HW 4.3 ROB NAVARRO

2. FIND $f(1), f(2), f(3), f(4), f(5)$ IF $f(0)=3$

a) $f(n+1) = -2f(n)$

$f(1) = -2f(0) = -6$

$f(2) = -2f(1) = 12$

$f(3) = -2f(2) = -24$

$f(4) = -2f(3) = 48$

$f(5) = -2f(4) = -96$

b) $f(n+1) = 3f(n)+7$

$f(1) = 3f(0)+7 = 16$

$f(2) = 3f(1)+7 = 3(16)+7 = 55$

$f(3) = 3f(2)+7 = 3(55)+7 = 172$

$f(4) = 3f(3)+7 = 3(172)+7 = 523$

$f(5)\ 3f(4)+7 = 3(523)+7 = 1576$

8. GIVE THE RECURSIVE DEF OF THE SEQ $\{a_n\}, n = 1,2,3 \ldots$ IF

a) $a_n = 4n-2$

BASE CASE: $a_1 = 2$

RECURSIVE CASE: $a_{n+1} = a_n + ?$

$? = a_{n+1} - a_n$

$= 4(n+1)-2 - (4n-2)$

$= 4n + 4 - 2 - 4n + 2$

$= 4$

$$\boxed{a_{n+1} = a_n + 4}$$

b) $a_n = 1 + (-1)^n$

BASE CASE: $a_1 = 0$

RECURSIVE CASE: $a_{n+1} = a_n + ?$

$$
\begin{aligned}
a_{n+1} &= 1 + (-1)^{n+1} \\
&= 1 + (-1)^n(-1) \\
&= 1 + (((-1)^n + 1) - 1)(-1) \\
&= 1 + (a_n - 1)(-1) \\
&= \boxed{2 - a_n}
\end{aligned}
$$

c) $a_n = n(n+1)$

base case: $a_1 = 2$

$a_{n+1} = a_n + ?$

$$
\begin{aligned}
? &= a_{n+1} - a_n \\
&= (n+1)((n+1)+1) - n(n+1) \\
&= (n+1)(n+1) + n + 1 - n^2 - n \\
&= n^2 + 2n + 1 - n^2 + 1 \\
&= 2n + 2
\end{aligned}
$$

$$\boxed{a_{n+1} = a_n + 2n + 2}$$

d) $a_n = n^2$

BASE CASE: $a_1 = 1$

RECURSIVE CASE: $a_{n+1} = a_n + ?$

$$
\begin{aligned}
? = a_{n+1} - a_n = (n+1)^2 - n^2 &= n^2 + 2n + 1 - n^2 \\
&= 2n + 1
\end{aligned}
$$

$$\boxed{a_{n+1} = a_n + 2n + 1}$$

24. GIVEN A RECURSIVE DEF OF:

a) $\overbrace{\text{THE SET OF ODD POSITIVE INTS}}^{S}$

BASE CASE $1 \in S$

RECURSIVE CASE: If $n \in S$ then $n+1 \in S$

b) $\overbrace{\text{THE SET OF POSITIVE INT POWERS OF 3}}^{S}$

BASE CASE: $3 \in 3$

RECURSIVE CASE: If $n \in S$ the $3n \in S$

c) $\overbrace{\text{THE SET OF POLYNOMIALS WITH INT COEFF.}}^{S}$

BASE CASE: $0 \in S$

RECURSIVE CASE If $p(x) \in S$, then $p(x) + cx^n \in S$,

where c & $n \in \mathbb{Z}$ and $n \geq 0$.

26. LET S BE THE SUBSET OF ORDERED PAIRS OF INTS DEFINED BY.

a) BASE CASE : $(0,0) \in S$

RECURSIVE CASE: If $(a,b) \in S$, then $(a+2, b+3) \in S$ and $(a+3, b+2) \in S$.

FIRST FIVE APPLICATIONS:

1. $(2,3), (3,2)$

2. $(4,5), (5,5), (6,4)$

3. $(6,8), (7,8), (8,7), (9,6)$

4. $(8,12), (9,11), (10,10), (11,9), (12,8)$

5. $(10,15), (11,14), (12,13), (13,12), (14,11), (15,10)$

28. GIVE A RECURSIVE DEF OF EACH OF THESE SETS OF ORDERED PAIRS OF POSITIVE INTS

a) $S = \{(a,b) \mid a \in Z^+, b \in Z^+, \text{ and } a+b \text{ is odd}\}$

BASE CASE: $(1,2), (2,1) \in S$

RECURSIVE CASE EITHER A OR B IS ODD

If $(a,b) \in S$ then $(a+2, b) \in S$ and $(a, b+2) \in S$

$(1,2) - (3,2) = 5$; $(1,4) = 5$ ✓

32) a) GIVE A RECURSIVE DEF OF THE FUNCTION ones(s), which COUNTS THE NUMBER ONES IN A BIT STRING S

BASE CASE: $ones(\lambda) = 0$

RECURSIVE CASE: If $x \in \Sigma$ and $w \in \Sigma^*$

then $ones(wx) = ones(w) + x$ where $x = 1$ or 0.