Exploring Dynamic Mode Decomposition for Robust System Identification: Applications to Adaptive Signalised Intersections (TRBAM-21-03451)

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Introduction

- This paper explores recent developments in DMD algorithm and the associated Koopman theory for application in signalized intersections.
- In this paper, we perform a detailed study on the impact of delay embedding in queue length system identification and prediction using DMDc and HDMDc.
- In addition, the issue of the optimum choice of data snapshots for system identification is also addressed.
- To our best knowledge, the choice of delay embedding and selection of the number of snapshots for system identification and prediction using DMD based algorithms have not been discussed in the existing literature.

Mathematical Background

- Dynamic mode decomposition (DMD) is a purely data-driven technique that can provide a locally linear description of any complex nonlinear dynamics.
- The noteworthy point about the technique is that it does not rely upon the system's internal physics to capture its dynamics which makes it comparable to a grey box models of system identification.
- Unlike purely data-driven sta-26tistical models and machine learning algorithms, DMD and related algorithms provide an approximate system identification.
- The identification of complex dynamical systems as an approximate linear dynamics has several benefits. Among them is the applicability of linear control algorithms.

DMDc and HDMDc

We can add control to DMD and form DMD with control (DMDc) problem. The main goal of DMDc is build up a relationship between present state x_k, future state x_{k+1} and control u_k

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

where, $x_k, x_{k+1} \in \mathbb{R}^n$ In matrix form (1) can be written as

$$X' = AX + B\Upsilon \tag{2}$$

With a simple matrix manipulation (2) can be written as

$$X' = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} X \\ \Upsilon \end{bmatrix} = G\Omega \tag{3}$$

where, the sequence of control action $\Upsilon = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \dots & u_m \\ | & | & & | \end{bmatrix}$

Here, $X \in \mathbb{R}^{n\times m}, X' \in \mathbb{R}^{n\times m}, u_k \in \mathbb{R}^q, A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times q}, G \in \mathbb{R}^{n\times(n+q)}, \Omega \in \mathbb{R}^{n\times(n+q)}$

Now we can take h time delay embedding of X and X' thus obtain $\tilde{X}$ and $\tilde{X}'$ and G becomes $\tilde{G}$

Taking SVD of $\tilde{X} = \tilde{U}\tilde{\Sigma}\tilde{V}^*$

$$\tilde{G} = \tilde{X}'(\tilde{U}\tilde{\Sigma}\tilde{V}^*)^{-1} = \tilde{X}'\tilde{V}\tilde{\Sigma}\tilde{U}^* \tag{4}$$

Here, $\tilde{X} \in \mathbb{R}^{nh\times(m-h)}, \tilde{X}' \in \mathbb{R}^{nh\times(m-h)}$. By spliting $\tilde{U}$ in two components vertically we can write $\tilde{U} = \begin{bmatrix} \tilde{U_1} \\ \tilde{U_2} \end{bmatrix}$ Now we can compute $\tilde{A}$ and $\tilde{A}$ by using the following equations

$$\tilde{A} = X\tilde{'}V\Sigma^{-1}\tilde{U_1}^* \tag{5}$$

$$\tilde{B} = X\tilde{'}V\Sigma^{-1}\tilde{U_2}^* \tag{6}$$

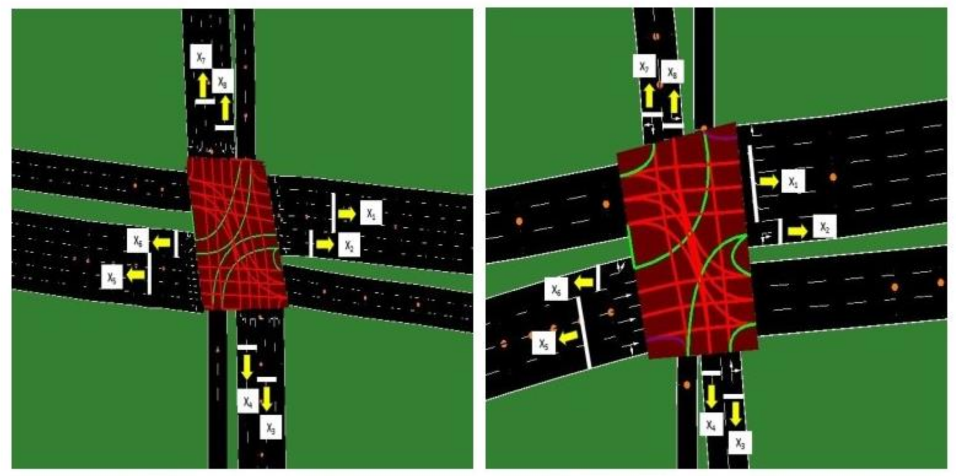
Study Area

- Study area - **Central Florida Region**
  - S.R. 50 at Murdock Boulevard and S.R. 50 at Rouse Road located in Orlando, Orange County Florida.
  - Total 2 Signalized Intersections.
- Data source: Regional Integrated Transportation Information System (RITIS) and Signal retiming report prepared by Faller, Davis & Associates, Inc. (FDA) for Florida Department of Transportation (FDOT) district 5.

(a) Rouse Intersection (b) Murdock Blvd. Intersection

Data

- **Types of Data:**
  - Signal timing data for S.R. 50 at Murdock Boulevard and S.R. 50 at Rouse Road located in Orlando, Orange County Florida.
  - The vehicle turning movement count aggregated for 15 minutes was collected from signal timing data.
  - The data of the vehicles' average speed in the network was collected from radar detectors.
- Data collection timing: The data was collected from 8:00 a.m. to 9:00 a.m. on February 17th, 2017.



(a) Rouse Intersection (b) Murdock Blvd. Intersection

Simulation Model Development and Calibration

- **Model Calibration:**
  - Traffic volume was used for calibrating the parameters in this study. In this study, Geoffrey E. Heavers (GEH) statistics were used for model calibration. This considers both the percentage difference and absolute value.
  - The equation of GEH:

$$GEH = \sqrt{\frac{2 * (V_{obs} - V_{sim})^2}{(V_{obs} + V_{sim})}}$$

  - In the above equation, V_{obs} are the traffic volumes travelling in different directions and aggregated for 15 minutes at each detector near the intersection. V_{sim} is the traffic volumes of the same 8 detectors in the simulation.
  - The value of GEH for the calibrated parameters was 0:78 for this study. This value shows that the simulated volume of vehicles replicate the real field volume. The calibrated values are displayed in the table 1.

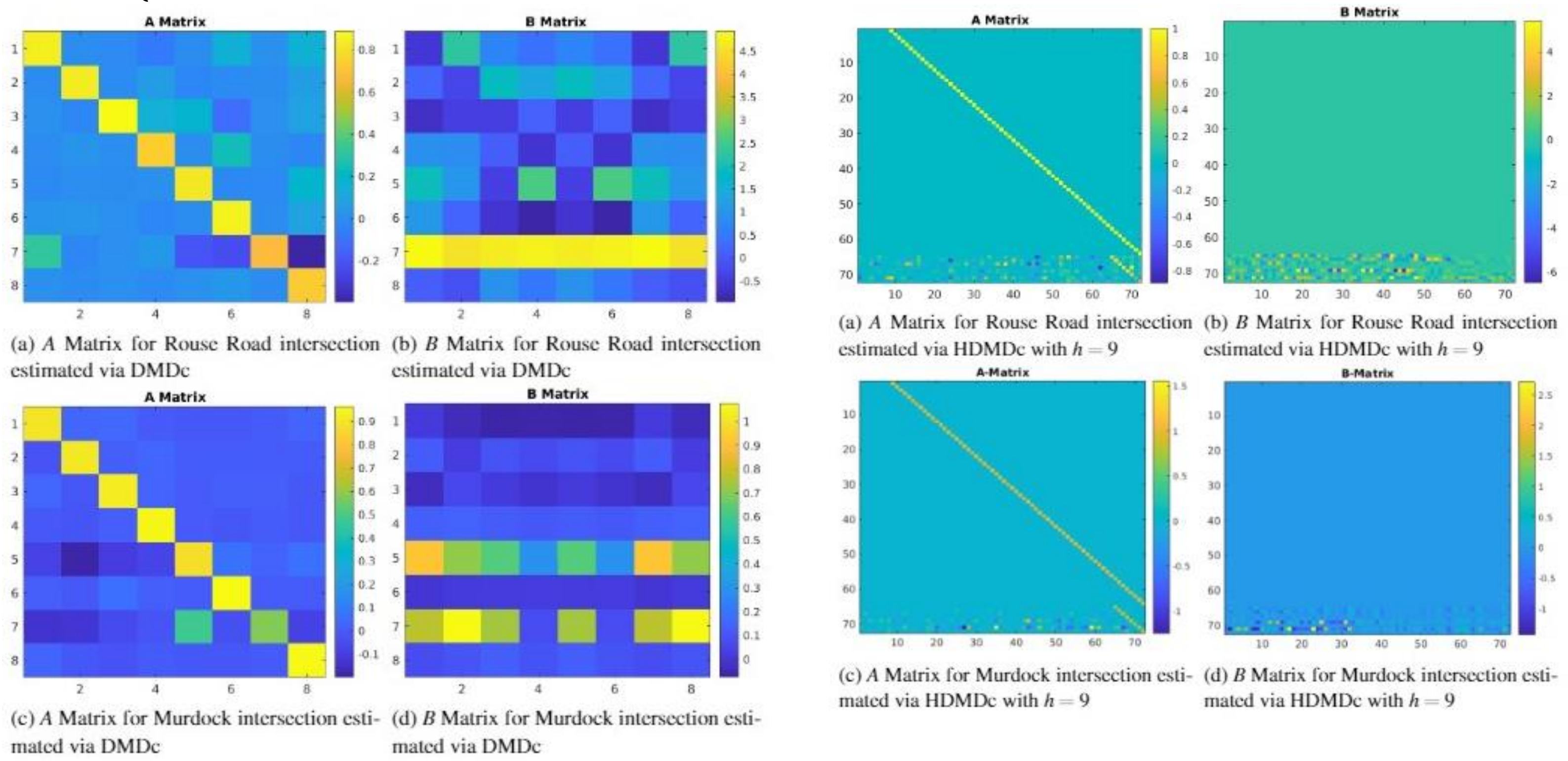
TABLE 1: SUMO calibration parameters

Parameters	Unit	Default value	Range	Calibrated value
Acceleration	m/s^2	2.6	$2.6-3.6$	3.4
Deceleration	m/s^2	4.5	$4.5-5.5$	4.5
Tau	N/A	1	$1-1.5$	1.5
Sigma	N/A	.5	$.1-.5$	.3

System Identification Procedure

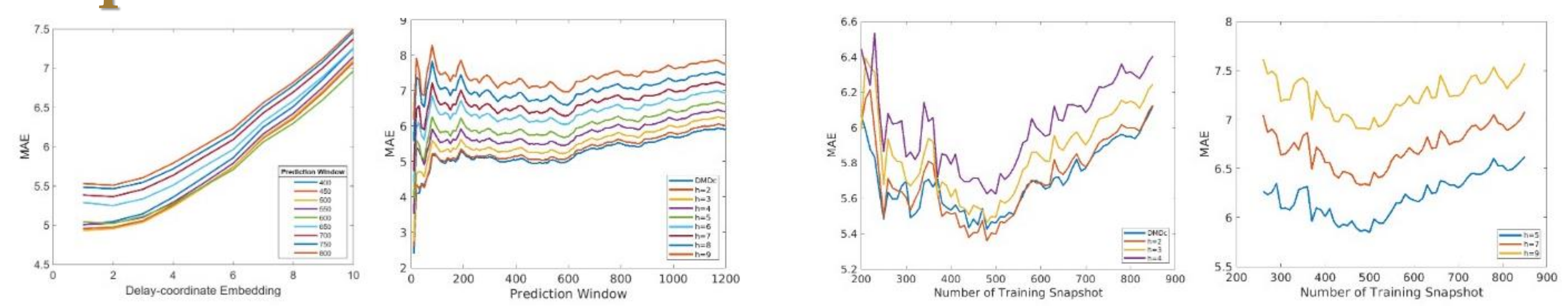
- Traffic lights at the intersections serve as the control input to the queue length dynamics. Therefore, the signal timings for red, green, and yellow lights can be converted to represent control inputs. We have modeled this phenomenon by considering binary control variable where green and yellow light is treated as 1 and the red light as 0. Mathematically it can be presented as follows:

- $u_k(i) = \begin{cases} 0 \; for \; red \; light \\ 1 \; for \; green \; and \; yellow \; light \end{cases}$



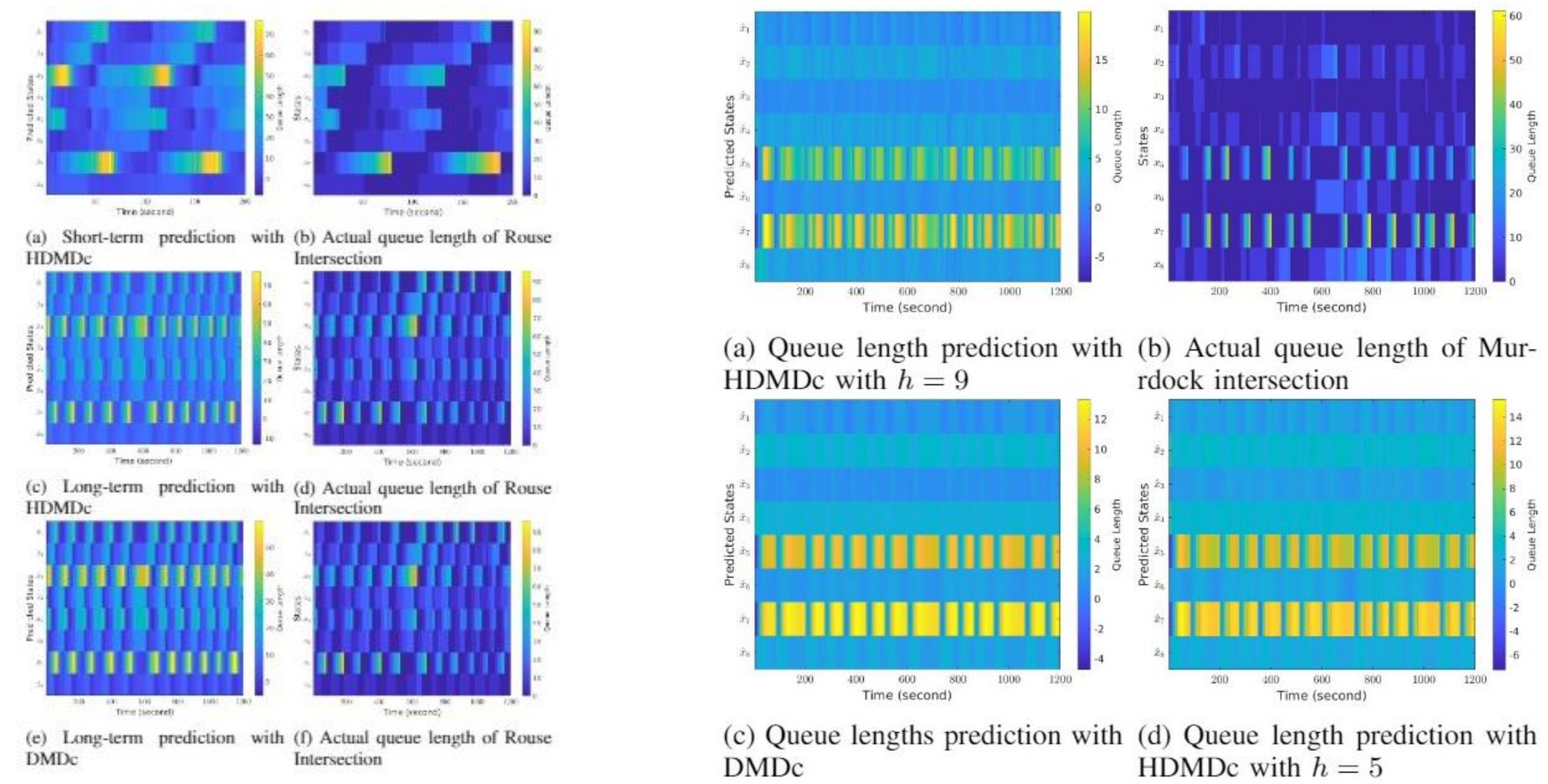
(a) A Matrix for Rouse Road intersection estimated via DMDc
(b) B Matrix for Rouse Road intersection estimated via DMDc
(a) A Matrix for Rouse Road intersection estimated via HDMDc with $h=9$
(b) B Matrix for Rouse Road intersection estimated via HDMDc with $h=9$
(c) A Matrix for Murdock intersection estimated via DMDc
(d) B Matrix for Murdock intersection estimated via DMDc
(c) A Matrix for Murdock intersection estimated via HDMDc with $h=9$
(d) B Matrix for Murdock intersection estimated via HDMDc with $h=9$

Impact of Prediction Window on Prediction Accuracy



(a) The impact of embedding on prediction results for different prediction windows
(b) The impact of prediction windows on prediction results for different embedding (h)
(c) The impact of the choice training snapshot on prediction results for lower number of embedding
(d) The impact of the choice of training snapshot on prediction for higher number of embedding

Impact of Delay-Coordinate Embedding on Prediction



(a) Short-term prediction with HDMDc
(b) Actual queue length of Rouse Intersection
(c) Long-term prediction with HDMDc
(d) Actual queue length of Rouse Intersection
(e) Long-term prediction with DMDc
(f) Actual queue length of Rouse Intersection

(a) Queue length prediction with HDMDc with $h=9$
(b) Actual queue length of Murdock intersection
(c) Queue lengths prediction with DMDc
(d) Queue length prediction with HDMDc with $h=5$

Validation of the Prediction Results

- The results in the table II show that DMDc was also able to predict the queue lengths with an accuracy close to LTSM for short-term prediction.
- So, this indicates, the system identification of the non-linear dynamics of the signalized traffic intersection using DMDc was performed to a satisfactory level.
- Finally, it can be inferred from the results that DMD based algorithms can perform effectively for non-linear system identification and short -term future state prediction for signalized traffic intersection.

TABLE II: Comparing Prediction Error between LSTM and DMDc

Intersection	Index	LSTM		DMDc	
Rouse Intersection	Training (seconds)	400	400	400	400
	Prediction (seconds)	15	30	15	30
	RMSE	1.23	1.61	5.09	5.16
	MAE	0.45	0.72	3.81	3.72
	MAPE (%)	23.38	12.57	22.87	25.74
Murdock Bvld. Intersection	Training (seconds)	400	400	400	400
	Prediction (seconds)	15	30	15	30
	RMSE	.54	0.11	1.54	2.38
	MAE	.03	0.09	1.01	1.79
	MAPE (%)	21.57	9.52	20.86	36.19

Conclusions

1. The queue length of all the incoming lanes in the intersection was extracted from SUMO simulation trajectories.
2. A locally linear system for queue length dynamics at the intersection was identified using DMDc and HDMDc.
3. Using the obtained system identification results, we predicted the queue lengths for short-term and long-term.
4. The prediction results were further analyzed against multiple factors - impact of delay embedding, effect of number of training snapshots, and the length of prediction window.
5. The prediction results were compared with state-of-the-art LSTM methods and the DMD based algorithm outperformed LSTM for the short-term prediction.

Limitations

- The queue lengths were not considered for each lane but aggregated for each moving direction.
- The intersections were adjacent to each other; however, we did not consider the mutual interactions among them (and with other nearby intersections). The yellow light was clubbed with the green light, which may not reflect reality.
- We only considered morning peak; however, it remains to be seen if the results are consistent during other times of the day or week.

Acknowledgment

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