An Overview of Decentralized Kalman Filter Techniques

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INTRODUCTION

An integrated navigation system typically consists of several sensors, each equipped with a microprocessor. A central computer is generally in control of the different sensors. Communication is achieved over a serial data bus, such as MIL-STD-1553. In general, the sensor configuration consists of an inertial system providing nearly continuous measurements of acceleration, velocity, position, and attitude plus other sensors providing discrete measurements of some of these parameters. Some of the other sensors contain Kalman Filters to improve some internal process. The Global Positioning System, for example, uses a Kalman Filter and inertial data to improve performance of it's code tracking loop during high dynamic maneuvers. As a result, a navigation system may contain several different Kalman Filtered solutions. Many different techniques have been suggested for combining data from multiple Kalman Filters. These include the Federated Kalman Filter [C1], several different Decentralized Kalman Filter techniques [S1], [HRL1], [G1], and the Cascaded Kalman Filter. In this paper the most promising of these candiates, the Federated Kalman Filter will be discussed.

FEDERATED FILTER THEORY

A block diagram of the Federated Kalman Filter is given in Figure 1.

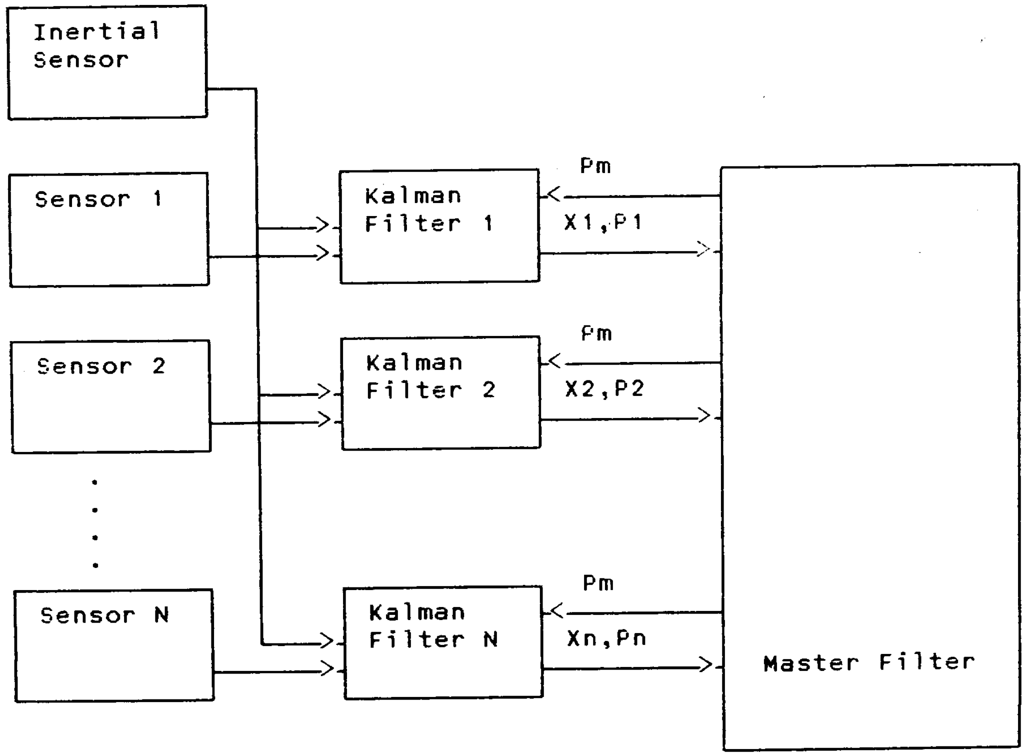

Figure 1 Federated Kalman Filter

Each local filter observes errors in the states of the inertial system. The Federated filter structure can be developed as follows. The inertial System is modeled as

$$X(k+1) = \Phi * X(k) + G * U \qquad \text{Eq. 1}$$

where Φ is the state transition matrix
U is uncertainty due to white noise
G is the noise distribution matrix
X are the states of the model

The value of U(k) is entirely independent from U(j) where j not equal k. The measurements in the system are given by N different sensors and modeled as:

$$Zi = Hi * X + vi \qquad \text{Eq 2}$$

where Hi is the observation matrix for sensor i
Zi is the measurement given by sensor i
vi is the random measurement error

The measurement error, v_i, is Normal$(0,\delta v)$ and independent of the noise in the process model. Each measurement, Z_i, is processed by a Kalman Filter residing in the ith sensor's microprocessor. Although the i measurements are independent of each other the Kalman Filter estimates from different sensors are correlated due to the common noise states of the inertial system model (eq 1). This correlation can be described as follows. The total system of equations, modeled across all the sensors, can be written as an augmented state equation.

$$\begin{bmatrix} X1 \\ X2 \\ \cdot \\ \cdot \\ \cdot \\ Xn \end{bmatrix}_{k+1} = \begin{bmatrix} \Phi 1 & & & \\ & \Phi 2 & 0 & \\ & & \ddots & \\ & 0 & & \Phi n \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ \cdot \\ \cdot \\ \cdot \\ Xn \end{bmatrix} + \begin{bmatrix} G1 \\ G2 \\ \cdot \\ \cdot \\ \cdot \\ Gn \end{bmatrix} U \qquad \text{Eq 3}$$

and each Xi

$$Xi = \begin{bmatrix} Xc1 \\ \cdot \\ Xcn \\ Xb1 \\ \cdot \\ Xbn \end{bmatrix} \begin{matrix} \text{Common States} \\ \\ \\ \\ \\ \text{Sensor i bias states} \end{matrix} \qquad \text{Eq 4}$$

The common states are due to the fact that each local Kalman filter models the inertial system. The covariance matrix associatted with the augmented system is of the form given in equation 5.

$$P = \begin{bmatrix} E[x1\ x1'] & \cdots\cdots & E[x1\ x2'] \\ \vdots & \ddots & \vdots \\ E[xn\ x1'] & \cdots\cdots & E[xn\ xn'] \end{bmatrix} \qquad \text{Eq 5}$$

Given the N local state estimates, Xi, and the augmented covariance matrix the optimal estimate of the common state vector is provided by minimizing the following cost function.

$$C = \left\| \begin{matrix} (X1 - Xm) \\ \vdots \\ (Xn - Xm) \end{matrix} \right\| P^{-1} \left\| \begin{matrix} (X1 - Xm) \\ \vdots \\ (Xn - Xm) \end{matrix} \right\|^{T} \qquad \text{Eq 6A}$$

If the cross correlation terms of the augmented covariance matrix were zero the cost function would reduce to that of equation 6B.

$$C = \Sigma\ (Xi - Xm)*Pii^{-1}*(Xi - Xm)' \qquad \text{Eq 6B}$$

Where Xm is the Combined state estimate
Pii is the covariance matrix associated with the common filter states of Filter i
Xi are the state estimates from filter i

The following property allows us to treat the cross terms as if they were zero.

Property 1

$$\begin{bmatrix} E[x1\ x1'] & \cdots & E[x1\ x2'] \\ \vdots & \ddots & \vdots \\ E[xn\ x1'] & \cdots & E[xn\ xn'] \end{bmatrix} <= \begin{bmatrix} \beta*E[x1\ x1'] & 0 & \cdots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \beta*E[xn\ xn'] \end{bmatrix}$$

This tells us that if the initial value of the covariance matrix and the process noise matrix of each local Kalman filter is multiplied by some scalar factor, β, the N Kalman Filtered outputs may be treated as independent. Minimizing the cost function of equation 6B yield the equations given in 7,8, and 9.

$$Pmm = \text{inverse}\ \{ (\beta*P11)^{-1} + \cdots\cdots + (\beta*Pnn)^{-1} \} \qquad \text{Eq 7}$$

$$= \beta*\text{inverse}\ \{ P11^{-1} + \cdots\cdots + Pnn^{-1} \} \qquad \text{Eq 8}$$

$$X_m = P_{mm}\,[(\beta * P_{11})^{-1} * X_1 + \ldots\ldots\ldots\ldots + (\beta * P_{nn})^{-1} * X_n] \quad \text{Eq 9}$$

The β in Eq 8 will cancel with the β in Eq 9. Therefore the β term does not affect the master estimate. The β term can always be factored out provided the common states of the local filters and their associated covariance terms are reset to $\beta * P_m$ and X_m at a rate equal to the highest local filter update rate in the system. In fact, the covariance matrix P_m is exactly equal to the covariance of a central Kalman Filter processing each of the independent measurements. Within each Local Kalman Filter the Local Covariance is updated as

$$P_i^{-1}(k|k) = [\,P_i^{-1}(k|k-1) + H_i^T R_i^{-1} H_i] \quad \text{Eq 10}$$

but

$$P_i(k|k-1) = \beta * P_m(k|k-1) \quad \text{Eq 11}$$

Equation 10 is the information form of the Kalman Filter measurement update equation associatted with the covariance matrix. Subsituting equation 10 in equation 7 yields

$$P_{mm}^{-1} = \{\,(P_{11})^{-1} + \ldots\ldots\ldots + (P_{nn})^{-1}\,\} \quad \text{Eq 12}$$

$$= \{\,P_i^{-1}(k|k-1) + \ldots + P_n^{-1}(k|k-1) + H_1^T R_1^{-1} H_1 + \ldots + H_n^T R_n^{-1} H_n\,\} \quad \text{Eq 13}$$

$$= \{\,P_m^{-1} * [1/\beta + \ldots + 1/\beta] + H_1^T R_1^{-1} H_1 + \ldots + H_n^T R_n^{-1} H_n\,\} \quad \text{Eq 14}$$

recall

$$\Sigma\,(1/\beta_1 + \ldots\ldots + 1/\beta_n) = 1$$

therefore

$$P_m(k|k) = \{\,P_m(k|k-1)^{-1} + H_1^T R_1^{-1} H_1 + \ldots + H_n^T R_n^{-1} H_n\,\} \quad \text{Eq 15}$$

but this is the same result that a central Kalman Filter processing the N independent measurements would provide. The Master Filter is thus the optimal solution to the problem.

SIMULATION RESULTS

An example navigation system is provided in Figure 1. This simulation assumes an inertial sensor which measures acceleration and by integration determines velocity and position. There are two other sensors in this example system. Sensor 1 provides a noisy measurement of velocity, once a second. Sensor 2 provides a noisy measure of position, once every 2 seconds. Both sensors contain a Kalman Filter that models the inertial system. Ideally, all sensor measurements would be processed in a single Kalman Filter to yield an optimal solution. In most real system, as with this simulation, each sensor contains an independent Kalman Filter. In Figure 2, the error in position and velocity estimated by Sensor 1 Kalman Filter is plotted versus time. The velocity error is reduced considerably from the initial value of 4 ft/sec. The position error is reduced, but sensor 1 measures velocity and can't provide a really good estimate of position. In Figure 3, the error in position and velocity estimated by Sensor 2 is plotted. The postion error is reduced from an initial value of 50 feet to about 7 feet. The velocity error is reduced, but not as well as the sensor 1 estimate. Choosing Sensor 1 provides the best source of velocity, but not position. Choosing Sensor 2 provides the best source of position, but not velocity. The system could be set up to use velocity from sensor 1 and position from sensor 2. A better technique would be to combine all the data from both filters to obtain the same results as single Kalman Filter that processed raw data from all the sensors. The Federated Filter provides a technique to combine estimates and covariance data from both Kalman Filters to yield the same result as a single Kalman Filter. The results are given in Figure 4. The filtered solution of velocity is slightly improved over the sensor 1 estimate and much improved over the sensor 2 estimate. The position error of the Federated solution is also better than either the sensor 1 or 2 estimate by themselves.

CONCLUSIONS

The Federated FIlter can provide performance equal to that of a single Kalman Filter that integrates all the independent sensor data in the system. The advantage is that a single filter is impracticle with existing sensors. The Federated Filter is practical, but for true optimal performance requires that all Kalman Filters contain the same process model and make their covariance matrixes available on the serial data bus. The Federated Filter can be reconfigured to provide a less optimal solution with a higher degree of fault tolerance. In the Fault tolerant mode the Kalman FIlters of Sensors 1 and 2 are isolated from each other. This maintains independent estimates of position and velocity. In the event of a Sensor Failure, a total reset is not required. The choice between full optimallity and Fault Tolerance must be decided based on the operational requirements of the navigation system.

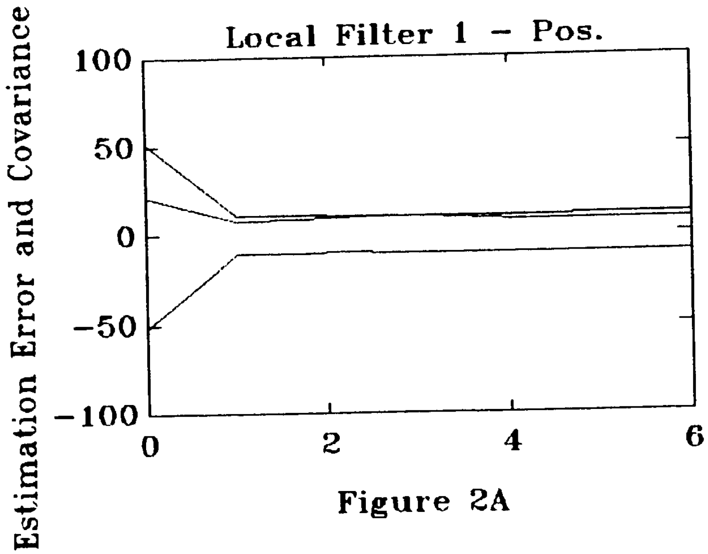

Figure 2A

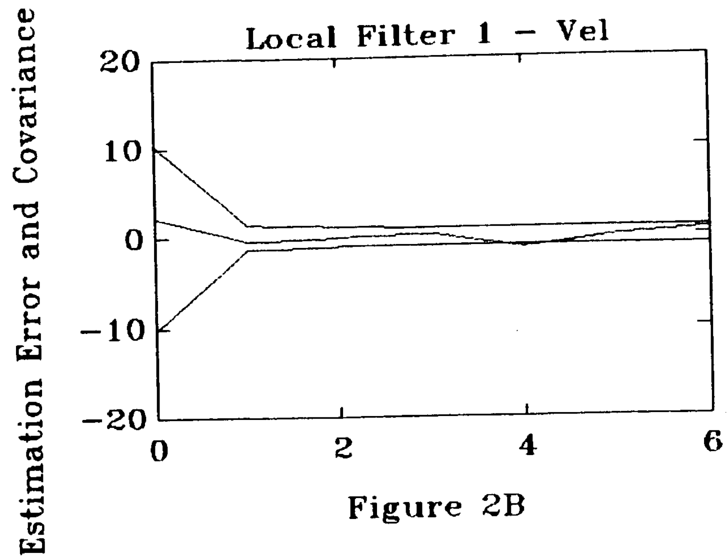

Figure 2B

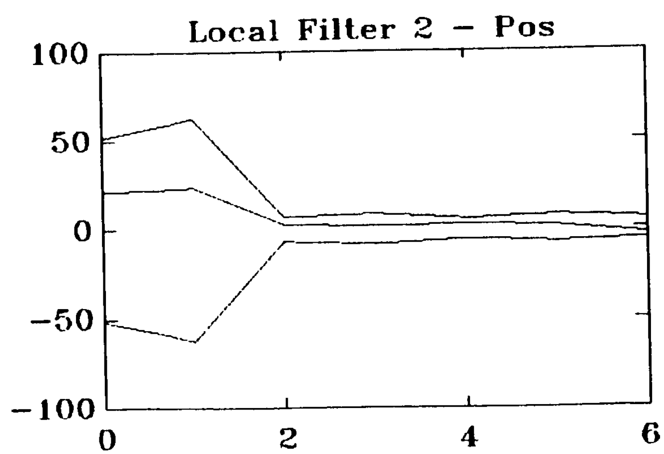

Figure 3A

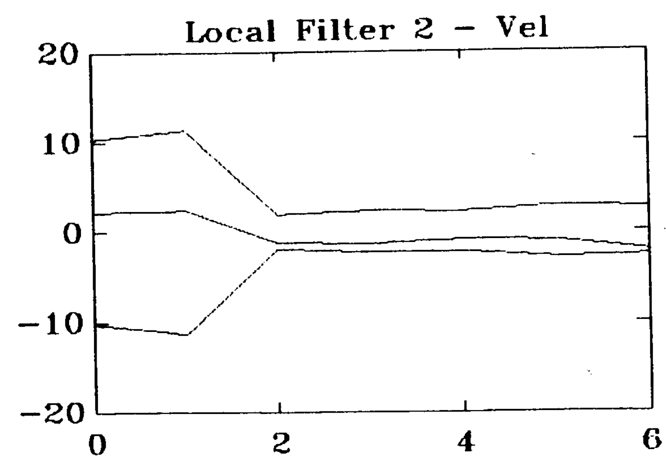

Figure 3B

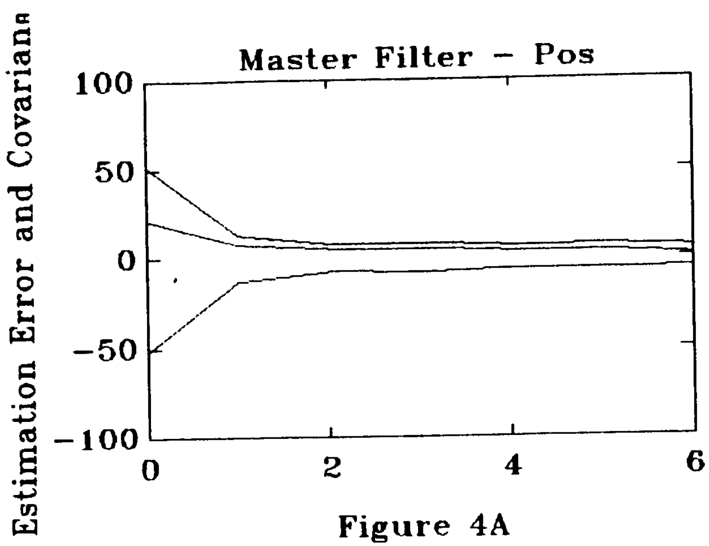

Figure 4A

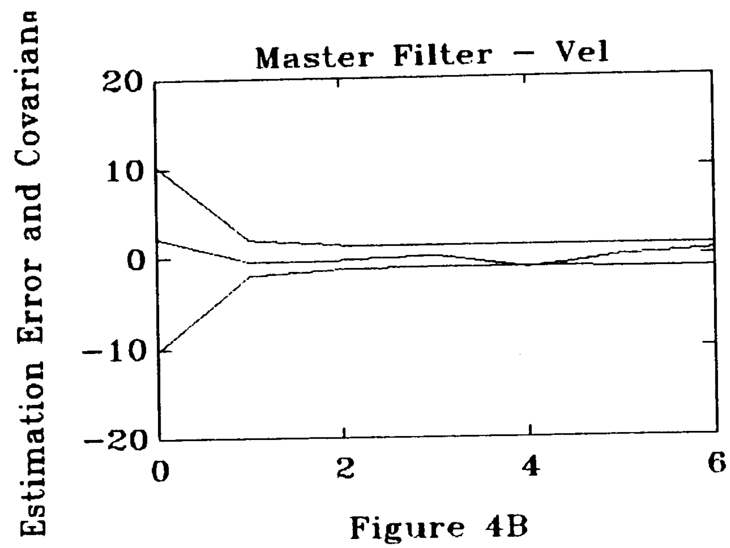

Figure 4B

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[HRL1] Hashemi, H.R., S. Roy, A. Laub, 'Parallel Structures for Kalman Filtering', Proceddings of the 26th Conference on Decision and Control, Dec 1987.

[S1] Speyer, J.L. , 'Computation and Transmission Requirements for a Decentralized Linear Quadratic Gaussian Control' IEEE Transactions on Automatic Control, AC-24:2, April 1979