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Problem # 1:

Suppose we have a population of four measurements 2, 4, 6 and 8. Draw a random sample of size 2 without replacement and demonstrate that

(i) The sample mean is an unbiased estimate of population mean.

(ii) $v(\bar{y}) = \frac{\sigma^2}{n}.\frac{N-n}{N-1}$

(iii) Verify is s^2 an unbiased estimate of σ^2?

(iv) Find 95% confidence interval for population mean and population total.

(v) Answer the above questions for sampling with replacement.

Problem #2:

Let X and Y denote the strength of concrete beams and cylinders. The following data are obtained

X:5.9,7.2,7.3,6.3,8.1,6.8,7.0,7.6,6.8,6.5,7.0,6.3,7.9,9.0,8.2,8.7,7.8,9.7,7.4,7.7,9.7,7.8,7.7,11.6,11.3,11.8,10.7.

Y:6.1,5.8,7.8,7.1,7.2,9.2,6.6,8.3,7.0,8.3,7.8,8.1, 7.4,8.5,8.9,9.8,9.7,14.1,12.6,11.2.

(i) Show that $\bar{X} - \bar{Y}$ is an unbiased estimator of$\mu_1 - \mu_2$. Calculate it for the given data.

(ii) Find the variance and standard deviation (standard error) of the estimator in Part(i), and then compute the estimated standard error.

(iii) Calculate an estimate of the ratio $\frac{\sigma_1}{\sigma_2}$ of the two standard deviations.

(iv) Suppose a single beam X and a single cylinder Y are randomly selected. Calculate an estimate of the variance of the difference X −Y.

Problem #3:

A farm grows grapes for jelly. The following data are measurements of sugar in a grapes of a sample taken from each of 30 truckloads.

16.0, 15.2, 12.0, 16.9, 14.4, 16.3, 15.6, 12.9, 15.3, 15.1, 15.8, 15.5, 12.5, 14.5, 14.9, 15.1, 16.0, 12.5, 14.3, 15.4, 15.4, 13.0, 12.6, 14.9, 15.1, 15.3, 12.4, 17.2, 14.7, 14.8

Assume that these observations of a random variable X that has mean μ and the standard deviation σ.

(i) Find point estimates of μ and σ.

(ii) Construct an approximate 90% / 95% / 80% confidence interval for μ.

Problem #4:

Draw random number of size 200 from (a) normal distribution with mean 50 and variance 26 and (b) exponential distribution with mean 60.

(i) Find the estimate of the parameters by maximum likelihood method.

(ii) Construct a 90% / 95% / 80% confidence interval for the parameter(s).

(iii) Estimate the variance using exponential distribution.

Problem #5:

The sample mean from population with pdf $f(x;\theta) = \theta e^{-\theta x}; x > 0, \theta > 0$ are given below

0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.58, 0.60, 0.73, 0.55, 0.23, 0.62, 0.38, 0.27, 0.36, 0.47, 0.49, 0.71.

(i) Find the estimate of θ by maximum likelihood method.

(ii) Construct a $90\% / 95\% / 80\%$ confidence interval for θ.
(iii)Estimate the variance of θ.

Problem#6:

According to a survey in 2008, the mean salary of MBA graduates in accounting was 37,000 Tk. per month. In a follow up study in June 2009, a sample of 48 MBA students graduating in accounting found a sample mean of 38,100 Tk. and a sample standard deviation of 5,200 Tk.

(i) Formulate the null and alternative hypothesis that can be used to determine whether the sample data support the conclusion that the MBA graduates in accounting have a mean salary greater than 37,000 Tk.
(ii) At 5% level of significance what is your conclusion?
(iii)Find the p-value and state your conclusion.
(iv)Find 95% confidence interval for mean salary of MBA graduates.

Problem#7:

The daily temperature (in degree Celsius) of two months during summer season are shown below:

Months	Daily temperature (in degree Celsius)
1	32, 34, 31, 33, 35, 36, 34, 34, 34, 35, 32, 33, 33, 33, 32, 32, 34, 33, 32, 34, 32, 31, 33, 34, 35, 34, 33, 33, 33, 34, 34
2	34, 34, 35, 35, 35, 35, 35, 35, 36, 37, 34, 33, 34, 35, 34, 34, 36, 34, 33, 34, 32, 33, 34, 36, 35, 35, 35, 34, 35, 34

(i) Input the two sets of data using R software and save this file in CSV format in desktop.
(ii) Formulate the null hypothesis and alternative hypothesis that can be used to determine that the temperature of both months are not similar?
(iii)Calculate the value of test statistic and state your conclusion.
(iv)What is the p-value of this test? Give your conclusion based on p-value.
(v) Construct box plots for these two sets of data. Do the box plots support your conclusion obtained in question (iv).

Problem #8: In a sample of 80 Americans, 44 wished that they were rich. In a sample of 90 Europeans, 41 wished that they were rich. Answer the following questions using R software:

(i) At $\alpha = 0.01$, is there a difference in the proportions?
(ii) What is the p-value of this test? What is your conclusion compared with p-value? Compare this conclusion with conclusion obtained in (i).
(iii) Find the 99% confidence interval for the difference of the two proportions.

Problem #9:

The number of students admitted in two departments in a university in different years are as follows:

Year	Statistics	Mathematics	Year	Statistics	Mathematics
2001	40	60	2011	37	55
2002	42	64	2012	38	54
2003	45	67	2013	43	69
2004	38	55	2014	42	65
2005	40	62	2015	39	59
2006	39	66	2016	46	70
2007	46	70	2017	42	68
2008	44	65	2018	41	62
2009	43	62	2019	42	64
2010	42	56	2020	38	58

The researcher claim that the variation in admission of students in different years are not same. Answer the following question using R software:

(i) Input the data in MS Excel and save this file in CSV format. Export this CSV file in R.

(ii) Formulate the null and alternative hypothesis.

(iii) Calculate the value of appropriate test statistic and comment on your result.

(iv) Find the p-value of this test and state your conclusion.

Problem #10: The following are the heights (X in cm) and weights (in kg) of 15 persons.

X	160	165	159	164	168	155	158	155	152	159	158	154	153	152	154
Y	70	72	64	63	72	65	62	56	56	60	58	58	55	56	60

(i) Input the dataset using R software and save this file in CSV format.

(ii) Test the hypothesis that the weight of animals significantly increased due to the increase in height? Conclusion your result using p-value method.

(iii) Test the significance of correlation between weight and height. Conclusion your result using p-value method.