Discrete Mathematics
Chapters 5.4 & Supplement A
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Exercise Set 5.4

2. Suppose $b_1, b_2, b_3, ...$ is a sequence defined as follows:

$$\begin{aligned} b_1 =& 4, b_2 = 12 \\ b_k =& b_{k-2} + b_{k-1} \text{ for all integers } k \geq 3 \end{aligned}$$

Prove that b_n is divisible by 4 for all integers $n \geq 1$.

7. Suppose $g_1, g_2, g_3, ...$ is a sequence defined as follows:

$$\begin{aligned} g_1 =& 3, g_2 = 5 \\ g_k =& 3g_{k-1} - 2g_{k-2} \text{ for all integers } k \geq 3 \end{aligned}$$

Prove that $g_n = 2^n + 1$ all integers $n \geq 1$.

9. Define a sequence $a_1, a_2, a_3, ...$ as follows: $a_1 = 1, a_2 = 3$, and $a_k = a_{k-1} + a_{k+2}$ for all integers $k \geq 3$. Use strong mathematical induction to prove that $a_n \leq \left(\frac{7}{4}\right)^n$ for all integers $n \geq 1$.

18. Compute $9^0, 9^1, 9^2, 9^3, 9^4$, and 9^5. Make a conjecture about the units digit of 9^n where n is a positive integer. Use strong mathematical induction to prove your conjecture.

19. Find the mistake in the following "proof" that purports to show that every nonnegative integer power of every nonzero real number is 1.
"Proof: Let r be any nonzero real number and let the property $P(n)$ be the equation $r^n = 1$.
Show that $P(0)$ **is true:** P(0) is true because $r^0 = 1$ by defintion of zeroth power.

Show that for all integers $k \geq 0$, if $P(i)$ is true for all integers i from 0 through k, then $P(k+1)$ is also true: Let k be any integer with $k \geq 0$ and suppose that $r^i = 1$ for all integers i from 0 through k. This is the inductive hypothesis. We must show that $r^{k+1} = 1$. Now

$$\begin{aligned} r^{k+1} &= r^{k+k-(k-1)} && (k+k-(k-1) = k+k-k+1 = k+1) \\ &= \frac{r^k \cdot r^k}{r^{k-1}} && \text{(by the laws of exponents)} \\ &= \frac{1 \cdot 1}{1} && \text{(by inductive hypothesis)} \\ &= 1. \end{aligned}$$

Thus $r^{k+1} = 1$ *[as was to be shown].*
[Since we have proved the basis step and the inductive step of the strong mathematical induction, we conclude that the given statement is true.]"

21. Use the well-ordering principle for the integers to prove the existence part of the unique factorization of integers theorem: Every integer greater than 1 is either prime or a product of prime numbers.

26. Suppose $P(n)$ is a property such that

    1. $P(0), P(1), P(2)$ are all true,
    2. for all integers $k \geq 0$, if $P(k)$ is true, then $P(3k)$ is true. Must it follow that $P(n)$ is true for all integers $n \geq 0$? If yes, explain why; if no, give a counterexample.