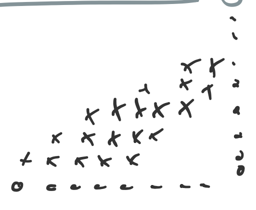
AT: session 2

Montag, 30. April 2018 17:08

About sheet 1:

Exercise 1: Dynamic Routing



output j

$y_i = \sum_j w_{ij} x_j$

$w_{ij} = \sum_k c_k \Gamma_{ijk}$

1. $y := x_{i+l} = \sum_j w_{ij} x_j$

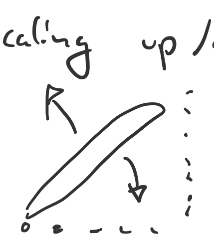
if $w_{ij} = \delta_{(i+l)j} = \begin{cases} 1 : & j = i+l \\ 0 : & \text{otherwise} \end{cases}$

then $y_i = \sum_j w_{ij} x_j =$

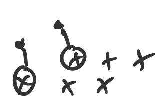
$\sum_j \delta_{(i+l)j} = 0 + \ldots + x_{i+l} + \ldots + 0 + \ldots$

$= x_{i+l} = y_i$

2. scaling up/down



3. • 10 shifts, 10 zooms $\to$ 100
   • real world unfeasible



large fan-out

way out:

$\to$ Ohlshausen et. al. 93.

4. $E = E_{match} + \beta E_{cons}$

$E_{match} = -\sum_i P_i y_i \qquad E_{cons} = \underbrace{\sum_{k, l \neq k}}_{\to \text{ double sum}} c_l c_k$

$y_i = \sum_j w_{ij} x_j = \sum_j x_j \sum_k c_k \Gamma_{ijk}$

$E = -\sum_i P_i \sum_j \left(x_j \sum_k c_k \Gamma_{ijk}\right) + \beta \sum_i \sum_{l \neq k} c_l c_k$

$\frac{\partial E}{\partial c_k} = -\sum_i \sum_j P_i x_j \Gamma_{ijk} + 2 \cdot \beta \cdot \sum_{l \neq k} c_l$

$\tau \frac{\partial}{\partial t} c_k + c_k = R\left(-\sum_i \sum_j P_i x_j \Gamma_{ijk} + 2 \beta \sum_{l \neq k} c_l\right)$

• E_{match} : max match between template and output
• E_{cons}

Exercise 2: optimal visual stimuli

simple cell : $y_s = \sigma(f \cdot x)$

complex cell : $y_c = \left[f^{(1)} \cdot x\right]^2 + \left[f^{(2)} \cdot x\right]^2$

$\|x\| = 1$

1. $\vec{f} \cdot \vec{x} = \|\vec{f}\| \cdot \underbrace{\|\vec{x}\|}_{=1} \cdot \underbrace{\cos \varphi}_{\to \max \text{ if } \ldots}$

$\vec{x} = \vec{f} \cdot b = \vec{f} \cdot \frac{1}{\|\vec{f}\|}$

2. $\frac{\partial L}{\partial \underline{x}} = 2\left(f^{(1)} \cdot \underline{x}\right) f^{(1)} + 2\left(f^{(2)} \underline{x}\right) f^{(2)} - 2\lambda x \stackrel{!}{=} 0$

$\lambda x = \underbrace{\left(f^{(1)} \underline{x}\right)}_{f^{(1)T} x} f^{(1)} + \underbrace{\left(f^{(2)} \underline{x}\right)}_{f^{(2)T} x} f^{(2)}$ (because its a scalar)

$= f^{(1)} f^{(1)T} \underline{x} + f^{(2)} f^{(2)T} \underline{x} =$

$= \underbrace{\left[f^{(1)} f^{(1)T} + f^{(2)} f^{(2)T}\right]}_{A : \text{ Projection matrix}} \underline{x}$

$\lambda \underline{x} = A \underline{x} \quad \to$ eigenvalue problem

$\to$ $\underline{x}$ is an eigenvector of A
$\to$ $\underline{x}$ is in the plane of $\underline{f}^{(1)}$ $\underline{f}^{(2)}$

$\underline{x} = \alpha \underline{f}^{(1)} + \beta \underline{f}^{(2)}$ with $\|x\|^2 = 1$

$\to \alpha^2 + \beta^2 = 1$

• stimulus unique for simple cell
• not for complex cell

Outlook for the sheet 2

$f(x) = x$

$F = \{1, x\} \to \{ax + b\}$ "Let F be the set of linear functions"

bases of all the functions with form $ax + b$

$\Delta_i = \langle \dot{y}_i^2 \rangle_t$

Δ_i smaller $\to$ y_i slower

Average over time:

$\langle \cdot \rangle_t = \frac{1}{b-a} \int_a^b \cdot \, dt \quad \text{if } t \in [a; b]$

average : $\langle x \rangle_t = \frac{1}{b-a} \int_a^b x \, dt$ (1st moment)

Variance : $Var(x) = \langle x^2 \rangle_t - \langle x \rangle_t^2$

$\langle x^2 \rangle_t \to \frac{1}{b-a} \int_a^b x^2 \, dt$ (2^{nd} moment)

proof by induction

$\to$ proof that sth. hold for $n = 1 \ldots N$
• base case : $n = 1$ show that "it" holds for $n = 1$
• assumption : assume $n = k$ $(*)$
• inductive step : show that if $n = k$ holds $\Rightarrow$ $n = k+1$ does for

example :

show : $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

base case : $n = 1$ $\quad \sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$ ✓

assumption : $\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \quad (*)$

induction : $\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1) \stackrel{(*)}{=} \frac{k(k+1)}{2} + (k+1)$

$= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+1+1)}{2}$

☐ $\dot{x} = M \dot{z} \qquad \dot{z} = \frac{\partial z}{\partial t}$

$\dot{x} \dot{x}^T = M \dot{z} \dot{z}^T M^T$

☐ trigonometric formulas

• $\cos(x) \cdot \cos(y) = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$
• $\sin(x) \cdot \sin(y) = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$
• $\cos^2(x) = \frac{1}{2}\left(1 + \cos(2x)\right)$
• $\sin^2(x) = \frac{1}{2}\left(1 - \cos(2x)\right)$