ROB NAVARRO
4.1 pg 151

whenever n is positive int $6! = 1\cdot2\cdot3\cdot4\cdot5\cdot6$

(6) $1\cdot1! + 2\cdot2! + \dots + n\cdot n! = (n+1)! - 1$

$P(n) = 1\cdot1! + 2\cdot2! \dots + n\cdot n! = (n+1)! - 1$ whenever n is a positive int

Base Case $P(1)$ $1 = 2! - 1$
$= 1$ ✓

Inductive $P(k) = 1\cdot1! + 2\cdot2! \dots k\cdot k! = (k+1)! - 1$

$P(k+1) = 1\cdot1! + 2\cdot2! \dots (k\cdot k!) + (k+1)\cdot(k+1)! = (k+1)!\cdot 1 + (k+1)!\cdot(k+1) - 1$

RHS: $(k+1)!\cdot 1 + (k+1)!\cdot(k+1) - 1$

$(k+1)!\,(k+2) - 1$

$(k+2)! - 1$

$P(k)$ is true so $P(k+1)$ is also true by induction.
$P(n)$ is true for all positive ints n by induction.

---

(8) $P(n)$: THE SUM OF THE FIRST n TERMS ON THE LHS IS CORRECT

$2 - 2\cdot7 + 2\cdot7^2 \dots + 2(-7)^n = \dfrac{1-(-7)^{n+1}}{4}$
WHEN N IS NON NEG

BASE CASE $P(0) = 2(-7)^0 = \dfrac{1-(-7)^1}{4}$
$= 2$ ✓

ASSUME $P(K)$ IS TRUE

$P(K+1): 2 - 2\cdot7 + 2\cdot7^2 + 2(-7)^k + 2(-7)^{(k+1)} = \dfrac{1-(-7)^{k+1}}{4} + 2\cdot(-7)^{k+1}$

$= \dfrac{1\cdot(-7)^{k+1}}{4} + \dfrac{8\cdot(-7)^{k+1}}{4}$

$= \dfrac{1 + 7(-7)^{k+1}}{4}$

$= \dfrac{1-(-7)^{k+2}}{4}$

$2 - 2\cdot7 + 2\cdot7^2 + \dots + 2\cdot(-7)^k + 2\cdot(-7)^{k+1} = \dfrac{1-(-7)^{k+2}}{4}$

SO IF $P(K)$ IS TRUE THEN $P(K+1)$ IS TRUE BY INDUCTION.
BY INDUCTION $P(n)$ is true for all non negative numbers.

(14.) EVERY POSITIVE INT N $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$

$P(1) = 2 = (1-1)\ 2^{1+1} + 2 = 2$

$P(k) = \sum_{i=1}^{k} i2^i = (k-1)2^{k+1} + 2$

$P(k+1) = \sum_{i=1}^{k} i2^i + (k+1) \cdot 2^{k+1} = (k-1)2^{k+1} + 2 + (k+1) \cdot 2^{k+1}$

$(k-1)2^{k+1} + 2 + (k+1) \cdot 2^{k+1}$

$2^{k+1} \cdot (k-1+k+1) + 2$

$2^{k+1} \cdot 2k + 2$

$\sum_{i=1}^{k+1} k2^k = k \cdot 2^{k+2} + 2$

THIS SHOWS THAT IF P(k) IS TRUE THEN P(k+1) IS TRUE BY INDUCTION.

SO BY INDUCTION P(n) IS TRUE FOR ALL POSITIVE INTS.

---

18. $P(n) \quad n! < n^n$

a) $P(2): \quad 2! < 2^2$

b) $P(2) \quad 2 < 4$ TRUE

c) $P(k) = k! < k^k$

d) WE WANT TO SHOW $(k+1)! < (k+1)^{(k+1)}$ FOR ANY INT > 1.

e) $k! \cdot (k+1) < k^k \cdot (k+1) < (k+1)^k(k+1) = (k+1)^{k+1}$

f) Since we have shown a basis and inductive step are true, by induction, then the statement is true for all ints > 1.

(29.) $n^2 - 7n + 12$ is non neg when $n \geq 3$

$P(n)$: $n^2 - 7n + 12$ is nonnegative when $n \geq 3$.

Base case: $P(3) = 9 - 21 + 12 = 0$, which is non-negative

Inductive $P(k+1) = (k+1)^2 - 7(k+1) + 12 = k^2 + 2k + 1 - 7k - 7 + 12$

$= (k^2 - 7k + 12) + (2k - 6)$

$= (k^2 - 7k + 12) + 2(k - 3)$

Since we know $P(k)$ is true and that $2(k-3) \geq 0$ when $k \geq 3$. THEN $P(k)$ AND $P(k+1)$ MUST BE TRUE BY INDUCTION.

(32) $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets that $A_j \subseteq B_j$ for $j = 1, 2, \ldots, n$ then $\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j$

BASE CASE $P(1)$ $A_1 \subseteq B_1 = \bigcup_{j=1}^{1} A_j \subseteq \bigcup_{j=1}^{1} B_j$

Inductive. $P(k)$ $A_j \subseteq B_j$ for $j = 1, 2, 3 \ldots k$

$\bigcup_{j=1}^{k+1} A_j \subseteq \bigcup_{j=1}^{k+1} B_j$

If some element x is $x \in \bigcup_{j=1}^{k}$ AND $x \in \bigcup_{j=1}^{k+1}$ we CAN ALSO ASSUME THAT $x \in \bigcup_{j=1}$ is, WE KNOW THIS BECAUSE $x \in A_{k+1}$ AND $A_{k+1} \subseteq B_{k+1}$. $x \in \left(\bigcup_{j=1}^{k} B_j\right) \cup B_{k+1} = \bigcup_{j=1}^{k+1} B$

BY INDUCTION $P(n)$ is true for all positive ints.

(40.) PROVE IF $A_1, A_2, \ldots A_n$ and B ARE SETS THEN:

$$P(n) = (A_1 \cap A_2 \cap \ldots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B)$$

$P(1)$: $A_1 \cup B = A_1 \cup B$ ✓

$P(k)$: $(A_1 \cap A_2 \cap \ldots \cap A_k) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_k \cap B)$

$P(k+1)$: $(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_k \cup B) \cap (A_{k+1} \cup B)$

$(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cup B$

$= ((A_1 \cap A_2 \cap \ldots \cap A_k) \cap A_{k+1}) \cup B$ ASSOCIATIVE

$((A_1 \cap A_2 \cap \ldots \cap A_k) \cup B) \cap (A_{k+1} \cup B)$

$(A_1 \cup B) \cap (A_2 \cup B) \cap \ldots (A_k \cup B) \cap (A_{k+1} \cup B)$

If $P(k)$ is true then $P(k+1)$ is true by induction. By induction $P(n)$ is true for all positive ints