Quiz 2

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1. Let the following predicates be given. The domain consists of all people.
   F(x) = x is friendly
   H(x) = x is helpful
   S(x) = x is a student

   Express each of the following English sentences in terms of F(x), H(x), S(x), quantifiers, and logical connectives.

   i. All students are not friendly.
   $\forall_x(S(x) \rightarrow \neg F(x))$
   ii. There are some students who aren't helpful.
   $\exists_x(S(x) \rightarrow \neg H(x))$
   iii. No student is friendly but not helpful.
   $\neg\forall_x(F(x) \wedge \neg H(x))$
   iv. Every friendly and helpful person is a student.
   $\forall_x((F(x) \wedge H(x)) \rightarrow S(x))$

2. Let B(x), S(x), and A(x) be the predicates

B(x) : x is a good basketball player
S(x) : x is a good soccer player
A(x) : x is a good athlete

Translate each of the following quantified logic expressions (provided in the file) into English considering the domain to consist of all people .

i. $\forall_x(A(x) \rightarrow (S(x) \vee B(x)))$
All good athletes are a good soccer player or basketball player.

ii. $\neg\forall_x(B(x))$
Not everyone is a good basketball player.
iii. $\exists_x((S(x) \wedge \neg B(x)) \vee \neg A(x)$
There exists a person who is a good soccer player and not a good basketball player, or is not a good athlete.
iv. $\exists_x \neg(S(x) \wedge \neg B(x))$
There exists a person who is not, a good soccer player and not a good basketball player.

3. Prove or disprove that $\forall_x(P(x) \rightarrow Q(x))$ and $\neg\exists_x\neg(\neg Q(x) \rightarrow \neg P(x))$ are logically equivalent.
$\forall_x(P(x) \rightarrow Q(x))$
$\equiv \neg(\neg\forall_x(P(x) \rightarrow Q(x)))$ (Double Negation)
$\equiv \neg(\exists_x\neg(\neg P(x) \rightarrow Q(x)))$ (5th Logical Equivalence table 7)
$\equiv \neg\exists_x\neg(\neg Q(x) \rightarrow \neg P(x))$ (2nd Logical Equivalence table 7)
This proves that $\forall_x(P(x) \rightarrow Q(x))$ and $\neg\exists_x\neg(\neg Q(x) \rightarrow \neg P(x))$ are logically equivalent.

8. Use a direct proof to show that the product of two nonzero rational numbers is also rational.
Let $x = \frac{p}{q}$ and $y = \frac{r}{s}$
Where p, q, r, and s are all integers and q and s do not equal 0.
$x * y = \frac{p}{q} * \frac{r}{s} = \frac{pr}{qs}$
Since p, q, r, and s are all integers, $qs \neq 0$ the product is rational.

9. Use a proof by contraposition to show that If $m^2 + n^2$ is an odd integer, then m is odd or n is odd.
Let: p = If $m^2 + n^2$ is an odd integer; q = m is odd or n is odd.
This gives us $p \rightarrow q$ and the contrapositive is $\neg q \rightarrow \neg p$
This states that "If neither m is odd or n is odd, then $m^2 + n^2$ is an even integer.
Knowing the definition of an even integer we will let $m = 2k$ and $n = 2l$
So, $m^2 + n^2 = (2k)^2 + (2l)^2 = 4k^2 + 4l^2 = 2(2k^2 + 2l^2)$
Since $(2k^2 + 2l^2)$ is just some integer this satisfies our contrapositive. This also solves the original proposition that if $m^2 + n^2$ is an odd integer, then m is odd or n is odd.