Programming tutorial

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Differential equations

(a) $\frac{\partial x}{\partial t} = -x \quad , \quad x_0 = 1 \quad \rightsquigarrow \quad x(t) = e^{-t}$

$\int \frac{\partial x}{-x} = \int -\partial t$

$\ln(x) = -t + c$

$x = \underbrace{c'}_{\exp(c)} \cdot \exp(-t)$

$\Rightarrow x(t) = \exp(-t)$

(b) $\frac{\partial x}{\partial t} = x^{-1} \quad , \quad x_0 = 1$

$\int x \, \partial x = \int dt$

$x = t + c \quad \Leftrightarrow \quad x = \pm\sqrt{2t + c'}$

$x(0) = \pm\sqrt{c'} = 1 \quad \Rightarrow \quad c' = 1$

$\Rightarrow x(t) = \sqrt{2t + 1}$

(c) $\frac{\partial x}{\partial t} = 1 - x \quad , \quad x_0 = 1$

$\int \frac{\partial x}{1-x} = \int dt$

$\ln(1-x) = -t - c$

$1 - x = c' \cdot e^{-t}$

$x = 1 - c' e^{-t}$

$x(0) = 1$

$\Rightarrow x(t) = 1 - e^{-t}$

(d) $\frac{\partial x}{\partial t} = x(1-x) \quad , \quad x_0 = 1/2$

$\int \frac{\partial x}{x(1-x)} = \int dt$

$\int \frac{x + (1-x)}{x(1-x)} \partial x = \int dt$

$\int \frac{x}{x(1-x)} + \frac{(1-x)}{x(1-x)} \partial x = \int dt$

$\int \frac{1}{1-x} + \frac{1}{x} \partial x = \int dt$

$\ln(x) - \ln(1-x) = t + c$

(...)

$\Rightarrow x(t) = \frac{e^t}{1 + e^t}$