

10/10/18

Tutorial-

Date _____

Page _____

Prob1: Find the cardinality of given set
 $A = \{1, 3, 5, 7, 9\}$.

Prob2: Consider set $X = \{2, 4, 6, 8, 10\}$. Find its power set, cardinality and cardinality of power set.

Prob3: Show that the following fuzzy sets satisfy De Morgan's law.

$$(a) M_A(x) = \frac{1}{1+5x}$$

$$(b) M_B(x) = \left[\frac{1}{1+5x} \right]^{1/2}$$

Prob4: Consider two fuzzy sets

$$A = \{1/2.0, 0.65/4.0, 0.5/6.0, 0.35/8.0, 0/10\}.$$

$$B = \{0/2.0, 0.35/4.0, 0.5/6, 0.65/8.0, 1/10\}.$$

Find the following:-

- (a) $A \cup B$ (b) $A \cap B$ (c) \bar{A} (d) \bar{B}
- (e) $\bar{A} \cap \bar{B}$ (f) $\bar{A} \cup \bar{B}$ (g) $\bar{A} \cap B$ (h) $\bar{A} \cup B$
- (i) $A \cup \bar{A}$ (j) $A \cap \bar{A}$ (k) $B \cup \bar{B}$ (l) $B \cap \bar{B}$

Prob5: The ~~disaggregated~~ discretized membership function (in non-dimensional unit) for UJT and BJT are given below:-

$$U_{T1} = \{0/0, 0.2/1, 0.3/2, 0.6/3, 0.9/4, 1/5\}$$

$$U_{T2} = \{0/0, 0.1/1, 0.2/2, 0.3/3, 0.4/4, 0.7/5\}$$

For the two fuzzy set perform the following operations

- i) $U_{T1} \vee U_{T2}$ ii) $U_{T1} \wedge U_{T2}$ iii) \bar{U}_{T1}
- iv) U_{T2} v) $\bar{U}_{T1} \wedge U_{T2}$ $\bullet = \bar{U}_{T1} \vee U_{T2}$

* AI → A guide to Intelligence System by Michel Nenniotsky.

Prob: Consider the following fractions over the given fuzzy sets

$$X = \{0.1/0, 0.2/1, 0.3/2, 0.4/3, 0.5/4\}$$

$$Y = \{0.5/0, 0.4/1, 0.3/2, 0.2/3, 0.1/4\}$$

- (a) $X \cup Y$ (b) $X \cap Y$ (c) \bar{X} (d) \bar{Y} (e) $X \cup \bar{Y}$
- (f) $\bar{X} \cap Y$ (g) $X \cup \bar{X}$ (h) $X \cap \bar{X}$ (i) $X \cup \bar{Y}$
- (j) $Y \cup \bar{X}$ (k) Algebraic sum (l) algebraic product (m) bounded sum (n) bounded difference

Prob: Draw fuzzy set 'around 10^{am} in the morning'

Solutions:

① 5.

$$\text{③ } H_A(x) = \frac{1}{1+5x} \quad H_B(x) = \left[\frac{1}{1+5x} \right]^{1/2}$$

~~100% 00% 00%~~

$$\text{AUB} = \left[\frac{1}{1+5x} \right]^{1/2} \text{ as it is larger}$$

$$\bar{AUB} = 1 - \left[\frac{1}{1+5x} \right]^{1/2}$$

$$\bar{A} = 1 - \underbrace{\frac{1}{1+5x}}_{\text{larger}}$$

$$\bar{B} = 1 - \underbrace{\left[\frac{1}{1+5x} \right]^{1/2}}_{\text{smaller}}$$

$$\bar{A} \cap \bar{B} = \text{smaller}$$

$$= 1 - \left[\frac{1}{1+5x} \right]^{1/2}$$

As, $\overline{A \cup B} = \overline{A} \cap \overline{B}$, so Demorgan's law hold. (✓).

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (To prove).

$$A \cap B = \frac{1}{1+5x} \quad (\text{As it is smaller}).$$

$$\overline{A \cap B} = 1 - \frac{1}{1+5x}.$$

$\overline{A} \cup \overline{B}$ = larger one

$$= 1 - \frac{1}{1+5x} \quad (\text{ie, } \overline{A}).$$

So $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (Proved).

②

$$X = \{2, 4, 6, 8, 10\}$$

Power set of $X = \{\emptyset, \{2\}, \{4\}, \{6\}, \{8\}, \{10\}, \{2, 4\}, \{2, 6\}, \{2, 8\}, \{2, 10\}, \{2, 4, 6\}, \{2, 4, 8\}, \{2, 4, 10\}, \{2, 6, 8\}, \{2, 6, 10\}, \{2, 8, 10\}, \{4, 6, 8\}, \{4, 6, 10\}, \{4, 8\}, \{4, 10\}, \{4, 8, 10\}, \{6, 8\}, \{6, 10\}, \{8, 10\}, \{6, 8, 10\}, \{2, 4, 6, 8\}, \{2, 4, 6, 10\}, \{2, 6, 8, 10\}, \{2, 4, 8, 10\}, \{6, 4, 8, 10\}, \{2, 4, 6, 8, 10\}\}.$

Cardinality of $X = 5$

Cardinality of power set of $X = 2^5 = 32$.

Prob 4.

$$A = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$$

hence

$$B = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$$

① $A \cup B = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$

② $A \cap B = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$

③ $\bar{A} = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$

④ $\bar{B} = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$

⑤ $\bar{A} \cap \bar{B} = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$

⑥ $\bar{A} \cup \bar{B} = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$

⑦ $\overline{A \cap B} = \bar{A} \cup \bar{B}$ (By DeMorgan's law)

⑧ $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (By DeMorgan's law)

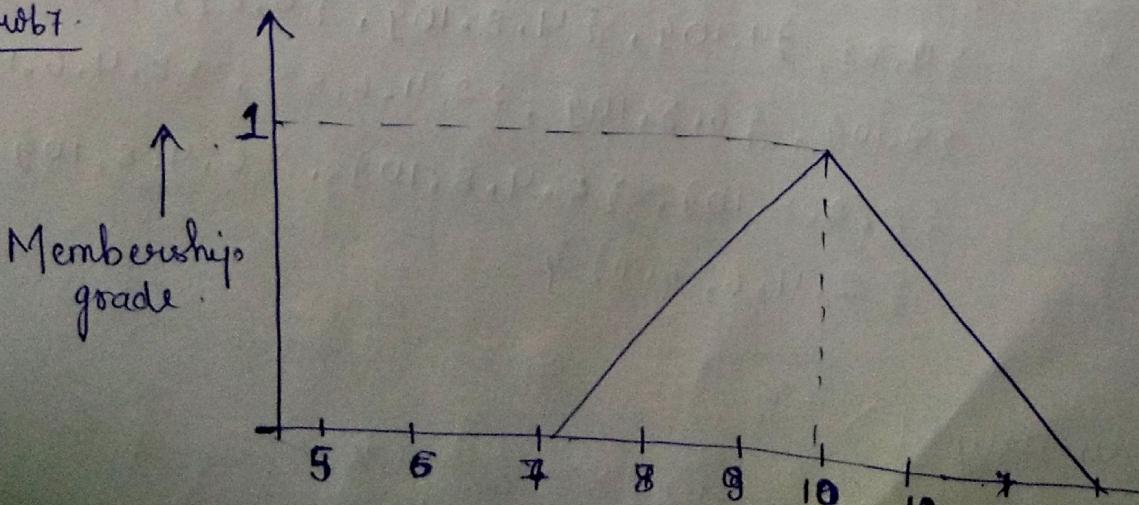
⑨ $A \cup \bar{A} = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$

⑩ $A \cap \bar{A} = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$

⑪ $B \cup \bar{B} = \{ \frac{1}{2.0}, \frac{0.65}{4.0}, \frac{0.5}{6.0}, \frac{0.65}{8.0}, \frac{1}{10} \}$

⑫ $B \cap \bar{B} = \{ \frac{0}{2.0}, \frac{0.35}{4.0}, \frac{0.5}{6.0}, \frac{0.35}{8.0}, \frac{0}{10} \}$

Prob 7.



$$\text{Prob. 6. } X = \{0.1/0, 0.2/1, 0.3/2, 0.4/3, 0.5/4\}$$

$$Y = \{0.5/0, 0.4/1, 0.3/2, 0.2/3, 0.1/4\}$$

$$\textcircled{a} \quad X \cup Y = \{0.5/0, 0.4/1, 0.3/2, 0.4/3, 0.5/4\}$$

$$\textcircled{b} \quad X \cap Y = \{0.1/0, 0.2/1, 0.3/2, 0.2/3, 0.1/4\}$$

$$\textcircled{c} \quad \overline{X} = \{0.9/0, 0.8/1, 0.7/2, 0.6/3, 0.5/4\}$$

$$\textcircled{d} \quad \overline{Y} = \{0.5/0, 0.6/1, 0.7/2, 0.8/3, 0.9/4\}$$

$$\textcircled{e} \quad \overline{X \cup Y} = \{0.5/0, 0.6/1, 0.7/2, 0.6/3, 0.5/4\}$$

$$\textcircled{f} \quad \overline{X \cap Y} = \{0.9/0, 0.8/1, 0.7/2, 0.8/3, 0.9/4\}$$

$$\textcircled{g} \quad X \cup \overline{X} = \{0.9/0, 0.8/1, 0.7/2, 0.6/3, 0.5/4\}$$

$$\textcircled{h} \quad X \cap \overline{X} = \{0.1/0, 0.2/1, 0.3/2, 0.4/3, 0.5/4\}$$

$$\textcircled{i} \quad X \cup \overline{Y} = \{0.5/0, 0.6/1, 0.7/2, 0.8/3, 0.9/4\}$$

$$\textcircled{j} \quad Y \cup \overline{X} = \{0.9/0, 0.8/1, 0.7/2, 0.6/3, 0.5/4\}$$

$$\textcircled{k} \quad X + Y = \cancel{\{H_X(x) + H_Y(x)\}} \quad \forall x \in X$$
$$= \{0.55/0, 0.52/1, 0.51/2, 0.52/3, 0.55/4\}$$

$$\textcircled{l} \quad X \cdot Y = \{H_X(x) \cdot H_Y(x) \quad \forall x \in X\}$$

$$= \{0.05/0, 0.08/1, 0.09/2, 0.08/3, 0.05/4\}$$

$$\textcircled{m} \quad X \oplus Y = \min(1, (H_X(x) + H_Y(x))) \quad \text{for } \forall x \in X$$

$$= \{0.6/0, 0.6/1, 0.6/2, 0.6/3, 0.6/4\}$$

(ii) $X \oplus Y = \max \{0, (H_A(x) - H_B(x))^2\}$

$$= \{0/0, 0/1, 0/2, 0.2/3, 0.4/4\}$$

(Prob 6). $U_{T1} = \{0/0, 0.2/1, 0.3/2, 0.6/3, 0.9/4, 1/5\}$

② $U_{T2} = \{0/0, 0.1/1, 0.2/2, 0.3/3, 0.4/4, 0.7/5\}$

③

④ (i) $H_{T1} \vee H_{T2} = \{0/0, 0.2/1, 0.3/2, 0.6/3, 0.9/4, 1/5\}$

⑤

(ii) $H_{T1} \wedge H_{T2} = \{0/0, 0.1/1, 0.2/2, 0.3/3, 0.4/4, 0.7/5\}$

⑥

⑦ (iii) $\overline{H_{T1}} = \{1/0, 0.8/1, 0.7/2, 0.4/3, 0.1/4, 0/5\}$

⑧

(iv) $\overline{H_{T2}} = \{1/0, 0.9/1, 0.8/2, 0.7/3, 0.6/4, 0.3/5\}$

⑨

⑩ (v) $\overline{H_{T1} \wedge H_{T2}} = \{1/0, 0.9/1, 0.8/2, 0.7/3, 0.6/4, 0.3/5\}$

⑪

⑫

— X —

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Degree of fuzziness:-

Let A be a fuzzy set defined over universe of discourse X (finite set).

$D_p(A, \bar{A}) \rightarrow$ distance b/w A and \bar{A}

In particular.

$D_1(A, \bar{A}) =$ distance of order 1, Hamming metric

$D_2(A, \bar{A}) =$ " " " " 2, Euclidian metric

$$D_p(A, \bar{A}) = \left[\sum_{i=1}^n |A(x_i) - \bar{A}(x_i)|^p \right]^{\frac{1}{p}} = \left[\sum_{i=1}^n |2A(x_i) - 1|^p \right]^{\frac{1}{p}}$$

- * Cardinality of X will define how many elements are in A .

$H_A(x_i)$ or $A(x_i) \rightarrow$ membership grade of x_i to A .

where n = cardinality.

In particular $p = 1, 2, 3, \dots$

$$D_1(A, \bar{A}) = \left[\sum_{i=1}^n |A(x_i) - \bar{A}(x_i)| \right] ; p = 1$$

$$D_2(A, \bar{A}) = \left[\sum_{i=1}^n |A(x_i) - \bar{A}(x_i)|^2 \right]^{\frac{1}{2}} ; p = 2$$

Notably we take $p = 1$ or 2 .

Degree of fuzziness of order p of fuzzy set A

$$= FUZ_p(A)$$

when A is a crisp set, it can either have value 0 or 1

$$D_p(A, \bar{A}) = \left[\sum_{i=1}^n |A(x_i) - \bar{A}|^p \right]^{\frac{1}{p}}$$

$$= n^{\frac{1}{p}}.$$

Pg Now: $FUZ_p(A) = \frac{n^{\frac{1}{p}} - D_p(A, \bar{A})}{n^{\frac{1}{p}}} = 1 - \frac{D_p(A, \bar{A})}{n^{\frac{1}{p}}}$

(i) $FUZ_1(A) = 1 - \frac{\left[\sum_{i=1}^n |\alpha_A(x_i) - 1| \right]}{n^{\frac{1}{p}}}.$

$$= 1 - \frac{\left[\sum_{i=1}^n |\alpha_A(x_i) - 1| \right]}{n}.$$

FUZ_2(A) = 1 - \frac{\left[\sum_{i=1}^n |\alpha_A(x_i) - 1|^2 \right]^{\frac{1}{2}}}{n^{\frac{1}{2}}}.

$$FUZ_p(A) = 1 - \frac{\left[\sum_{i=1}^n |\alpha_A(x_i) - 1|^p \right]^{\frac{1}{p}}}{n^{\frac{1}{p}}}$$

When ^{A is} crisp set either value 0 or 1

then $FUZ_p(A) = 1 - \frac{n^{\frac{1}{p}}}{n^{\frac{1}{p}}} = 0$ (minimum).

$FUZ_p(A) = 1 - 0 = 1$ (maximum)

when all $\alpha_A(x_i) = 0.5$.

This expression is true, for finite set of elements in A.

Now if we have infinite universe of discourse, then how the expression will be modified, so that it still satisfies De Luca and Termini criterion.

$$FUZ_p(A) = 1 - \frac{\left[\int_a^b |2A(x) - 1|^p dx \right]^{1/p}}{(b-a)^{1/p}}$$

Mamdani Inference System

Sugeno Inference System

→ By making use of this we make some inference.

Inputs can be single or multiple and similarly O/P also.

SISO → Single Input Single O/P.

MISO → Multiple " " " "

MIMO → Multiple " " Multiple " "

Rule 1. If U is A. then V is B.

Here U is Input.

V is O/P. (or control action).

A, B are fuzzy set.

B is a fuzzy set defined over universe of discourse V.

Degree of firing of rule! - is on higher side then rule is very relevant in the rule base.

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Mamdani Inference System or Mamdani linguistic model (SISO)

Single Input Single Output

- We will have a rule base comprising of m rules.
- We will have an input U .

Rule 1

If U is B_1 Then V is D_1

$$\tau_1 = V_x (A(x) \wedge B_1(x)) \rightarrow f_1(y) = \tau_1 \wedge D_1(y)$$

$\tau_1 = B_1(x^*)$ (when U is a crispset).

Rule 2

If U is B_2 Then V is D_2

$$\tau_2 = V_x (A(x) \wedge B_2(x)) \rightarrow f_2(y) = \tau_2 \wedge D_2(y)$$

$\tau_2 = B_2(x^*)$

Rule m .

If U is B_m Then V is D_m

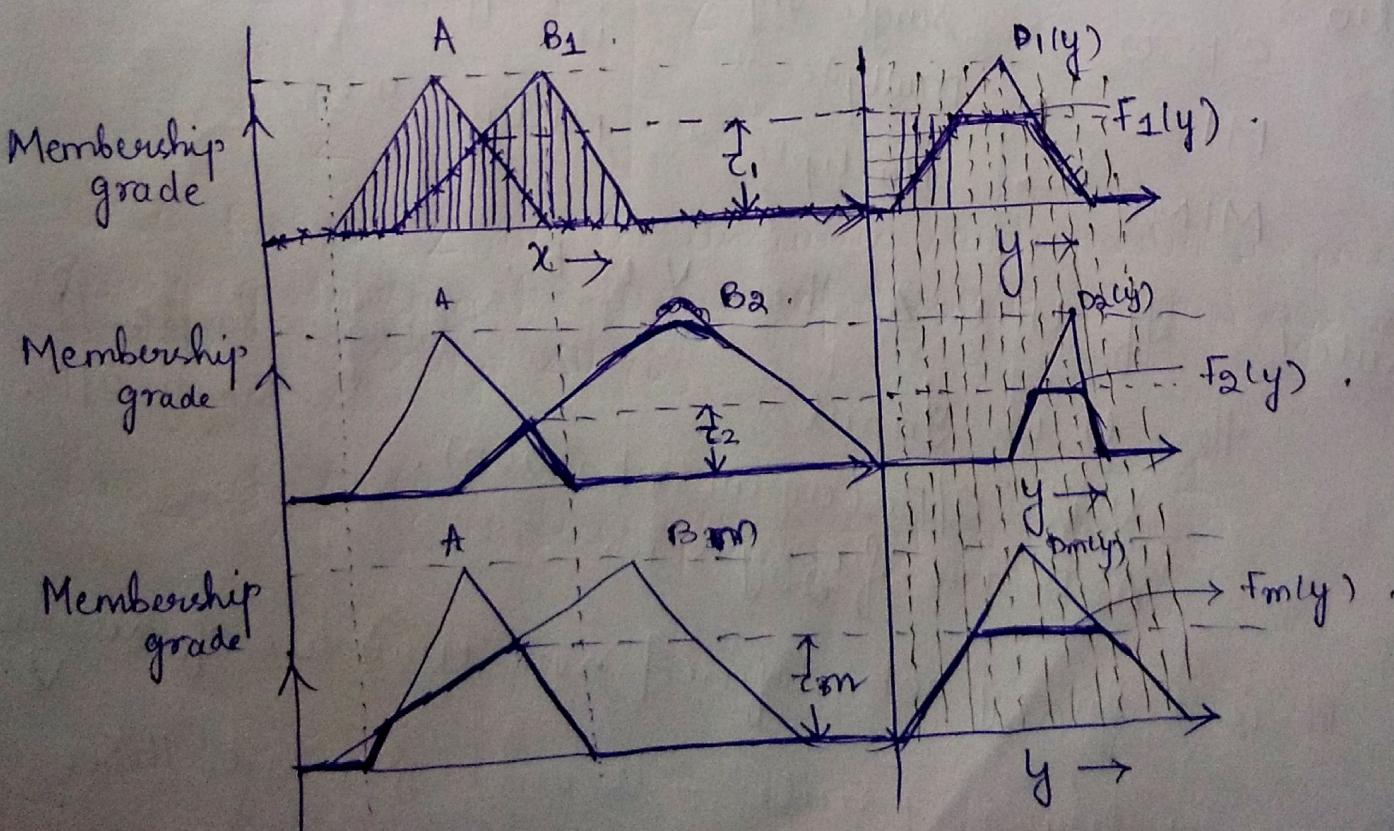
$$\tau_m = V_x (A(x) \wedge B_m(x)) \rightarrow f_m(y) = \tau_m \wedge D_m(y)$$

$\tau_m = B_m(x^*)$

 $f_1(y)$ $f_2(y)$ $f_m(y)$ $F(y)$

$F(y) \rightarrow$ inferred
fuzzy outcome.

$U = A$
 x^*
(crisp value
if U is
a crispset)



- U is one ^{single} input and its domain is X .
- + Y is defining the domain of the output (\mathbb{B})
- U is defined over the range of the input i.e., X .
- B is defined over the entire domain of U i.e., \mathbb{B} .
- There are fuzzsets B_1, B_2, \dots, B_m defined over U .
- U could be a fuzzy set or a crisp value.
- We are supposed to calculate the ~~the~~ degree of relevance of the rules for the current state of the system i.e., $U=A$. (Relevance of a particular state of system is known as degree of firing)
- Degree of firing is calculated as T .
- $f_1(y) \rightarrow$ Inferred fuzzy outcome from Rule 1
All these inferred fuzzy outcomes have to be aggregated.
- $T_1 \rightarrow$ Take intersection of $A(x)$ and $B_1(x)$ ^{for all value of x} and then taking the maximum of all
- $D_1 \rightarrow$ fuzzy set defined or positioned over y .
(Output)
- $f_2(y) \rightarrow$ Inferred fuzzy outcome of Rule 2
Finally we will take maximum of all this $f_1(y), f_2(y), \dots$
- $f_m(y) \rightarrow$ which will give us the aggregate inferred fuzzy outcome.
which will then go through the process of defuzzification

→ Rule should be written so that it caters to the entire (all points) operating domain of the system.

As the state (operating) will change accordingly the no. of rules relevant to the system changes.

- If resolution is very good (define more linguistic values), the number of rules is on higher side.
- If you increase the number of inputs, the number of rules is on higher side.

Tutorial

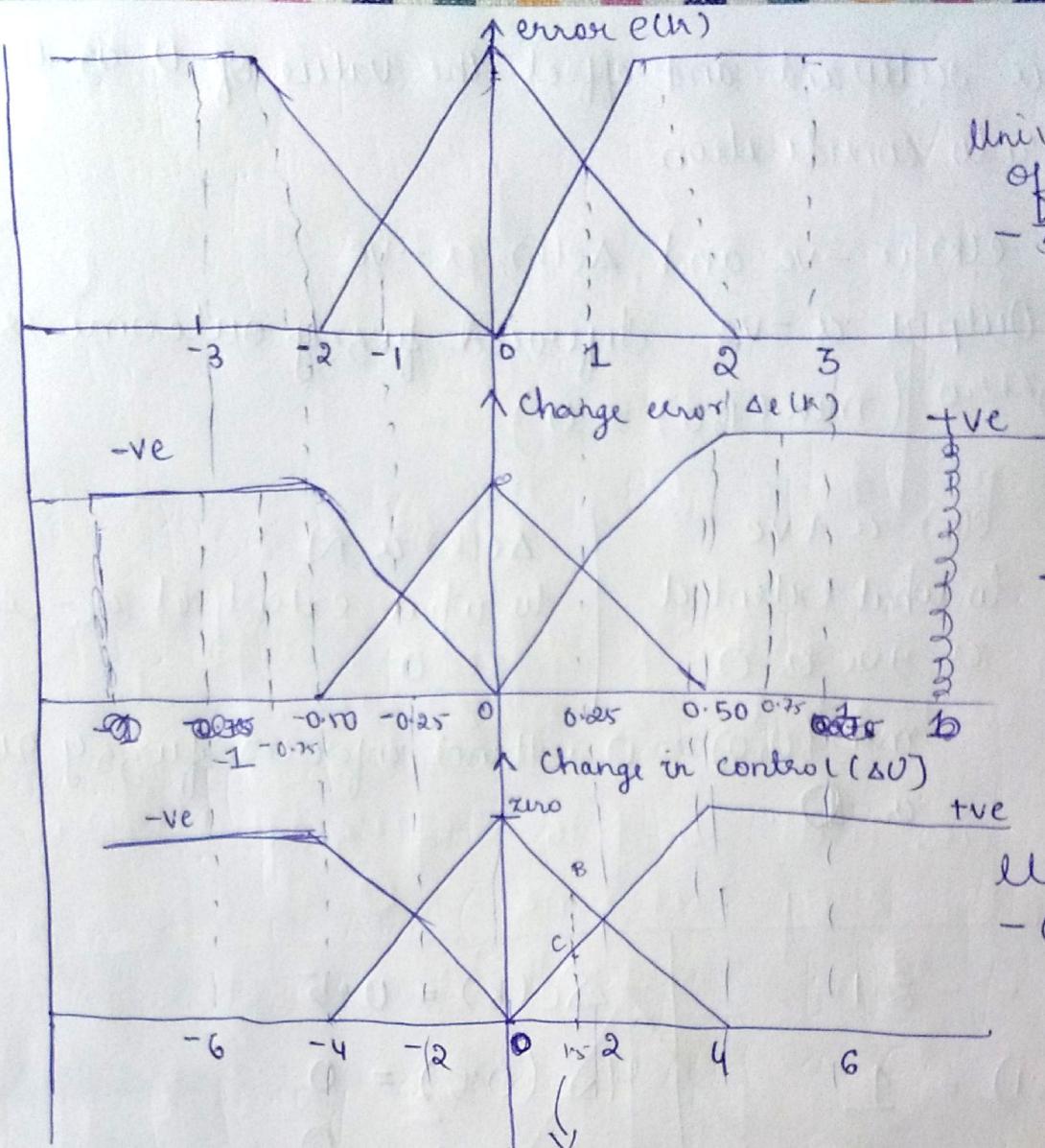
Q Consider an FLC (Fuzzy logic controller) of Mamdani type with a rule base defined by the matrix

$\Delta e(k)$	$e(k) \rightarrow$		
$\Delta e(k)$	N	P	Z
N			
P			
Z			

Where P, N and Z are the linguistic levels positive, negative and zero respectively of the term set of error, e error change Δe and control change $\Delta u(k)$.

Find out $\Delta u(k)$ for $e(k) = -2.1$ and $\Delta e(k) = 0.5$

(i) $\Delta u(k)$ for $e(k) = -0.9$ and $\Delta e(k) = 0.2$



Universe of discourse
-3 to 3

Universe of discourse
-1 to 1

Universe of discourse
-6 to 6

at 1.5 membership grade is

$1 - \frac{1}{4} \times 1.5$ gives value at c.

$\frac{3}{4} \times 1.5$ give B value

Sol- $e(k) \rightarrow$

	N	P	Z
N	N	Z	N
Δe(k)	P	Z	P
↓	Z	N	P

at an instant k the values are given

$$(i) \quad e(k) = -2.1 \quad \Delta e(k) = 0.5$$

We require output close to the desired output the change in control value.

Each rule is taken into consideration and the rule

which is relevant and effect the value of U is taken into consideration.

Rule 1 :- $e(k)$ is -ve and $\Delta e(k)$ is -ve

Output is -ve. Inferred fuzzy outcome is zero ($\min(1,0) = 0$) .

Rule 2 :- $e(k)$ is +ve

to what extent it is +ve is 0

$\Delta e(k)$ is N

to what extent it is -ve i.e., 0

$\min(0,0) = 0$. Hence inferred fuzzy outcome is 0.

$$e(k) = -2.1$$

$$\Delta e(k) = 0.5$$

$$H_N(-2.1) = 1$$

$$H_N(0.5) = 0$$

$$H_Z(-2.1) = 0$$

$$H_Z(0.5) = 0$$

$$H_P(-2.1) = 0$$

$$H_P(0.5) = 1$$

w

Ze

Rule 1 :- If $e(k)$ is N AND $\Delta e(k)$ is N, THEN
 $\Delta U(k)$ is N.

-2.1 $e(k)$ is N upto what extent $\rightarrow 1$.

0.5 $\Delta e(k)$ is N upto what extent $\rightarrow 0$

AND, $\min(1,0) = 0$.

Now, $\Delta U(k)$ is N for 0 upto what extent = 0
so, Inferred fuzzy outcome = 0.

Rule 2: If $e(k)$ is Z and $\Delta e(k)$ is N THEN
 $\Delta U(k)$ is N .

$$e(-2.1) \text{ is } Z = 0$$

$$e(0.5) \text{ is } N = 0$$

$\min(0, 0) = 0$. Hence, Inferred fuzzy outcome $= 0$.

Rule 3: If $e(k)$ is P and $\Delta e(k)$ is N THEN
 $\Delta U(k)$ is Z .

$$e(-2.1) \text{ is } P = 0$$

$$e(0.5) \text{ is } N = 0$$

$$\min(0, 0) = 0$$

Hence, Inferred Fuzzy outcome $= 0$

($\Delta U(k)$ is chopped at 0, so our inferred Fuzzy outcome $= 0$)

Rule 4: - If $e(k)$ is N AND $\Delta e(k)$ is Z THEN
 $\Delta U(k)$ is N .

$$e(-2) \text{ is } N = 1$$

$$e(0.5) \text{ is } Z = 0$$

$$\min(1, 0) = 0$$

Hence, Inferred Fuzzy outcome $= \emptyset$ (Fuzzy set).

Rule 5: - If $e(k)$ is Z AND $\Delta e(k)$ is Z THEN
 $\Delta U(k)$ is Z . (Inferred fuzzy outcome $\neq 0$).

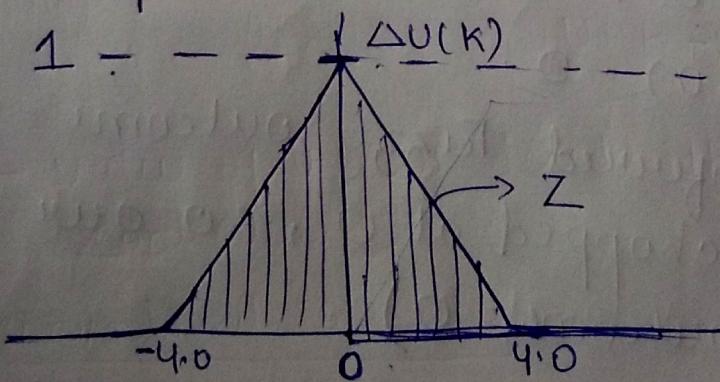
likewise for all rules, we have calculated

		e(k)		
		N	Z	P
N		0	0	0
Z		0	0	0
P		1	0	0

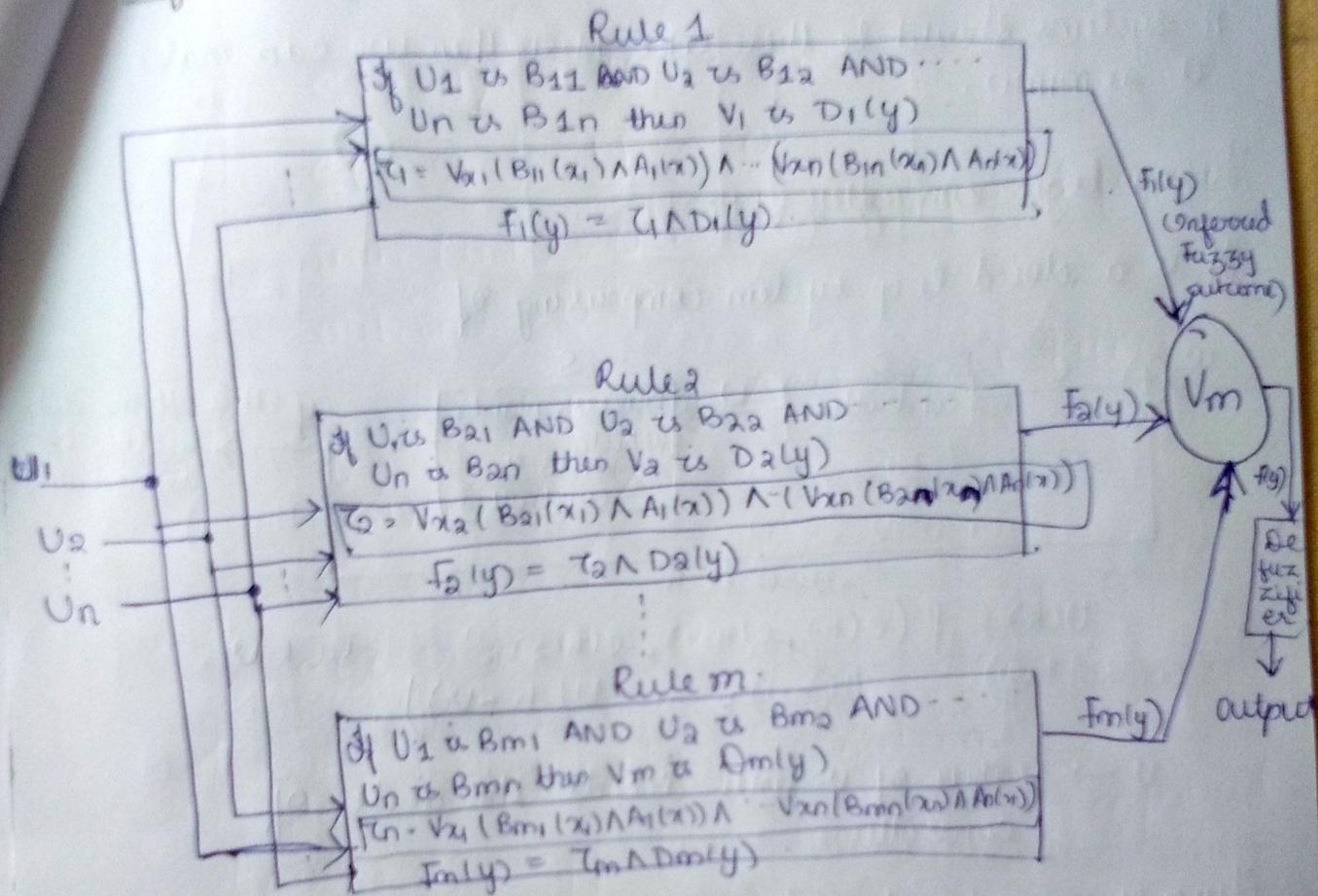
$\Delta e(k)$

When we will do aggregation of these then we will get the zero (z) fuzzy set

Now we will go for Defuzzification,
then the crisp outcome = 0.



Multiple input and single output Mamdani fuzzy Inference system :- (MISO)



- $f_1(y)$ is aggregated fuzzy outcome
- A rule having higher degree of firing then the rule is relevant for the particular output.

→ where $A_1, A_2 \dots A_n$ are the fuzzy value of $U_1, U_2 \dots U_n$.

→ $B_{11}, B_{12} \dots B_{1n}$ are the fuzzy sets positioned over the range of U_1 and $B_{21}, B_{22}, B_{23} \dots B_{2n}$ are fuzzy sets positioned over the range of U_2 and so one ...

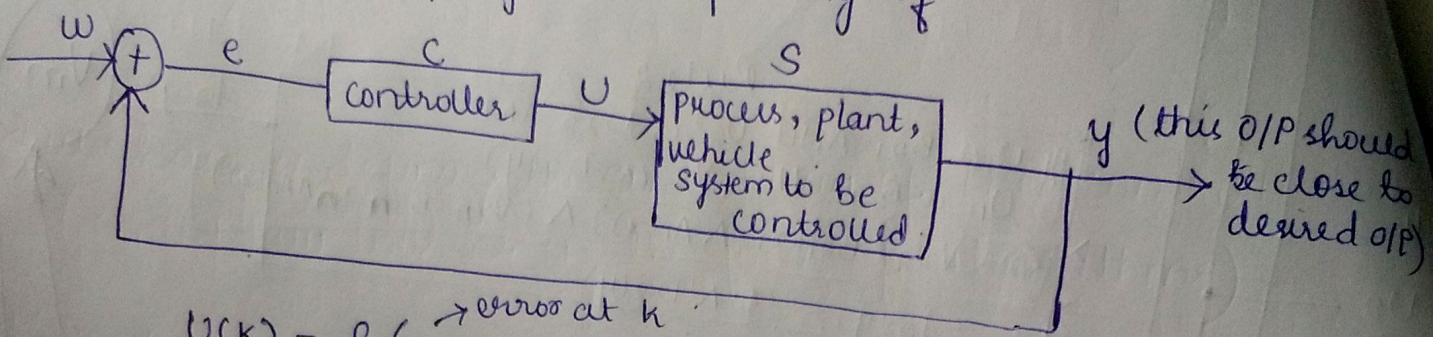
→ All the fuzzy sets are positioned over the universe of discourse U .

multiple input and multiple output Mamdani System :-

If systems should be there like MISO then we can make it as MIMO.

Fuzzy logic controller (FLC) :-

It is a closed loop system comprising of



$$U(k) = f(e(k), e(k-1), e(k-2), \dots, e(k-v), U(k-1), \dots, U(k-v))$$

(control law)

$v \rightarrow$ value of v will decide the order of controller.

Types of conventional controller :-

P \rightarrow proportional controller.

PI \rightarrow proportional & integral controller.

PID \rightarrow proportional, integral & derivative controller.

\rightarrow In conventional controller we have a mathematical model whose output is the output of controller, thus

is a classical controller.

\rightarrow FLC is different from conventional controller as FLC have logical rule base not the mathematical model.

In FLC we can get rid of mathematical complexity, also time of computation is decreased.

→ FLC has a disadvantage over classic controller as FLC is entirely based on experience and knowledge of domain expert. It may not give the desired output.

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Fuzzy logic controller

① PI-like ② PD-like ③ PID-like

These are analogue of mathematical controller.

How those controller are different from each other?

⇒ Job of the controller is to ensure the O/P of the system closed to desired O/P in presence of noise, etc. This is done by → we have to sense the state of system in terms of variables ~~and~~ along with the rule base to bring ~~desired~~ O/P close to desired O/P.

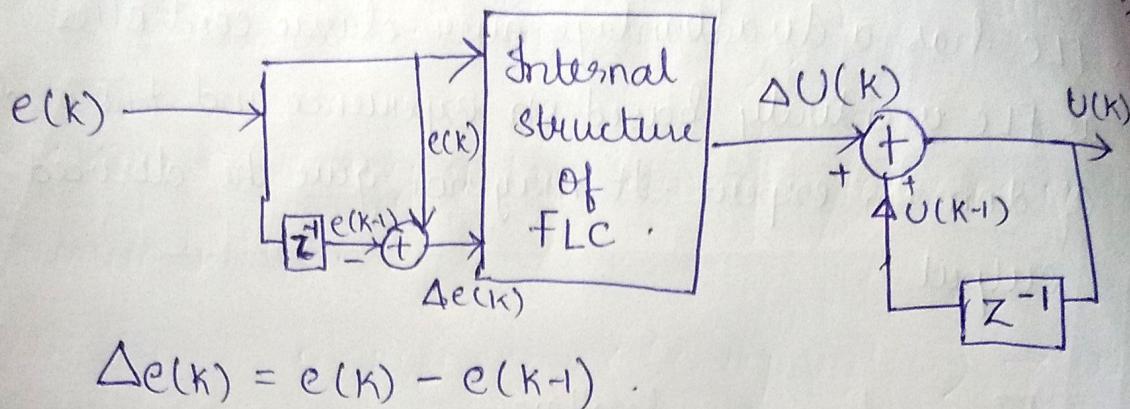
⇒ In all these controllers there will be error and change in error (Δe)

PI-like controller $\rightarrow \Delta U(k) = f(e(k), \Delta e(k))$.
(Normally it is called FLC)

PD-like controller $\rightarrow U(k) = f(e(k), \Delta e(k))$.

PID-like controller $\rightarrow U(k) = f(e(k), \Delta e(k), \sum e(k))$

PI-like controllers or FLC (generally)



→ Rule base is defining the control law of this controller
 (control law will be in the form of algorithms).

Rule 1: (Define some relation b/w input and output)
 If error, $e(k)$ is approximately zero AND
 change in error, $\Delta e(k)$ is positive THEN

Also $\Delta U(k)$ is +ve.

{ Fuzzy set
 defined over
 $\Delta U(k)$ }

{ Fuzzy set
 defined over
 universe of discourse $e(k)$ }

{ Fuzzy set
 defined over
 $\Delta e(k)$ }

There can be many combinations, but we have to consider only that combination relevant to the operating state of the system.

Rule 2:- If error, $e(k)$ is approx. zero AND change in error, $\Delta e(k)$ is -ve Then $\Delta U(k)$ is -ve.

Also

Rule 3: - If error, $e(k)$ is approx. zero AND change in error, $\Delta e(k)$ is approx. zero THEN $\Delta u(k)$ is approx. zero.

also

Rule 4: - If error, $e(k)$ is +ve AND change in error, $\Delta e(k)$ is approx. zero THEN $\Delta u(k)$ is +ve.

also

Rule 5: - If error, $e(k)$ is -ve AND change in error, $\Delta e(k)$ is approx. zero THEN $\Delta u(k)$ is -ve.

→ Number of rules will increase for the cardinality of the term set (will be more).

Here → cardinality of term set for $e(k)$ is 3 (+ve, -ve, approx. zero).
" " " " " " " " $\Delta e(k)$ is 3.
" " " " " " " " $\Delta u(k)$ is 3.

→ Increase in resolution can be like → increase the cardinality of the error or change in error, etc. (i.e., introducing more linguistic value).

→ Same kind of rules can be written for PD-like controller but here instead of $\Delta u(k)$ there will be directly the control action $u(k)$.

- +10 min
- If I have the values of $e(k)$ and $\Delta e(k)$; then we should be able to tell the control action $U(k)$ and $\Delta U(k)$. This is the role of the Controller (or which controller will tell us).
- Error can have any value b/w its minimum and maximum value.
- ↳ We have been given error (crisp value) → we have to change error and $\Delta e(k)$ to 'fuzzified' value (fuzzification) → We will get inferred value from the rule base → then we will go for the defuzzification of the $U(k)$ value.

Strengths of Fuzzy controller

- Fuzzy controllers are very fast as compared to conventional controllers (as there is many mathematical calculation involved in conventional controller).
- It provides us a mean to exploit the knowledge of the domain expert.
- You can exploit the knowledge of the domain experts to find out the value of the control action.

→ ANN are Capable of Learning

→ Just by combining the capability of learning of ANN models and exploiting the knowledge of domain expert we can develop some hybrid model.

- ① Fuzzification of the input variables
- ② Rule evaluation
- ③ Aggregation of the rule outputs
- ④ Defuzzification

Q

Rule 1

If $x \in A_3$
OR $y \in B_1$
THEN $z \in C_1$

Rule 1

If project-funding is adequate
OR project-staffing is small
THEN risk is low

Rule 2

If $x \in A_2$
AND $y \in B_2$
THEN $z \in C_2$

Rule 2

If project-funding is marginal
AND project-staffing is large
THEN risk is normal

Rule 3

If $x \in A_3$
THEN $z \in C_3$

If project-funding inadequate
THEN risk is high

→ ANN are capable of Learning

→ Just by combining the capability of learning of ANN models and exploiting the knowledge of domain expert we can develop some hybrid model.

30/10/18

- ① Fuzzification of the input variables
- ② Rule evaluation
- ③ Aggregation of the rule outputs
- ④ Defuzzification

Eg:

Rule: 1

If x is A3
OR y is B1
THEN z is C1

Rule: 1

IF project-funding is adequate
OR project-staffing is small
THEN risk is low.

Rule: 2

IF x is A2
AND y is B2
THEN z is C2

Rule: 2

IF project-funding is marginal
AND project-staffing is large
THEN risk is normal.

Rule: 3

If x is A3
THEN z is C3

If project-funding is inadequate
THEN risk is high