(40.) PROVE IF $A_1, A_2, \ldots A_n$ and B ARE SETS THEN:

$$P(n) = \begin{cases} (A_1 \cap A_2 \cap \ldots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B) \end{cases}$$

$P(1)$: $A_1 \cup B = A_1 \cup B$ ✓

$P(k)$: $(A_1 \cap A_2 \cap \ldots \cap A_k) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_k \cup B)$

$P(k+1)$: $(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_k \cup B) \cap (A_{k+1} \cup B)$

$(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cup B$

$= ((A_1 \cap A_2 \cap \ldots \cap A_k) \cap A_{k+1}) \cup B$ ASSOCIATIVE

$((A_1 \cap A_2 \cap \ldots \cap A_k) \cup B) \cap (A_{k+1} \cup B)$

$(A_1 \cup B) \cap (A_2 \cup B) \cap \ldots (A_k \cup B) \cap (A_{k+1} \cup B)$

IF $P(k)$ is true then $P(k+1)$ is true by induction. By induction $P(n)$ is true for all positive ints