

Discrete Mathematics

Homework 3

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1. A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.
- (a) Every animal in the menagerie is brown or gray or black.
 - (b) There is an animal in the menagerie that is neither a cat nor a dog.
 - (c) No animal in the menagerie is blue.
 - (d) There are in the menagerie a dog, a cat, and a bird that all have the same color.

Ans:

- (a) False. There is a blue bird.
- (b) True.
- (c) False. There is a blue bird.
- (d) True.

2. Find a counterexample to show that the statement below is false.

$$\forall a \in \mathbb{Z}, (a - 1)/a \text{ is not an integer.}$$

Ans: The negation of the statement would be $\exists a \in \mathbb{Z}, (a - 1)/a$ is an integer. If $a = 0$, $a - 1/1 = 0$ which is $\in \mathbb{Z}$. Since the negation is true for $a = 0$ then the original statement is false.

3. Rewrite each of the following statements in the form " \forall _____ x , _____".
- (a) Every real number is positive, negative, or zero.

- (b) No logicians are lazy.
 (c) The number -1 is not equal to the square of any real number.

Ans: A3. (double check a)

- (a) \forall real numbers $x, x > 0 \vee x < 0 \vee x = 0$
 (b) \forall logicians l, l is not lazy
 (c) \forall real numbers $x, x^2 \neq -1$

4. In any mathematics or computer science text other than this book, find an example of a statement that is universal but is implicitly quantified. Copy the statement as it appears and rewrite it making the quantification explicit. Give a complete citation for your example, including title, author, publisher, year, and page number.

Ans: Theorem 9.1: Convergence of a Monotone, Bounded Sequence

If a sequence s_n is bounded and monotone, it converges.

For every sequence s_n , if s_n is bounded and monotone, s_n converges.

Citation: Calculus 6th edition by Hughes-Hallett, Gleason, McCallum et al., Wiley & Sons, Inc., 2013, page 495

5. Let \mathbb{R} be the domain of the predicate variable x . Which of the following are true and which are false? Give counterexamples for the statements that are false.
- (a) $ab = 0 \implies a = 0$ or $b = 0$
 (b) $a < b$ and $c < d \implies ac < bd$

Ans:

- (a) True.
 (b) False.
 Example (1)
 $a = -2, b = -1, c = -5, d = -3$
 $a < b$ and $c < d$
 $ac \geq bd$

Example (2)

$$a = 1, b = 2, c = -5, d = -4$$

$$a < b \text{ and } c < d$$

$$ac \geq bd$$

6. Which of the following is a negation for "All dogs are loyal"? More than one answer may be correct.
- (a) All dogs are loyal.
 - (b) No dogs are loyal.
 - (c) Some dogs are disloyal.
 - (d) Some dogs are loyal.
 - (e) There is a disloyal animal that is not a dog.
 - (f) There is a dog that is disloyal.
 - (g) No animals that are not dogs are loyal.
 - (h) Some animals that are not dogs are loyal.

Ans: (c) and (f)