(14.) EVERY POSITIVE INT N $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$

$P(1) = 2 = (1-1)\ 2^{1+1} + 2 = 2$

$P(k) = \sum_{i=1}^{k} i2^i = (k-1)2^{k+1} + 2$

$P(k+1) = \sum_{i=1}^{k} i2^i + (k+1)\cdot 2^{k+1} = (k-1)2^{k+1} + 2 + (k+1)\cdot 2^{k+1}$

$(k-1)2^{k+1} + 2 + (k+1)\cdot 2^{k+1}$

$2^{k+1}\cdot(k-1+k+1) + 2$

$2^{k+1}\cdot 2k + 2$

$\sum_{i=1}^{k+1} k2^k = k\cdot 2^{k+2} + 2$

THIS SHOWS THAT IF P(K) IS TRUE THEN P(K+1) IS TRUE BY INDUCTION.

SO BY INDUCTION P(n) IS TRUE FOR ALL POSITIVE INTS.

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18. P(n) $n! < n^n$

a) $P(2)$: $2! < 2^2$

b) $P(2)$ $2 < 4$
TRUE

c) $P(k) = k! < k^k$

d) WE WANT TO SHOW $(k+1)! < (k+1)^{(k+1)}$ FOR ANY INT > 1.

e) $k!\cdot(k+1) < k^k\cdot(k+1) < (k+1)^k(k+1) = (k+1)^{k+1}$

f) Since we have shown a basis and inductive step are true, by induction, then the statement is true for all ints > 1.