HW 3.2

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2. Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a) The set of sophomores taking discrete mathematics in your school.
$A \cap B$

b) The set of sophomores at your school who are not taking discrete mathematics.
$A - B$

c) The set of students at your school who either are sophomores or are taking discrete mathematics.
$A \cup B$

d) The set of students at your school who either are not sophomores or are not taking discrete mathematics.
$\overline{\mathrm{A}} \cup \overline{\mathrm{B}}$

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

a) $A \cup B$
$\{a, b, c, d, e, f, g, h\}$

b) $A \cap B$
$\{a, b, c, d, e\}$

c) $A - B$
$\{\}$

d) $B - A$
{f, g, h}

12. Prove the first absorption law from Table 1 by showing that if A and B are sets, then $A \cup (A \cap B) = A$.
In order to prove this we must prove that $x \in A \cup (A \cap B)$ and $x \in A$. We know that $x \in A$ and $x \in B$ based on the definition of a union. Either way we know that if $x \in A \cup (A \cap B)$ then $x \in A$. We can conclude $A \cup (A \cap B) \subseteq A$ and $A \subseteq A \cup (A \cap B)$. This completes the proof.

16. Let A and B be sets. Show that:

a) $(A \cap B) \subseteq A$
Let $x \in (A \cup B)$, then $x \in A$ and $x \in B$ by the definition of intersection. Based on this information we can conclude that $(A \cap B) \subseteq A$.

b) $A \subseteq (A \cup B)$
Let $x \in A$ then by the definition of a union $x \in (A \cup B)$

c) $A - B \subseteq A$
Let $x \in A - B$, then by the definition of difference we know that $x \in A$ and $x \notin B$. This proves that $A - B \subseteq A$.

d) $A \cap (B - A) = \varnothing$
Using proof by contradiction, there exists an $x \in A \cap (B - A)$. We know that $x \in A$ and $x \in (B - A)$, but by definition of difference we also have $x \notin A$. This leads to a contradiction that $x \in A$ and $x \notin A$. This means that the assumption was false and we proved that $A \cup (B - A) = \varnothing$.

e) $A \cup (B - A) = A \cup B$

A	B	$B - A$	$A \cup (B - A)$	$A \cup B$
1	1	0	1	1
1	0	0	1	1
0	1	1	1	1
0	0	0	0	0

18. Let A, B, and C be sets, show that:

a) $(A \cup B) \subseteq (A \cup B \cup C)$
Let $x \in (A \cup B)$, then by the definition of a union $x \in A$ or $x \in B$. Then by the definition of union we can also conclude that $x \in (A \cup B \cup C)$. Thus, we can conclude that that $(A \cup B) \subseteq (A \cup B \cup C)$.

b) $(A \cap B \cap C) \subseteq (A \cap B)$
By the definition of intersection we know that $x \in A, x \in B$, and $x \in C$. We also know by the definition of intersection that $x \in (A \cap B)$. Thus, we can conclude that $(A \cap B \cap C) \subseteq (A \cap B)$

c) $(A - B) - C \subseteq A - C$
Let $x \in (A - B) - C$, then by the definition of difference we know that $x \in A, x \notin B$ and $x \notin C$. Then by the definition of difference we know that $x \in A - C$. Thus, we can conclude that $(A - B) - C \subseteq A - C$.

d) $(A - C) \cap (C - B) = \varnothing$
Using proof by contradiction, there exists an $x \in (A - C) \cap (C - B)$. By the definition of intersection we then know that $x \in (A - C)$ and $x \in (C - B)$. Then by the definition of difference we know that $x \in A, x \notin C, x \in C$, and $x \notin B$. This creates a contradiction with $x \in C$ and $x \notin C$. This means that the assumption was false and we proved that $(A - C) \cap (C - B) = \varnothing$.

e)$(B - A) \cup (C - A) = (B \cup C) - A$

A	B	C	$C - A$	$B - A$	$B \cup C$	$(C - A) \cup (B - A)$	$(B \cup C) - A$
1	1	1	0	0	1	0	0
1	1	0	0	0	1	0	0
1	0	1	0	0	1	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	0	1	1	0	1	1	1
0	0	0	0	0	0	0	0

20. Show that if A and B are sets, then $(A \cap B) \cup (A \cap \overline{\mathrm{B}}) = A$

A	B	$A \cap B$	$\overline{\mathrm{B}}$	$(A \cap \overline{\mathrm{B}})$	$(A \cap B) \cup (A \cap \overline{\mathrm{B}})$
1	1	1	0	0	1
1	0	0	1	1	1
0	1	0	0	0	0
0	0	0	1	0	0