Multi-variable Calculus
Problem Set 1

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1. What is wrong with each of the following statements? Explain briefly.
   (a) "In 3-space $y = 1$ is a line parallel to the x-axis."
   (b) "The graph of the function $f(x, y) = x^2 + y^2$ is a c

Ans: A1.

(a) In 2-space $y = 1$ is a line parallel to the x-axis, however in 3-space $y = 1$ is not a line but a plane that is parallel to the x-axis.

(b) For the function $f(x, y) = x^2 + y^2$ to be a circle it has to be a single variable function with value of x or y or z set to a constant c. This will then be a graph in 2-space of a circle centered at $(0, 0)$ with radius $\sqrt{c}$. However, the multi-variable function $f(x, y) = x^2 + y^2$ is not a circle but it is bowl shaped with contour diagrams showing circles, with radii that vary as $f(x, y)$ is set to different constants c where $c \geq 0$, concentric at $(0, 0)$

2. Sketch a graph of the surface $x^2 + y^2 + z^2 = 9$ and briefly describe it in words, geometrically. Make sure you mark the coordinates of at least one point in your sketch to reflect the scale of the sketch.

Ans: A2. The graph $x^2 + y^2 + z^2 = 9$ is a sphere with radius 3 units centered at the origin

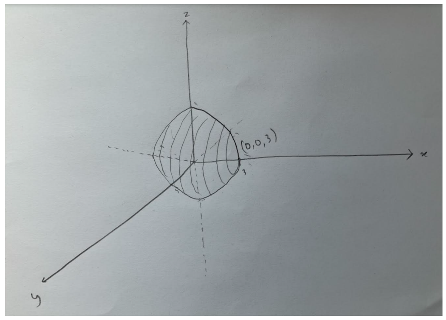

Figure 1

3. The following problems concern the concentration, C, in mg/liter, of a drug in the blood as a function of x, the amount, in mg, of the drug given and t, the time in hours since the injection. For $1 \leq x \leq 4$ and $t \geq 0$ we have $C = f(x,t) = te^{-t(5-x)}$.
   (a) Find $f(3,2)$. Give units and interpret in terms of drug concentration.
   (b) Graph the single variable function $f(4,t)$ (in the variable t) and explain its significance in terms of drug concentration.
   (c) Graph $f(a,t)$ for $a = 1,2,3,4$ on the same axes. Describe how the graph changes as a increases and explain what this means in terms of drug concentration.

Ans: A3.

(a) $C = f(x,t) = te^{-t(5-x)}$
$f(3,2) = ?$
$x = 3$ meaning that 3 mg of the drug was given
$t = 2$ meaning that 2 hours have passed since the injection
C or concentration of drug in mg/liter of 3 mg after 2 hours will be:
$C = 2e^{-2(5-3)} = 0.04$ mg/liter

The concentration in blood of 3 mg of the drug after 2 hours is 0.04 mg/liter

(b) Drug concentration (mg/l) as a function of x, the amount in mg of the drug given, and t, the time in hours since the injection with x held constant at 4 mg

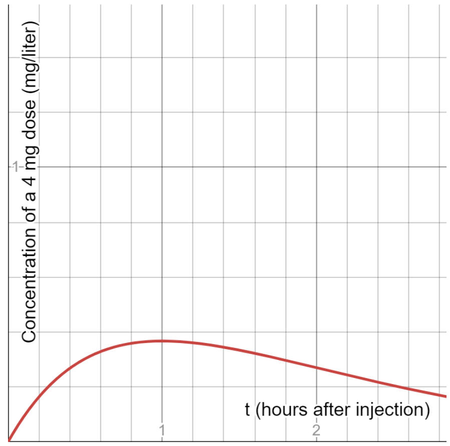

Figure 2

(c) As more mg of the drug is given, the maximum concentration of the dose in the blood in mg/l increases. This can be seen on the graphs as the peaks of the functions $f(4,t)$, $f(3,t)$, $f(2,t)$, and $f(1,t)$. The highest peak is at $a = 4$ and the lowest peak is at $a = 1$. The time t hours since the injection for the drug to reach its maximum concentration increases as the value of a or the the value of the dose given increases.

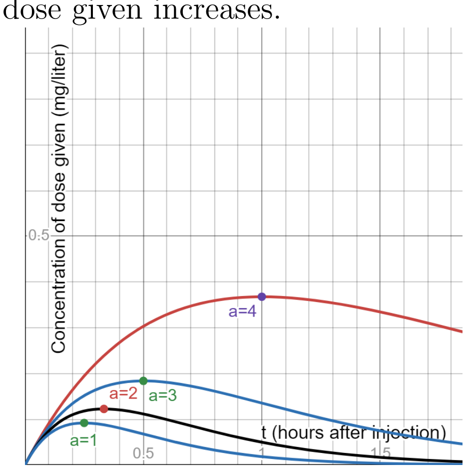

Figure 3

4. Without a calculation or computer, match the functions (a)-(f) with their cross-sections with x fixed in the Figure 3. Explain your reasoning

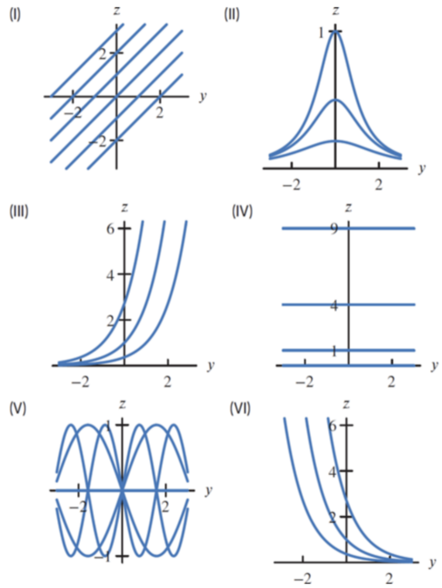

Figure 4

Ans: A4.

(a) (II) shows the cross-sections of function $z = \frac{1}{(1+x^2+y^2)}$ with x fixed as the cross-sections are parabolic.

(b) (I) shows the cross-sections of function $z = 1+x+y$ as the function is linear and will have contour lines that are parallel and equally spaced.

(c) We are looking for cross-sections showing an increasing value for z as y increases for any fixed value of x. This is the case in (IV) so these must be the cross-sections for the function $z = e^{-x+y}$.

(d) We are looking for cross-sections showing a decreasing value for z as y increases for any fixed value of x. This is the case in (IV) so these must be the cross-sections for the function $z = e^{x-y}$.

(e) (V) shows periodic behavior so it has to be the graph showing cross-sections of the function $z = sin(xy)$.

(f) (IV) shows contour lines where z is not changed by y. The z-intercept is always a square number so these are the cross-sections of the function $z = x^2$.