Quiz 3

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1. Use proof by contraposition to prove the following statement -
Suppose $a, b \in Z$. If $a^2(b^2 - 2b)$ is odd, then a and b are odd.

Let: p = If $a^2(b^2 - 2^b)$ is odd; q = then a and b are odd
$p \to q$; contraposition: $\neg q \to \neg p$ This states that if a or b is even, then $a^2(b^2 - 2b)$ is even.
If a is positive number then let: $a = 2k$:
$(2k)^2(b^2 - 2b) = 4k^2(b^2 - 2b) = 4k^2b^2 - 8k^2b = 2(2k^2b^2 - 4k^2b)$
This proves that if a is a positive number then $a^2(b^2 - 2b)$ is even.
If b is a positive number then let $b = 2k$:
$a^2(2k^2 - 2^{2k}) = 2(a^2k^2 - a^2)$ This proves that if b is a positive number then $a^2(b^2 - 2b)$ is even.
From these results we can conclude that if $a^2(b^2 - 2b)$ is odd, then a and b are odd by contraposition.

2. Use proof by contradiction to show that If a and b are rational numbers with $b \neq 0$ and x is an irrational number, then $a + bx$ is irrational.

For proof by contradiction let $a + bx$ be a rational number.
Let: $a + bx = \frac{c}{d}$, $a = \frac{p}{q}$ and $b = \frac{s}{t}$. Where d, q and $t \neq 0$
Then: $x = (\frac{c}{d} - \frac{p}{q}) * \frac{t}{s} = \frac{cqt - dpt}{dqs}$ Thus, x is a rational number, which is a contradiction.
Therefore, $a + bx$ is irrational.