Assignment 1.7

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2. Use a direct proof to show that the sum of two even integers is even.
Let: $n_1 = 2k$ and $n_2 = 2k$ where k is an integer
$n_1 + n_2 = 2k + 2k = 4k = 2(2k)$
Using the definition of an even integer we can see that the some of two even integers is a even integer.

4. Show that the additive inverse, or negative, of an even number is an even number using a direct proof.
Let: $n = 2k$ where k is an integer
The inverse of n is: $-n = -(2k)$ This shows that through the definition of an even integer, that the inverse of an even number is still an even number.

6. Use a direct proof to show that the product of two odd numbers is odd.
Let: $n_1 = 2k_1 + 1$ and $n_2 = 2k_2 + 1$ where k is an integer
The product: $n_1 * n_2 = (2k_1 + 1) * (2k_2 + 1) = 4k_1k_2 + 2k_1 + 2k_2 + 1$
After simplifying we get: $2 * (2k_1k_2 + k_1 + k_2) + 1$, which shows that the product of 2 odd numbers is indeed an odd number by definition of an odd number.

14. Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.
Since x is rational we know, by the definition of a rational number that $x = p/q$ where $q \neq 0$. Since $x \neq 0$ it is safe to assume that $p \neq 0$, since this would cause $x = 0$. Based on this information we can assume that $1/x$ is a rational number by definition of a rational number.

16. Prove that if m and n are integers and mn is even, then m is even or n is even.
Let:
$p = m$ and n are integers and mn is even
$q = m$ is even or n is even
This gives us $p \to q$
If we use the contrapositive for this implication we would change all of the evens in p and q to odds. We know that the contrapositive is true based on the work done in question 6. With this in mind we can conclude that the original implication is true since the contra-

positive is true.

18a. Prove that if n is an integer and $3n + 2$ is even, then n is even using a proof by contraposition.
Let:
$p = n$ is an integer and $3n + 2$ is even
$q = n$ is even
This gives us $p \to q$
Using the contrapositive we can show that $3n + 2$ is odd
Using the definition of an odd number we know that $n = 2k + 1$
So: $3n + 2 = 3(2k + 1) + 2 = 6k + 5 = (6k + 4) + 1 = 2(3k + 2) + 1$
Based on the definition of an odd number, $n = 2k+1$, we can assume that $3n+2$ is indeed an odd integer. The solves the contrapositive and proves that if n is an integer and $3n+2$ is even, then n is even