HW 4.2

Rob Navarro

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2. Use strong induction to show that all dominoes fall in an infinite arrangement of dominoes if you know that the first three dominoes fall, and that when a domino falls, the domino three farther down in the arrangement also falls.

$P(n)$: the dominoes at the $n, n+1$, and $n+2$ position fall.
Base Case: We know that $P(1)$ is true since we are told the first three dominoes fall.
Inductive Case: We can assume that $P(1), P(2)$, and $P(3)$ are true based on what we are told. We are also told that the domino three farther down falls so if $P(1)$ is true we know that $P(4)$ is also true. To prove that $P(m+1)$ is true we need to look back at $P(m-2)$, so if $m > 3$ we know that $n-2, n-1$, and n position fall. It is then safe to say that n, n + 1, and n + 2 will all fall for all possible integer n in an infinite set of dominoes. We have proven $P(n)$ is true for all positive integers.

Let P(n) be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of the exercise outline a strong induction proof that P(n) is true for n ? 18.

a) Show statements $P(18), P(19), P(20), P(21)$ are true, completing the basis step of the proof.
$P(18)$: 2 7-cent stamps and 1 4-cent stamp.
$P(19)$: 1 7-cent stamp and 3 4-cent stamps.
$P(20)$: 5 4-cent stamps.
$P(21)$: 3 7-cent stamps.
b) What is the inductive hypothesis of the proof?
$P(m)$: We can form any postage with 4-cent and 7-cent stamps when $m \geq 18$.
c) What do you need to prove in the inductive step?
If $P(m)$ is true then we need to show that P(m + 1) can be formed with just 4 and 7 cent stamps. d) Complete the inductive step for m ? 21. We already know that P(m - 3) is true based on the work done in step a. Based on this fact we know that P(m + 1) can be created by adding an additional 4-cent stamp to P(m - 3). Thus we have proven that P(m + 1) is true.

e) Explain why these steps show that this statement is true whenever n ? 18.
We have proven both the base case and inductive case, so by strong induction we know that the statement is true for every integer n greater than or equal to 18.

12. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on.

P(n): the positive integer n can be written as a sum of distinct powers of two.
Base Case: $P(1) : 1 = 2^0$ and $P(2) : 2 = 2^1$
Inductive Case: Assume that $P(j)$ whenever $j \leq k$

Even: When $k + 1$ is even we also know that $\frac{k+1}{2}$ is an integer, and that $\frac{k+1}{2} \leq k$. We know that $P(j)$ is true when $j \leq k$ and since $\frac{k+1}{2} \leq k$ we can assume that $\frac{k+1}{2}$ can be represented as a sum of distinct powers of two. So if k + 1 is an even number P(k + 1) is true.

Odd: If k + 1 is odd then k must be even. The only power of 2 that is odd is 2^0, and since k is even it can't have 2^0 as part of the summation. We then know that $k + 1 = k + 2^0$, which is a sum of distinct powers of two.

We have proven that P(k + 1) can be represented as a sum of distinct powers of two, which shows that the hypothesis was true. By strong induction P(n) is true for all positive integers $n \geq 3$.

30. Find out the flaw with the following "proof" that "$a^n = 1$" for all nonnegative integers n, whenever a is nonzero real number.

The flaw in this proof is that we can't assume that $a^1 = 1$. In the inductive proof steps, it involves a^k and a^{k-1}, which is incorrect.