperatures for the rooms and the walls are 20°C and 18°C respectively, and they are enforced through an equality constraint.
- **System Model**: The implicit Euler formulation of the system model from (5), along with the model ODEs from (1), (2), and (3), are formulated as equality constraints for optimization problem for the whole horizon to enforce the dynamics of the system.

2) Objective Function: There are two types of objective functions investigated:

Pseudo-economic cost function:

$$\begin{aligned} l_{\text{pseudo}}(x_k, u_k, u_{k-1}) = {} & (x_k - x_{\text{ref}})^T Q (x_k - x_{\text{ref}}) \\ & + (\Delta u_k)^T R (\Delta u_k) + c^T u_k + \rho(\varepsilon_{\text{ub}}^2 - \varepsilon_{\text{lb}}^2) \end{aligned} \tag{6}$$

Economic cost function:

$$l_{eco}(x_k, u_k, u_{k-1}) = \Delta u_k^T R \Delta u_k + c^T u_k + \rho(\varepsilon_{ub}^2 - \varepsilon_{lb}^2) \tag{7}$$

where:

x_{ref} is the desired setpoint (20°C).

Q and R are positive semi-definite weighting matrices.

c is a cost factor for using the heating and cooling devices.

$\Delta u_k = u_k - u_{k-1}$ represents changes in the control input.

ρ is the slack variable.

ε_{ub} is the slack variable for upper bound of room temperatures.

ε_{lb} is the slack variable for lower bound of room temperatures.

3) Optimization Variables: The decision variables of the optimization are all the states and input trajectory throughout the horizon, as well as the slack variable for the upper and lower bounds of the room temperatures.

The optimization problem for our MPC looks as follows:

$$
\begin{aligned}
\underset{u_1,\dots,u_{N-1},x_1,\dots,x_N,\varepsilon_{ub},\varepsilon_{lb}}{\text{minimize}} \quad & \sum_{k=0}^{N-1} l(x(k),u(k),u(k-1),\varepsilon) \\
\text{subject to} \quad & x(k+1) = x(k) + hf(x(k+1),u(k),d(k)) \\
& 19-\varepsilon_{lb} \leq T_{R,i} \leq 23+\varepsilon_{ub} (^\circ\text{C}) \\
& 5 \leq T_{W,i} \leq 40 (^\circ\text{C}) \\
& 0 \leq \dot{Q}_{heat,i} \leq 3(kW) \\
& 0 \leq \dot{Q}_{cool,i} \leq 3(kW) \\
& -\infty \leq \varepsilon_{ub} \leq \infty \\
& -\infty \leq \varepsilon_{lb} \leq \infty \\
& x(0) = x_{\text{init}}
\end{aligned} \tag{8}
$$

It is worth mentioning that during each iteration of solving the optimization problem, warm-starting the optimizer was utilized by giving the solution of the previous iteration as an initial guess to the current one. This greatly reduced the time for solving the problem and finding an optimal sequence of control inputs.

In order to fully exploit the capabilities of MPC prediction throughout the horizon, the weather forecast of environmental parameters used (environmental temperature and solar radiation) was adapted throughout the prediction horizon. The forecasted environmental parameters were assumed to be without uncertainty.

Different combinations between the discretization stepsize h and prediction horizon N should be explored, while taking into consideration computational complexity, closed-loop cost and constraints violations.

III. Results [M. Elbery O. Mostafa]

In this section, the plot results of the autonomous behaviour of the system, as well as, after applying MPC control are discussed.

When investigating different discretization times (h), it was clear that having a small value for h will greatly increase the complexity of the problem, whereas, having a large value for h will simplify the problem, but, will slow down the system response which results in a controller that does not adapt well to changes.

On the other hand, the choice of a good prediction horizon N is an important aspect that requires investigating and tuning. A too short horizon might result in an unstable closed-loop controller, whereas, a too long horizon will have a more effective controller, but, will cause the computational complexity to be too high and the problem might become intractable.

After exploring different combinations, a prediction horizon (N_{hours}) of 10 hours and a discretization step (h) of 15 minutes (0.25 hours) proved to have the best balance between computational complexity, closed-loop cost and constraints violations. Note that, when using an actual horizon for the MPC, the prediction horizon will be $N = N_{hours}/h = 40$ steps.

A. Autonomous System Behavior [M. Khan O. Mostafa]

For extended warm periods, the room temperatures tend to rise, above the external temperature. On the other hand, during long cold periods, the room temperatures decrease, below the external temperature. This behavior can be attributed to the latent thermal potential withing the building structure and the walls. However, the system's behavior stabilizes after some time. The simulation of the system for seven days both in summer and winter are shown in Figures 4 and 5.

It is clear from the autonomous behaviour of the system during both seasons, that the system is stable.

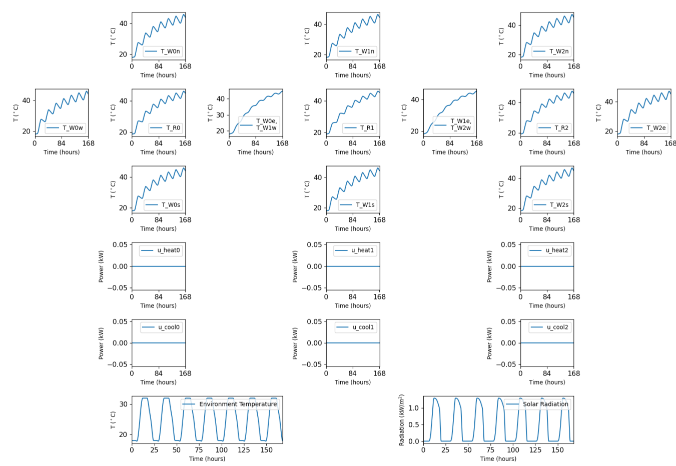

Fig. 4. Autonomous system behavior in summer

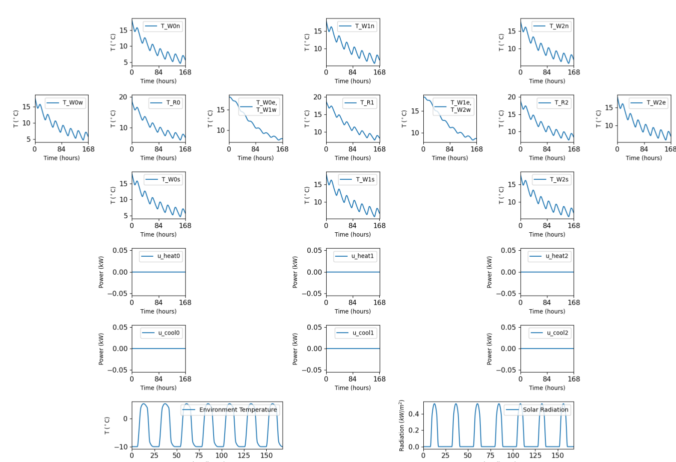

Fig. 5. Autonomous system behavior in winter

B. MPC Results [M. Elbery S.U. Ali]

The results for controlling the system for seven days during both seasons are shown in this section. The penalty used for tracking error (Q) is 20, the penalty for changing the input (R) is 10, and the cost factor for using the heating and cooling devices (c) is 10. The slack penalty (ρ) is 100, a high slack penalty for breaking the soft constraints was chosen to prevent significant violations.

1) Setpoint Tracking: In this section, the pseudo-economic cost function (6) was used to softly track a room temperature of 20°C. Figures 6 and 7 shows the results of tracking-based MPC.

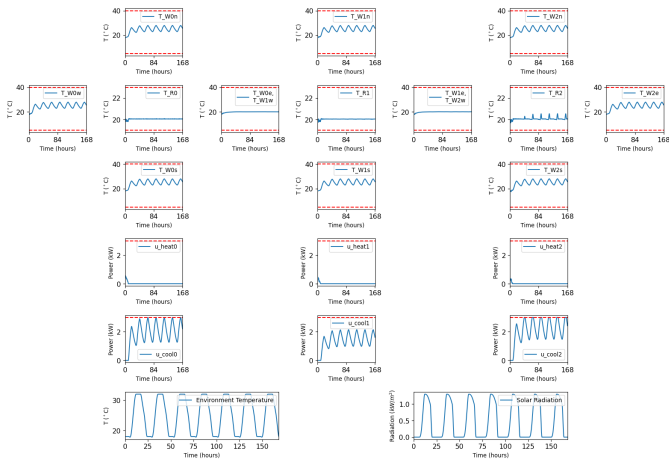

Fig. 6. Tracking MPC in summer.

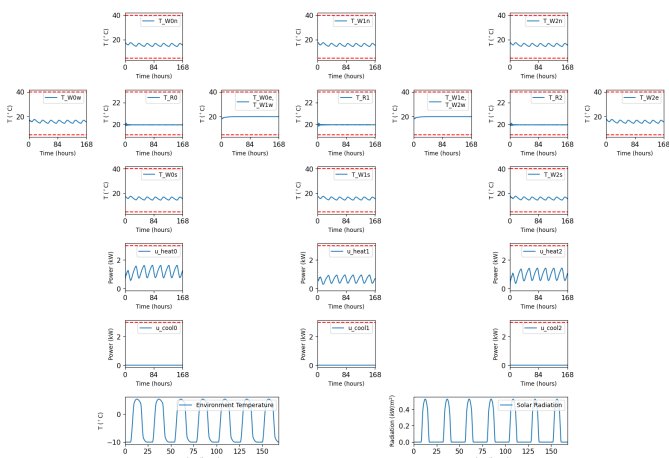

Fig. 7. Tracking MPC in winter.

2) Economic MPC Adaptation: In this section, the economic cost function (7) was used to maintain the room temperature within the bounds. Figures 8 and 9 shows the results of economic MPC.

C. Performance Comparison of different MPCs [M. Elbery]

Tracking-based MPC when using the pseudo-economic cost function is trying to always keep the room temperatures stable at the reference point, which it succeeds in doing. The constraint are not violated when doing tracking MPC. It should be mentioned that it might be impossible to perfectly track all of the rooms to exactly 20°C during the summer, because of limitations regarding the cooling devices.

On the other hand, economic-based MPC is trying to keep the system within bounds. During economic MPC, the soft constraints might be minimally violated, this is because the system is moving very close to the bounds and there

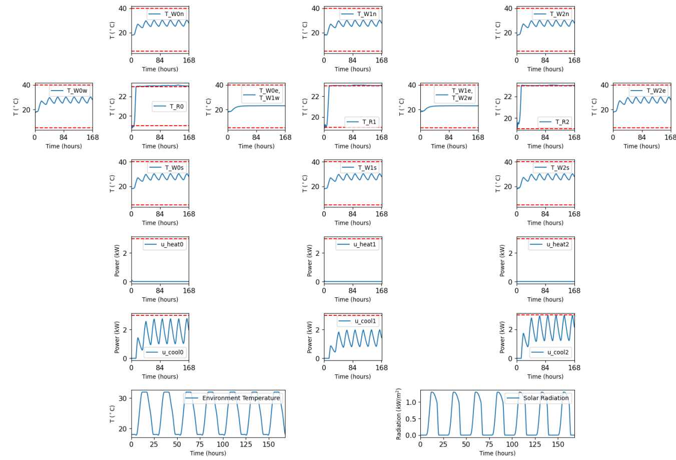

Fig. 8. Economic MPC in summer.

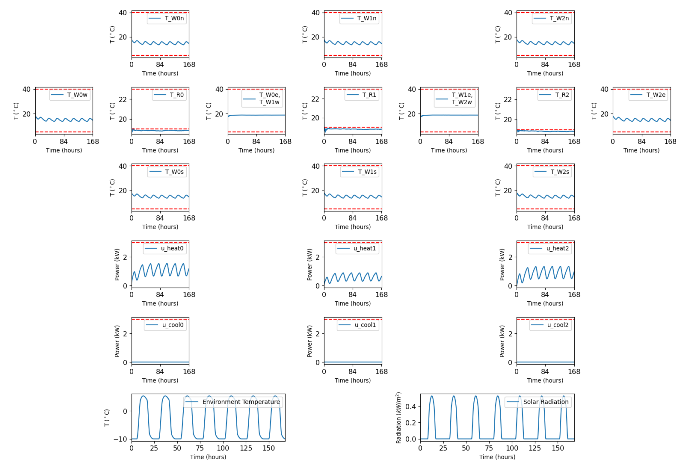

Fig. 9. Economic MPC in winter.

is process and measurement noise during state estimation causing perturbations that might break the constraints.

The cost of using the input devices and changing them is lower for the economic MPC than in the case of tracking MPC. However, it should be noted that the total cost of economic MPC might be higher than the tracking case, because of the minimal constraint violations caused by state, which adds to the total cost through the slack cost.

IV. Conclusion [M. Khan]

In conclusion, MPC proves to be the optimal controller for room temperature control in order to achieve sustainable building management with energy efficiency. Discretizing using Implicit Euler and integrating MPC with a Kalman Filter Estimator is highly effective to simulate a real world scenario. Applying an economic cost function for MPC optimization is more cost effective with regards to input devices. Employing soft constraints especially for the economic case is important so that the problem is not infeasible when constraints are slightly violated. Adapting weather forecasts throughout the prediction horizon greatly improves our MPC performance.

References

[1] J. Rawlings and D. Mayne, *Model Predictive Control: Theory and Design*. Nob Hill Pub., 2009. [Online]. Available: https://books.google.de/books?id=3$_r fQQAACAAJ$

[2] D. Lee and C.-C. Cheng, “Energy savings by energy management systems: A review,” *Renewable and Sustainable Energy Reviews*, vol. 56, pp. 760–777, 2016.

[3] A. Ern and J.-L. Guermond, *Implicit and explicit Euler schemes*. Cham: Springer International Publishing, 2021, pp. 147–160.

[4] G. Welch, G. Bishop, *et al.*, “An introduction to the kalman filter,” 1995.

[5] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl, “CasADi – A software framework for nonlinear optimization and optimal control,” *Mathematical Programming Computation*, 2018.