Discrete Mathematics
Extra Problems

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1. Prove or disprove the following. If true, use logical equivalences and specify the laws at each step.

$$(p \longrightarrow q) \leftrightarrow q \equiv p \vee q$$

Ans: True.

$$\begin{aligned}
&(p \longrightarrow q) \equiv \neg p \vee q && \text{(By representation of If-Then as or)}\\
&\neg p \vee q \rightarrow q \wedge q \rightarrow \neg p \vee q && \text{(By definition of the biconditional)}\\
&((p \wedge \neg q) \vee q) \wedge (\neg q \vee (\neg p \vee q)) && \\
& && \text{(By representation of If-Then as or and DeMorgaan's)}\\
&((p \vee q) \wedge \neg q \vee q) \wedge ((\neg q \vee \neg p) \vee (\neg q \vee q)) && \text{(By distributive laws)}\\
&(p \vee q \wedge T) \wedge (\neg q \vee \neg p \vee T) && \text{(By negation laws)}\\
&p \vee q && \text{(By identity and universal bound laws)}
\end{aligned}$$

8. Write the following statement and its negation without using English phrases. Indicate which is true
If x and y are integers where $x - y = 8$ then $xy + 16$ is a perfect square

Ans:

$$\forall x, y \in \mathbb{Z}, x - y = 8 \longrightarrow \exists k \in \mathbb{Z},\ xy + 16 = k^2$$

The negation would then be

$$\exists x, y \in \mathbb{Z}, x - y = 8 \wedge \forall k \in \mathbb{Z},\ xy + 16 \neq k^2$$

Proof. Because $x - y = 8$ we can rearrange to get $y = x - 8$. Substituting this into $xy + 16$ we get

$$x(x-8) + 16 \tag{1}$$

$$x^2 - 8x + 16 = (x-4)(x-4) = (x-4)^2 \tag{2}$$

Because $xy + 16$ can be written as the square integer $(x-4)^2$ then it will always be a perfect square by definition. $\square$

9. Write the following statement as a universal conditional without using English phrases. Assume that the domain is the real numbers.
A sufficient condition for a set to contain only perfect squares is that it be empty.

Ans:

$$\forall X \subseteq \mathbb{R}, |X| = 0 \longrightarrow \forall x \in X, \exists k \in \mathbb{Z}, x = k^2$$