43. USE STRUCTURAL INDUCTION TO SHOW THAT $n(T) \geq 2h(T) + 1$, WHERE T IS A FULL BINARY TREE, $n(T)$ EQUALS THE # OF VERTICES OF T, AND $h(T)$ IS the HEIGHT OF T.

BASE CASE: THE ROOT r IS A LEAF OF THE FULL BINARY TREE WITH EXACTLY ONE VERTEX r. THIS TREE HAS NO INTERNAL VERTICES.

RECURSIVE CASE: THE SET OF LEAVES OF THE TREE $T = T_1 \cdot T_2$ IS THE UNION OF THE SETS OF LEAVES OF T_1 AND OF T_2. THE INTERNAL VERTICES OF T ARE THE ROOT r of T AND THE UNION OF THE SET OF INTERNAL OF T_1 AND THE SET VERTICIES OF T_2

PROOF

BASE CASE: $n(T) = 1$ AND $h(T) = 0$; $1 \geq 2 \cdot 0 + 1$

RECURSIVE CASE: ASSUME THE RESULTS HOLD FOR ALL BINARY TREES SMALLER THAN T.

$n(T) \geq 2h(T) + 1$

$n(T_1) \geq 2h(T_1) + 1$ $\quad$ $n(T_2) \geq 2h(T_2) + 1$

$n(T) = 1 + n(T_1) + n(T_2)$ and $1 + \max(h(T_1), h(T_2))$

$$\begin{aligned} n(T) &= 1 + n(T_1) + n(T_2) \\ &\geq 1 + 2h(T_1) + 1 + 2h(T_2) + 1 \\ &\geq 1 + 2\max(h(T_1), h(T_2)) + 2 \\ &= 1 + 2(\max(h(T_1), h(T_2)) + 1) \\ &= 1 + 2h(T) \checkmark \end{aligned}$$

44. USE STRUCTURAL INDUCTION TO SHOW THAT $l(T)$, the number OF LEAVES OF A FULL BINARY TREE T, IS ONE MORE THAN $i(T)$, the NUMBER OF INTERNAL VERTICES OF T.

BASE CASE: A SINGLE root r HAS NO INTERNAL VERTICIES ✓

INDUCTIVE CASE: ASSUME THAT RESULTS HOLD FOR ALL FULL BINARY TREES SMALLER THAN T.

$l(T_1) = i(T_1) + 1$ $\quad$ $l(T_2) = i(T_2) + 1$ $\quad$ $l(T) = l(T_1) + l(T_2)$

$i(T) = i(T_1) + i(T_2) + 1$

$$\begin{aligned} l(T) &= l(T_1) + l(T_2) \\ &= i(T_1) + 1 + i(T_2) + 1 \\ &= i(T) + 1 \checkmark \end{aligned}$$