HW 3.3

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April 26, 2015

4. What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals:

a) $(-2)^n : 1, -2, 4, -8$
b) $3 : 3, 3, 3, 3$
c) $7 + 4^n : 8, 11, 23, 71$
d) $2^n + (-2)^n : 2, 0, 8, 0$

29. What are the values of these sums?

a) $\sum_{k=1}^{5}(k+1) = 2+3+4+5+6 = 20$
b) $\sum_{j=0}^{4}(-2)^j = 1-2+4-8+16 = 11$
c) $\sum_{i=1}^{1} 03 = 10 * 3 = 30$
d) $\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8} 2^j = \frac{2^9 - 2^0}{2-1} = 511$

31. What is the value of each of these sums of terms of geometric progression?

a) $\sum_{j=0}^{8} 3 * 2^j = 3 * \frac{2^9-2^0}{2-1} = 1533$
b) $\sum_{j=1}^{8} 2^j = \sum_{j=0}^{8} 2^j - \sum_{j=0}^{0} 2^j = \frac{2^9-2^0}{2-1} - \frac{2^1-2^0}{2-1} = 511 - 1 = 510$
c) $\sum_{j=2}^{8}(-3)^j = \sum_{j=0}^{8}(-3)^j - \sum_{j=0}^{1}(-3)^j = \frac{(-3)^9-(-3)^0}{-3-1} - \frac{(-3)^2-(-3)^0}{-3-1} = 4923$
d) $\sum_{j=2}^{8} 2 * (-3)^j = 2\sum_{j=2}^{8}(-3)^j = 2 * \frac{(-3)^9-(-3)^0}{-3-1} = 2 * 4921 = 9842$

32. Find the value of each of these sums.

a) $\sum_{j=0}^{8}(1+(-1)^j) = 2+0+2+0+2+0+2+0+2 = 10$

b) $\sum_{j=0}^{8}(3^j - 2^j) = \sum_{j=0}^{8} 3^j - \sum_{j=0}^{8} 2^j = \frac{(3)^9-(3)^0}{3-1} - \frac{(2)^9-(2)^0}{2-1} = 9841 - 511 = 9330$

c) $\sum_{j=0}^{8}(2*3^j+3*2^j) = 2*\sum_{j=0}^{8} 3^j+3*\sum_{j=0}^{8} 2^j = 2*\frac{(3)^9-(3)^0}{3-1} - 3*\frac{(2)^9-(2)^0}{2-1} = 19682+1533 = 21215$

d) $\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8} 2^j(2-1) = \sum_{j=0}^{8} 2^j = \frac{2^9-2^0}{2-1} = 511$

34. Compute each of these double sums.

a) $\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j) = \sum_{i=1}^{3}(\sum_{j=1}^{2} i - \sum_{j=1}^{2} j) = 2*\sum_{i=1}^{3} i - \sum_{i=1}^{3} 3 = 2(1+2+3) - 3*3 = 3$

b) $\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j) = \sum_{i=0}^{3}(3*\sum_{j=0}^{2} i + 2*\sum_{j=0}^{2} j) = \sum_{i=0}^{3}(9i+6) = 9*(0+1+2+3)+6*4 = 78$

c) $\sum_{i=0}^{3}\sum_{j=0}^{2} j = \sum_{i=0}^{3} 3 = 9$

d) $\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3 = \sum_{i=0}^{2} i^2 \sum_{j=0}^{3} j^3 = 5*36 = 180$

36. Use the identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ and exercise 19 to compute $\sum_{k=1}^{n} \frac{1}{k(k+1)}$.

$\sum_{k=1}^{n} \frac{1}{k(k+1)} = (1-\frac{1}{2}) + (\frac{1}{2} - \frac{2}{3})... + (\frac{1}{n} - \frac{1}{n+1})$

Let $a_k = \frac{1}{k}$ Then using the conclusion from exercise 19 we know $\sum_{j=1}^{n}(a_j - a_{n+1}) = a_1 - a_{n+1}$.

With this in mind we now know $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

40. Find $\sum_{k=99}^{2} 00k^3$.

$\sum_{k=99}^{200} k^3 = \sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3 = \frac{200^2*201^2}{4} - \frac{98^2*99^2}{4} = 380477799$