Discrete Mathematics
Chapter 9.5 Homework

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Exercise Set 9.5

4. Write an equation relating $P(8,3)$ and $\binom{8}{3}$.

> **Ans:** $\frac{P(8,3)}{3!}$

6. Use Theorem 9.5.1 to compute each of the following
   (c) $\binom{6}{2}$
   (d) $\binom{6}{3}$
   (e) $\binom{6}{4}$
   (f) $\binom{6}{5}$
   (g) $\binom{6}{6}$

> **Ans:**
> (c) $\binom{6}{2} = \frac{6!}{2!(6-2)!} = 15$
>
> (d) $\binom{6}{3} = \frac{6!}{3!(6-3)!} = 20$
>
> (e) $\binom{6}{4} = \frac{6!}{4!(6-4)!} = 15$
>
> (f) $\binom{6}{5} = \frac{6!}{5!(6-5)!} = 6$
>
> (g) $\binom{6}{6} = \frac{6!}{6!(6-6)!} = 1$

20. (a) How many distinguishable ways can the letters of the word MILLIMICRON be arranged in order?
    (b) How many distinguishable orderings of the letters of MILLIMICRON begin with U and end with L?

(c) How many distinguishable orderings of the letters of MILLIMICRON contain the two letters HU next to each other in order?

Ans:
(a) $11! = 39916800$ ways
(b) $9! = 362880$ orderings
(c) $9! = 362880$ orderings

21. In Morse code, symbols are represented by variable-length sequences of dots and dashes. How many different symbols can represented by sequences of seven or fewer dots and dashes?

Ans: $7! + 6! + 5! + 4! + 3! + 2! + 1! = 5913$ different symbols.

22. Each symbol in the Braille code is represented by a rectangular arrangement of six dots, each of which may be raised or flat against a smooth background. Given that at least one of the six dots must be raised, how many symbols can be represented in the Braille code?

Ans: $2^6 - 1 = 63$ symbols.

24. The number 42 has the prime factorization $2 \cdot 3 \cdot 7$. Thus 42 can be written in four ways as a product of two positive integer factors (without regard to the order of the factors): 1· 42, 2· 21, 3· 14 and 6· 7. Answer a,c,&d below without regard to the order of the factors.
    (a) List the distinct ways the number 210 can be written as a product of two positive integer factors.
    (c) If $n = p_1p_2p_3p_4p_5$, where the p_i are distinct prime numbers, how many ways can n be written as a product of two positive integer factors?
    (d) If $n = p_1p_2....p_k$, where the p_i are distinct prime numbers, how many ways can n be written as a product of two positive integer factors?

Ans:

(a) $210 = 2 \cdot 3 \cdot 5 \cdot 7$. Thus 210 can be written as $1 \cdot 210, 2 \cdot 105, 3 \cdot 70, 5 \cdot 42, 7 \cdot 30, 10 \cdot 21, 14 \cdot 15$.

(c) Let $S = \{p_1, p_2, p_3, p_4, p_5\}$. Let $P = p_1p_2p_3p_4p_5$, and let f_1f_2 be any factorization of P. The product of the numbers in any subset $A \subseteq S$ can be used for f_1, with the product of the numbers in A^c being f_2. There are many ways to write f_1f_2 as there are subsets of S, namely $2^5 = 32$ (by Theorem 6.3.1). But given any factors f_1 and f_2, $f_1f_2 = f_2f_1$. Thus counting the number of ways to write f_1f_2 counts each factorization twice, so the answer is $\frac{32}{2} = 16$.

(d) In $\frac{2^k}{2}$ ways.

27. Let A be a set with eight elements.

(b) How many relations on A are reflexive?

(d) How many relations on A are both reflexive and symmetric?

Ans:

(b) The total set of relations is the number of subsets of the set $A \times A$ and so there are 2^{64} possible relations. For a relation to be reflexive, there must be n ordered pairs of the form (a, a) for each element a in the set. This gives 2^{64-8} or 2^{56} relations.