04_Head_Models
Mittwoch, 30. Mai 2018 10:23

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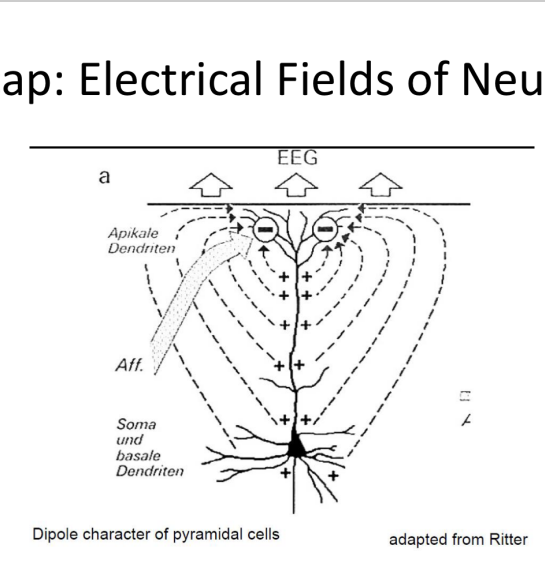
Brain Computer Interfacing

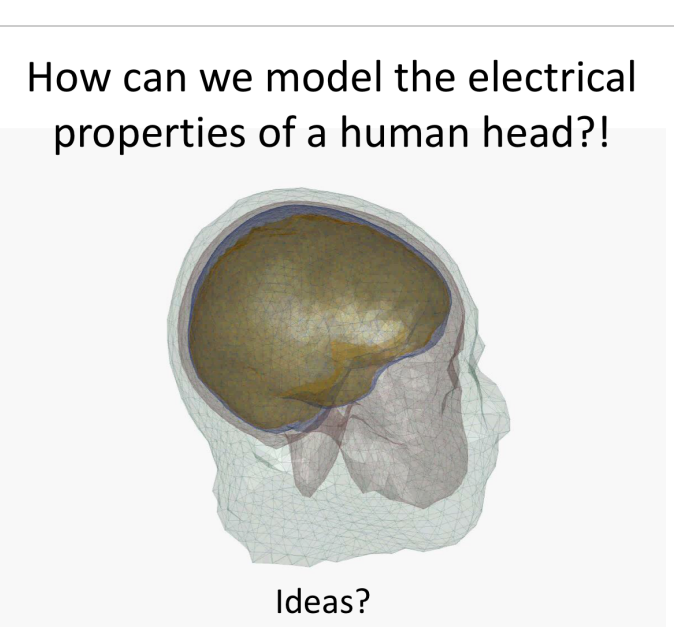
From Neurons To Data
Session 4: Head Models

Recap: Electrical Fields of Neurons



Dipole character of pyramidal cells adapted from Ritter

How can we model the electrical properties of a human head?!



Ideas?

- Resistance:
  R → Integration over space
- U = R · I
- Ions / fluids [CSF] for electricity
  ↳ polarized current
- Na^{2+} Cl^{-}

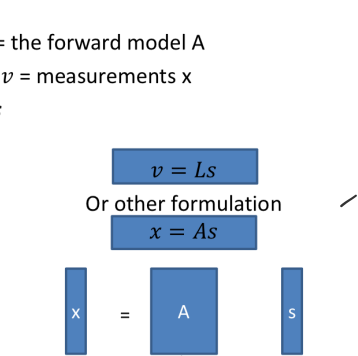
Volume conduction for most researchers:

- The leadfield L = the forward model A
- Sensor voltages v = measurements x
- Source activity s

$v = Ls$

Or other formulation

$x = As$



At the end we arrive at linear equations

→but let's dig a little deeper!

Formalism for differential field equations in cartesian coordinates

Nabla $\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$ taking derivatives to different directions Scalar function f(x,y,z) Vectorfield $\vec{F} = \begin{pmatrix} F_x(x,y,z) \\ F_y(x,y,z) \\ F_z(x,y,z) \end{pmatrix}$

- Gradient $\nabla f = \begin{pmatrix} \frac{\partial}{\partial x} f \\ \frac{\partial}{\partial y} f \\ \frac{\partial}{\partial z} f \end{pmatrix}$ direction of strongest ascend
- Divergence $\nabla \cdot \vec{F} = \frac{\partial}{\partial x} F_x + \frac{\partial}{\partial y} F_y + \frac{\partial}{\partial z} F_z$ source density
- Rotor $\nabla \times \vec{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y \\ \frac{\partial}{\partial z} F_x - \frac{\partial}{\partial x} F_z \\ \frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x \end{pmatrix}$ rotation

All you need is Maxwell

magnetic field is source free

Maxwell equations: (local/differential form)

$$\boxed{\begin{aligned} \nabla \cdot \vec{D} &= \rho & (1)\\ \nabla \cdot \vec{B} &= 0 & (2)\\ \nabla \times \vec{E} &= -\delta_t \vec{B} & (3)\\ \nabla \times \vec{H} &= \vec{j} + \delta_t \vec{D} & (4) \end{aligned}}$$

System of differential equation that describes an electric field

Rotor electric field

Electric Field Strength $\vec{E}$
Electric Flux $\vec{D}$
Magnetic Field Strength $\vec{H}$
Magnetic Flux $\vec{B}$
Charge Density ρ
Current Density $\vec{j}$

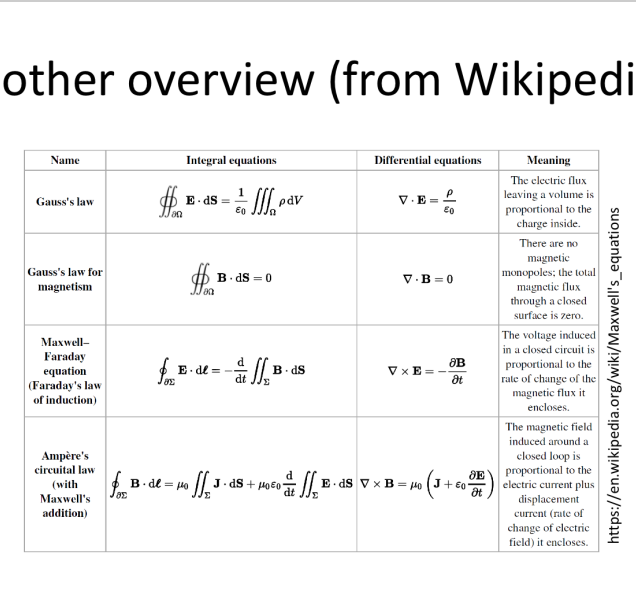
(1) is called Gauss's Law
(2) is called Gauss's Law for magnetic fields
(3) is called Maxwell-Faraday equation (Induction Law)
(4) is called Ampere's circuital law (with Maxwell's addition)

This describes the more general case: Electromagentism/Electrodynamics

Maxwell's equations

- Describes simple and complex fields up to wave propagation
- Can be simplified to e.g. (quasi-) electrostatics for volume conduction of the human head
- Also have a volumetric/integral form

Another overview (from Wikipedia)



Name	Integral equations	Differential equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_\Omega \rho \, dV$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric flux leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux through a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \iint_\Sigma \mathbf{B} \cdot d\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage induced in a closed circuit is proportional to the rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 \iint_\Sigma \mathbf{J} \cdot d\mathbf{S} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint_\Sigma \mathbf{E} \cdot d\mathbf{S}$	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$	The magnetic field induced around a closed loop is proportional to the electric current plus displacement current (rate of change of electric field) it encloses.

https://en.wikipedia.org/wiki/Maxwell's_equations

From Maxwell to electrostatics

- the charge retention $\nabla \cdot \vec{j} = -\delta_t \vec{D}$ is implicitly included in the last equation
- Further we get the possibility of describing the electric field with a simple potential : $\vec{E} = -\nabla\varphi$ as the curl is $\nabla \times \vec{E} = -\delta_t \vec{B} = 0$ (the field is conservative).
- If we allow time changing electric charges and only look at the steady state, we are in a (quasi-) stationary regime which can be treated similarly. This is especially in low frequency (LF) observations as for EEG of interest.
- As the space charge is assumed to be static $\delta_t \rho = 0$ and so $\nabla \cdot \vec{j} = 0$, a divergence free current is found.

$$\boxed{\begin{aligned} \nabla \cdot \vec{D} &= \rho \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\delta_t \vec{B} \\ \nabla \times \vec{H} &= \vec{j} + \delta_t \vec{D} \end{aligned}}$$

(Quasi-)Electrostatic Equations

Maxwell:

$$\boxed{\begin{aligned} \nabla \cdot \vec{D} &= \rho \\ \cancel{\nabla \cdot \vec{B} = 0} \\ \cancel{\nabla \times \vec{E} = -\delta_t \vec{B}} \\ \nabla \times \vec{H} &= \vec{j} + \cancel{\delta_t \vec{D}} \end{aligned}}$$

- **static field**
  $\delta_t \vec{D} = \delta_t \varepsilon_r \varepsilon_0 \vec{E} = 0$
- **magnetic induction insignificant**
  $\nabla \times \vec{E} = -\delta_t \vec{B} = 0$

→ $\vec{E} = -\nabla\varphi$: scalar potential sufficient to describe field

Approach for EEG head modeling

- Basis mostly: (quasi-) electrostatics
- Macroscopic model for the current denisities:

$\vec{J} = \sigma\vec{E} + \vec{J}_P$ $\vec{J}_P$ primary currents (in cell)
  $\sigma\vec{E}$ secondary currents (volume conduction)

$\rightarrow \nabla \cdot \vec{J} = \nabla \cdot (\sigma\vec{E} + \vec{J}_P) = \nabla \cdot (-\sigma\nabla\varphi + \vec{J}_P) = 0$

Reordered: Poisson's equation

$$\nabla \cdot (\sigma\nabla\varphi) = \nabla \cdot \vec{J}_P$$

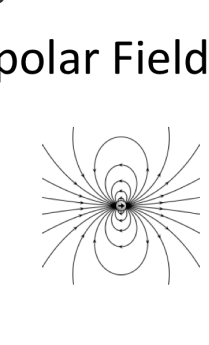
The field that is produced   Stable by all fields coming from outside

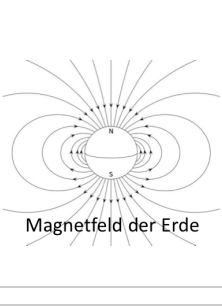
$\frac{\partial^2}{\partial x^2} f(x) = 0$ Laplace

$\frac{\partial^2}{\partial x^2} f(x) = g(x)$ Poisson

Primary Currents J_P : Dipolar Field

- current dipole (Neuronal activity)
  - analytic solution $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3}$
    $\vec{p} = q \cdot \vec{d}$
  - Current density $\vec{J}_P = \sigma\vec{E} = -\sigma\nabla\varphi$





Magnetfeld der Erde

Laplace

$\nabla \cdot (\sigma\nabla\varphi) = \sigma\nabla^2\varphi = \nabla \cdot \vec{J}_P$

- The basic solution $G(r;r')$ to a Laplace operator ∇^2 solves $\nabla^2 G(r;r') = -\delta(|r - r'|)$
- where $\nabla^2 = \nabla \cdot \nabla$ and $\delta(x)$ is the Dirac delta.
- The Green function is such a solution. It also vanishes at infinity, which is in correspondence with the physical intuition that a static field has no effect far away from it's sources [3]. In 3 dimensions the Green function G is:
  $$G(\vec{r};\vec{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}$$
- This solution is does in general not fulfill the boundary conditions.

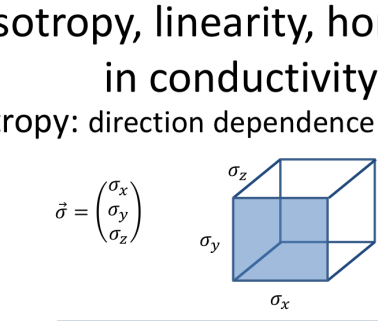
Surprise!

- $\nabla \cdot (\sigma\nabla\varphi) = \sigma\nabla^2\varphi = \nabla \cdot \vec{J}_P = f$
- $G(\vec{r};\vec{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}$
- Point charge potential:
  $$\varphi(r - r') = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\vec{r} - \vec{r}'|}$$

(an)isotropy, linearity, homogeneity in conductivity

- Anisotropy: direction dependence

$\vec{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$



Example: white matter

- Linearity

$U = RI$ $\vec{J}(\vec{r}) = \sigma(\vec{r})\vec{E}(\vec{r})$

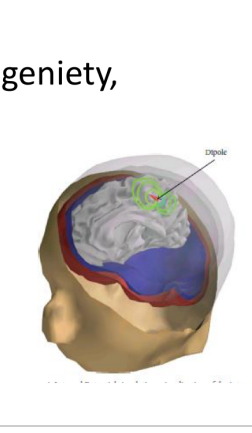
R, σ ... independent of amplitudes of $\vec{J}, \vec{E}$

- Homogeneity: constant over (sub-)space

$\vec{J}(\vec{r}) = \sigma\vec{E}(\vec{r})$
- Biological tissue in general inhomogeneous
- Homogeneity is an approximation

Example: Boundary Element Method

- Basic field equations solved for every compartment individually
- Within compartments: homogeniety, anisotropy, linearity



From Gramfort 2011

INDIVIDUAL HEAD

Overview

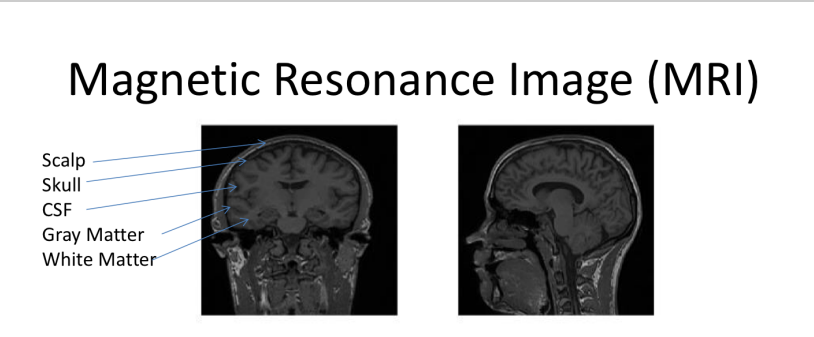
- Human heads are similar

→many people use generic models (e.g. averages)

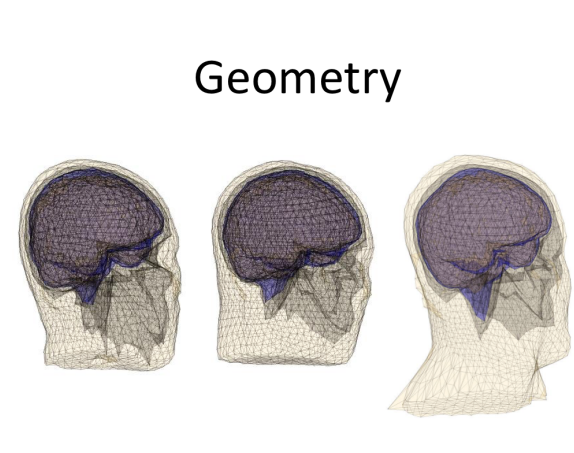
- Human heads are distinct

→For better models individual anatomies are extracted

Magnetic Resonance Image (MRI)



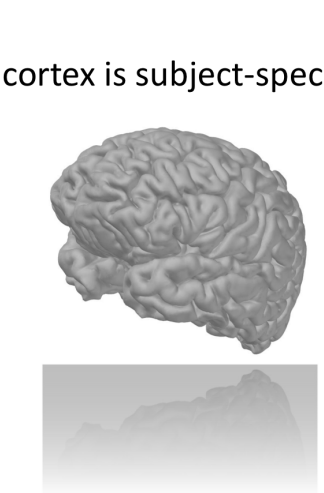
Geometry



- Overall size/shape
- Skull thickness
- CSF/brain position
- Scalp thickness

Individual brain

- Folding of cortex is subject-specific

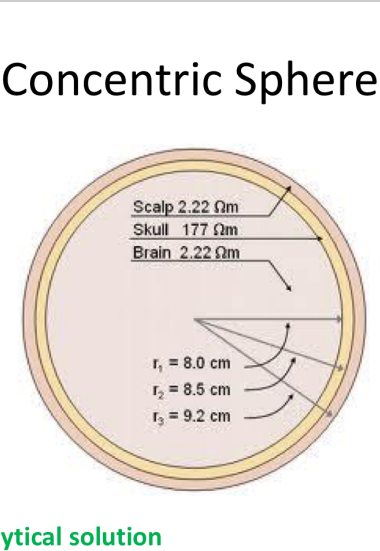


Individual head model is needed when you want to know where a signal came from

(Realistic) forward models

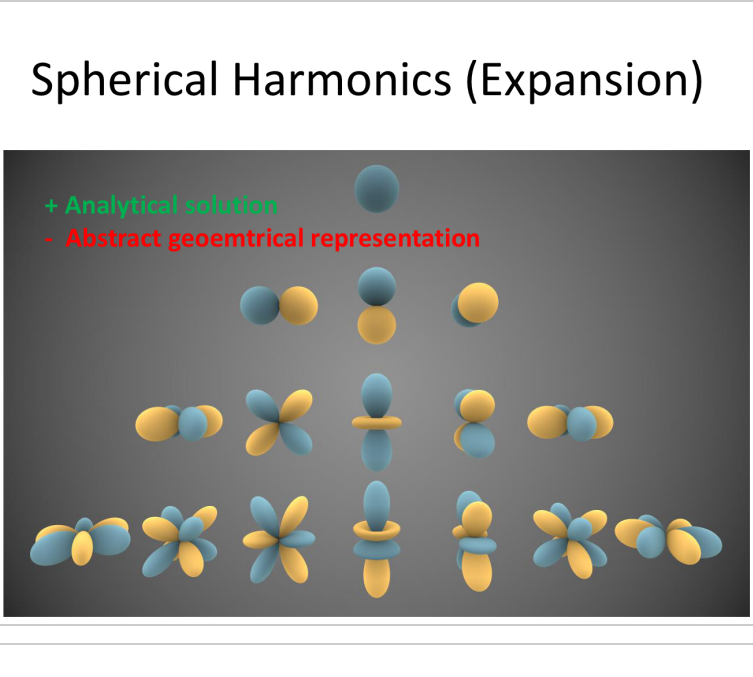
- Analytical solutions: spherical, spheroid, spherical harmonics
- Real head geometries: numerical solutions
  - Boundary Element Method (BEM)
  - Finite Element Methods (FEM)
  - Finite Differences Method (FDM)

Concentric Spheres



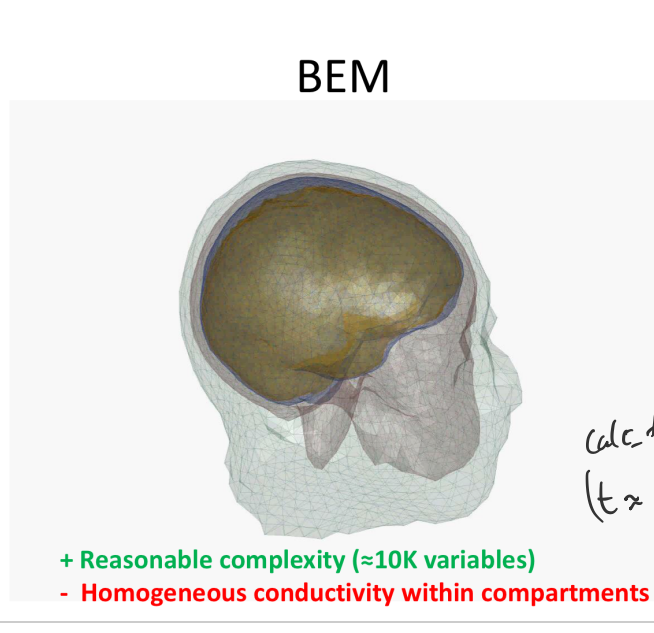
+ Analytical solution
- Only geometrical parameters: radii of spheres

Spherical Harmonics (Expansion)



+ Analytical solution
- Abstract geometrical representation

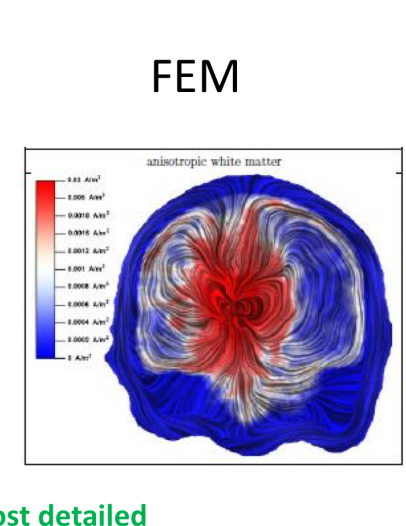
BEM



+ Reasonable complexity (≈10K variables)
- Homogeneous conductivity within compartments

calc time (t ≈ 1h)

FEM

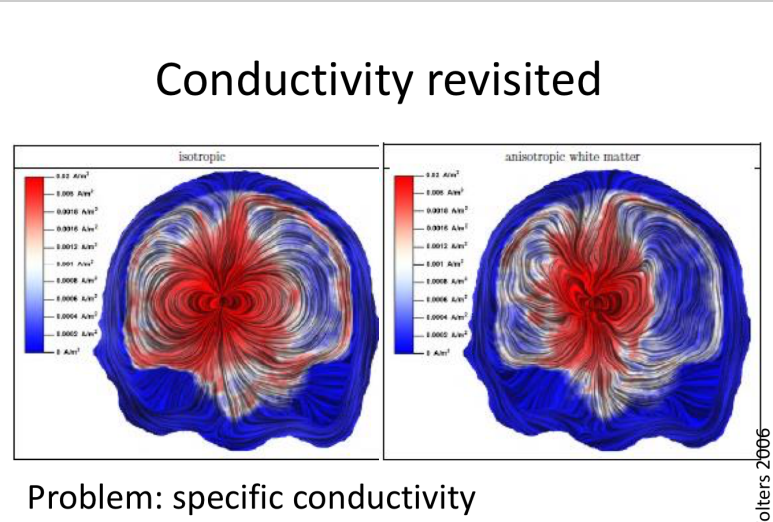


+ Most detailed
- High dimensionality (≈1M variables)

(t ≈ 12h)

From Wolters 2006

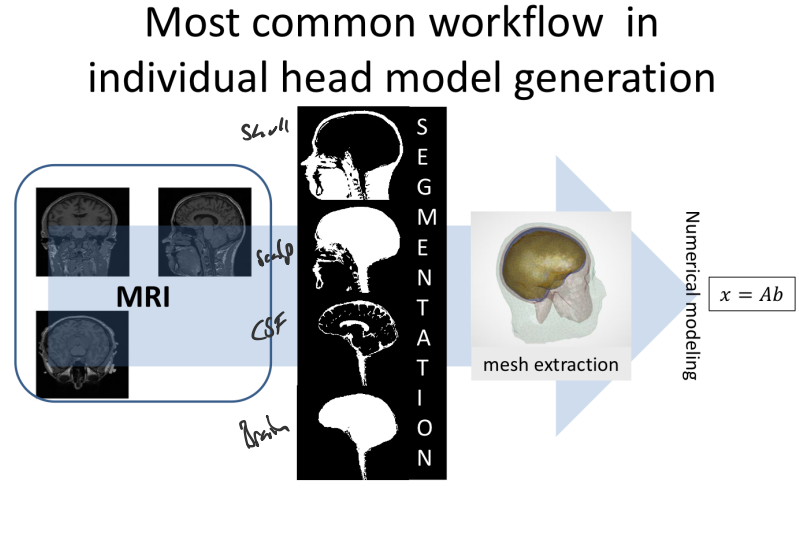
Conductivity revisited



Problem: specific conductivity
variance inhomogeneity anisotropy

From Wolters 2006

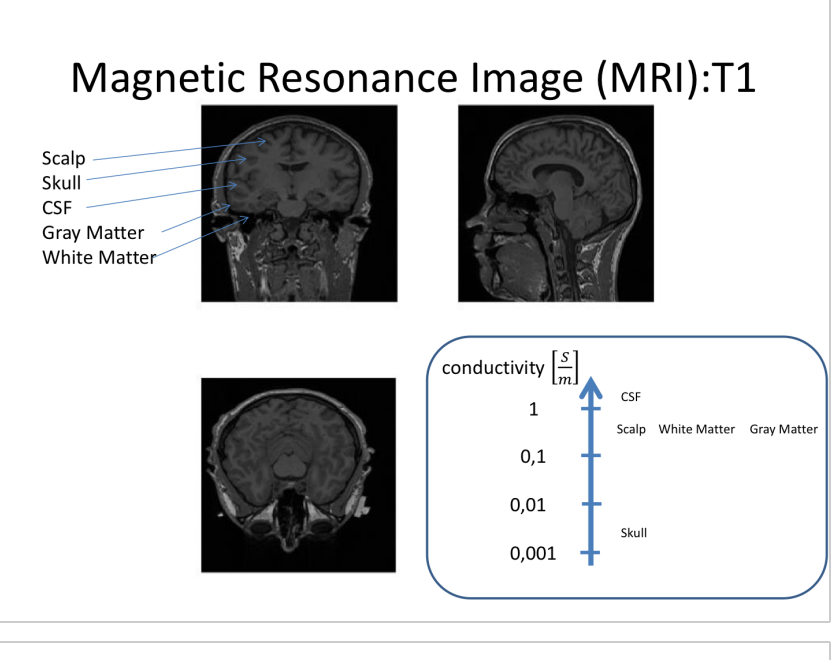
Most common workflow in individual head model generation



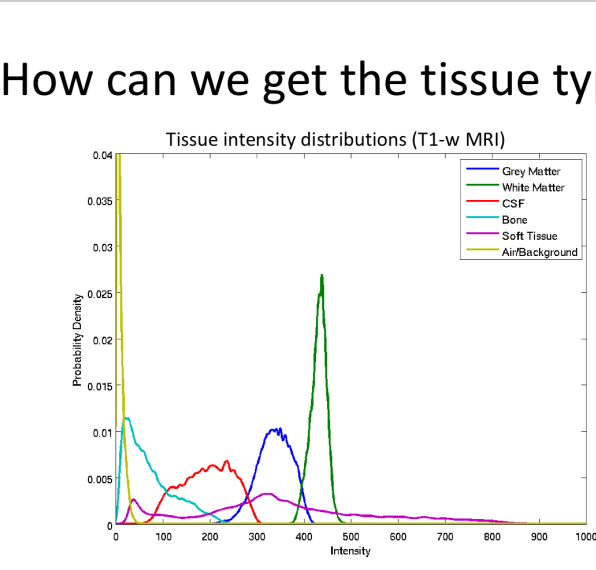
From MRI

EXTRACTING THE INDIVIDUAL GEOMETRY

Magnetic Resonance Image (MRI):T1



How can we get the tissue type?



Borders are overlapping → hard for deciding

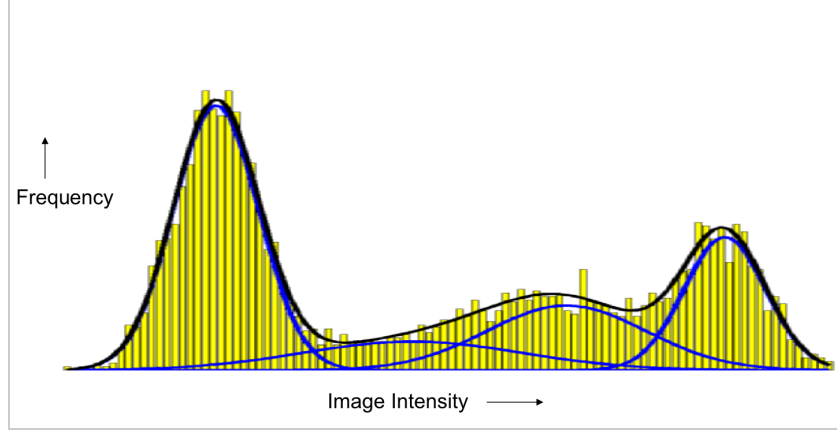
x segmente this pic in homework

Most widely used segmentation toolbox: SPM

- SPM: statistical parametric mapping
  http://www.fil.ion.ucl.ac.uk/spm/
- Principled Bayesian probabilistic formulation
- Gaussian mixture model segmentation with deformable tissue probability maps (priors)
  - The inverse of the transformation that aligns the TPMs can be used to normalize the original image
- Bias correction is included within the model

Mixture of Gaussians

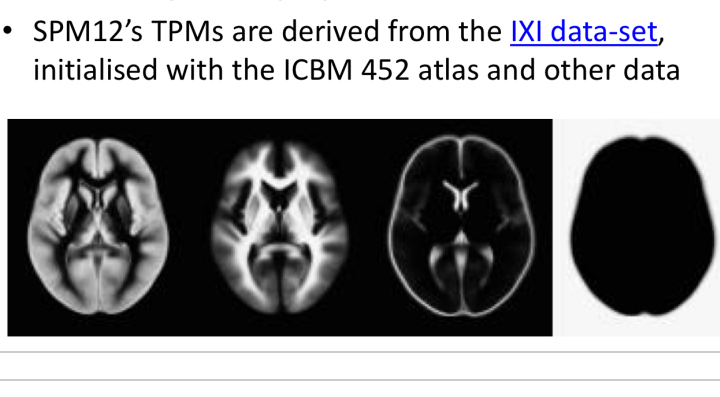
- Models intensities



Tissue Probability Maps as prior

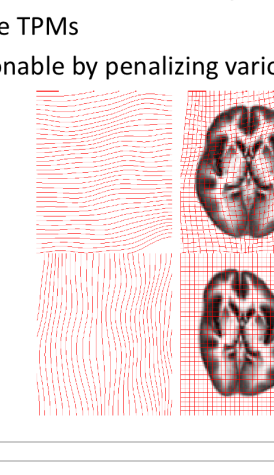
SPM: statistical parametric mapping

- Tissue probability maps (TPMs) are used as the prior, instead of just the proportion of voxels in each class
- SPM12's TPMs are derived from the IXI data-set, initialised with the ICBM 452 atlas and other data



Non-linear warp

- Tissue probability images are warped to match the subject
- The inverse transform warps to the TPMs
- Warps are constrained to be reasonable by penalizing various distortions (energies)



4-shell BEM modeling in detail

SO WHAT ARE THE RELEVANT EQUATIONS?

Approach for EEG head modeling

- Basis mostly: (quasi-) electrostatics
- Macroscopic model for the current denisities:

$\vec{J} = \sigma\vec{E} + \vec{J}_P$ $\vec{J}_P$ primary currents (in cell)
  $\sigma\vec{E}$ secondary currents (volume conduction)

$\rightarrow \nabla \cdot \vec{J} = \nabla \cdot (\sigma\vec{E} + \vec{J}_P) = \nabla \cdot (-\sigma\nabla\varphi + \vec{J}_P) = 0$

Reordered: Poisson equation

$$\nabla \cdot (\sigma\nabla\varphi) = \nabla \cdot \vec{J}_P$$

...some notations

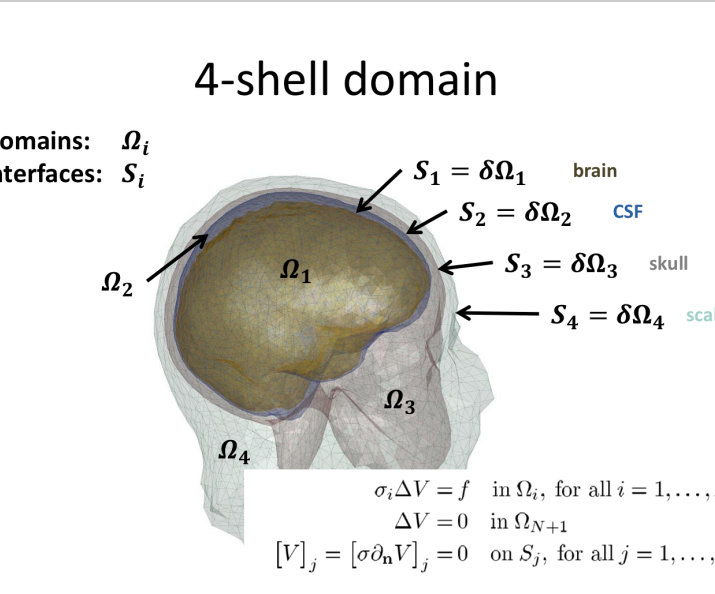
- $\delta_{\boldsymbol{n}} V = \boldsymbol{n} \cdot \nabla V$ (normal derivative)
- $[f]_j = f_{S_j}^- - f_{S_j}^+$ (jump of function f at interface S_j)

the functions f^- and f^+ on S_j being, respectively, the interior and exterior limits of f

$$\text{for } \mathbf{r} \in S_j, \qquad f_{S_j}^{\pm}(\mathbf{r}) = \lim_{\alpha \to 0^{\pm}} f(\mathbf{r} + \alpha\mathbf{n}).$$

4-shell domain

Domains: $\boldsymbol{\Omega_i}$
Interfaces: $\boldsymbol{S_i}$


$\boldsymbol{S_1 = \delta\Omega_1}$ brain
$\boldsymbol{S_2 = \delta\Omega_2}$ CSF
$\boldsymbol{S_3 = \delta\Omega_3}$ skull
$\boldsymbol{S_4 = \delta\Omega_4}$ scalp
$\boldsymbol{\Omega_1}$
$\boldsymbol{\Omega_2}$
$\boldsymbol{\Omega_3}$
$\boldsymbol{\Omega_4}$

$$\begin{aligned} \sigma_i \Delta V &= f && \text{in } \Omega_i, \text{ for all } i = 1, \ldots, N \\ \Delta V &= 0 && \text{in } \Omega_{N+1} \\ [V]_j = [\sigma\partial_{\mathbf{n}} V]_j &= 0 && \text{on } S_j, \text{ for all } j = 1, \ldots, N. \end{aligned}$$

In general relevant equations

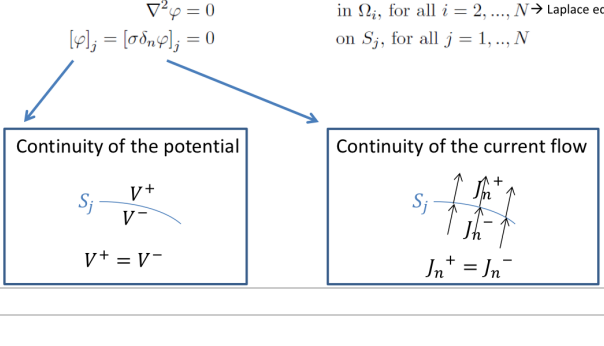
$$J = \sigma E + J^P \qquad (1)$$

$$\nabla \cdot (\sigma\nabla\varphi) = f = \nabla \cdot J^P. \qquad (2)$$

If we assume the conductivity to be piecewise constant in subspaces Ω_i we get a set of Poisson equations for the EEG case:

$\sigma_i \nabla^2 \varphi = f$ in Ω_1 → Poisson's equation inside the brain
$\nabla^2 \varphi = 0$ in Ω_i, for all $i = 2, \ldots, N$ → Laplace equation within domains
$[\varphi]_j = [\sigma\delta_n\varphi]_j = 0$ on S_j, for all $j = 1, \ldots, N$

Continuity of the potential   Continuity of the current flow



$V^+ = V^-$ $J_n^+ = J_n^-$

Potential operators

$$\begin{aligned} (\mathcal{D}f)(\mathbf{r}) &= \int_{\partial\Omega} \partial_{\mathbf{n}'} G(\mathbf{r} - \mathbf{r}') f(\mathbf{r}') \, \mathrm{d}s(\mathbf{r}') \\ (\mathcal{S}f)(\mathbf{r}) &= \int_{\partial\Omega} G(\mathbf{r} - \mathbf{r}') f(\mathbf{r}') \, \mathrm{d}s(\mathbf{r}') \\ (\mathcal{N}f)(\mathbf{r}) &= \int_{\partial\Omega} \partial_{\mathbf{n},\mathbf{n}'} G(\mathbf{r} - \mathbf{r}') f(\mathbf{r}') \, \mathrm{d}s(\mathbf{r}') \\ (\mathcal{D}^* f)(\mathbf{r}) &= \int_{\partial\Omega} \partial_{\mathbf{n}} G(\mathbf{r} - \mathbf{r}') f(\mathbf{r}') \, \mathrm{d}s(\mathbf{r}') \,. \end{aligned}$$

$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}$$

→Convolution

Single/Double-layer formulations

- Potential equations can be treated in different ways
  - Classical potential theory: mostly either single or double layer formulation
  - Symmetric formulation: joint single and double layer formulation for numerical stability and convenience in symmetric problem

Symmetric formulations

We base our symmetric integral formulation on Clerk et al. [1]
Similarly we consider for $i = 1, 2, 3$ a potential function which is harmonic in $\mathbb{R}^3 \backslash \delta\Omega_i$:

$$u_i = \begin{cases} V & \text{in } \Omega_i \\ 0 & \text{in } \mathbb{R}^3 \backslash \overline{\Omega_i}. \end{cases}$$

Additionally we define $p_{S_i} = \sigma_i [\delta_n V]_{S_i} = \sigma_i (\delta_n V)_{S_i}^-$.
Representation theorem:

$$\begin{array}{lll} -p = +\mathcal{N} & [u] - \mathcal{D}^* & [p], \text{ for } \notin \delta\Omega \\ -u = -\mathcal{D} & [u] + \mathcal{S} & [p], \text{ for } \notin \delta\Omega \\ -p^{\pm} = +\mathcal{N} & [u] + \left(\pm\frac{\mathcal{I}}{2} - \mathcal{D}^* \right) & [p], \text{ for } r \in \delta\Omega \\ -u^{\pm} = +\left(\mp\frac{\mathcal{I}}{2} - \mathcal{D} \right) & [u] + \mathcal{S} & [p], \text{ for } r \in \delta\Omega \end{array}$$

(4-shell) symmetric BEM for EEG

Based on the set of linear equations defined in [1, 2], we get for the 4-shell BEM:

$$\begin{array}{l} \text{brain} \{ \\ \text{csf} \{ \\ \text{skull} \{ \\ \text{scalp} \{ \end{array} \quad A \begin{bmatrix} V_{S_1} \\ p_{S_1} \\ V_{S_2} \\ p_{S_2} \\ V_{S_3} \\ p_{S_3} \\ V_{S_4} \end{bmatrix} = \mathbf{b},$$

$$A = \begin{bmatrix} (\sigma_1+\sigma_2)\mathcal{N}_{11} & -2\mathcal{D}^*_{11} & -\sigma_2\mathcal{N}_{12} & \mathcal{D}^*_{12} & 0 & 0 & 0 \\ -2\mathcal{D}_{11} & (\sigma_1^{-1}+\sigma_2^{-1})\mathcal{S}_{11} & \mathcal{D}_{12} & -\sigma_2^{-1}\mathcal{S}_{12} & 0 & 0 & 0 \\ -\sigma_2\mathcal{N}_{21} & \mathcal{D}^*_{21} & (\sigma_2+\sigma_3)\mathcal{N}_{22} & -2\mathcal{D}^*_{22} & -\sigma_3\mathcal{N}_{23} & \mathcal{D}^*_{23} & 0 \\ \mathcal{D}_{21} & -\sigma_2^{-1}\mathcal{S}_{21} & -2\mathcal{D}_{22} & (\sigma_2^{-1}+\sigma_3^{-1})\mathcal{S}_{22} & \mathcal{D}_{23} & -\sigma_3^{-1}\mathcal{S}_{23} & 0 \\ 0 & 0 & -\sigma_3\mathcal{N}_{23} & \mathcal{D}^*_{32} & (\sigma_2+\sigma_3)\mathcal{N}_{33} & -2\mathcal{D}^*_{33} & -\sigma_4\mathcal{N}_{34} \\ 0 & 0 & \mathcal{D}_{23} & -\sigma_3^{-1}\mathcal{S}_{32} & -2\mathcal{D}_{33} & (\sigma_3^{-1}+\sigma_4^{-1})\mathcal{S}_3 & \mathcal{D}_{34} \\ 0 & 0 & 0 & 0 & -\sigma_3\mathcal{N}_{34} & \mathcal{D}^*_{43} & \sigma_4\mathcal{N}_{44} \end{bmatrix}$$

$$b = \begin{bmatrix} (\sigma_1\delta_n v)_{S_1} \\ (v)_{S_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

f refers to the analytic dipolar solution!

where $v = -\frac{1}{\sigma_1} f * G$ is the analytical solution of $\sigma_1 \nabla^2 v = f$. G is the radius dependent Green function.

Recommended Papers

[1] M. Clerc, J. Badier, G. Adde, J. Kybic, and T. Papadopoulo. Boundary element formulation for electrical impedance tomography. *ESAIM: Proceedings*, 14:63–71, 2005.

[2] A. Gramfort, T. Papadopoulo, E. Olivi, and M. Clerc. Forward field computation with OpenMEEG. *Computational Intelligence and Neuroscience*, Article ID 923703, 2011.

[3] J. Kybic, M. Clerc, T. Abboud, O. Faugeras, R. Keriven, and T. Papadopoulo. A common formalism for the integral formulations of the forward EEG problem. *IEEE Trans. Med. Img.*, 24:12–28, 2005.