Discrete Mathematics
Chapters 9.1,9.2,9.3 & 9.9 Homework

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Exercise Set 9.1

24. If the largest of 87 consecutive integers is 326, what is the smallest?

Ans:

$$87 = 326 - m + 1$$
$$m = 240$$

The smallest integer is 240.

Exercise Set 9.2

17. (a) How many integers are there from 1000 through 9999?
    (b) How many odd integers are there from 1000 through 9999?

Ans:
(a) 9000
(b) $9 \cdot 9 \cdot 9 \cdot 5 = 3645$

36. Prove that for all integers $n \geq 3$,

$$P(n+1, 3) - P(n, 3) = 3P(n, 2)$$

Ans:

Proof. Suppose n is an integer greater than or equal to 3, then

$$P(n+1,3) = \frac{(n+1)!}{(n-2)!} = (n+1)(n)(n-1) = n^3 - n$$

$$P(n,3) = \frac{n!}{(n-3)!} = n(n-1)(n-2) = n^3 - 3n^2 + 2n$$

$$P(n+1,3) - P(n,3) = n^3 - n - n^3 + 3n^2 - 2n = 3n^2 - 3n$$

but $3P(n,2)$ is equal to $3 \cdot \frac{n!}{(n-2)!}$ or $3(n(n-1)) = 3n^2 - 3n$. Therefore $P(n+1,3) - P(n,3) = 3P(n,2)$. □

37. Prove that for all integers $n \geq 2$, $P(n,n) = P(n,n-1)$.

Ans:

Proof. Suppose n is an integer greater than or equal to 2.

$$P(n,n) = \frac{n!}{(n-n)!} = n!$$

But $P(n,n-1)$ is equal to $\frac{n!}{1!}$ or $n!$. Therefore $P(n,n) = P(n,n-1)$. □

Exercise Set 9.3

23. (a) How many integers from 1 through 1,000 are multiples of 2 or multiples of 9?
    (b) Suppose an integer from 1 through 1,000 is chosen at random. Use the result of part (a) to find the probability that the integer is a multiple of 2 or a multiple of 9.
    (c) How many integers from 1 through 1,000 are neither multiples of 2 nor multiples of 9?

Ans:

(a) Let A be the set of multiples of 2 and B be the set of multiples of 9. $N(A) = 500$ and $N(B) = 111$. The intersection, or the set of multiples of 18 will then have $N(A \cap B) = 55$. Therefore, there are $500 + 111 - 55$ or 556 integers that are multiples of 2 or multiples of 9.

(b) $\frac{556}{1000} = 0.556$

(c) $(1 - 0.556) \cdot 1000 = 444$

Exercise Set 9.9

24. A company uses two proofreaders X and Y to check a certain manuscript. X misses 12% of typographical errors and Y misses 15%. Assume that the proofreaders work independently.

    (a) What is the probability that a randomly chosen typographical error will be missed by both proofreaders?

    (b) If the manuscript contains 1,000 typographical errors, what number can be expected to be missed?

Ans:

(a) $0.12 \cdot 0.15 = 0.018$ or 18%

(b) $1000 \cdot 0.018$ or 18 errors.