Chapter 5 (Sep. 9)
Indirect proof of conditional statements

$$p \longrightarrow q$$

1. Proof by contradiction
2. Contra-positive Proof

Proposition. *If P, then Q*

Proof. Suppose $\neg Q$
(details)
Therefore $\neg P$ $\square$

P	Q	$P \implies Q$	$\neg Q$	$\neg P$	$\neg Q \implies \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Chapter 5, Exercise 2

$$\text{Suppose } x \in \mathbb{Z}, \text{If } x^2 \text{ is odd, then } x \text{ is odd}$$

Two options:
1. Direct proof

$$x^2 \text{ is odd } (P) \implies$$
$$x^2 = 2a + 1, a \in \mathbb{Z} \implies$$
$$?$$
$$x = 2b + 1, b \in \mathbb{Z} \implies$$
$$x \text{ is odd } (Q)$$

2. Indirect proof

$$x \text{ is even } (\neg Q) \implies$$
$$x = 2a, a \in \mathbb{Z} \implies$$
$$x^2 = (2a)^2 = 4a^2 = 2(2a^2)$$
$$b = 2a^2, b \in \mathbb{Z} \implies$$
$$x^2 \text{ is even } (\neg P)$$

Proof. We suppose the contra-positive of the given statement. This means we suppose x is not odd, and we argue that x^2 is not odd. So suppose x is even. Then $x = 2a$ for some $a \in \mathbb{Z}$ by definition of even. This means

$$x^2 = (2a)^2 = 4a^2 = 2(2a^2).$$

Let $b = 2a$. Then $x^2 = 2b$, where $b \in \mathbb{Z}$, so we can conclude that x^2 is even by definition. $\square$

Chapter 5, Exercise 4

Suppose $a, b, c \in \mathbb{Z}$. If a does not divide bc, then a does not divide b.

Recall

$$a \mid b \Leftrightarrow b = a \cdot c, a \in \mathbb{Z}$$

Two options:
1. Direct proof

$$a \nmid c \ (P) \Rightarrow$$

$$?$$

$$a \nmid b \ (Q)$$

2. Indirect proof:

$$a \mid b \ (\neg Q) \Rightarrow$$

$$b = ad, d \in \mathbb{Z} \Rightarrow$$

$$bc = (a \cdot d) \cdot c = a \cdot (d \cdot c)$$

$$e = d \cdot c, e \in \mathbb{Z} \Rightarrow$$

$$a \mid bc \ (\neg P)$$

Definition of congruent modulo n

$$a \equiv b \Leftrightarrow n \mid a - b$$

$$\Leftrightarrow a - b = n \cdot c, c \in \mathbb{Z}$$

$$\Leftrightarrow a \text{ and } b \text{ have the same remainder when divided by } n$$

Chapter 8 (Oct. 9) - Proofs involving sets
Example 8.8: Prove that if A and B are sets then $\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$

Proof.

$$X \in \mathscr{P}(A) \cup \mathscr{P}(B) \Longrightarrow$$

$$X \in \mathscr{P}(A) \text{ or } \mathscr{P}(B) \Longrightarrow$$

$$X \subseteq A \text{ or } X \subseteq B \Longrightarrow$$

$$X \subseteq A \cup B \implies$$
$$X \in \mathscr{P}(A \cup B)$$
$$\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$$

□

Example 8.9: Suppose A and B are sets. If $\mathscr{P}(A) \subseteq \mathscr{P}(B)$, then $A \subseteq B$. (Page 162) How do we get proofs with aligned equations on LaTeX(like Example 8.13)?

Proof.

$$2x - 5y = y$$
$$3x + 9y = 3$$

□