(29.) $n^2 - 7n + 12$ is non neg when $n \geq 3$

$P(n)$: $n^2 - 7n + 12$ is nonnegative when $n \geq 3$.

Base case: $P(3) = 9 - 21 + 12 = 0$, which is non-negative

Inductive $P(k+1) = (k+1)^2 - 7(k+1) + 12 = k^2 + 2k + 1 - 7k - 7 + 12$

$= (k^2 - 7k + 12) + (2k - 6)$

$= (k^2 - 7k + 12) + 2(k - 3)$

Since we know $P(k)$ is true and that $2(k-3) \geq 0$ when $k \geq 3$. THEN $P(k)$ AND $P(k+1)$ MUST BE TRUE BY INDUCTION.

(32) $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets that $A_j \subseteq B_j$ for $j = 1, 2, \ldots, n$ then $\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j$

BASE CASE $P(1)$ $A_1 \subseteq B_1 = \bigcup_{j=1}^{1} A_j \subseteq \bigcup_{j=1}^{1} B_j$

Inductive. $P(k)$ $A_j \subseteq B_j$ for $j = 1, 2, 3, \ldots, k$

$\bigcup_{j=1}^{k+1} A_j \subseteq \bigcup_{j=1}^{k+1} B_j$

If some element x is $x \in \bigcup_{j=1}^{k}$ AND $x \in \bigcup_{j=1}^{k+1}$ we CAN ALSO ASSUME THAT $x \in \bigcup_{j=1}^{k}$ is, WE KNOW THIS BECAUSE $x \in A_{k+1}$ AND $A_{k+1} \subseteq B_{k+1}$. $x \in (\bigcup_{j=1}^{k} B_j) \cup B_{k+1} = \bigcup_{j=1}^{k+1} B$

BY INDUCTION $P(n)$ is true for all positive ints.