

Discrete Mathematics

Chapter 5.1 Homework

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Exercise Set 5.1

9. Find explicit formulas for the sequence of the form a_1, a_2, a_3, \dots with the initial terms below

$$1 - \frac{1}{2}, \frac{1}{2} - \frac{1}{3}, \frac{1}{3} - \frac{1}{4}, \frac{1}{4} - \frac{1}{5}, \frac{1}{5} - \frac{1}{6}, \frac{1}{6} - \frac{1}{7}$$

Ans:

$$a_k = \frac{1}{k} - \frac{1}{k+1} \text{ for all integers } k \geq 0$$

29. Evaluate the summation for the indicated values of the variable

$$1(1!) + 2(2!) + 3(3!) + \dots + m(m!); m = 2$$

Ans:

$$= 1(1!) + 2(2!)$$

$$= 1 + 4$$

$$= 5$$

43. Write using summation or product notation

$$(1-t) \cdot (1-t^2) \cdot (1-t^3) \cdot (1-t^4)$$

Ans:

$$\prod_{k=1}^4 (1 - t^k)$$

52. Transform by making the change of variable $j = i - 1$

$$\sum_{i=1}^{n-1} \frac{i}{(n-1)^2}$$

Ans:

$$i = j + 1$$

$$j = i - 1$$

When $i = 1$, then $j = 1 - 1 = 0$ so the lower limit is 0. When $i = n - 1$, $j + 1 = n - 1$, $j = n - 2$ so the upper limit is $n - 2$.

Substituting $i = j + 1$ we then have

$$\sum_{i=1}^{n-1} \frac{i}{(n-i)^2} = \sum_{j=0}^{n-2} \frac{j+1}{(n-j-1)^2}$$

55. Write as a single summation or product

$$2 \cdot \sum_{k=1}^n (3k^2 + 4) + 5 \cdot \sum_{k=1}^n (2k^2 - 1)$$

Ans: By Theorem 5.1.1 (2)

$$2 \cdot \sum_{k=1}^n (3k^2 + 4) + 5 \cdot \sum_{k=1}^n (2k^2 - 1) = \sum_k^n = (6k^2 + 8) + \sum_{k=1}^n (10k^2 - 5)$$

By Theorem 5.1.1 (1)

$$\begin{aligned}
 &= \sum_{k=1}^n (6k^2 + 8 + 10k^2 - 5) \\
 &= \sum_{k=1}^n 16k^2 + 3
 \end{aligned}$$

Compute 59 & 62. Assume the values of the variables are restricted so the functions are defined.

59.

$$\frac{4!}{0!}$$

Ans: $0! = 1$ by definition and $4! = 4 \cdot 3 \cdot 2 \cdot 1$

$$\begin{aligned}
 &= \frac{24}{1} \\
 &= 24
 \end{aligned}$$

62.

$$\frac{n!}{(n-2)!}$$

Ans:

$$\begin{aligned}
 &= \frac{n(n-1)(n-2)!}{(n-2)!} \\
 &= n(n-1) \\
 &= n^2 - n
 \end{aligned}$$

74. Prove that if p is a prime number and r is an integer with $0 < r < p$, then $\binom{p}{r}$ is divisible by p .

Ans: Insert Solution Here...