Chapter 5, 10 - Prove using contrapositive proof

Proposition. *Suppose $x, y, z \in \mathbb{Z}$ and $x \neq 0$. If $x \nmid yz$, then $x \nmid y$ and $x \nmid z$*

Proof. We employ contrapositive proof. Suppose $x \mid y$ or $x \mid z$. By definition $x \mid y$, $y = xq$ for some integer q. By definition $x \mid z, z = ax$ for some integer a. From these two definitions of y and z we can write yz equals

$$yz = xq \cdot ax$$

We then rearrange this into $aqx \cdot x$ and because the products of integers are integers we can write $k = aqx$ where $k \in \mathbb{Z}$

$$yz = kx$$

We have written yz as k multiples of x. Therefore $x \mid yz$. □

Chapter 5, 25 - Prove using either direct or contrapositive proof

Proposition. *Let $n \in \mathbb{N}$. If $2^n - 1$ is prime, then n is prime.*

Proof. Suppose $2^n - 1$ is prime, then by definition there are exactly two positive divisors of $2^n - 1$ which are $2^n - 1$ and 1. □