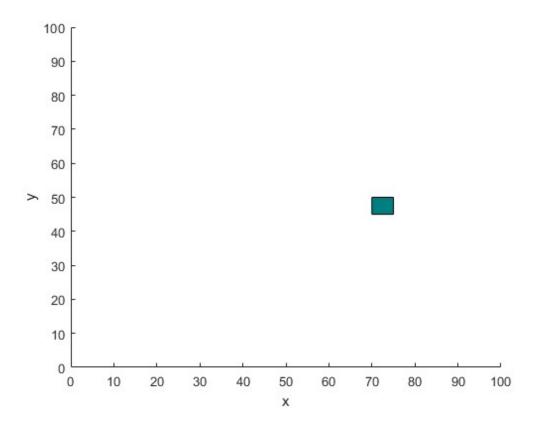
- **q**1
- grow the tree
- total number of node in the tree
- number of nodes in the sequence that reaches goal area

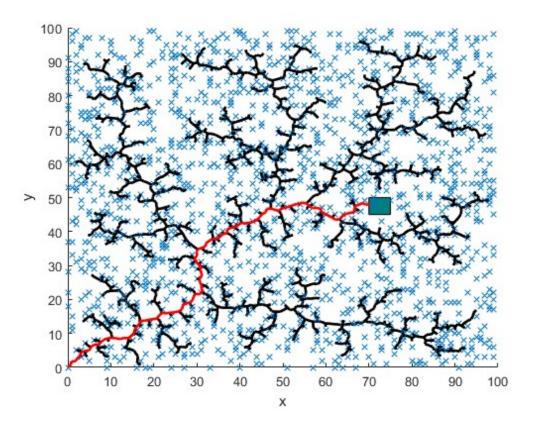
q1

```
clearvars
close all
\% x and y axises limit from 0 to x_max and 0 to y_max respectively.
x_max = 100; %;
y_max = 100; %;
% distance of moving at every step
EPS = 1;
% maximum iterations
numNodes = 3000;
% attributions of starting point
q_start.coord = [0 0];
q_start.cost = 0;
q_start.parent = 0; %.parent means the index of parent node
% initialize the tree
nodes(1) = q_start;
% plot the goal area
figure(1)
axis([0 x_max 0 y_max])
goal_area = rectangle('Position',[70,45,5,5],'FaceColor',[0 .5 .5]);
xlabel('x')
ylabel('y')
hold on
```



```
for i = 1:1:numNodes
   % generate the random points in the given safe area and plot the points
   q_rand = [floor(rand(1)*x_max) floor(rand(1)*y_max)];
   plot(q_rand(1), q_rand(2), 'x', 'Color', [0 0.4470 0.7410])
   % Find the nearest point existing on the tree to the random point
   ndist = [];
   for j = 1:1:length(nodes)
        n = nodes(j);
       tmp = dist(n.coord, q_rand);
       ndist = [ndist tmp];
   end
    [mini_distance, idx] = min(ndist);
   q_nearest = nodes(idx);
   % move to the random point with distance of eps if distance between
   % random point and nearest point is bigger than eps.
   q_new.coord = steer(q_rand, q_nearest.coord, mini_distance, EPS);
   line([q_nearest.coord(1), q_new.coord(1)], [q_nearest.coord(2), q_new.coord(2)],...
        'Color', 'k', 'LineWidth', 2);
   drawnow
   hold on
   q_new.cost = dist(q_new.coord, q_nearest.coord) + q_nearest.cost;
   q_new.parent = idx;
   % Append to nodes
   nodes = [nodes q_new];
```

```
% Break if the link from second to last node to last node intersects any of
    % the four edges of the goal area
    if ~noCollision(q_nearest.coord, q_new.coord, [70,45,5,5])
        break
    end
end
q_end = q_new;
num_node_path = 1;
while q_end.parent ~= 0
    start = q_end.parent;
    line([q_end.coord(1), nodes(start).coord(1)], [q_end.coord(2), nodes(start).coord(2)],...
        'Color', 'r', 'LineWidth', 2);
    hold on
    q_end = nodes(start);
    num_node_path = num_node_path+1;
end
```



total number of node in the tree

```
num_node_tree = length(nodes)
```

```
num_node_tree =
```

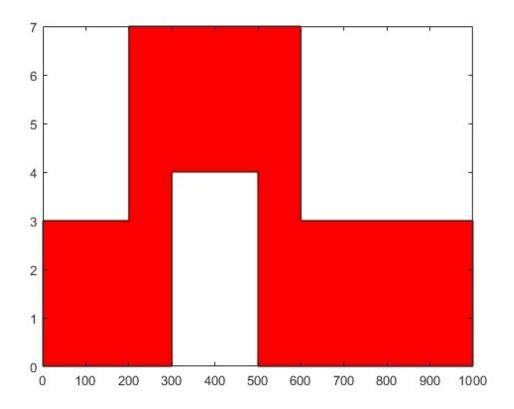
1917

number of nodes in the sequence that reaches goal area

num_node_path
<pre>num_node_path =</pre>
111

- grow the tree
- total number of node in the tree
- number of nodes in the sequence that reaches goal area

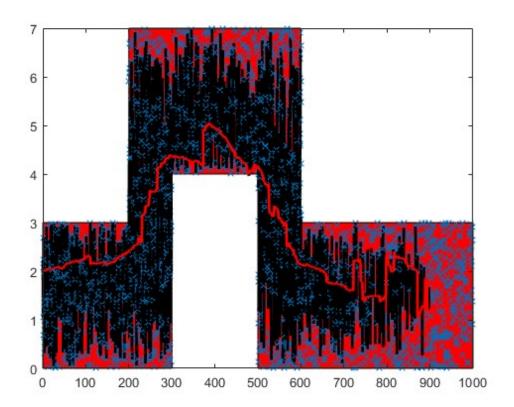
```
clearvars
close all
\% x and y axises limit from 0 to x_max and 0 to y_max respectively.
x_{max} = 1000;
y_max = 7;
% distance of moving at every step
EPS = 1;
% maximum iterations
numNodes = 6000;
\% attributions of starting point
q_start.coord = [0 2];
q_start.cost = 0;
q_start.parent = 0; %.parent means the index of parent node
% initialize the tree
nodes(1) = q_start;
% plot the safe area
figure(1)
x=[0\ 300\ 300\ 500\ 500\ 1000\ 1000\ 600\ 600\ 200\ 0]; %x coordinates of all the vertices
y=[0\ 0\ 4\ 4\ 0\ 0\ 3\ 3\ 7\ 7\ 3\ 3]; %y coordinates of all the vertices
X=[x,x(1)]; %????????????????????
Y=[y,y(1)]; %??
plot(X,Y,'k') %?????
fill(x,y,'r') % fill the safe zone with color
hold on
% plot the goal area
```



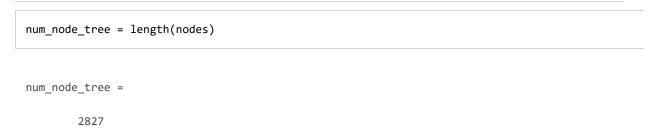
. . .

```
for i = 1:1:numNodes
    pan = 0;
    \% generate the random points in the given safe area and plot the points
    while ~pan
         q_rand = [rand*x_max rand*y_max];
         pan = inpolygon(q_rand(1),q_rand(2),X,Y);
    plot(q_rand(1), q_rand(2), 'x', 'Color', [0 0.4470 0.7410])
    % Find the nearest point existing on the tree to the random point
    ndist = [];
    for j = 1:1:length(nodes)
         n = nodes(j);
         tmp = dist(n.coord, q_rand);
         ndist = [ndist tmp];
    end
    [mini_distance, idx] = min(ndist);
    q_nearest = nodes(idx);
    \ensuremath{\text{\%}} move to the random point with distance of eps if distance between
    % random point and nearest point is bigger than eps.
    q_new.coord = steer(q_rand, q_nearest.coord, mini_distance, EPS);
    \label{line} line([\texttt{q\_nearest.coord}(\texttt{1}), \, \texttt{q\_new.coord}(\texttt{1})], \, [\texttt{q\_nearest.coord}(\texttt{2}), \, \texttt{q\_new.coord}(\texttt{2})], \dots
         'Color', 'k', 'LineWidth', 2);
    drawnow
```

```
hold on
    q_new.cost = dist(q_new.coord, q_nearest.coord) + q_nearest.cost;
    q_new.parent = idx;
    InorOn = inpolygon(q_new.coord(1),q_new.coord(2),X,Y);
    % Append to nodes
    if InorOn == 1
        nodes = [nodes q_new];
    end
    % Break if the link from second to last node to last node intersects any of
    % the four edges of the goal area
    if ~noCollision(q_nearest.coord, q_new.coord, [900,1,50,0.5])
    end
end
q_end = q_new;
num_node_path = 1;
while q_end.parent ~= 0
    start = q_end.parent;
    line([q_end.coord(1), nodes(start).coord(1)], [q_end.coord(2), nodes(start).coord(2)],...
        'Color', 'r', 'LineWidth', 2);
    hold on
    q_end = nodes(start);
    num_node_path = num_node_path+1;
end
```





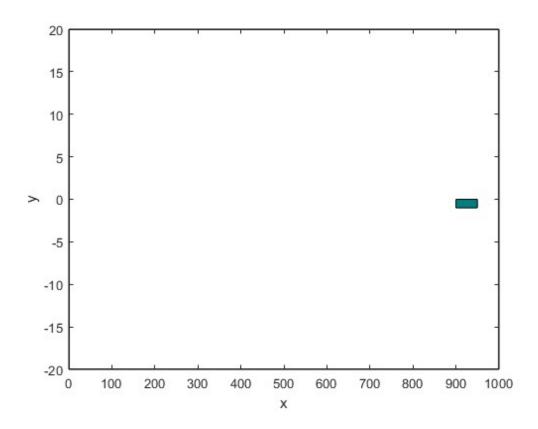


number of nodes in the sequence that reaches goal area

```
num_node_path
num_node_path =
   906
```

- define the vehicle parameters
- grow the tree
- total number of node in the tree
- number of nodes in the sequence that reaches goal area

```
clearvars
close all
% x and y axises limit from 0 to x_max and 0 to y_max respectively.
x_max = 1000;
y_max = 20;
y min = -20;
phi_max = pi;
phi_min = -pi;
% time of moving at every step
EPST = 0.1;
% maximum iterations
numNodes = 6000;
% attributions of starting point
q_start.coord = [0 0 0]';
q_start.cost = 0;
q_start.parent = 0; %.parent means the index of parent node
% initialize the tree
nodes(1) = q_start;
% plot the safe area
figure(1)
x=[0\ 0\ 1000\ 1000]; %x coordinates of all the vertices
y=[-20\ 20\ 20\ -20]; %y coordinates of all the vertices
X=[x,x(1)]; %??????????????????????
Y=[y,y(1)]; %??
plot(X,Y,'k') %?????
fill(x,y,'w') % fill the safe zone with color
hold on
% plot the goal area
figure(1)
axis([0 x_max y_min y_max])
goal_area = rectangle('Position',[900,-1,50,1],'FaceColor',[0.5.5]);
xlabel('x')
ylabel('y')
hold on
```

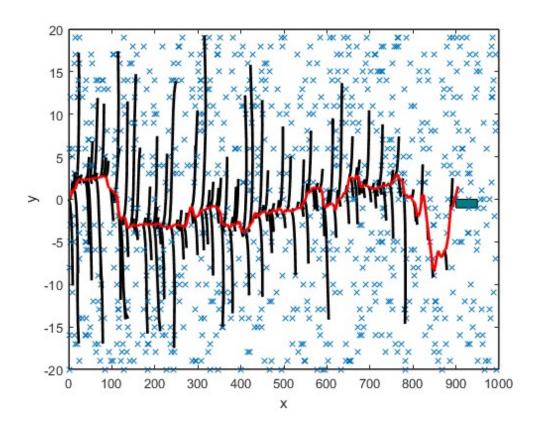


define the vehicle parameters

```
vx = 30;
L = 3;
```

```
for i = 1:1:numNodes
   \ensuremath{\text{\%}} generate the random points in the given safe area and plot the points
   max]';
   plot(q_rand(1), q_rand(2), 'x', 'Color', [0 0.4470 0.7410])
   % Find the nearest point existing on the tree to the random point
   ndist = [];
   for j = 1:1:length(nodes)
       n = nodes(j);
       tmp = dist(n.coord(1:2), q_rand);
       ndist = [ndist tmp];
   end
   [mini_distance, idx] = min(ndist);
   q_nearest = nodes(idx);
   %brute force to check all the possible steering angles, and assign the
   %closet to q_new
   k = 1;
   tempdist = [];
   q_newPossible = [];
   for delta = -20:2:20
```

```
deltaRad = delta*pi/180;
                    dxdt = @(t,x) \text{ kinematicsModel}(x, deltaRad, vx, L);
                    [tsol, xsol] = ode45(dxdt,[0,EPST],q_nearest.coord);
                    InorOn = all(inpolygon(xsol(:,1),xsol(:,2),X,Y)) && all(xsol(:,3)<=pi) && all(xsol(:,3
3)>=-pi);
                    k=k+1;
                    if InorOn == 1
                              q_newPossible = [q_newPossible; xsol(end,:)];
                              tempdist = [tempdist dist(q_newPossible(end,1:2), q_rand)];
                    end
          end
          [mini_distance2, idx2] = min(tempdist);
          if isempty(q_newPossible(idx2,:))
                    continue;
          end
          q_new.coord = q_newPossible(idx2,:);
          line([q_nearest.coord(1), q_new.coord(1)], [q_nearest.coord(2), q_new.coord(2)],...
                     'Color', 'k', 'LineWidth', 2);
          drawnow
          hold on
          q_new.cost = dist(q_new.coord, q_nearest.coord) + q_nearest.cost;
          q new.parent = idx;
          InorOn = inpolygon(q_new.coord(1),q_new.coord(2),X,Y);
          % Append to nodes
          if InorOn == 1
                    nodes = [nodes q new];
          end
          % Break if the link from second to last node to last node intersects any of
          % the four edges of the goal area
          if ~noCollision(q_nearest.coord, q_new.coord, [900,1,50,0.5]) && q_new.coord(3)>=-pi/6 &&
  q_new.coord(3)<=pi/6
                    break
          end
end
q_{end} = q_{new};
num_node_path = 1;
while q_end.parent ~= 0
          start = q_end.parent;
          line([q_end.coord(1), nodes(start).coord(1)], [q_end.coord(2), nodes(start).coord(2)],...
                     'Color', 'r', 'LineWidth', 2);
          hold on
          q_end = nodes(start);
          num_node_path = num_node_path+1;
end
```



total number of node in the tree

num_node_tree = length(nodes)

num_node_tree =

866

number of nodes in the sequence that reaches goal area

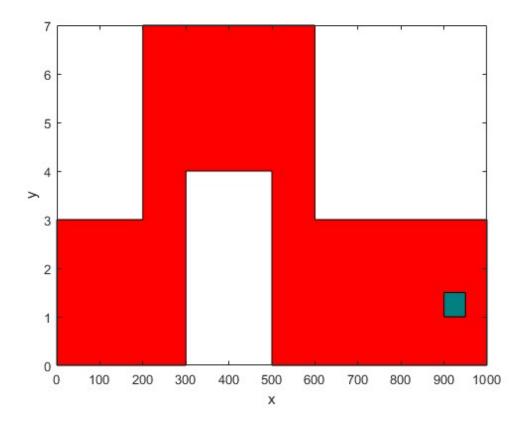
 ${\tt num_node_path}$

num_node_path =

306

- define the vehicle parameters
- grow the tree
- total number of node in the tree
- number of nodes in the sequence that reaches goal area

```
%
clearvars
close all
% x and y axises limit from 0 to x_max and 0 to y_max respectively.
x_max = 1000;
y_max = 7;
phi max = pi;
phi_min = -pi;
% time of moving at every step
EPST = 0.1;
% maximum iterations
numNodes = 6000;
% attributions of starting point
q_start.coord = [0 2 0]';
q_start.cost = 0;
q_start.parent = 0; %.parent means the index of parent node
% initialize the tree
nodes(1) = q_start;
% plot the safe area
figure(1)
x=[0\ 300\ 300\ 500\ 500\ 1000\ 1000\ 600\ 600\ 200\ 200\ 0]; %x coordinates of all the vertices
y=[0\ 0\ 4\ 4\ 0\ 0\ 3\ 3\ 7\ 7\ 3\ 3]; %y coordinates of all the vertices
X=[x,x(1)]; %???????????????????
Y=[y,y(1)]; %??
plot(X,Y,'k') %?????
fill(x,y,'r') % fill the safe zone with color
hold on
% plot the goal area
figure(1)
axis([0 x_max 0 y_max])
goal_area = rectangle('Position',[900,1,50,0.5],'FaceColor',[0 .5 .5]);
xlabel('x')
ylabel('y')
hold on
```



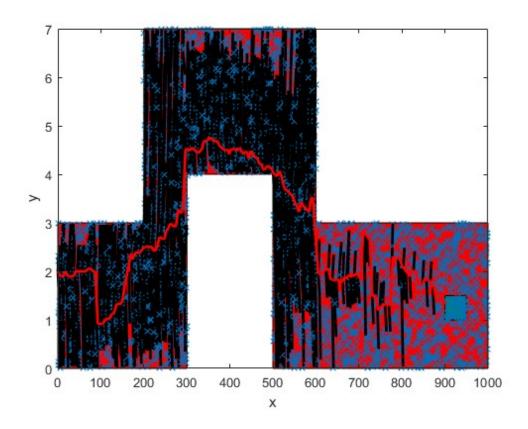
define the vehicle parameters

```
vx = 30;
L = 3;
```

```
for i = 1:1:numNodes
    %q_rand = [floor(rand(1)*x_max) floor(rand(1)*y_max*2)-y_max floor(rand(1)*2*phi_max)-phi
_max]';
    %plot(q_rand(1), q_rand(2), 'x', 'Color', [0 0.4470 0.7410])
    pan = 0;
    % generate the random points in the given safe area and plot the points
    while ~pan
        q_rand = [rand*x_max rand*y_max floor(rand(1)*2*phi_max)-phi_max];
        pan = inpolygon(q_rand(1),q_rand(2),X,Y);
    end
    plot(q_rand(1), q_rand(2), 'x', 'Color', [0 0.4470 0.7410])
    % Find the nearest point existing on the tree to the random point
    ndist = [];
    for j = 1:1:length(nodes)
        n = nodes(j);
        tmp = dist(n.coord(1:2), q_rand);
        ndist = [ndist tmp];
    [mini_distance, idx] = min(ndist);
    q_nearest = nodes(idx);
```

```
%brute force to check all the possible steering angles, and assign the
    %closet to q_new
    k = 1;
    tempdist = [];
    q newPossible = [];
    for delta = -20:2:20
        deltaRad = delta*pi/180;
        dxdt = @(t,x) kinematicsModel(x, deltaRad, vx, L);
        [tsol, xsol] = ode45(dxdt,[0,EPST],q_nearest.coord);
        InorOn = all(inpolygon(xsol(:,1),xsol(:,2),X,Y)) && all(xsol(:,3)<=pi) && all(xsol(:,</pre>
3)>=-pi);
        k=k+1;
        if InorOn == 1
            q_newPossible = [q_newPossible; xsol(end,:)];
            tempdist = [tempdist dist(q_newPossible(end,1:2), q_rand)];
        end
    [mini_distance2, idx2] = min(tempdist);
    if isempty(q_newPossible(idx2,:))
        continue;
    end
    q_new.coord = q_newPossible(idx2,:);
    line([q_nearest.coord(1), q_new.coord(1)], [q_nearest.coord(2), q_new.coord(2)],...
        'Color', 'k', 'LineWidth', 2);
    drawnow
    hold on
    q_new.cost = dist(q_new.coord, q_nearest.coord) + q_nearest.cost;
    q_new.parent = idx;
    InorOn = inpolygon(q_new.coord(1),q_new.coord(2),X,Y);
    % Append to nodes
    if InorOn == 1
        nodes = [nodes q_new];
    \% Break if the link from second to last node to last node intersects any of
    % the four edges of the goal area
    if ~noCollision(q_nearest.coord, q_new.coord(1:2), [900,1,50,0.5]) && q_new.coord(3)>=-pi
/6 && q_new.coord(3)<=pi/6
        break
    end
end
q_{end} = q_{new};
num_node_path = 1;
while q end.parent ~= 0
    start = q_end.parent;
    line([q_end.coord(1), nodes(start).coord(1)], [q_end.coord(2), nodes(start).coord(2)],...
        'Color', 'r', 'LineWidth', 2);
    hold on
```

```
q_end = nodes(start);
num_node_path = num_node_path+1;
end
```



total number of node in the tree

```
num_node_tree = length(nodes)
num_node_tree =
```

2345

number of nodes in the sequence that reaches goal area

```
num_node_path
num_node_path =
302
```

```
clc
clear
% Francesco Borrelli ME C231A 2015
% Kinematic Navigation
N=50;
sampling=10;
%Var Defintions
z = sdpvar(2,N);
%Initial and terminal condition
z0 = [0;1];
zT = [850;1];
dzmin=-[20;2];
dzmax=[20;2];
zmin = [0;0];
zmax = [1000;7];
%Obstacle list
i=1;
obs{i}.center=[400;1];
obs{i}.LW=[200;2];
obs{i}.theta=0; %(in radiants)
i=i+1;
obs{i}.center=[800;5];
obs{i}.LW=[400;4];
obs{i}.theta=0; %(in radiants)
% some obtacle postprocessing
for j=1:length(obs)
   t=obs{j}.theta;
   % generate T matrix for each obstacle
   obs{j}.T=[cos(t), -sin(t);sin(t) cos(t)]*diag(obs{j}.LW/2);
   % polyehdral representaion
   obs{j}.poly=obs{j}.T*unitbox(2)+obs{j}.center;
end
%try to remove/add this one
%Constraints
%Setup Optimization Problem
cost = 0;
Q=eye(2);
constr = [z(:,1)==z0,z(:,N)==zT];
for t = 2:N
     cost=cost+(z(:,t)-z(:,t-1))'*Q*(z(:,t)-z(:,t-1));
     constr = constr +[dzmin<= z(:,t)-z(:,t-1)<=dzmax];</pre>
```

```
constr = constr + [zmin<=z(:,t)<= zmax];</pre>
                    for k = 0:sampling-1
                                  for j=1:length(obs)
                                               xs=z(:,t-1)+k/sampling*(z(:,t)-z(:,t-1));
                                               \label{eq:constr} \mbox{$<$} constr = constr + [norm(inv(obs{j}.T)*(xs - obs{j}.center),2) > = sqrt(2)];
                                               constr = constr + [(xs-obs{j}.center)'*inv(obs{j}.T)'*inv(obs{j}.T)*(xs-obs{j}.center)'*inv(obs{j}.T)'*inv(obs{j}.T)*(xs-obs{j}.center)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*inv(obs{j}.T)'*in
enter)>=2];
                    end
options = sdpsettings('solver', 'ipopt');
%options.ipopt=ipoptset('linear_solver','MUMPS');
solvesdp(constr,cost,options);
z_vec = double(z);
% Plotting Functions % to add title and labels
th = 0:pi/50:2*pi;
for j=1:length(obs)
             for l=1:length(th)
                           z=[cos(th(1));sin(th(1))]*sqrt(2);
                          y=obs{j}.T*z+obs{j}.center;
                           xobs{j}(1) = y(1);
                          yobs{j}(1) = y(2);
             end
end
 ************************************
```

This program contains Ipopt, a library for large-scale nonlinear optimization. Ipopt is released as open source code under the Eclipse Public License (EPL).

For more information visit http://projects.coin-or.org/Ipopt

```
Total number of variables....:
                                                       96
                  variables with only lower bounds:
                                                        0
              variables with lower and upper bounds:
                                                       96
                  variables with only upper bounds:
                                                        0
Total number of equality constraints....:
                                                        0
Total number of inequality constraints....:
                                                     1176
       inequality constraints with only lower bounds:
  inequality constraints with lower and upper bounds:
                                                        0
       inequality constraints with only upper bounds:
                                                     1176
```

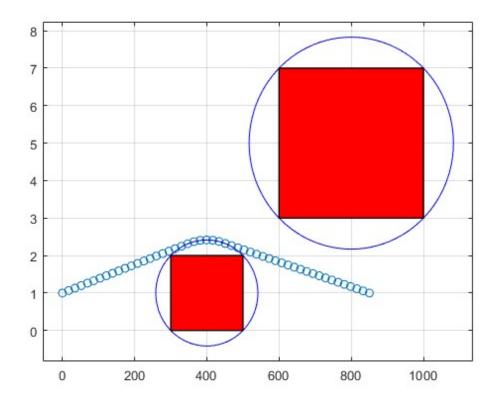
Number of Iterations....: 332

```
(scaled)
                                                        (unscaled)
Objective....:
                         1.4745068842705905e+03
                                                  1.4745068842705905e+04
                                                  3.3529033117656133e-07
Dual infeasibility....:
                          3.3529033117656135e-08
Constraint violation...:
                          0.0000000000000000e+00
                                                  0.0000000000000000e+00
                         1.00000000000000003e-11
                                                  1.0000000000000002e-10
Complementarity....:
Overall NLP error....: 3.3529033117656135e-08
                                                  3.3529033117656133e-07
```

```
Number of objective function evaluations
                                                   = 1001
Number of objective gradient evaluations
                                                   = 333
Number of equality constraint evaluations
                                                   = 0
Number of inequality constraint evaluations
                                                   = 1001
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 333
                                                   = 0
Number of Lagrangian Hessian evaluations
Total CPU secs in IPOPT (w/o function evaluations) =
                                                          1.324
Total CPU secs in NLP function evaluations
                                                          0.736
```

EXIT: Optimal Solution Found.

```
figure
axis([zmin(1) zmax(1) zmin(2) zmax(2)])
plot(z_vec(1,:),z_vec(2,:),'o')
hold on
for j=1:length(obs)
plot(xobs{j}, yobs{j},'b');
plot(obs{j}.T*unitbox(2)+obs{j}.center);
end
```



Contents

```
clc
clear
% Francesco Borrelli ME C231A 2015
% Kinematic Navigation
sampling=10;
%Var Defintions
z = sdpvar(2,N);
%Initial and terminal condition
z0 = [0;1];
zT = [850;1];
dzmin=-[20;2];
dzmax=[20;2];
zmin = [0;0];
zmax = [1000;7];
%Obstacle list
%Obstacle list
i=1;
obs{i}.center=[400;1];
obs{i}.LW=[200;2];
obs{i}.theta=0; %(in radiants)
i=i+1;
obs{i}.center=[800;5];
obs{i}.LW=[400;4];
obs{i}.theta=0; %(in radiants)
% integer variables
d = binvar(4*length(obs),(N-1)*sampling);
% bigM constant
bM=1000;
% some obstacle postprocessing
for j=1:length(obs)
   t=obs{j}.theta;
   % generate T matrix for each obstacle
   obs\{j\}.T=[cos(t), -sin(t);sin(t) cos(t)]*diag(obs\{j\}.LW/2);
   % polyehdral representaion
   obs{j}.poly=obs{j}.T*unitbox(2)+obs{j}.center;
end
%try to remove/add this one
```

```
z obs{4}=[3;7];
dot{4}=8;
%Qobs{4}=diag([1,10]);
%Constraints
%Setup Optimization Problem
cost = 0;
constr = [z(:,1)==z0;z(:,N)==zT];
Q=eye(2);
constr = [zmin <= z(:,N) <= zmax, z(:,1) == z0,z(:,N) == zT];
for t = 2:N
     cost=cost+(z(:,t)-z(:,t-1))'*Q*(z(:,t)-z(:,t-1));
     constr = constr +[dzmin <= z(:,t)-z(:,t-1) <= dzmax];
     constr = constr +[zmin<= z(:,t)<=zmax];</pre>
     for k = 0:sampling-1
         for j=1:length(obs)
             zs=z(:,t-1)+k/sampling*(z(:,t)-z(:,t-1));
             [H,K]=double(obs{j}.poly);
             constr = constr +[H(1,:)*(zs)]>=K(1)-(1-d((j-1)*4+1,(t-2)*sampling+k+1))*bM ...
                              H(2,:)*(zs)>=K(2)-(1-d((j-1)*4+2,(t-2)*sampling+k+1))*bM ...
                              H(3,:)*(zs)>=K(3)-(1-d((j-1)*4+3,(t-2)*sampling+k+1))*bM ...
                              H(4,:)*(zs)>=K(4)-(1-d((j-1)*4+4,(t-2)*sampling+k+1))*bM ...
                              d((j-1)*4+1,(t-2)*sampling+k+1)+d((j-1)*4+2,(t-2)*sampling+k+1
+d((j-1)*4+3,(t-2)*sampling+k+1)+d((j-1)*4+4,(t-2)*sampling+k+1)>=1;
         end
     end
end
options = sdpsettings('solver', 'gurobi');
%options.ipopt=ipoptset('linear_solver','MUMPS');
solvesdp(constr,cost,options);
z_{vec} = double(z);
% Plotting Functions % to add title and labels
```

```
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Optimize a model with 5296 rows, 4020 columns and 15880 nonzeros
Model has 198 quadratic objective terms
Variable types: 100 continuous, 3920 integer (3920 binary)
Coefficient statistics:
                   [5e-04, 1e+03]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e+00, 4e+00]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+00, 1e+03]
Presolve removed 4328 rows and 3388 columns
Presolve time: 0.03s
Presolved: 968 rows, 632 columns, 2893 nonzeros
Presolved model has 190 quadratic objective terms
Variable types: 96 continuous, 536 integer (536 binary)
```

Found heuristic solution: objective 15613.676716 Found heuristic solution: objective 14853.329930

Root relaxation: objective 1.474490e+04, 810 iterations, 0.01 seconds

Nodes			Current Node			0	Objective Bounds				Work		
]	Expl	Unexpl	Obj	Depth	Int	Inf	Incum	nbent	в Ве	estBd	Gap	It/Node	Time
	0	0	14744.8	980	0	328	14853.3	3299	14744.	8980	0.73%	-	0s
	0	0	14744.8	980	0	374	14853.3	3299	14744.	8980	0.73%	-	0s
	0	0	14744.8	980	0	306	14853.3	3299	14744.	8980	0.73%	-	0s
	0	0	14744.8	980	0	306	14853.3	3299	14744.	8980	0.73%	-	0s
	0	0	14744.8	980	0	255	14853.3	3299	14744.	8980	0.73%	-	0s
	0	0	14744.8	980	0	248	14853.3	3299	14744.	8980	0.73%	-	0s
	0	2	14744.8	980	0	248	14853.3	3299	14744.	8980	0.73%	-	0s
*	203	77		1	06	1	4853.313	3802	14744.	9943	0.73%	11.7	0s
Н	267	22				1	4745.018	3326	14744.	9943	0.00%	10.8	0s
Н	278	17				1	4745.006	5375	14744.	9943	0.00%	10.5	0s

Cutting planes: Clique: 311

MIR: 12

Explored 287 nodes (4744 simplex iterations) in 0.30 seconds Thread count was 4 (of 4 available processors)

Solution count 5: 14745 14745 14853.3 ... 15613.7

Optimal solution found (tolerance 1.00e-04)
Best objective 1.474500637480e+04, best bound 1.474499433966e+04, gap 0.0001%

```
figure
plot(z_vec(1,:),z_vec(2,:),'o')
hold on
for j=1:length(obs)
plot(obs{j}.T*unitbox(2)+obs{j}.center);
end
```

