17 Optimal Distributed Kalman Filtering Fusion with Singular Covariances of Filtering Errors and Measurement Noises

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CONTENTS

17.1 INTRODUCTION

Multiple sensors estimation fusion has been used pervasively from civil to military fields, for example, target tracking and localization, fault diagnosis, surveillance and monitoring, air traffic control, and so forth and showed some merits for estimation fusion, for example, increasing reliability and survivability, improving estimation accuracy, reducing communication burden, and so on.

Generally speaking, there are two basic fusion architectures [1]: *centralized* and *decentralized/distributed* (with respect to *measurement fusion* and *track fusion*, respectively) depending on whether the raw measurements are sent to the fusion center or not. More precisely, centralized fusion means that the fusion center receives all raw measurements whereas distributed fusion implies that each sensor sends preprocessed measurements to the fusion center. It is well known that Kalman filtering is one of the most popular recursive least mean square error (LMSE) algorithms to optimally estimate the unknown state or process of a dynamic system. Centralized *Kalman filtering fusion* means that the fusion center can use all raw measurements from the local sensors in time to obtain globally optimal state estimates in the sense of LMSE. However, sending raw measurements needs more communication bandwidth, computation, and power consumption. Consequently, centralized fusion has a poor survivability of the system (in particular, in a war situation). For the case of distributed fusion, every local sensor first implements a Kalman filter based on its own observations for local requirements, and then sends the processed data–local state estimate to a fusion center. The

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fusion center fuses all received local estimates to yield an optimal state estimate in terms of LMSE. This distributed processing seems more preferred for many practical issues.

When the estimation error and sensor noise covariance matrices are invertible, an optimal Kalman filtering fusion formula (Equation 17.23) in Section 17.2, has been proposed in Refs. [2–6]. More importantly, it was proved to be globally optimal in the sense that the fused Kalman filtering is equivalent to the centralized Kalman filtering using all sensor measurements. Moreover, a Kalman filtering fusion with feedback was also proposed there. Moreover, a rigorous performance analysis for Kalman filtering fusion with feedback was provided in Refs. [6,7].

In this chapter, we consider the distributed Kalman filtering fusion for the case where covariances of estimation errors and measurement noises are singular, which is motivated by the following reasons. First, the existing fusion algorithms proposed in Refs. [2–6] strongly depend on invertible estimation error covariance matrices (see Equations 17.22 and 17.23). However, this condition is not always guaranteed to hold. Hence, the proposed fusion formula here could be applied to more general and extensive cases than the results there. Specific practical applications in Refs. [8–14] are Kalman filtering for linear dynamic systems with state equality constraints. In particular, in Refs. [9,10,12], the authors provided good reasons why one should not use a reduced state space for treating the constrained system in some practical problems. Specifically, Ref. [9] considers the biped locomotion problem and reduced it to a state estimation with equality constraint. Also, Refs. [10] and [12] consider a linear system and measurements describing movement of a land-based vehicle. Furthermore, the vehicle moving on a road is described by a linear equation. Similarly, there are many practical dynamic systems with constrained movement trajectories in space and air vehicle systems. The preceding examples imply that the state belongs to a subspace of whole space with respect its original dimension space, which leads to the deduction that the state is a degenerated random vector and its covariance must be singular. Second, our proposed fused state estimate is still equivalent to the centralized Kalman filtering using all sensor measurements, which means that the centralized Kalman filtering can also be obtained by fusing local estimates even if covariances of estimation errors and measurement noises are singular. In addition, our result is not same as the result in Ref. [15], where the authors investigated the problem of estimation fusion based on local transformation of raw measurements (not on the local estimate) under the condition that the filtering error covariances are singular. Furthermore, because our globally optimal distributed fusion is of the form of convex linear combination of one-step prediction of centralized Kalman filtering and all the local Kalman filtering and its corresponding one-step prediction, this obviously provides a theoretical support to global optimality of the convex combination fusion algorithm in Ref. [16]. In Ref. [16], the global optimality of algorithm was only supported by numerical examples without rigorous theoretical analysis. As stated in the last paragraph of Section III on page 67 Ref. [16], "this opens a hope that it is possible to derive a globally optimal distributed Kalman filtering fusion. Of course, a rigorous analysis for the equivalence is worth studying in the future." A more detailed explanation for the theoretical significance of our result here is provided in Remark 17.7 in Section 17.5.

Although the the convex combination fusion algorithm in Ref. [16] can be theoretically proved via our result here to have the same performance as *centralized Kalman filtering*, numerical examples here show that our proposed algorithm could save computation significantly compared with the fusion algorithm in Ref. [16]. For more details, see Table 17.1 in Section 17.6.

To derive globally optimal distributed Kalman filtering fusion equivalent to the centralized Kalman filtering, as in Refs. [2–6], a key skill is to find sufficient statistics of all sensor observations that can be equivalently expressed in terms of two-step sensor estimates. If this is done well, the centralized Kalman filtering can be easily rewritten as the corresponding distributed Kalman filtering fusion. To the best of our knowledge, so far, without the assumption of matrix invertibility there has not been any work similar to that done 20 years ago in Refs. [2–5]. In fact, what we technically are doing in this chapter is finding such sufficient statistics even without invertible covariances of filtering errors and measurement noises.

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In addition, when there is a *feedback*, we obtain performance analysis results similar to those given in Refs. [6,7] for the case in which the *estimation error covariance matrices* are singular, that is, the corresponding fusion formula with feedback is, like the fusion without feedback, exactly identical to the corresponding centralized Kalman filtering fusion formula using all sensor measurements. Moreover, the various P matrices in the feedback Kalman filtering at both local filters and the fusion center are still the covariance matrices of tracking errors. Furthermore, the feedback can reduce the covariance of each local tracking error.

The rest of the chapter is organized as follows. Section 17.2 describes the system model of Kalman filtering and presents the existing fusion formula. Section 17.3 presents a new distributed Kalman filtering fusion formula with singular estimation error covariance matrices and we prove the optimality of the proposed fusion formula. In Section 17.4, we also prove the optimality of the proposed fusion formula for Kalman filtering fusion with feedback when covariances of filtering errors are singular. We consider optimal Kalman filtering fusion with both singular covariances of filtering errors and measurement noises in Section 17.5. In Section 17.6, we demonstrate through several examples that our proposed fusion formula not only is globally optimal, but also consumes less computation than the algorithm in Ref. [16]. Finally, Section 17.7 contains some concluding remarks.

Throughout this chapter, we adopt the following notations. I denotes the identity matrix of appropriate dimension, $(\cdot)'$ denotes the transpose of the corresponding matrix, and $(\cdot)^*$ denotes the Hermitian conjugate of the corresponding matrix.

17.2 PROBLEM FORMULATION

Assume the l-sensor distributed linear dynamic system is given by

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + v_k, \quad k = 0,1,\ldots, \tag{17.1}$$

$$\mathbf{y}_k^i = H_k^i \mathbf{x}_k + w_k^i, \quad i = 1,\ldots,l, \tag{17.2}$$

where Φ_k is a matrix of order $(r \times r)$, $\mathbf{x}_k, v_k \in \mathbb{R}^r$, $H_k^i \in \mathbb{R}^{N_i \times r}$, $\mathbf{y}_k^i, w_k^i \in \mathbb{R}^{N_i}$. The process noise v_k and measurement noise w_k^i are both zero-mean random variables independent of each other temporally and are not cross correlated.

The stacked measurement equation can be expressed as

$$\mathbf{y}_k = H_k \mathbf{x}_k + w_k, \tag{17.3}$$

where

$$\begin{aligned} \mathbf{y}_k &= \left(\mathbf{y}_k^{1'},\ldots,\mathbf{y}_k^{l'}\right)', \\ H_k &= \left(H_k^{1'},\ldots,H_k^{l'}\right)', \\ w_k &= \left(w_k^{1'},\ldots,w_k^{l'}\right)'. \end{aligned} \tag{17.4}$$

Moreover, the covariances of sensor noises are given by

$$R_k^i = \mathrm{Cov}\left(w_k^i\right), \quad i = 1,\ldots,l, \tag{17.5}$$

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and

$$R_k = \mathrm{Cov}(w_k) = \mathrm{diag}\left(R_k^1, \ldots, R_k^l\right), \tag{17.6}$$

where R_k and R_k^i are both invertible for all i, that is, R_k could be any positive definite matrix with on-diagonal blocks R_k^i of full rank.

Due to results in Kalman filtering Refs. [17–19], it is well known that the centralized Kalman filtering by using all sensor observations is expressed in the following form:

$$\begin{aligned} \mathbf{x}_{k/k} &= \mathbf{x}_{k/k-1} + K_k(\mathbf{y}_k - H_k\mathbf{x}_{k/k-1}) \\ &= (I - K_kH_k)\mathbf{x}_{k/k-1} + K_k\mathbf{y}_k, \end{aligned} \tag{17.7}$$

$$\begin{aligned} K_k &= P_{k/k}H_k'R_k^{-1} \\ &= P_{k/k-1}H_k'\left(H_kP_{k/k-1}H_k' + R_k\right)^{-1} \end{aligned} \tag{17.8}$$

with covariance of filtering error given by

$$\begin{aligned} P_{k/k} &= (I - K_kH_k)P_{k/k-1} \\ &= P_{k/k-1} - P_{k/k-1}H_k'\left(H_kP_{k/k-1}H_k' + R_k\right)^{-1} H_kP_{k/k-1} \end{aligned} \tag{17.9}$$

or

$$P_{k/k}^{\dagger} = P_{k/k-1}^{\dagger} + P_{k/k-1}^{\dagger}P_{k/k-1}H_k'R_k^{-1}H_kP_{k/k-1}P_{k/k-1}^{\dagger}. \tag{17.10}$$

Equation 17.10 comes from Lemma 17.4 in Appendix I, where

$$\mathbf{x}_{k/k-1} = \Phi_k\mathbf{x}_{k-1/k-1} \tag{17.11}$$

$$P_{k/k} = E[(\mathbf{x}_{k/k} - \mathbf{x}_k)(\mathbf{x}_{k/k} - \mathbf{x}_k)' \mid \mathbf{y}_0, \ldots, \mathbf{y}_k] \tag{17.12}$$

$$P_{k/k-1} = E[(\mathbf{x}_{k/k-1} - \mathbf{x}_k)(\mathbf{x}_{k/k-1} - \mathbf{x}_k)' \mid \mathbf{y}_0, \ldots, \mathbf{y}_{k-1}] \tag{17.13}$$

Remark 17.1

Equation 17.10 shows that even if $P_{k/k}$ may be singular, similarly, the pseudo-inverse of $P_{k/k}$ still can be expressed in terms of the pseudo-inverse of $P_{k/k-1}$. The result could be regarded as an extension of

$$P_{k/k}^{-1} = P_{k/k-1}^{-1} + H_k'R_k^{-1}H_k, \tag{17.14}$$

when $P_{k/k}$ and $P_{k/k-1}$ are both invertible. ■

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Remark 17.2

As stated in Section 17.1, Kalman filtering for linear dynamic system with state equality constraints in Refs. [11,12] certainly leads to the deduction that Cov(v_k) and Cov($\mathbf{x}_k$) must be singular, which means that $P_{k/k}$ and $P_{k/k-1}$ may be singular. To the best of our knowledge, Equation 17.10 is new and will play an important role in our results. For details, see Equation 17.24. ■

Likewise, the local Kalman filtering at the ith sensor is

$$\begin{aligned} \mathbf{x}^i_{k/k} &= \mathbf{x}^i_{k/k-1} + K^i_k\left(\mathbf{y}^i_k - H^i_k\mathbf{x}^i_{k/k-1}\right) \\ &= \left(I - K^i_kH^i_k\right)\mathbf{x}^i_{k/k-1} + K^i_k\mathbf{y}^i_k, \end{aligned} \tag{17.15}$$

$$\begin{aligned} K^i_k &= P^i_{k/k}H^{i'}_kR^{i^{-1}}_k \\ &= P^i_{k/k-1}H^{i'}_k\left(H^i_kP^i_{k/k-1}H^{i'}_k + R^i_k\right)^{-1} \end{aligned} \tag{17.16}$$

with covariance of filtering error given by

$$\begin{aligned} P^i_{k/k} &= \left(I - K^i_kH^i_k\right)P^i_{k/k-1} \\ &= P^i_{k/k-1} - P^i_{k/k-1}H^{i'}_k\left(H^i_kP^i_{k/k-1}H^{i'}_k + R^i_k\right)^{-1}H^i_kP^i_{k/k-1} \end{aligned} \tag{17.17}$$

or

$$P^{i^\dagger}_{k/k} = P^{i^\dagger}_{k/k-1} + P^{i^\dagger}_{k/k-1}P^i_{k/k-1}H^{i'}_kR^{i^{-1}}_kH^i_kP^i_{k/k-1}P^{i^\dagger}_{k/k-1} \tag{17.18}$$

where

$$\mathbf{x}^i_{k/k-1} = \Phi_k\mathbf{x}^i_{k-1/k-1} \tag{17.19}$$

$$P^i_{k/k} = E\left[\left(\mathbf{x}^i_{k/k} - \mathbf{x}_k\right)\left(\mathbf{x}^i_{k/k} - \mathbf{x}_k\right)' \middle| \mathbf{y}^i_0,\ldots,\mathbf{y}^i_k\right] \tag{17.20}$$

$$P^i_{k/k-1} = E\left[\left(\mathbf{x}^i_{k/k-1} - \mathbf{x}_k\right)\left(\mathbf{x}^i_{k/k-1} - \mathbf{x}_k\right)' \middle| \mathbf{y}^i_0,\ldots,\mathbf{y}^i_{k-1}\right] \tag{17.21}$$

We first recall some known results about Kalman filtering fusion. In Refs. [2–6], if covariances of filtering errors are invertible, the centralized filtering and error matrix can be expressed by the local filtering and error matrices as follows:

$$P^{-1}_{k/k} = P^{-1}_{k/k-1} + \sum_{i=1}^{l}\left(P^{i^{-1}}_{k/k} - P^{i^{-1}}_{k/k-1}\right) \tag{17.22}$$

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and

$$P_{k/k}^{-1}\mathbf{x}_{k/k} = P_{k/k-1}^{-1}\mathbf{x}_{k/k-1} + \sum_{i=1}^{l}\left(P_{k/k}^{i^{-1}}\mathbf{x}_{k/k}^{i} - P_{k/k-1}^{i^{-1}}\mathbf{x}_{k/k-1}^{i}\right) \tag{17.23}$$

Moreover, Ref. [7] provided a rigorous performance analysis for the case in which there is feedback from the fusion center to sensors, that is, the one-step predictions $\mathbf{x}_{k/k-1}^{i}$ and $P_{k/k-1}^{i}$ at every local sensor in Equations 17.22 and 17.23 are replaced by the feedback $\mathbf{x}_{k/k-1}$ and $P_{k/k-1}$, respectively. Zhu et al. [7] have shown that although the feedback cannot improve the performance at the fusion center, the feedback can reduce the covariance of each local tracking error. Obviously, for Equations 17.22 and 17.23 to be true, invertible error covariance matrices are necessary.

From then on, the assumption of invertibility of estimation error covariance matrices has been a restrictive condition for the preceding results.

Thus, when covariances of filtering errors are not invertible,

- Can we find sufficient statistics of sensor observations similar to

$$H_k' R_k^{-1}\mathbf{y}_k = \sum_{i=1}^{l} H_k^{i'} R_k^{i^{-1}}\mathbf{y}_k^{i}$$

  so that the centralized Kalman filtering can still be expressed in terms of the local filtering?
- Can we yet obtain a similar performance analysis for the distributed Kalman filtering with feedback?

The positive answers to the preceding questions are presented below.

17.3 DISTRIBUTED KALMAN FILTERING FUSION WITH SINGULAR COVARIANCES OF FILTERING ERRORS

First, we focus on how to express the centralized filtering as a combination of local filtering when only covariances of filtering errors are singular.

Equations 17.10 and 17.18 and Lemma 17.7 in Appendix I lead to the deduction that the estimation error covariance of the centralized Kalman filtering can be expressed in terms of the estimation error covariances of all local filters as follows:

$$P_{k/k}^{\dagger} = P_{k/k-1}^{\dagger} + \sum_{i=1}^{l} P_{k/k-1}^{\dagger}P_{k/k-1}\left(P_{k/k}^{i^{\dagger}} - P_{k/k-1}^{i^{\dagger}}\right)P_{k/k-1}P_{k/k-1}^{\dagger} \tag{17.24}$$

Remark 17.3

To the best of our knowledge, the updated formula (Equation 17.24) is new and it could deal with the case in which the filtering error covariance is singular. It is worth noting that in this case, the error covariance of centralized Kalman filtering can still be expressed in terms of the estimation error covariances of all local filters. Compared with Equation 17.22, we not only replaced $P_{k/k}^{-1}$ and $P_{k/k}^{i^{-1}}$

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by $P^{\dagger}_{k/k}$ and $P^{i^{\dagger}}_{k/k}$, respectively, but also multiplied an orthogonal projection operator $P^{\dagger}_{k/k-1}P_{k/k-1}$ at the both sides of $\left(P^{i^{\dagger}}_{k/k} - P^{i^{\dagger}}_{k/k-1}\right)$. Of course, the new updated formula includes Equation 17.22 as a special case when error covariances are invertible. The orthogonal projection operator $P^{\dagger}_{k/k-1}P_{k/k-1}$ means the information matrices (error covariances) are concentrated in some subspace (in fact, the space is the column space of $P^{\dagger}_{k/k-1}P_{k/k-1}$) instead of the whole space because the centralized and local information matrices (error covariances) of the previous step are not invertible. ■

To derive globally optimal distributed Kalman filtering fusion equivalent to the centralized Kalman filtering, as done in Refs. [2–6], a key skill is to find sufficient statistics of all sensor observations $\left\{\mathbf{y}^1_k,\ldots,\mathbf{y}^l_k\right\}$ that can be equivalently expressed in terms of two-step sensor estimates $\left\{\mathbf{x}^1_{k/k},\mathbf{x}^1_{k/k-1};\ldots;\mathbf{x}^l_{k/k},\mathbf{x}^l_{k/k-1}\right\}$. If this is done well, the centralized Kalman filtering can be easily rewritten as the corresponding distributed Kalman filtering fusion, which naturally is globally optimal. When the covariances of filtering errors and sensor noises are both invertible, such sufficient statistics for the centralized Kalman is simply $H'_kR^{-1}_k\mathbf{y}_k = \sum_{i=1}^{l} H^{i'}_k R^{i^{-1}}_k \mathbf{y}^i_k$ (for details, see $K_k\mathbf{y}_k$ in Equation 17.7), where $H^{i'}_k R^{i^{-1}}_k \mathbf{y}^i_k = P^{i^{-1}}_{k/k}\mathbf{x}^i_{k/k} - P^{i^{-1}}_{k/k-1}\mathbf{x}^i_{k/k-1}$ (see Equation 17.23).

Remark 17.4

If the covariances of filtering errors and sensor noises are not invertible, it is difficult to find the aforementioned sufficient statistics. Hence, to the best of our knowledge, so far, there has not been any work without the assumption of matrix invertibility as done 20 years ago in Refs. [2–5]. In fact, what we are doing technically in this chapter is just to find such sufficient statistic even without invertible covariances of filtering errors and measurement noises. Specifically, we carefully apply the matrix analysis technique (it equivalently analyzes information matrices corresponding to the state of the system) to obtain the globally optimal distributed fusion formula. ■

Equations 17.4 through 17.7 imply that

$$
\begin{aligned}
K_k\mathbf{y}_k &= P_{k/k}H'_kR^{-1}_k\mathbf{y}_k \\
&= \sum_{i=1}^{l} P_{k/k}H^{i'}_kR^{i^{-1}}_k\mathbf{y}^i_k \\
&= \sum_{i=1}^{l} P_{k/k}P^{i^{\dagger}}_{k/k}P^{i}_{k/k}H^{i'}_kR^{i^{-1}}_k\mathbf{y}^i_k \\
&= \sum_{i=1}^{l} P_{k/k}P^{i^{\dagger}}_{k/k}K^i_k\mathbf{y}^i_k,
\end{aligned}
\tag{17.25}
$$

where the third equality is due to Lemma 17.6 in Appendix I, and the fourth equality uses Equation 17.16.

Remark 17.5

From Equations 17.7 and 17.15, it is easy to see that Equation 17.25 plays a very important role in the key technique for the distributed Kalman filtering fusion equivalent to the centralized one because $K^i_k\mathbf{y}^i_k$ can be expressed simply in terms of $\mathbf{x}^i_{k/k-1}$ and $\mathbf{x}^i_{k/k}$ in following Equation 17.26. ■

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To express the centralized filtering $\mathbf{x}_{k/k}$ in terms of the local filtering, we use Equations 17.15 and 17.25 to eliminate $\mathbf{y}_k$ from 17.7. Note that Equation 17.15 means that

$$K_k^i \mathbf{y}_k^i = \mathbf{x}_{k/k}^i - \left(I - K_k^i H_k^i\right)\mathbf{x}_{k/k-1}^i. \tag{17.26}$$

Thus, substituting Equations 17.25 and 17.26 into 17.7 yields

$$\begin{aligned}
\mathbf{x}_{k/k} &= (I - K_k H_k)\mathbf{x}_{k/k-1} + \sum_{i=1}^{l} P_{k/k} P_{k/k}^{i^\dagger}\left[\mathbf{x}_{k/k}^i - \left(I - K_k^i H_k^i\right)\mathbf{x}_{k/k-1}^i\right] \\
&= (I - K_k H_k)\mathbf{x}_{k/k-1} + \sum_{i=1}^{l} P_{k/k} P_{k/k}^{i^\dagger}\mathbf{x}_{k/k}^i - \sum_{i=1}^{l} P_{k/k} P_{k/k}^{i^\dagger}\left(I - K_k^i H_k^i\right)\mathbf{x}_{k/k-1}^i \\
&= \left(I - P_{k/k} H_{k'} R_k^{-1} H_k\right)\mathbf{x}_{k/k-1} + \sum_{i=1}^{l} P_{k/k} P_{k/k}^{i^\dagger}\mathbf{x}_{k/k}^i + \sum_{i=1}^{l}\left[-P_{k/k} P_{k/k}^{i^\dagger}\left(I - P_{k/k}^i H_k^{i'} R_k^{i^{-1}} H_k^i\right)\right]\mathbf{x}_{k/k-1}^i \\
&= \left(I - P_{k/k} H_{k'} R_k^{-1} H_k\right)\mathbf{x}_{k/k-1} + \sum_{i=1}^{l} P_{k/k} P_{k/k}^{i^\dagger}\mathbf{x}_{k/k}^i + \sum_{i=1}^{l}\left[-P_{k/k}\left(P_{k/k}^{i^\dagger} - H_k^{i'} R_k^{i^{-1}} H_k^i\right)\right]\mathbf{x}_{k/k-1}^i,
\end{aligned} \tag{17.27}$$

where the third equality uses Equation 17.16, and the last equality is again thanks to Lemma 17.6 in Appendix I.

Equations 17.24 and 17.27 mean that the centralized filtering and its error matrix are explicitly expressed in terms of the filtering and error matrices of the local filtering, respectively. Consequently, the distributed Kalman filtering fusion given in Equations 17.24 and 17.27 has the same performance as that of the centralized Kalman filtering.

In addition, when error matrices of the fusion center and local sensor, $P_{k/k}$ and $P_{k/k}^i$, are invertible, we can easily see that Equations 17.24 and 17.27 now reduce to 17.22 and 17.23.

17.4 OPTIMALITY OF KALMAN FILTERING FUSION WITH FEEDBACK WHEN COVARIANCES OF FILTERING ERRORS ARE SINGULAR

In this section, we consider the optimality of Kalman filtering fusion with feedback under the condition that covariances of filtering errors are singular. When there is feedback, all the local sensors receive the latest estimate of the fusion center. Thus, similarly to the results in Ref. [4], the following local and global one-stage predictions were modified naturally as

$$\hat{\mathbf{x}}_{k/k-1}^i = \mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1/k-1} = \hat{\mathbf{x}}_{k/k-1} \tag{17.28}$$

$$\hat{P}_{k/k-1}^i = \hat{P}_{k/k-1}, \quad \forall i \tag{17.29}$$

where $\widehat{(\cdot)}$ denotes an estimated vector or covariance matrix with feedback after this modification. Therefore, similarly, for Equations 17.24 and 17.27, we consider the following filtering fusion with feedback,

$$\hat{P}_{k/k}^\dagger = \hat{P}_{k/k-1}^\dagger + \sum_{i=1}^{l} \hat{P}_{k/k-1}^\dagger \hat{P}_{k/k-1}\left(\hat{P}_{k/k}^{i\dagger} - \hat{P}_{k/k-1}^{i\dagger}\right)\hat{P}_{k/k-1}\hat{P}_{k/k-1}^\dagger \tag{17.30}$$

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$$\hat{\mathbf{x}}_{k/k} = \left(I - \hat{P}_{k/k} H_k' R_k^{-1} H_k\right)\hat{\mathbf{x}}_{k/k-1} + \sum_{i=1}^{l} \hat{P}_{k/k} \hat{P}_{k/k}^{i^\dagger} \hat{\mathbf{x}}_{k/k}^{i}$$
$$+ \sum_{i=1}^{l} \left[-\hat{P}_{k/k}\left(\hat{P}_{k/k}^{i^\dagger} - H_k^{i'} R_k^{i^{-1}} H_k^{i}\right)\right]\hat{\mathbf{x}}_{k/k-1}^{i}. \tag{17.31}$$

Because all the results and the mathematical deductions with feedback are similar to those in Ref. [7], we directly provide the results without detailed proof as follows:

1. The filtering fusion Equations 17.28 to 17.31 with feedback also can achieve the same performance as that of the centralized filtering fusion, that is,

$$\hat{\mathbf{x}}_{k/k} = \mathbf{x}_{k/k}, \qquad \hat{P}_{k/k} = P_{k/k} \tag{17.32}$$

2. The matrices $\hat{P}_{k/k}$ and $\hat{P}_{k/k}^{i}$ are still the covariances of the global and local filtering errors respectively, that is to say,

$$\hat{P}_{k/k} = E[(\hat{\mathbf{x}}_{k/k} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k/k} - \mathbf{x}_k)' | \mathbf{y}_0, \ldots, \mathbf{y}_k] \tag{17.33}$$

and

$$\hat{P}_{k/k}^{i} = E[(\hat{\mathbf{x}}_{k/k}^{i} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k/k}^{i} - \mathbf{x}_k)' | \mathbf{y}_0^{i}, \ldots, \mathbf{y}_k^{i}] \tag{17.34}$$

3. The feedback can benefit the local filters in the sense of reducing the covariance of each local tracking error, that is,

$$\hat{P}_{k/k}^{i} \preceq P_{k/k}^{i}, \qquad i = 1, 2, \ldots, l \tag{17.35}$$

17.5 OPTIMAL KALMAN FILTERING FUSION WITH SINGULAR COVARIANCES OF FILTERING ERRORS AND MEASUREMENT NOISES

In this section, we consider the case where the covariances of both filtering errors and measurement noise are singular. Obviously, it is more challenging than the previous case where the covariances of filtering errors are singular and the covariances of the measurement noise are invertible. Similarly, according to the standard results in Kalman filtering Refs. [17–19],

$$K_k = P_{k/k} H_k' R_k^{-1} \tag{17.36}$$

and

$$K_k^{i} = P_{k/k}^{i} H_k^{i'} R_k^{i^{-1}} \tag{17.37}$$

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do not hold in general because R_k and R_k^i may be singular. Fortunately,

$$K_k = P_{k/k-1}H_k'\left(H_k P_{k/k-1}H_k' + R_k\right)^{\dagger} \tag{17.38}$$

and

$$K_k^i = P_{k/k-1}^i H_k^{i'}\left(H_k^i P_{k/k-1}^i H_k^{i'} + R_k^i\right)^{\dagger} \tag{17.39}$$

still hold true. The preceding analysis implies that we cannot use the technique as Equations 17.25 to 17.27.

In what follows, we deal with the problem by using another trick.

We first give the limit formula for the Moore–Penrose inverse (see e.g., Ref. [20]). Assume that $A \in \mathbb{C}^{m\times n}$, then

$$\lim_{\epsilon\to 0^+}(A^*A + \epsilon I)^{-1}A^* = A^{\dagger}, \tag{17.40}$$

which plays a key role in the following analysis. From Remark 17.5 and Equation 17.25, we know that the key is to express the centralized sufficient statistics $K_k\mathbf{y}_k$ by linear combination of sensor sufficient statistics $K_k^i\mathbf{y}_k^i$, $i \le l$. Therefore, we give the following result.

Proposition 17.1

For system 17.1, assume $\mathbf{y}_k^i$ and $\mathbf{y}_k$ are defined by Equations 17.2 and 17.3 respectively. Moreover, under the assumptions that both covariances of filtering errors and measurement noises are singular, K_k and K_k^i are defined by Equations 17.38 and 17.39, respectively, for all $k = 0,1,\ldots,$

$$K_k\mathbf{y}_k = \sum_{i=1}^{l} K_k(i)K_k^{i^{\dagger}}K_k^i\mathbf{y}_k^i, \tag{17.41}$$

where

$$K_k = \begin{pmatrix} K_k(1) & K_k(2) & \ldots & K_k(l) \end{pmatrix} \tag{17.42}$$

is an appropriate partition of matrix K_k such that $K_k\mathbf{y}_k = \sum_{i=1}^{l} K_k(i)\mathbf{y}_k^i$. ■

Proof 17.1

The proof is lengthy and has been relegated to Appendix II. ■

In the following, we give the optimal Kalman filtering fusion formula with singular covariances of filtering errors and measurement noises.

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Proposition 17.2

Under the assumptions of Proposition 17.1, for all $k = 0, 1, \ldots,$

$$\begin{aligned}
\mathbf{x}_{k/k} &= \mathbf{x}_{k/k-1} + K_k(\mathbf{y}_k - H_k\mathbf{x}_{k/k-1}) \\
&= (I - K_k H_k)\mathbf{x}_{k/k-1} + K_k\mathbf{y}_k \\
&= (I - K_k H_k)\mathbf{x}_{k/k-1} + \sum_{i=1}^{l} K_k(i) K_k^{i^\dagger} K_k^i \mathbf{y}_k^i \\
&= (I - K_k H_k)\mathbf{x}_{k/k-1} \\
&\quad + \sum_{i=1}^{l} K_k(i) K_k^{i^\dagger} \left(\mathbf{x}_{k/k}^i - \left(I - K_k^i H_k^i\right)\mathbf{x}_{k/k-1}^i\right),
\end{aligned} \tag{17.43}$$

where $K_k(i)$ defined by Equation 17.42. ■

Proof 17.2

Clearly, Equation 17.7 implies that the second equality holds. It is easily seen that the third equality holds because of Proposition 17.1. Moreover, it follows from Equation 17.26 that the last equality is true. ■

Remark 17.6

Equation 17.43 shows that centralized Kalman filtering can still be expressed as a linear combination in terms of local Kalman filtering even if both covariance matrices of filter errors and measurement noises are singular. Clearly, it is a generalization of fusion formula 17.23. However, so far, we have not derived a simple distributed recursive formula for $P_{k/k}$ similar to Equation 17.24, and it is not certain if such a recursive formula really exists. ■

Remark 17.7

Furthermore, because our globally optimal distributed fusion is of the form of convex linear combination of $\left\{\mathbf{x}_{k/k-1}, \mathbf{x}_{k/k}^1, \mathbf{x}_{k/k-1}^1; \ldots; \mathbf{x}_{k/k}^l, \mathbf{x}_{k/k-1}^l\right\}$, this obviously provides a theoretical support to global optimality of the convex combination fusion algorithm of $\left\{\tilde{\mathbf{x}}_{k/k-1}, \mathbf{x}_{k/k}^1, \mathbf{x}_{k/k-1}^1; \ldots; \mathbf{x}_{k/k}^l, \mathbf{x}_{k/k-1}^l\right\}$ in Ref. [16], where $\tilde{\mathbf{x}}_{k/k-1}$ is the prediction of convex linear combination fusion algorithm. Our result implies that the algorithm in Ref. [16] is equivalent to the centralized Kalman filtering provided that $\tilde{\mathbf{x}}_{k/k-1}$ there is identical to the prediction $\mathbf{x}_{k/k-1}$ of the centralized Kalman filtering. This can be easily implemented while one takes the same initial values for the two recursive algorithms. ■

17.6 NUMERICAL EXAMPLES

As we have emphasized in the introduction, the three recursive algorithms—centralized Kalman filtering fusion (CKF), convex linear minimum mean square error (LMMSE) combination optimal distributed Kalman filtering fusion (CLDKF) in Ref. [16], and globally optimal distributed Kalman filtering fusion (GODKF) (Equation 17.27)—are theoretically the same as globally optimal LMSE estimation provided that they have the same initial values. In this section, we provide several

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numerical examples that demonstrate that our GODKF not only provides theoretical support to the CLDKF, but also uses less computation quantity than the CLDKF.

In what follows, we present two types of dynamic systems modeled as two objects moving on a circle and a straight line with process noise and measurement noise, respectively. Moreover, similar to the models proposed in Ref. [10], the two dynamical systems subject to state equality constraints are the models with singular estimation error covariance and measurement noise covariance matrices. All codes are written by MATLAB 7.8® and performed on a laptop computer with Intel CPU 2.00GHZ processor and 2.98GB memory.

All the following examples under the assumptions that the original object dynamics and measurement equations are modeled as follows:

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + v_k, \tag{17.44}$$

$$\mathbf{y}_k^i = H_k^i \mathbf{x}_k + w_k^i, \quad i = 1,2, \tag{17.45}$$

where the state $\mathbf{x}_k$ is known to actually be constrained in the null space of D_k:

$$\mathcal{N}(D_k) \triangleq \{\mathbf{x} : D_k \mathbf{x} = \mathbf{0}\} \tag{17.46}$$

For example, the objects are actually moving on piecewise straight road in the following examples. Besides, $v_k, w_k^i, k = 0,1,2,\ldots$, satisfy the assumptions of standard Kalman filtering.

Example 17.1: Circle Moving Model with Singular Estimation Error Covariance Only

In this example, for program simplicity, D_k is assumed to be constant matrix $D = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$. More specifically, the system (Equations 17.44 and 17.45) subject to constraint (Equation 17.46) can be converted into the following form:

$$\begin{aligned} \mathbf{x}_{k+1} &= P\Phi_k \mathbf{x}_k + Pv_k, \\ \mathbf{y}_k^i &= H_k^i \mathbf{x}_k + w_k^i, \quad i = 1,2, \end{aligned} \tag{17.47}$$

where

$$P = I - D^{\dagger} D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$P\Phi_k = P \begin{pmatrix} \cos(2\pi/300) & \sin(2\pi/300) \\ -\sin(2\pi/300) & \cos(2\pi/300) \end{pmatrix}$$

and

$$R_{Pv_k} = \begin{pmatrix} 0 & 0 \\ 0 & 15 \end{pmatrix}$$

are all constant matrices.

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The two measurement matrices and measurement noise covariance matrices are given by

$$H_k^1 = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}, \quad H_k^2 = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix},$$

and

$$R_k^i = R_{w_k^i} = \begin{pmatrix} 15 & 0 \\ 0 & 15 \end{pmatrix}, \quad i = 1,2$$

The stacked measurement equation is written as

$$\mathbf{y}_k = H_k \mathbf{x}_k + w_k,$$

where

$$\mathbf{y}_k = \begin{pmatrix} \mathbf{y}_k^1 \\ \mathbf{y}_k^2 \end{pmatrix}, \quad H_k = \begin{pmatrix} H_k^1 \\ H_k^2 \end{pmatrix}, \quad w_k = \begin{pmatrix} w_k^1 \\ w_k^2 \end{pmatrix},$$

and the covariance of the noise w_k is given by

$$R_k = \mathrm{diag}\left(R_k^1, R_k^2\right)$$

The initial values are given as follows: $Ex_0 = (50,0)'$, $x_{0|0} = (50,0)'$, $P_{0|0}^i = \mathrm{Var}(x_0) = \begin{pmatrix} 0 & 0 \\ 0 & 15 \end{pmatrix}$.

Using a Monte Carlo method of 1000 runs, time index k = 1, 2, …, 300, that is, we assume that Kalman filtering implements 300 steps. We evaluate tracking performance of an algorithm by

$$E_k = \frac{1}{1000} \sum_{j=1}^{1000} \left\| \mathbf{x}_{k/k}^{(j)} - \mathbf{x}_k^{(j)} \right\|^2, \quad k = 1,2,\ldots,300$$

The numerical results of the three algorithms are given in Figure 17.1.

Example 17.2: Circle Moving Model with Singular Estimation Error and Measurement Noise Covariances

In this example, we only change the two measurement matrices and singular measurement noise covariance matrices in Example 17.1 to

$$R_k^i = R_{Pw_k^i} = \begin{pmatrix} 0 & 0 \\ 0 & 15 \end{pmatrix}, \quad i = 1,2, \quad H_k^1 = \begin{pmatrix} 2.5 & 0 \\ 2.5 & -2.5 \end{pmatrix},$$

and

$$H_k^2 = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}.$$

The numerical results are given in Figure 17.2.

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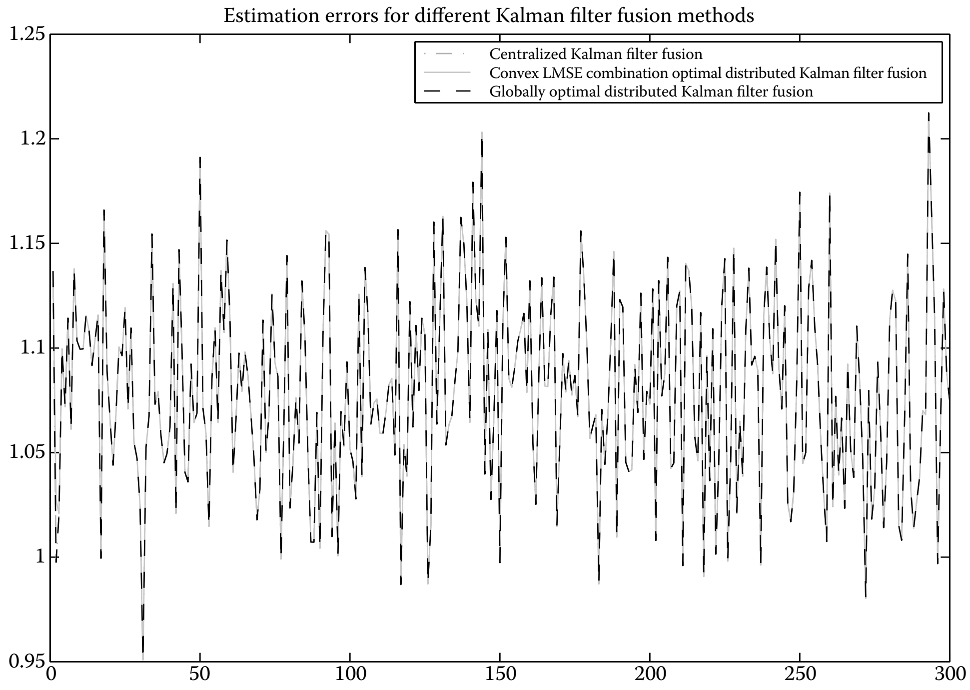

FIGURE 17.1 The average tracking error of three algorithms in Example 17.1.

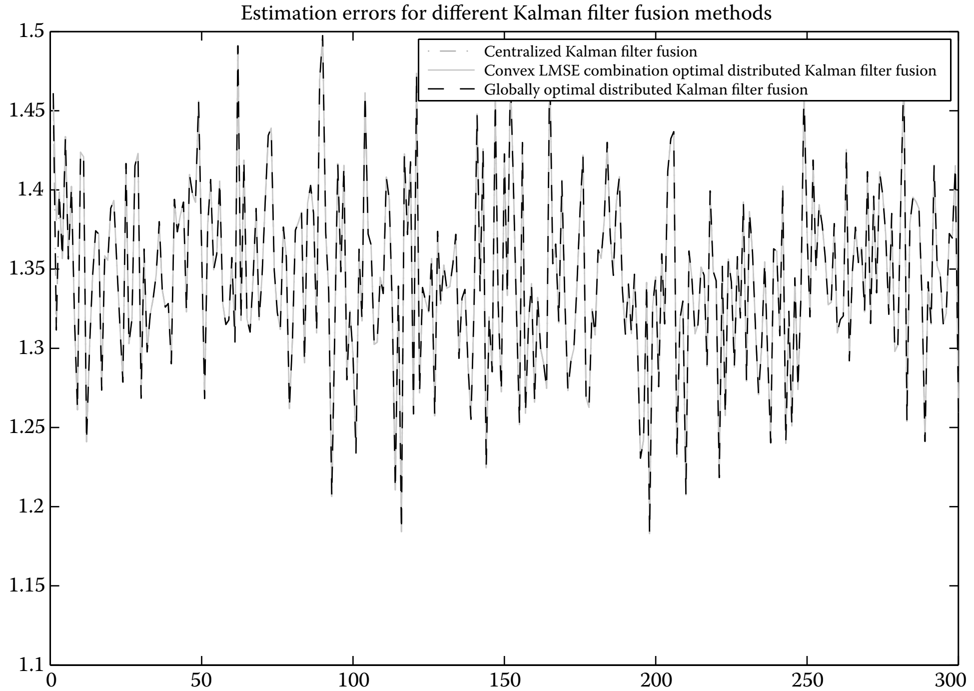

FIGURE 17.2 The average tracking errors of three fusion algorithms in Example 17.2.

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Example 17.3: Straight Line Moving Model with Singular Estimation Error and Measurement Noise Covariances

In this example, we let

$$D = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad P = I - D^{\dagger}D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

Consequently,

$$P\Phi_k = P\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad R_{Pv_k} = \begin{pmatrix} 15 & 0 \\ 0 & 0 \end{pmatrix}.$$

The two measurement matrices and singular measurement noise covariance matrices are given by:

$$R_k^i = R_{Pw_k^i} = \begin{pmatrix} 15 & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, \quad H_k^1 = \begin{pmatrix} 2.5 & 2.5 \\ 0 & -2.5 \end{pmatrix}$$

and

$$H_k^2 = \begin{pmatrix} 3 & 6 \\ 0 & -6 \end{pmatrix}.$$

The numerical results are shown in Figure 17.3.

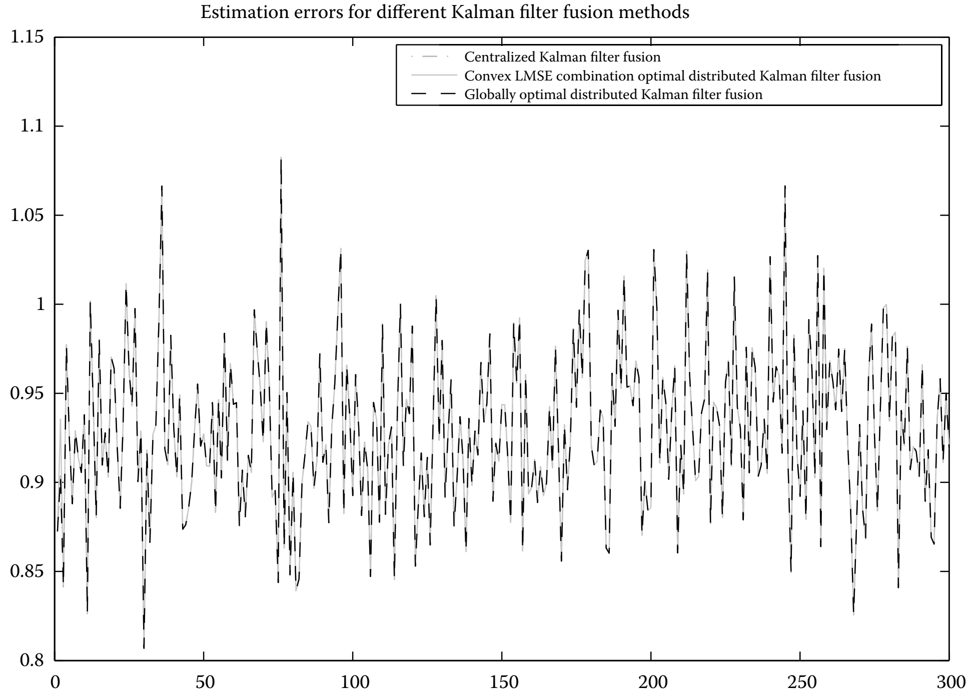

FIGURE 17.3 The average tracking errors of the three fusion algorithms in Example 17.3.

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TABLE 17.1
Comparison of Time Consumption of the Two Distributed Fusion Algorithms

	Example 17.1	Example 17.2	Example 17.3
GODKF	133.0454(s)	129.3726(s)	130.4649(s)
CLDKF	216.0183(s)	217.1235(s)	218.1664(s)
$\frac{\text{GODKF}}{\text{CLDKF}}$	61.59%	59.58%	59.80%

Although Figures 17.1 to 17.3 show that the performances of both GODKF and CLDKF are the same as that of CKF, their computational burdens are not the same. To show this difference, we provide computation times of the two distributed fusion algorithms as follows.

From Table 17.1 and Figures 17.1 to 17.3 in our numerical results, we have the following three observations

1. Figures 17.1, 17.2, and 17.3 show that the numerical results of GODKF and CLDKF are both generally the same as those of CKF, which is consistent to the theoretical analysis in this paper.
2. Although the two distributed fusion algorithms work well in all of the preceding examples, Table 17.1 shows that, compared with CLDK in Ref. [6], our proposed GODKF algorithm could save computation significantly.
3. Roughly speaking, at each iteration, the main difference of computation burdens of the fusion coefficients of GODKF and CLDKF is that the former need to compute K_k and $K_k^i, i \leq l$ (e.g., see Equation 17.43), i.e., computations of P_k and P_k^i, totally $(l + 1)$ $(r \times r)$ matrices (see Equations 17.38 and 17.39); however, the latter needs to compute totally $\frac{1}{2}(2l+1)(2l+2)$ $(r \times r)$ correlation matrices of all $(2l + 1)$ local estimation errors, that is, every sub-block of the covariance matrix C_k of $(2l + 1)$ local estimation errors (C_k was defined in Ref. [16]).

17.7 CONCLUSIONS

We have presented in this chapter distributed Kalman filtering fusion algorithms for linear dynamic systems with singular covariances of filtering errors and measurement noises and rigorously showed that the fused state estimate is equivalent to the centralized Kalman filtering using all sensor measurements. Moreover, we have also proved that the covariances of centralized filtering error can still be expressed as a combination of covariances of local filtering errors when the covariances of centralized filtering errors are singular and the covariances of measurement noises are nonsingular. Furthermore, we have also presented a modified Kalman filtering fusion formula with feedback for the same dynamic system. Then, the proposed fusion formula with feedback is also exactly equivalent to the corresponding centralized Kalman filtering fusion formula using all sensor measurements. Our proposed fusion formula reveals a new aspect that even if information matrix (covariance of filtering error) is singular, the performance of the centralized Kalman filtering could be still obtained by fusing local sensor state estimate. Several numerical examples have demonstrated the theoretical significance and computational advantage of our results.

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APPENDIX I

Lemma 17.1

For $A \in \mathbb{C}^m_{\succeq}$, $B \in \mathbb{C}^m_{\succeq}$, if $A \succeq B$, then,

$$\mathcal{R}(A) \supseteq \mathcal{R}(B). \tag{17.48}$$

■

Proof 17.3

For an arbitrary $x \in \mathcal{R}(A)^{\perp} = \mathcal{N}(A)$, it follows from $A \succeq B$ that

$$0 = x^* Ax \geq x^* Bx \geq 0,$$

which further implies that

$$0 = x^* Bx = \left(B^{\frac{1}{2}}x\right)^* B^{\frac{1}{2}}x$$

That is to say, $x \in \mathcal{N}(B) = \mathcal{R}(B)^{\perp}$. Note that $\mathcal{R}(A)^{\perp} \subseteq \mathcal{R}(B)^{\perp}$ amounts to $\mathcal{R}(A) \supseteq \mathcal{R}(B)$, the lemma has been verified. ■

Lemma 17.2

For $A \in \mathbb{C}^m_{\succeq}$, $B \in \mathbb{C}^m_{\succeq}$, if $A \succeq B$, then,

$$AA^{\dagger} B = B = BA^{\dagger}A. \tag{17.49}$$

■

Proof 17.4

Due to Lemma 17.1, there must exist a matrix Y such that

$$AY = B, \tag{17.50}$$

which means that

$$AA^{\dagger} B = AA^{\dagger} AY = B \tag{17.51}$$

and

$$\begin{aligned} BA^{\dagger}A &= B^*A^{\dagger}A = (AY)^* A^{\dagger}A \\ &= Y^*A^*A^{\dagger}A = Y^*AA^{\dagger}A \\ &= Y^*A = Y^*A^* = (AY)^* \\ &= B^* = B. \end{aligned} \tag{17.52}$$

The following lemma is attributable to Radoslaw Kala and Krzysztof Klaczyński [21]. ■

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Lemma 17.3

For $A \in \mathbb{C}^m_{\succeq}$, $D \in \mathbb{C}^r_{\succeq}$ and $B \in \mathbb{C}^{m\times r}$ define

$$\tilde{A} = A + BDB^* \tag{17.53}$$

and

$$V = B^*\ GB + D^\dagger. \tag{17.54}$$

Assume

$$\mathcal{R}(B^*) \subseteq \mathcal{R}(D) \tag{17.55}$$

and

$$\mathcal{R}(B) \subseteq \mathcal{R}(A) \tag{17.56}$$

are satisfied. Then

$$\tilde{A}^\dagger = A^\dagger - A^\dagger BV^\dagger B^* A^\dagger \tag{17.57}$$

■

Lemma 17.4

Suppose that $H_k^i \in \mathbb{C}^{N_i\times r}$, $R_k^i \in \mathbb{C}^{N_i}_{\succeq}$ and $P^i_{k/k-1} \in \mathbb{C}^r_{\succeq}$, $i = 1,2,\cdots,l$. Moreover, let $H_k = \left(H_k^{1*}, H_k^{2*}, \cdots, H_k^{l*}\right)^*$, $R_k = \text{diag}\left(R_k^1, R_k^2, \cdots, R_k^l\right)$,

$$P_{k/k} = P_{k/k-1} - P_{k/k-1}H_k'\left(H_k P_{k/k-1} H_k' + R_k\right)^\dagger H_k P_{k/k-1} \tag{17.58}$$

and

$$P^i_{k/k} = P^i_{k/k-1} - P^i_{k/k-1}H_k^{i'}\left(H_k^i P^i_{k/k-1} H_k^{i'} + R_k^i\right)^\dagger H_k^i P^i_{k/k-1} \tag{17.59}$$

Then

$$P^\dagger_{k/k} = P^\dagger_{k/k-1} + P^\dagger_{k/k-1} P_{k/k-1} H_{k'} R_k^{-1} H_k P_{k/k-1} P^\dagger_{k/k-1} \tag{17.60}$$

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and

$$P_{k/k}^{i^\dagger} = P_{k/k-1}^{i^\dagger} + P_{k/k-1}^{i^\dagger} P_{k/k-1}^{i} H_k^{i'} R_k^{i^{-1}} H_k^{i} P_{k/k-1}^{i} P_{k/k-1}^{i^\dagger} \tag{17.61}$$

■

Proof 17.5

It is obvious that

$$\mathcal{R}\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_k'\right) \subseteq \mathcal{R}\left(P_{k/k-1}^{\dagger}\right) \tag{17.62}$$

Note that $R_k \in \mathbb{R}^{\left(\sum_{i=1}^{l} N_i\right)\times\left(\sum_{i=1}^{l} N_i\right)}$ and is invertible, which leads to $\mathcal{R}\left(R_k^{-1}\right)$ is the entire linear space $\mathbb{R}^{\sum_{i=1}^{l} N_i}$. Thus, together with the relation $H_k \in \mathbb{R}^{\left(\sum_{i=1}^{l} N_i\right)\times r}$, it is obvious that

$$\begin{aligned}
\mathcal{R}\left(\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_{k'}\right)^{*}\right) &= \mathcal{R}\left(\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_{k'}\right)'\right) \\
&= \mathcal{R}\left(H_k \left(P_{k/k-1}^{\dagger} P_{k/k-1}\right)'\right) \subseteq \mathcal{R}(H_k) \subseteq \mathbb{R}^{\sum_{i=1}^{l} N_i} \\
&= \mathcal{R}\left(R_k^{-1}\right)
\end{aligned} \tag{17.63}$$

Furthermore, according to property of Moore–Penrose generalized inverse of matrix, we have

$$P_{k/k-1}\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_k'\right) = P_{k/k-1} H_k' \tag{17.64}$$

and

$$\begin{aligned}
&\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_k'\right)^{*} P_{k/k-1}\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_k'\right) \\
&= \left(H_k P_{k/k-1} P_{k/k-1}^{\dagger}\right) P_{k/k-1}\left(P_{k/k-1}^{\dagger} P_{k/k-1} H_k'\right) \\
&= H_k P_{k/k-1} H_k'.
\end{aligned} \tag{17.65}$$

Note that

$$P_{k/k} = P_{k/k-1} - P_{k/k-1} H_k' \left(H_k P_{k/k-1} H_k' + R_k\right)^{\dagger} H_k P_{k/k-1}, \tag{17.66}$$

combined with Equations 17.62 through 17.65 and Lemma 17.3, we know Equation 17.60 holds. Consequently, Equation 17.61 also holds. ■

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Lemma 17.5

For any i, $1 \le i \le l$, and k, $k = 2, 3, \ldots$, we have

$$P_{k/k-1} \preceq P^i_{k/k-1} \tag{17.67}$$

and

$$P_{k/k} \preceq P^i_{k/k} \tag{17.68}$$

where $P_{k/k-1}$, $P_{k/k}$, $P^i_{k/k-1}$ and $P^i_{k/k}$ are defined as Equations 17.12, 17.13, 17.20, and 17.21, respectively. ■

Proof 17.6

The local filtering $\mathbf{x}^i_{k/k}$ can be regarded as a particular multisensor filtering fusion that only uses measurements $\{\mathbf{y}^i_1,\ldots,\mathbf{y}^i_k\}$ and the combination coefficients of all other sensor measurements are zeros. However, the centralized filtering $\mathbf{x}_{k/k}$ is optimal by using all the measurements $\{\mathbf{y}^1_1,\ldots,\mathbf{y}^l_1;\ldots;\mathbf{y}^1_k,\ldots,\mathbf{y}^l_k\}$. Moreover, it implies that $\mathbf{x}_{k/k}$ is the best one in the sense that its estimation error matrix $P_{k/k}$ is the smallest with respect to matrix Löwner partial order (matrix positive semi-definite order). Consequently, the lemma is true. ■

Lemma 17.6

For any i, $1 \le i \le$ l, we have

$$P_{k/k-1} P^{i^\dagger}_{k/k-1} P^i_{k/k-1} = P_{k/k-1} \tag{17.69}$$

and

$$P_{k/k} P^{i^\dagger}_{k/k} P^i_{k/k} = P_{k/k}, \tag{17.70}$$

where $P_{k/k-1}$, $P_{k/k}$, $P^i_{k/k-1}$ and $P^i_{k/k}$ are defined as Equations 17.12, 17.13, 17.20, and 17.21, respectively. ■

Proof 17.7

It is obvious because Lemmas 17.2 and 17.5 hold. ■

Lemma 17.7

Under the assumptions of Lemma 17.4, we have

$$P^\dagger_{k/k} = P^\dagger_{k/k-1} + \sum_{i=1}^{l} P^\dagger_{k/k-1} P_{k/k-1} \left(P^{i^\dagger}_{k/k} - P^{i^\dagger}_{k/k-1} \right) P_{k/k-1} P^\dagger_{k/k-1} \tag{17.71}$$

■

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Proof 17.8

According to Lemma 17.4, we have

$$
\begin{aligned}
P_{k/k}^{\dagger} &= P_{k/k-1}^{\dagger} + P_{k/k-1}^{\dagger}P_{k/k-1}H_k'R_k^{-1}H_kP_{k/k-1}P_{k/k-1}^{\dagger} \\
&= P_{k/k-1}^{\dagger} + \sum_{i=1}^{l} P_{k/k-1}^{\dagger}P_{k/k-1}H_k^{i'}R_k^{i^{-1}}H_k^{i}P_{k/k-1}P_{k/k-1}^{\dagger} \\
&= P_{k/k-1}^{\dagger} + \sum_{i=1}^{l}\left\{\left[P_{k/k-1}^{\dagger}P_{k/k-1}\left(P_{k/k-1}^{i^{\dagger}}P_{k/k-1}^{i}\right)\right]H_k^{i'}R_k^{i^{-1}}H_k^{i}\right. \\
&\qquad \left.\times\left[\left(P_{k/k-1}^{i}P_{k/k-1}^{i^{\dagger}}\right)P_{k/k-1}P_{k/k-1}^{\dagger}\right]\right\} \\
&= P_{k/k-1}^{\dagger} + \sum_{i=1}^{l}\left\{P_{k/k-1}^{\dagger}P_{k/k-1}\left[\left(P_{k/k-1}^{i^{\dagger}}P_{k/k-1}^{i}\right)H_k^{i'}R_k^{i^{-1}}H_k^{i}\right.\right. \\
&\qquad \left.\left.\times\left(P_{k/k-1}^{i}P_{k/k-1}^{i^{\dagger}}\right)\right]P_{k/k-1}P_{k/k-1}^{\dagger}\right\} \\
&= P_{k/k-1}^{\dagger} + \sum_{i=1}^{l} P_{k/k-1}^{\dagger}P_{k/k-1}\left(P_{k/k}^{i^{\dagger}} - P_{k/k-1}^{i^{\dagger}}\right)P_{k/k-1}P_{k/k-1}^{\dagger},
\end{aligned}
\tag{17.72}
$$

where the third equality uses the fact that

$$
\begin{aligned}
P_{k/k-1}^{\dagger}P_{k/k-1}\left(P_{k/k-1}^{i^{\dagger}}P_{k/k-1}^{i}\right) &= \left(P_{k/k-1}^{i}P_{k/k-1}^{i^{\dagger}}\right)P_{k/k-1}P_{k/k-1}^{\dagger} \\
&= P_{k/k-1}^{\dagger}P_{k/k-1},
\end{aligned}
\tag{17.73}
$$

thanks to Lemmas 17.2 and 17.5, and the last equality uses Lemma 17.4 again. ■

Lemma 17.8

Suppose that $P \in \mathbb{C}_{\succeq}^{r}$, $H \in \mathbb{C}^{N\times r}$, $R \in \mathbb{C}_{\succeq}^{N}$, then

$$
\lim_{\epsilon\to 0^+} PH^*(HPH^* + R + \epsilon I_N)^{-1} = PH^*(HPH^* + R)^{\dagger}
\tag{17.74}
$$

■

Proof 17.9

Let $A = \begin{pmatrix} P^{\frac{1}{2}}H^* \\ R^{\frac{1}{2}} \end{pmatrix}$, where $P^{\frac{1}{2}} \succeq \mathbf{0}$ and $R^{\frac{1}{2}} \succeq \mathbf{0}$ denote the square roots of P and R respectively, because $P \succeq \mathbf{0}$ and $R \succeq \mathbf{0}$. Therefore,

$$
A^*A = HPH^* + R.
\tag{17.75}
$$

Equation 17.40 implies that

$$
\lim_{\epsilon\to 0^+} A(A^*A + \epsilon I_N)^{-1} = (A^*)^{\dagger} = A(A^*A)^{\dagger}
\tag{17.76}
$$

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Moreover, Equations 17.75 and 17.76 lead to

$$\lim_{\epsilon\to 0^+}\begin{pmatrix} P^{\frac{1}{2}}H^*(HPH^*+R+\epsilon I_N)^{-1} \\ R^{\frac{1}{2}}(HPH^*+R+\epsilon I_N)^{-1}\end{pmatrix} = \lim_{\epsilon\to 0^+}\begin{pmatrix} P^{\frac{1}{2}}H^* \\ R^{\frac{1}{2}}\end{pmatrix}(HPH^*+R+\epsilon I_N)^{-1}$$

$$= \begin{pmatrix} P^{\frac{1}{2}}H^* \\ R^{\frac{1}{2}}\end{pmatrix}(HPH^*+R)^{\dagger} \tag{17.77}$$

$$= \begin{pmatrix} P^{\frac{1}{2}}H^*(HPH^*+R)^{\dagger} \\ R^{\frac{1}{2}}(HPH^*+R)^{\dagger}\end{pmatrix}.$$

It immediately follows that

$$\lim_{\epsilon\to 0^+} P^{\frac{1}{2}}H^*(HPH^*+R+\epsilon I_N)^{-1} = P^{\frac{1}{2}}H^*(HPH^*+R)^{\dagger} \tag{17.78}$$

Note that

$$PH^*(HPH^*+R+\epsilon I_N)^{-1} = P^{\frac{1}{2}}P^{\frac{1}{2}}H^*(HPH^*+R+\epsilon I_N)^{-1} \tag{17.79}$$

and the proof is complete. ■

Lemma 17.9

Assume that $(\Omega,\mathcal{F},P)$ is a probability space, $\mathbf{y}:\Omega\to\mathcal{R}^n$ is a random vector with zero mean and invertible covariance R. If there exists a matrix $A\in\mathcal{R}^{m\times n}$ such that

$$AY(\omega)=\mathbf{0},\ \forall\ \omega\in\Omega, \tag{17.80}$$

Then,

$$A=\mathbf{0} \tag{17.81}$$

■

Proof 17.10

Note $E\,(Y)=\mathbf{0}$ and by Equation 17.80, we have

$$\mathbf{0}=\mathrm{Cov}(\mathbf{0})=\mathrm{Cov}(AY)=ARA'. \tag{17.82}$$

This implies that $A=\mathbf{0}$ because R is invertible. ■

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Lemma 17.10

Suppose that $P \in \mathbb{C}^r_{\succeq}$, $H \in \mathbb{C}^{N\times r}$, $R \in \mathbb{C}^N_{\succeq}$; then $\forall\, \epsilon \geq 0$,

$$\operatorname{rank}\left(PH^*(HPH^* + R + \epsilon I_N)^{\dagger}\right) = \operatorname{rank}(PH^*) \tag{17.83}$$

■

Proof 17.11

For $\forall\, \epsilon \geq 0$, it is obviously that

$$HPH^* + R + \epsilon I_N \succeq HPH^* \tag{17.84}$$

It follows from Equation 17.84 and Lemma 17.1 that

$$\mathcal{R}(HPH^* + R + \epsilon I_N) \supseteq \mathcal{R}(HPH^*), \tag{17.85}$$

which further implies that

$$HPH^*(HPH^* + R + \epsilon I_N)^{\dagger}(HPH^* + R + \epsilon I_N) = HPH^*. \tag{17.86}$$

On the one hand, we have

$$\begin{aligned}
&\operatorname{rank}\left(PH^*(HPH^* + R + \epsilon I_N)^{\dagger}(HPH^* + R + \epsilon I_N)\right)\\
&\geq \operatorname{rank}\left(HPH^*(HPH^* + R + \epsilon I_N)^{\dagger}(HPH^* + R + \epsilon I_N)\right)\\
&= \operatorname{rank}(HPH^*) = \operatorname{rank}\left(HP^{\frac{1}{2}}P^{\frac{1}{2}}H^*\right)\\
&= \operatorname{rank}\left(\left(P^{\frac{1}{2}}H^*\right)^* P^{\frac{1}{2}}H^*\right) = \operatorname{rank}\left(P^{\frac{1}{2}}H^*\right)\\
&\geq \operatorname{rank}\left(P^{\frac{1}{2}}P^{\frac{1}{2}}H^*\right) = \operatorname{rank}(PH^*),
\end{aligned} \tag{17.87}$$

where the first equality holds because of Equation 17.86, and the second equality is due to $P^{\frac{1}{2}} \succeq \mathbf{0}$ denotes the square root of P because $P \succeq \mathbf{0}$.

On the other hand, it is well known that

$$\begin{aligned}
&\operatorname{rank}\left(PH^*(HPH^* + R + \epsilon I_N)^{\dagger}(HPH^* + R + \epsilon I_N)\right)\\
&\leq \operatorname{rank}(PH^*).
\end{aligned} \tag{17.88}$$

Equations 17.87 and 17.88 mean that Lemma 17.10 holds.

The following lemma is due to Ref. [22]. ■

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Lemma 17.11

Assume that $A, B \in \mathbb{C}^{m\times n}$, where B is regarded as a variable. Then

$$\lim_{B\to A} B^{\dagger} = A^{\dagger}$$

is equivalent to

$$\text{rank}\ (B) = \text{rank}\ (A)$$

when B is approaching sufficiently close to A. ■

APPENDIX II

Proof of Proposition 17.1

Proof 17.12

For an arbitrary given integer $t \geq 0$ and $\varepsilon > 0$, we consider the l-sensor distributed linear dynamic system as follows:

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + v_k, \quad k = 0,1,\ldots, \tag{17.89}$$

$$\mathbf{y}_k^i = H_k^i \mathbf{x}_k + w_k^i, \quad i = 1,\ldots,l, \quad k = 0,1,\ldots t-1, \tag{17.90}$$

$$\mathbf{y}_k^i(\epsilon) = H_k^i \mathbf{x}_k + w_k^i + \sqrt{\epsilon}\breve{w}_k^i, \quad i = 1,\ldots,l, \quad k = t, \tag{17.91}$$

where Φ_k is a matrix of order $(r \times r)$, $\mathbf{x}_k, v_k \in \mathbb{R}^r$, $H_k^i \in \mathbb{R}^{N_i \times r}$, $\mathbf{y}_k^i, w_k^i \in \mathbb{R}^{N_i}$. The process noise v_k and measurement noise w_k^i ($w_k^i + \sqrt{\epsilon}\breve{w}_k^i$ when $k = t$) are both zero-mean random variables independent of each other temporally and are not cross correlated. Compared with system (Equations 17.1 and 17.2), here we have replaced $\mathbf{y}_t^i$ and w_t^i by $\mathbf{y}_t^i(\epsilon)$ and $w_t^i + \sqrt{\epsilon}\breve{w}_t^i$ when $k = t$, respectively; moreover, w_t^i and $\sqrt{\epsilon}\breve{w}_t^i$ are independent random variables with zero mean and covariance

$$\text{Cov}\left(w_t^i\right) = R_t^i \tag{17.92}$$

and

$$\text{Cov}\left(\breve{w}_t^i\right) = I_{N_i} \tag{17.93}$$

Consequently, similar to the definition of Equations 17.5 and 17.6, we let

$$\begin{aligned} R_t^i(\epsilon) &\triangleq \text{Cov}\left(w_t^i + \sqrt{\epsilon}\breve{w}_t^i\right) = \text{Cov}\left(w_t^i\right) + \epsilon\text{Cov}\left(\breve{w}_t^i\right) \\ &= R_t^i + \epsilon I_{N_i}, \quad i = 1,\ldots,l. \end{aligned} \tag{17.94}$$

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and

$$
\begin{aligned}
R_t(\epsilon) &\triangleq \mathrm{Cov}\left(w_t + \sqrt{\epsilon}\breve{w}_t\right) \\
&= \mathrm{Cov}\left(\left(w_k^{1'},\ldots,w_k^{l'}\right)' + \sqrt{\epsilon}\left(\breve{w}_k^{1'},\ldots,\breve{w}_k^{l'}\right)'\right) \\
&= \mathrm{diag}\left(R_t^1,\ldots,R_t^l\right)' + \mathrm{diag}\left(\epsilon I_{N_1},\ldots,\epsilon I_{N_l}\right)' \\
&= R_t + \epsilon I_N
\end{aligned}
\tag{17.95}
$$

where

$$
N = \sum_{i=1}^{l} N_i. \tag{17.96}
$$

Denote

$$
\mathbf{y}_t(\epsilon) \triangleq \left(\mathbf{y}_t^{1'}(\epsilon),\ldots,\mathbf{y}_t^{l'}(\epsilon)\right)' \tag{17.97}
$$

Consequently, according to Kalman filtering theory,

$$
\begin{aligned}
K_t(\epsilon) &= P_{t/t-1}H_t'\left(H_t P_{t/t-1} H_t' + R_t + \epsilon I_N\right)^{-1} \\
&= P_{t/t}(\epsilon) H_t'(R_t + \epsilon I_N)^{-1}
\end{aligned}
\tag{17.98}
$$

and

$$
\begin{aligned}
K_t^i(\epsilon) &= P_{t/t-1}^i H_t^{i'}\left(H_t^i P_{t/t-1}^i H_t^{i'} + R_k^i + \epsilon I_N\right)^{-1} \\
&= P_{t/t}^i(\epsilon) H_t^{i'}\left(R_t^i + \epsilon I_N\right)^{-1},
\end{aligned}
\tag{17.99}
$$

where

$$
P_{t/t}(\epsilon) = (I - K_t(\epsilon)H_t)P_{t/t-1} \tag{17.100}
$$

and

$$
P_{t/t}^i(\epsilon) = \left(I - K_t^i(\epsilon)H_t^i\right)P_{t/t-1}^i \tag{17.101}
$$

Lemma 17.8 in Appendix I implies that

$$
\lim_{\epsilon\to 0^+} K_t(\epsilon) = P_{t/t-1}H_t'\left(H_t P_{t/t-1} H_t' + R_t\right)^{\dagger} = K_t \tag{17.102}
$$

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and

$$\lim_{\epsilon\to 0^+} K_t^i(\epsilon) = P_{t/t-1}^i H_t^{i'} \left(H_t^i P_{t/t-1}^i H_t^{i'} + R_t^i \right)^\dagger = K_t^i \tag{17.103}$$

also hold.

Let

$$K_t(\epsilon) = \begin{pmatrix} K_t(\epsilon,1) & K_t(\epsilon,2) & \ldots & K_t(\epsilon,l) \end{pmatrix} \tag{17.104}$$

be an appropriate partition of matrix $K_t(\varepsilon)$ such that $K_t(\epsilon)\mathbf{y}_t(\epsilon) = \sum_{i=1}^{l} K_t(\epsilon,i)\mathbf{y}_t^i(\epsilon)$.

Because $\varepsilon > 0$, Equation 17.94 means we are dealing with the case where measurement noises possess invertible covariances. Thus, by the similar analysis in Equation 17.25, it follows that

$$\begin{aligned} K_t(\epsilon)\mathbf{y}_t(\epsilon) &= \left(K_t(\epsilon,1) K_t(\epsilon,2) \ldots K_t(\epsilon,l) \right) \mathbf{y}_t(\epsilon) \\ &= \sum_{i=1}^{l} K_t(\epsilon,i)\mathbf{y}_t^i(\epsilon) = \sum_{i=1}^{l} P_{t/t}(\epsilon) P_{t/t}^{i^\dagger}(\epsilon) K_t^i(\epsilon)\mathbf{y}_t^i(\epsilon) \\ &= \left(P_{t/t}(\epsilon) P_{t/t}^{1^\dagger}(\epsilon) K_t^1(\epsilon) \cdots P_{t/t}(\epsilon) P_{t/t}^{l^\dagger}(\epsilon) K_t^l(\epsilon) \right) \mathbf{y}_t(\epsilon). \end{aligned} \tag{17.105}$$

Because $\mathbf{y}_t(\varepsilon)$ denotes a random variable with an invertible covariance $H_t \mathrm{Cov}(\mathbf{x}_t) H_t' + R_t + \epsilon I_N$, combined with Lemma 17.9 in Appendix I, Equations 17.104 and 17.105 lead to the deduction that

$$K_t(\epsilon,i) = P_{t/t}(\epsilon) P_{t/t}^{i^\dagger}(\epsilon) K_t^i(\epsilon), \quad i = 1,2,\ldots,l \tag{17.106}$$

Furthermore, according to the property of linear matrix equation, it follows from Equation 17.106 that the row space of $K_t(\varepsilon, i)$ is contained in the row space of $K_t^i(\epsilon)$. Consequently,

$$K_t(\epsilon,i) = K_t(\epsilon,i) K_t^{i^\dagger}(\epsilon) K_t^i(\epsilon), \quad i = 1,2,\ldots,l. \tag{17.107}$$

It follows from Lemma 17.10 in Appendix I that $\forall \epsilon \geq 0$,

$$\begin{aligned} \mathrm{rank}\left(K_t^i(\epsilon) \right) &= \mathrm{rank}\left(P_{t/t-1}^i H_t^{i'} \left(H_t^i P_{t/t-1}^i H_t^{i'} + R_t^i + \epsilon I_{N_i} \right)^\dagger \right) \\ &= \mathrm{rank}\left(P_{t/t-1}^i H_t^{i'} \right). \end{aligned} \tag{17.108}$$

Because Moore–Penrose generalized inverse is continuous when rank is unchanged (see Lemma 17.11 in Appendix I), Equations 17.103 and 17.108 mean that

$$\lim_{\epsilon\to 0^+} \left(K_t^i(\epsilon) \right)^\dagger = \left(\lim_{\epsilon\to 0^+} K_t^i(\epsilon) \right)^\dagger = K_t^{i^\dagger} \tag{17.109}$$

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Note that $K_t(i)$ has been defined in Equation 17.42, taking the limits for both hands of Equation 17.106, and combining Equations 17.102–17.104, and 17.109, we achieve

$$\begin{aligned} K_t(i) &= \lim_{\epsilon\to 0^+} K_t(\epsilon,i) = \lim_{\epsilon\to 0^+}\left[K_t(\epsilon,i)K_t^{i^\dagger}(\epsilon)K_t^i(\epsilon)\right] \\ &= \left[\lim_{\epsilon\to 0^+} K_t(\epsilon,i)\right]\times\left[\lim_{\epsilon\to 0^+} K_t^{i^\dagger}(\epsilon)\right]\times\left[\lim_{\epsilon\to 0^+} K_t^i(\epsilon)\right] \\ &= K_t(i)K_t^{i^\dagger}K_t^i, \quad i = 1,2,\ldots,l. \end{aligned} \tag{17.110}$$

Thus we have finished the proof [8,13,14]. ■

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