

Discrete Mathematics

Chapters 5.9, 6.1 & 6.2 Homework

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Exercise Set 5.9

6. Define a set S recursively as follows:

- I. BASE: $a \in S$
- II. RECURSION: If $s \in S$, then,
 - a. $sa \in S$
 - b. $sb \in S$
- III. RESTRICTION: Nothin is in S other than objects defined in I and II above.

Use structural induction to prove that every string in S begins with an a .

11. Define a set S recursively as follows:

- I. BASE: $0 \in S$
- II. RECURSION: If $s \in S$, then,
 - a. $s + 3 \in S$
 - b. $s - 3 \in S$
- III. RESTRICTION: Nothin is in S other than objects defined in I and II above.

Use structural induction to prove that integer in S is divisible by 3.

16. Give a recursive definition for the set of all strings of 0's and 1's for which all the 0's precede all the 1's.

18. Give a recursive definition for the set of all strings of a 's and b 's that contain an odd number of a 's.

25. Student C tries to define a function $G : \mathbb{Z}^+ \rightarrow \mathbb{Z}$ by the rule

$$G(n) = \begin{cases} 1 & \text{if } n \text{ is } 1 \\ G\left(\frac{n}{2}\right) & \text{if } n \text{ is even} \\ 2 + G(3n - 5) & \text{if } n \text{ is odd and } n > 1 \end{cases}$$

for all integers $n \geq 1$. Student D claims that G is not well defined. Justify student D 's claim.

Exercise Set 6.1

Exercise Set 6.2