task 5

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \text{ Constant}$$

$$\vec{J} = \sigma \begin{pmatrix} \frac{d\phi}{dx}\left(\phi(\vec{r})\right) \\ \frac{d\phi}{dy}\left(\phi(\vec{r})\right) \\ \frac{d\phi}{dy}\left(\phi(\vec{r})\right) \end{pmatrix}$$

$$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\phi(\vec{r}) = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)$$

$$= \underbrace{\left(\frac{1}{4\pi\varepsilon_0}\right)}_{u}\underbrace{\left(\frac{x.p_x + y.p_y + z.p_z}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)}_{v}$$

$$\frac{d\phi}{dx} = 0 + \left(\frac{1}{4\pi\varepsilon_0}\right)\left(v'\right)$$

$$v' = \frac{(x^2+y^2+z^2)^{\frac{3}{2}}(p_x) - (x.p_x + y.p_y + z.p_z)\left(\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{\frac{1}{2}}(\cancel{2}x)}{(x^2+y^2+z^2)^3}$$

$$v' = \frac{(p_x)(x^2+y^2+z^2)^{\frac{3}{2}}}{(x^2+y^2+z^2)^3} - \frac{(3x)(x.p_x + y.p_y + z.p_z)(x^2+y^2+z^2)^{\frac{1}{2}}}{(x^2+y^2+z^2)^3}$$

$$= \frac{p_x}{\quad} - \frac{(3x)(x.p_x + y.p_y + z.p_z)}{\quad}$$

$$\frac{d\phi}{dx} = \frac{(P_x)(x^2+y^2+z^2)^{\frac{3}{2}}}{(x^2+y^2+z^2)^3} - \frac{(3x)(x.P_x+y.P_y+z.P_z)(x^2+y^2+z^2)^{\frac{1}{2}}}{(x^2+y^2+z^2)^3}$$

$$= \frac{P_x}{(\sqrt{x^2+y^2+z^2})^3} - \frac{3x\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}P_x\\P_y\\P_z\end{pmatrix}(x^2+y^2+z^2)^{\frac{1}{2}}}{(x^2+y^2+z^2)^3}$$

$$\boxed{\begin{aligned}\vec{p} &= \begin{pmatrix}P_x\\P_y\\P_z\end{pmatrix}\\ \vec{r} &= \begin{pmatrix}x\\y\\z\end{pmatrix}\\ |\vec{r}| &= \sqrt{x^2+y^2+z^2}\end{aligned}}$$

$$= \frac{P_x}{|\vec{r}-\vec{r_0}|^3} - \frac{3x(\vec{r}-\vec{r_0})(\vec{p})(x^2+y^2+z^2)^{\frac{1}{2}}}{(x^2+y^2+z^2)^3}$$

$$= \frac{P_x}{|\vec{r}-\vec{r_0}|^3} - \frac{3x(\vec{r}-\vec{r_0})(\vec{p})}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

$$\frac{d\phi}{dx} = \frac{P_x}{|\vec{r}-\vec{r_0}|^3} - 3\cdot\frac{(\vec{r}-\vec{r_0})(\vec{p})}{|\vec{r}-\vec{r_0}|^5}\cdot x$$

$$\frac{d\phi}{d\begin{pmatrix}x\\y\\z\end{pmatrix}} = \frac{\begin{pmatrix}P_x\\P_y\\P_z\end{pmatrix}}{|\vec{r}-\vec{r_0}|^3} - 3\cdot\frac{(\vec{r}-\vec{r_0})(\vec{p})}{|\vec{r}-\vec{r_0}|^5}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$\vec{E} = \nabla\phi = \frac{\vec{p}}{|\vec{r}-\vec{r_0}|^3} - 3\cdot\frac{(\vec{r}-\vec{r_0})(\vec{p})}{|\vec{r}-\vec{r_0}|^5}(\vec{r}-\vec{r_0})$$