

HOMEWORK #4

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Answer of Question 1)

n zero text, 0010

Algorithm makes 2 successful compare and 1 unsuccessful compare because of input 1. So number of character comparison is $3 \cdot n = 3n$

For the worst case of input 3-bit pattern, worst input is 001 in 3-bit. So the worst case $O(3n)$

Answer of Question 2)

A to E = 3, E to B = 1, B to C = 6, C to D = 2

$$3 + 1 + 6 + 2 = 12$$

Upper side is an example of applying brute-force algorithm.

There is a lot of road to reach every city.

The answer is = A to D = 4, D to C = 2, C to E = 4, E to B = 1

$$4 + 2 + 4 + 1 = 11$$

the smallest reached number

Answer of Question 3)

Algorithm - Q3 (n):

if $n == 1$
return 0

else
return $1 + \text{Algorithm-Q3}(\text{floor}(n/2))$

end

Time complexity of this algorithm =

$$T(n) = \begin{cases} 0 & n=1 \\ 1 + T(\text{floor}(n/2)) & n > 1 \end{cases}$$

If we use Master Theorem

$$a=1 \quad b=2 \quad d=0$$

$$a = b^d$$

$$1 = 2^0 \text{ then } O(n^d \log n)$$

So Time complexity is $O(n^0 \log n) = \boxed{O(\log n)}$

Answer of Question 4)

Algorithm - Q4 (array bottles):

while length of bottles > 1 :

$n = \text{length of bottles} / 2$

$L1 = \text{bottles}[0, 1, \dots, n-1]$

$L2 = \text{bottles}[n, \dots, 2n-1]$

if (weight of $L1 < \text{weight of } L2$)

$\text{check} = 1$

end if

else if (weight of $L1 > \text{weight of } L2$)

$\text{check} = -1$

end if

else

$\text{check} = 0$

end else

if ($\text{check} == 1$)

$\text{bottle} = L1$

end if

else if ($\text{check} == -1$)

$\text{bottle} = L2$

end if

else if ($\text{check} == 0$)

$L3 = \text{bottle}[2n]$

$\text{bottle} = L3$

end if

end while

return bottle[0]

end function

The Time Complexity for this algorithm =

The best case = $\boxed{O(1)}$ \rightarrow if length of bottle = 1

The Average case - and the worst case = $\boxed{O(\log_3 n)}$

Because if we use master theorem

$$T(n) = T(n/3) + 6$$

$$a=1 \quad b=3 \quad d=0$$

$$a = b^d \Rightarrow 1 = 3^0 \Rightarrow O(n^d \log_3 n) = \boxed{O(\log_3 n)}$$

Explanation =

In my algorithm, I split the bottles 0 to length/2 and length/2 to length. Then I check weight of them. To be incorrect bottle, it must be less weight than the others because there is no way to fill the bottle more weight than its volume. So I compare them and if weight of first one is less than the other then, result in the first one. if more than the other one then the result in the second one. if equals, then result is the last one. These all in while part. It loops until one lasts. Then it returns first element of the list and this is result.

Answer of Question 5)

h-algorithm-Question 5(array1, n, array2, m, x):

```
if (x > n+m)
    return -1 // fail
```

```
end if
```

```
if (x ≤ 0)
    return -1
```

```
end if
```

```
if (n > m)
    return h-algorithm-Question 5(array2, m, array1,
                                   n, x)
```

```
end if
```

```
if (n == 0)
    return array2[x-1]
```

```
end if
```

```
if (x == 1)
    if (array[0] < array2[0])
```

```
        return array[0]
```

```
    end if
```

```
    else return array2[0]
```

```
    end else
```

```
end if
```

```
if (n < x/2)
```

```
    i = n
```

```
end if
```

```
else
```

```
    i = x/2
```

```
end else
```


if ($m < x/2$)

$J = m$

end if

else

$J = x/2$

end else

if ($\text{array}[i-1] > \text{array}[J-1]$)

return $\text{h-algorithm-Question 5}(\text{array 1}, 1, \text{array 2} + J, m - J, x - J)$

end if

else

return $\text{Algorithm - Question 5}(\text{array 1} + i, m - i, \text{array 2}, m, x - i)$

end else

end function

$\text{Algorithm - Question 5}(\text{array}, \text{array 2}, x) :$

$n = \text{length of array}$

$m = \text{length of array 2}$

MergeSort(array)

MergeSort(array 2)

$h - \text{algorithm - Question 5}(\text{array}, n, \text{array 2}, m, x)$

end function.

The worst case = $\overbrace{O(n \log n)}^{\text{MergeSort}} + \overbrace{O(n \log n)}^{\text{MergeSort}} + \overbrace{O(\log(n+m))}^{h\text{-algorithm-Question 5}} = \boxed{O(n \log n)}$

↳ Because At most it can take $n+m$, in code first if part.