Discrete Mathematics
Chapters 5.2 & 5.3 Homework

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Exercise Set 5.2

2. Use mathematical induction to show that any postage of at least 12¢ can be obtained using 3¢ and 7¢ stamps.

4. For each integer n with $n \geq 2$, let $P(n)$ be the formula

$$\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$$

   (a) Write $P(2)$. Is $P(2)$ true?

   (b) Write $P(k)$.

   (c) Write $P(k+1)$.

   (d) In a proof by mathematical induction that the formula holds for all integers $n \geq 2$, what must be shown in the inductive step?

   Prove by mathematical induction. Do not derive from Theorem 5.2.2 or Theorem 5.2.3.

7. For all integers $n \geq 1$,

$$1 + 6 + 11 + 16 + ... + (5n - 4) = \frac{n(5n-3)}{2}.$$

   Prove by mathematical induction.

12. $$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}, \text{ for all integers } n \geq 1$$

   Use the formula for the sum of the first n integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 23 & 29.

23.

$$7 + 8 + 9 + 10 + ... + 600$$

29.

$$1 - 2 + 2^2 - 2^3 + ... + (-1)^n 2^n, \text{ where } n \text{ is a positive integer}$$

Find the mistake in the proof fragment below.

35. **Theorem:** For any integer $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

"Proof (by mathematical induction): Let the property $P(n)$ be $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$

Show that $P(1)$ **is true:** When $n = 1$

$$\sum_{i=1}^{1} i(i!) = (1+1)! - 1$$

So $1(1!) = 2! - 1$

and $1 = 1$

Thus $P(1)$ is true."

Exercise Set 5.3

2. Experiment with computing values of the product $(1+\frac{1}{2})+(1+\frac{1}{3})...(1+\frac{1}{n})$ for small values of n to conjecture a formula for this product for general n. Prove your conjecture by mathematical induction.

5. Evaluate the sum $\sum_{k=1}^{n} \frac{k}{(k+1)!}$ for $n = 1, 2, 3, 4$, and 5.
Make a conjecture about a formula for this sum for general n, and prove your conjecture by mathematical induction.

12. For any integer $n \geq 0, 7^n - 2^n$ is divisible by 5.

28. Prove that for all integers $n \geq 1$,

$$\frac{1}{3} = \frac{1+3}{5+7} = \frac{1+3+5}{7+9+11} = ...$$

$$= \frac{1 + 3 + ... + (2n - 1)}{(2n + 1) + ... + (4n - 1)}.$$

35. Let m and n be any integers that are greater than or equal to 1.

    (a) Prove that a necessary condition for an $m \times n$ checkerboard to be completely coverable by L-shaped trominoes is that mn be divisible by 3.

    (b) Prove that having mn divisible by 3 is not a sufficient condition for an $m \times n$ checkerboard to be completely coverable by L-shaped trominoes.