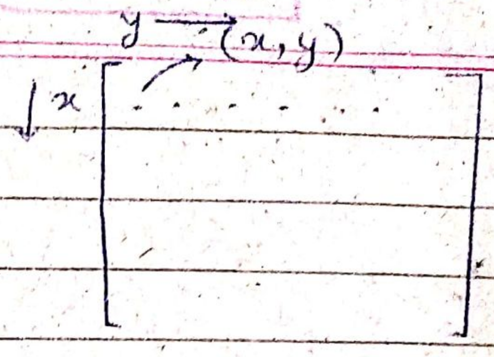
2/01/19



$f(x,y)$ → spatial domain

f → Amplitude (intensity Gray level)

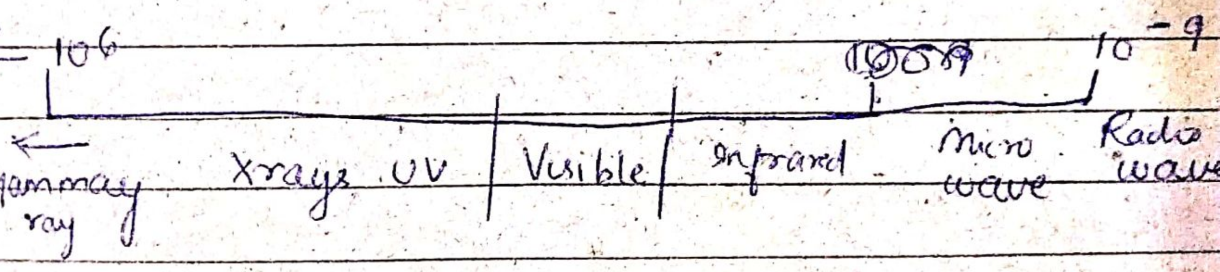
→ The particular value at (x,y) gives the intensity at that particular position.

→ The intensity value can have values between 0-255

→ What is image?

→ Types of image

EM wave



→ There are some problems related to x-rays or UV images (there are some modalities involves here)

→ X-rays
- Low dose x-ray (preferred for human use)
- High dose x-ray

The dose given in low dose x-ray is even less than the parameters defined for low-dose.
At that point, we require to enhance the image (or improve the image).

Gamma rays images → Captured from satellites
↳ In cardiology fields (to detect heart blockage or infection)

arious steps in Image Processing :-
d two categories :-
Giving Image as input and O/P is full image also
Giving Image as input and getting some image parameter / segment image / etc as O/P

1 Image Acquisition (Just giving image as I/P, to identify the modalities or parameters)
Image Filtering and Enhancement
Image Restoration
Color Image Processing
Wavelets & Multiresolution Processing
Image Compression
Morphological Image Processing
Segmentation
Representation and description
Object Recognition.

age Filtering → Subjective performance
age Restoration → Objective "

bjective Performance → Zoom, rotate, transform etc

Objective Performance → Restricted to a particular area.
(Enhance the area of interest)

4) Color Image processing :-
Types of color models → RGB
→ HSI , etc

5) Wavelets :-
We have divide the image in four parts :-

LL	HL
LH	HH

We are only interested in this part
(we get max info from here).

(Pyramidal Structure)

6) Image Compression
→ For image transferring (we reduce the resolution)

Till Step 6 → We get Image as O/P

But from Step 7 to 10 → Output is not a complete image.

→ this is not clearly visible

So for this we use Morphological operators like (Erosion, open, close, thinning, dilation, etc)

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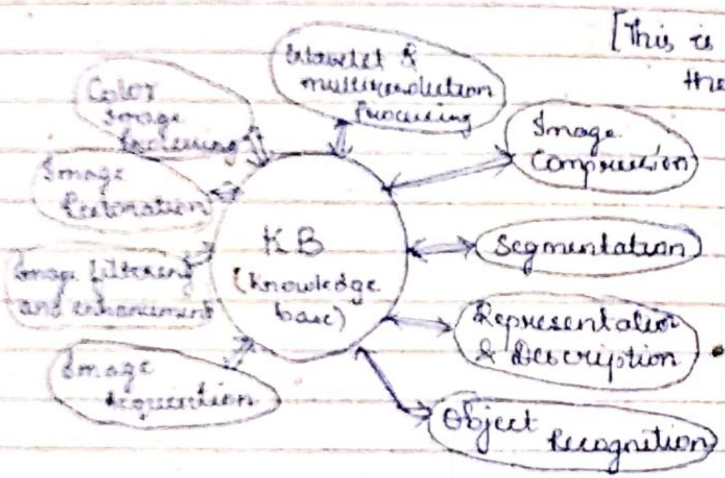
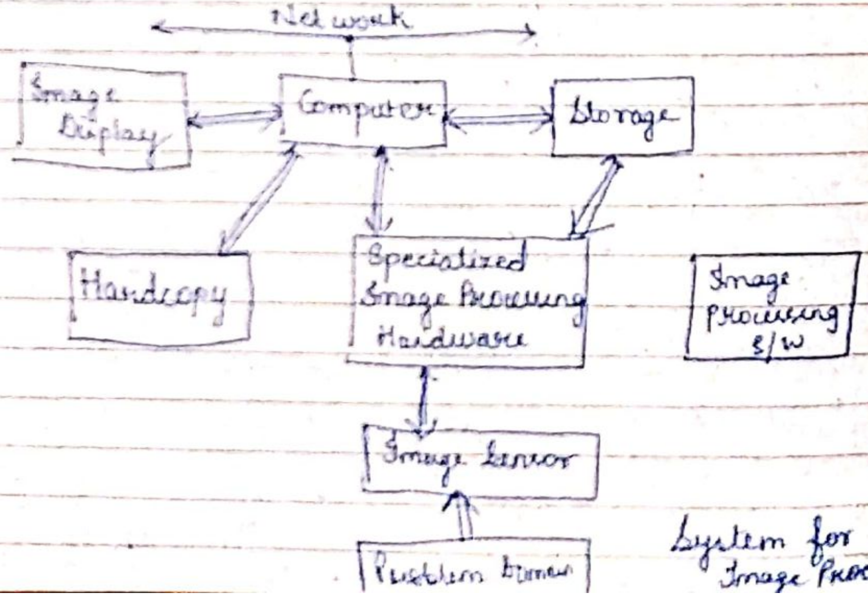
[This is just for theoretical background]





System for Image Processing.

Sensor can be of types like
- Single sensor
- Sensor strip
  - Linear
  - Circular.

Sometimes sensor move and Object remains stationary (Scanner)

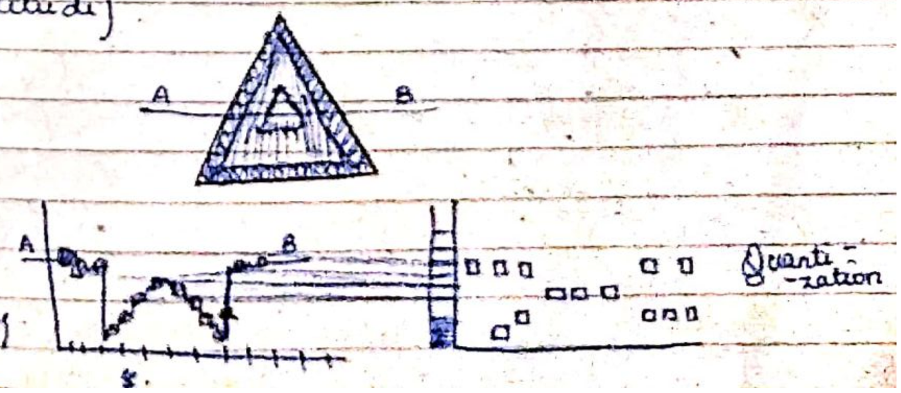
Sometimes sensor is stationary and object moves (MRI m/c).

* Human eye and image formation (Read yourself (Not imp))

$f(x,y)$

amp ⇒ f — To digitalize the amplitude ⇓ Quantization (Digitilization of amplitude)

(x,y) → To digitalize the spatial co-ordinate ⇓ sampling



If sensor captures any image → it is in continuous wave form → we then digitalize the image using sampling and quantization.

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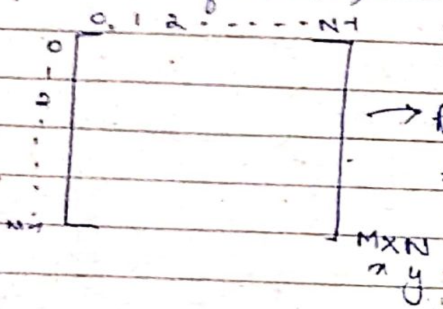
Representation of Digital Image

intensity level (L)
No. of bit (K)



→ representation of image in matrix form.

$\boxed{V = \{0, 1\}}$ For this image we have two intensity level 0, 1 {binary image}

Binary image means actual black & white image.

$L = 2^k$

When we have two level = 0, 1
$L = 2$
$K = 1$ (1 bit to represent)

For gray scale image level = 0 to 255
$L = 256$
$= 2^8$ (8 bits required to represent a pixel).

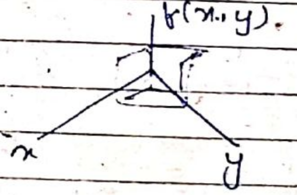
No:- of pixels per inch (DPI) → when our bits increases then DPI also increases.

No:- of bits required to represent / store a image of M.XN form ~~when~~ = $M \times N \times K$.

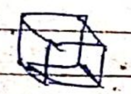
When square image, No:- of bits required to store an image $= N^2 \times K$.

To represent an image we have,

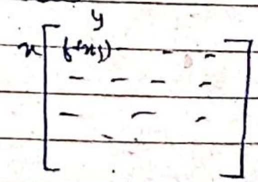
① 3-D view



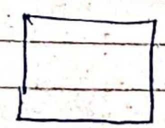
If we have an image



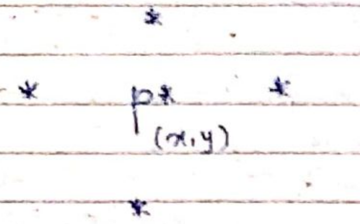
② Matrix-view



③ 2-D view



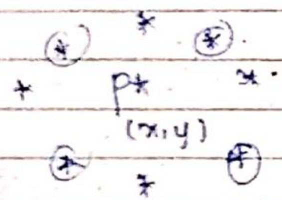
Relationship between pixels



$\longrightarrow$ 4-neighbours.
(We are taking vertical & horizontal pixels)

$N_4(p) \{ (x, y+1) . (x, y-1) \ (x+1, y) \ (x-1, y) \}$

when we are taking diagonal pixels, ~~also~~



$N_D(p) \begin{cases} (x-1, y+1) \ (x-1, y-1) \\ (x+1, y+1), (x+1, y-1) \end{cases}$

When we are taking 8 neighbouring pixels for a particular pixels.

$$\boxed{N_8(p) = N_4(p) \cup N_D(p)}$$

A particular pixel always have 8 neighbouring pixel (4 vertical and horizontal.
4 diagonal).

Adjacency of pixels.

4-adjacency = $q \in N_4(p)$
8-adjacency = $q \in N_8(p)$
m-adjacency (mixed pixels) = ① $q \in N_4(p)$ or ② $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \phi$

→ 2 pixel p & q are 4 adjacent if q belongs to $N_4(p)$ $(q \in N_4(p))$ and q and $p \in V$

Def:
2 pixel p & q with values from V are 4 adjacent if q is in set $N_4(p)$.

→ We, if draw m-connected components in 8-adjacency then multiple path exist (i.e, there is ambiguity). But in m-adjacency no such ambiguity exist i.e, there is only a single path.

→ If a path exist in m-adjacency then it will exist in 8-adjacency also.

Eg:-

	0	1	2	3	4
0	0	1	1	2	3
1	1	1	2	0	1
2	2	2	$\overset{q_4}{3}$ (2,2)	$\overset{q_1}{3}$ (2,3)	$\overset{q_2}{2}$ q(2,4)
3	1	3	$q_3$0 (3,2)	0 p(3,3)	1 q(3,4) q_3
4	0	0	$q_6$2 (4,2)	$q_7$0 (4,3)	$q_8$1 (4,4)

$V = \{0, 2, 3\}$

$N_4(p) = \{(2,3), (4,3), (3,2), (3,4)\}$

↳ does not belong to V

~~Thus it is not $N_4(p)$.~~

Thus we have only 3 pixels in $N_4(p)$

$N_4(p) = \{(2,3), (4,3), (3,2)\}$

$N_D(p) = \{(2,2), (4,2), (2,4)\}$

$N_8(p) = N_4(p) \cup N_D(p)$

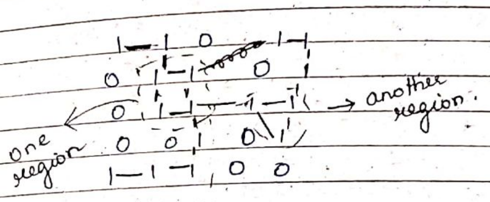
$= \{(2,3), (4,3), (3,2), (2,2), (4,2), (2,4)\}$

Check adjacency for :-

1) p & q_1 → 4 adj
2) p & q_3 → Not 4 adj because $q \notin N_4(p)$
3) p & q_5 → 4 adj
4) p & q_8 → not 4 adj, not 8 adj

If our pixel is 8-adj, we will check for m-adjacent if m-adjacent path exist then it is good as ambiguity is removed.

The region boundary is defined with the help of connected components.



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	0	1	2	1 ← q
	1 (C)	0	1	3
	0 (B)	2	1	2
p →	1 (A)	0	1	1

$V = \{0, 1\}$

4 path - Vertical & Horizontal movement
8-path = " along with diagonal
m-path ⇒ V & H & D but with cond. neighbour of P & Q is ϕ

If length is not given then we will consider unit length b/w 2 pixel

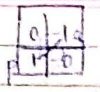
4 path doesn't exist b/w p and q

8-path exist in 3 way with length

$D_A = 5$

$D_B = 6$

$D_C = 5$

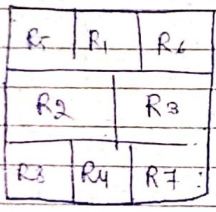


If we want to find m-path b/w p & q

We will see the 4-neighbours of p = {0,0} and ∈ V

" " " " of q = {0,0}

Thus 4-neigh of p ∩ 4-neigh of q ≠ ϕ
Thus ~~the~~ cannot consider diagonal path in m-path



→ If 4-path does not exist b/w two regions then regions are not connected

→ Here whenever we talk about distance → we are talking about eucledian distance

$p(x,y)$ $q(s,t)$ $z(u,v)$

① $D(p,q) \geq 0$ $D(p,q) = 0$ iff $(p = q)$

(ii) $D(p,q) = D(q,p)$

(iii) $\boxed{D(p,z) \leq D(p,q) + D(q,z)}$

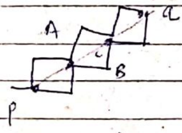
3- distances :-

① Eucledian distance, $D_e = \sqrt{(x-s)^2 + (y-t)^2}$

② City block distance $\equiv D_4 = |x-s| + |y-t|$

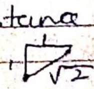
③ Chess board distance $\equiv D_8 = Max(|x-s|, |y-t|)$

City block distance.



Rath than moving diagonally from p to q, we will move in a fashion that will block the cities {i.e., ~~big~~ path A ~~and~~ or B)

→ city-block distance over-estimate the eucledian distance

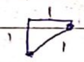


By ecludian: $\sqrt{2} = 1.414$
By city block $= 1 + 1$
$= 2$

3 3
3 2 2 3
3 2 1 2 3
3 2 1 0 1 2 3
3 2 1 2 3
3 2 2 3
3 3

Chess-board Distance

→ In chess-board, queen can move vertically, horizontally and diagonally.



→ So it is underestimating the euclidean distance which is 1.414.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Enhancement by using point processing of image

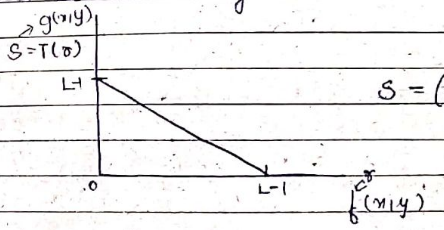
Given an image $f(x,y)$.
On applying transformation function we get
$$g(x,y) = T f(x,y)$$

Image Enhancement Technique

1. Image Negative
2. Image Thresholding — Linear / Logarithmic
3. Clipping
4. Bit plane slicing
5. ~~Controlled~~ contrast stretching

Image Negative

↳ Brighter pixel becomes darker and darker pixel becomes brighter.



$$S = (L-1) - r$$

Maximum intensity level is known as L.

1	2	4	5
3	3	4	6
1	4	0	7
3	2	4	3

$L = 2^3 = 8$

$L = \{0-7\}$ } {Pixel intensity lies b/w 0 to 7}

Here our L = 8

$S = 7 - r$

$r=0$ $S=7$; $r=1$ $S=6$; $r=2$ $S=5$.

2/1/19.

Image Thresholding

- Image clipping
- Intensity level slicing with background
- Intensity level slicing without background

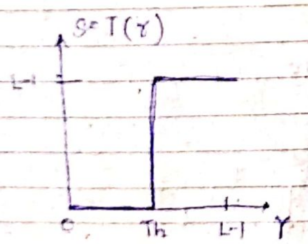
$$s = \begin{cases} L-1 & r \geq Th \\ 0 & \text{otherwise} \end{cases}$$

2	3	4	6
0	1	5	2
3	2	5	6
4	2	7	3

Initial image

$Th = 4$ $\Longrightarrow$

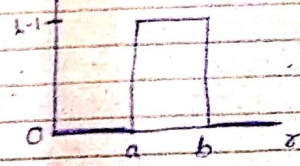
0	0	7	7
0	0	7	0
0	0	7	7
7	0	7	0



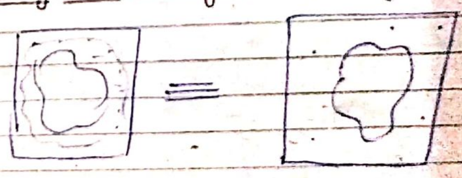
If we are taking a particular segment of image (rather than whole image), suppose we have highest intensity in that segment is 5 then we can take $L-1 = 5$ (rather than 7) but not necessarily

Clipping (without background slicing)

$$s = \begin{cases} L-1 & a \leq r \leq b \\ 0 & \text{otherwise} \end{cases}$$



Intensity level slicing with background



Intensity level slicing without background

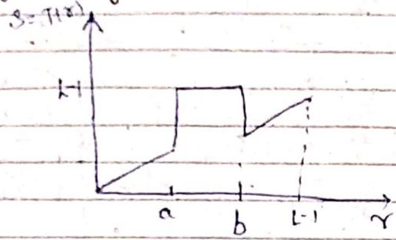
Eg:-

2	3	4	6
0	1	5	2
3	2	5	6
4	2	6	3

$a = 2$
$b = 5$
$\longrightarrow$

7	7	7	0
0	0	7	7
7	7	7	0
7	7	0	7

Image slicing with background



$$s = \begin{cases} L-1 ; & a \le r \le b \\ r ; & \text{otherwise} \end{cases}$$

Eg:-

2	3	4	6
0	1	5	2
3	2	5	6
4	2	6	3

$a = 2$
$b = 5$
$\Rightarrow$

7	7	7	6
0	1	7	7
7	7	7	6
7	7	6	7

Bit plane slicing

2	13	4	6
10	1	15	2
3	12	5	6
4	12	6	3

$\longrightarrow$

0010	1101	0100	0110
1010	0001	1111	0010
0011	1100	0101	0110
0100	0100	0110	0011

The number of bits can be divided into that much planes:-
i.e, no:- of planes = no:- of bits.

For a gray scal ~~pixel~~ image number of bits $(2^8) = 8$
So we can have 8 planes.

Here in our example we can divide the bits into 4 planes (4 bits).

MSB

0	1	0	0
1	0	1	0
0	1	0	0
0	0	0	0

Central pixel

0	1	1	1
0	0	1	0
0	1	1	1
1	1	1	0

Central pixel

1	0	0	1
1	0	1	1
1	0	0	1
0	0	1	1

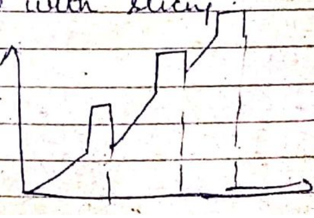
0	1	0	0
0	1	1	0
1	0	1	0
0	0	0	1

LSB.

★ This is useful when info is not clear so we slice the image. Used in MRI or FMRI.

We can also do multiple clipping ~~is~~ (multiple times) and same with slicing.

Eg:



Applying slicing multiple times.

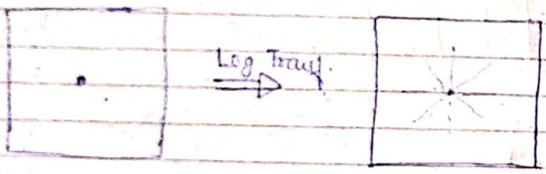
Log Transformation

$$S = c \cdot \log(1+r)$$

Gamma | Power | Exponential Transformation.

$$S = c \cdot r^{\gamma}$$

* generally in both the cases c is considered as 1.

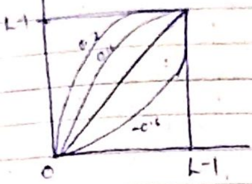


→ Narrow range low input gray level value mapped with wider range of O/P values.

→ If we apply first fourier transform and then log transf. then also we get same O/P but not that efficient

Gamma | Power | Exponential

→ Narrow range of darker I/P value mapped with wider range of darker O/P value



Stretching of actual image with respect to value of gamma to enhance or transform the image.

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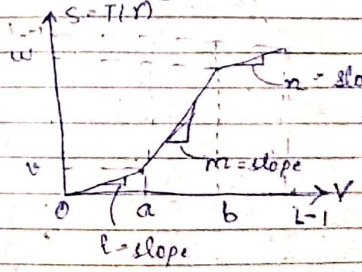
→ Histogram is used to determine the threshold value (but not accurate).

· Contrast stretching.



$$S = \begin{cases} l \cdot r & 0 \le r \le a \\ m \cdot (r-a) + v & a \le r \le b \\ n(r-b) + w & b \le r \le L-1 \end{cases}$$

The intensity level between a and b ~~is very~~ has very less difference.

So we will ~~create~~ give them all one intensity level.

Q. 40, 100.

Eg:-

7	12	2	3
10	15	1	1
12	4	6	15
8	2	7	15

$a = 8$
$b = 12$
$v = 4$
$w = 8$

$l = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4}{8} = 0.5$

$m = \frac{8-4}{12-8} = \frac{4}{4} = 1$

$n = \frac{15-8}{15-12} = \frac{7}{3} = 2.33$

(i) $s = 0.5 \cdot r$ if $0 \le r \le 8$
$= 0 \le s \le 4$

(ii) $s = (r-8)+4$ $\quad 8 < r \le 12$
$s = 4 < s \le 8$

(iii) $s = \frac{7}{3}(r-12)+8$ $\quad 12 < r \le 15$
$8 < s \le 15$

Value of transformed image.

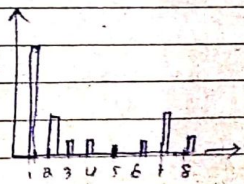
3.5	8	1	1.5
6	15	0.5	0.5
8	2	3	15
4	1	3.5	15

Histogram

→ Statistical analysis of image.

eg:

7	1	2	3
1	1	1	1
1	4	6	1
8	2	7	1



→ With respect to contrast, bifurcate the image properties.

→ There are various methods to find ~~histogram~~ threshold ~~form~~ histogram: or use histogram -

① By finding mean
② Clipping
③ Segmentation
④ Gray level slicing.
} etc.

→ If there is a flower with black background, but it is not clearly bifurcating so here we can do contrast stretching using histogram (or by normalization of image).

1st Assignment:

Q1: Explain the cross-sectional view of human-eye and describe image formation in human eye.

Q2: Consider the image segment S and assume $V = \{1, 2\}$. Compute the length of shortest 4, 8 and m path between pixel p and q. Repeat the ques. for $V = \{0, 1\}$.

Q3: Explain the fundamental steps involve in digital image processing.

—— X ——

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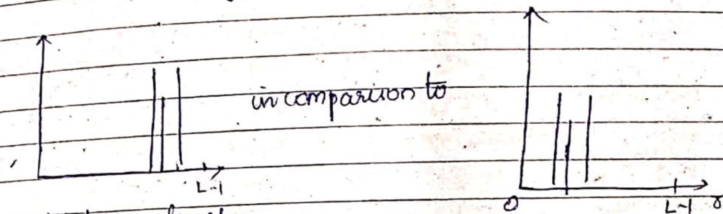
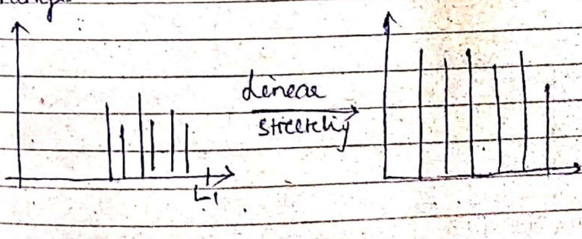
Histogram:
→ Histogram stretching / Linear stretching
→ Histogram Equilization.



High contrast image

Low contrast image

→ If there is a very dark image, and we want the image to be little bit dim, then we will stretch it over a range → we are doing linear stretching.
→ We are not changing the frequencies of the pixel, rather we are increasing their range.



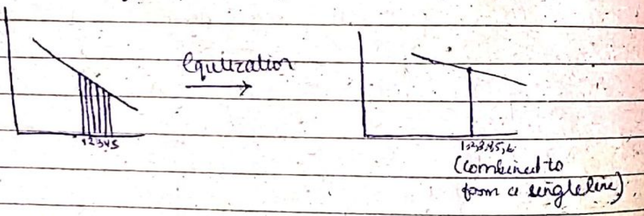
→ The slope won't be changed, only the range will increase.

Histogram stretching

$$\boxed{S = T(r) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} (r - r_{min}) + S_{min}}$$

Histogram Equilization.

→ In equilization, stretching will be there at first and then some of the pixels will equilize or combine to form a single intensity level.



→ Here also frequency remains same.

Q.

5	5	3	4	5
5	4	3	4	2
4	2	3	4	6
5	4	2	4	2
3	4	5	6	3

I.L	0	1	2	3	4	5	6	7
fry	0	0	3	5	8	6	2	0
changed freq (after stretching)	4	0	5	0	8	6	0	2

vary b/w 2 to 6

If we want to stretch the range will be increased to 0-7.

For O/P image max intensity = 7
min " = 0

$S_{max} = 7$
$S_{min} = 0$

$r_{min} = 2$
$r_{max} = 6$ } For original image.

$$T(r) = \frac{7-0}{6-2}(r-2) + 0$$

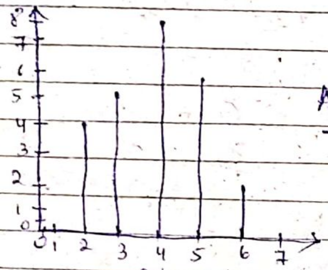
$$S = T(r) = \frac{7}{4}(r-2)$$

5.25	5.25	1.75	3.5	5.25
5.25	3.5	1.75	3.5	[illegible]
3.5	[illegible]	1.75	3.5	7
5.25	3.5	[illegible]	3.5	[illegible]
1.75	3.5	5.25	7	1.75

$r = 2, s = 0$
$r = 3; s = 1.75 \approx 2$
$r = 4; s = 3.5 \approx 4$
$r = 5; s = 5.25 \approx 5$
$r = 6; s = 7 \approx 7$

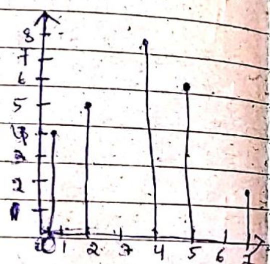
Resultant Image

5	5	2	4	5
5	4	2	4	0
4	0	2	4	7
5	4	0	4	0
2	4	5	7	2



given image.

After linear stretching



Linear stretching.

Histogram Equilization

N = 25

GL	pixel n_k	PDF $r = \frac{n_k}{N}$	CDF $\Sigma p = s$	$s_k = s*(L-1)$	Round off
0	0	0	0	0	0
1	0	0	0	0	0
2	4	4/25 = 0.16	0.16	1.12	1
3	5	5/25 = 0.2	0.36	2.52	3
4	8	8/25 = 0.32	0.68	4.76	5
5	6	6/25 = 0.24	0.92	6.44	6
6	2	2/25 = 0.08	1.00	7.0	7
7	0	0	0	0	0

Initial Image

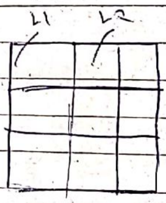
5	5	3	4	5
5	4	3	4	2
4	2	3	4	6
5	4	2	4	2
3	4	5	6	3

Final image

6	6	3	5	6
6	5	3	5	1
5	1	3	5	7
6	5	1	5	1
3	5	6	7	3

31/1/19

Histogram Specification/Matching } → Generalized form of HE.



Dividing image into different parts and localizing each of the sub-part separately.
This is known as Local Histogram Equilization.

Enhancement can be done in 2 ways:-
① point processing (till now all the techniques)
② neighbourhood processing. (masking).

In equilization, the nearest pixels which have slight difference in intensity level then we combine them all and assign a new intensity level.

→ We can apply histogram equilization locally and also globally too.

→ Globally → when we apply equilization on whole image.

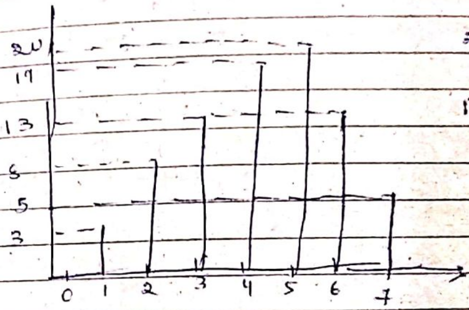
→ locally → when we apply equilization on some part of an image.

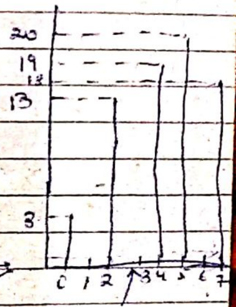
0

6	5	4	7	5	3	5	5	3
6	5	4	7	6	5	3	2	6
5	4	3	6	7	4	2	4	7
5	4	3	6	4	5	2	3	2
5	4	3	5	4	6	4	1	5
6	4	3	5	5	5	3	6	1
6	5	2	4	5	4	5	7	3
4	5	2	4	6	5	2	3	4
4	3	4	6	6	3	4	1	2

N = 9×9
= 81.

GL (gray level)	0	1	2	3	4	5	6	7
freq real img	0	3	8	13	19	20	13	5.





GL	freq	PDF $P_r = \frac{freq}{N}$	CPDF $S_k = \sum P$	$(L-1) * S_K$	round off
0	0	0	0	0	0
1	3	0.037	0.037	0.259	0
2	8	0.098	0.135	0.945	1
3	13	0.160	0.295	2.065	2
4	19	0.235	0.530	3.710	4
5	20	0.248	0.778	5.446	5
6	13	0.160	0.938	6.559	7
7	5	0.062	1.000	7	7

New image mapping
↓

0	1	2	3	4	5	6	7
3	8	13	0	19	20	0	18

$$CDF = \int_0^{r} P_r(x)$$

→ Any probability distribution f^n (Here it is normal)

So, P_i can be anything. Here it is normally distributed

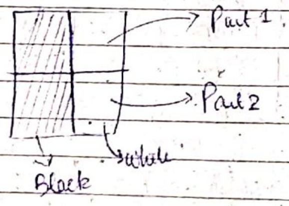
Steps of transformation.

1. Calculate frequency of each intensity
2. Find prob. using any probability distribution function
3. Find the cumulative ~~PD~~ PDF.
4. Then multiply the highest intensity level to CPDF
5. Then round off it to find the new intensity level.

Noise → Additive noise
→ Multiplicative noise (Difficult to remove)

Histogram of 2 region or image can be same.

eg



Local histogram of part 1 and part 2 will be same.

Q. How to draw a comparative histogram of image A as compare to image B.

1. First do HE of image A.
2. Then " " " image B.
3. Then draw a similar gray level mapping of both image A and image B.

GL	GL_A (after eq.)	GL_B (after eq.)	GLfinal
0	1	0	4
1	2	0	4
2	3	0	5
3	3	0	5
4	5	2	6
5	6	4	6
6	7	6	7
7	7	7	7

GL_A ~~[illegible]~~

	0	1	2	3	4	5	6	7
freq	8	10	10	2	12	16	4	2

Given image frequency table

HE

is ~~[illegible]~~ G_{LA}

There will be another image B.

G_{LB} is HE of that image.

We have to find comparative HE of image A with respect to image B.

$(G_{LB} >= G_{LA})$

(for each G_{LA}, we will see at which G_{LB} $G_{LB} >= G_{LA}$) then GLfinal = GL where condition satis-

Histogram table for final image.

0	1	2	3	4	5	6	7
0	0	0	0	18	12	28	6

If the characteristics of target/desired image is combined with characteristic of source image then final image will shift/concentrate towards the characteristics of target image.

Multi Image Operation.

① $g(x,y) = \min\left(f_1(x,y) + f_2(x,y),\ 255\right)$

② $g(x,y) = \left| f_1(x,y) - f_2(x,y) \right|$

③ $g(x,y) = \lambda \cdot f(x,y)$ {Scalar product}

④ $g(x,y) = f_1(x,y) * f_2(x,y)$ {No matrix multiplication}

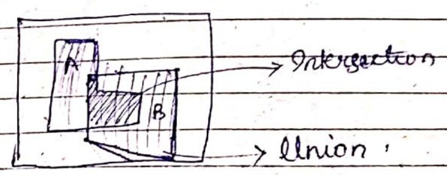
Here we will multiply pixel by pixel.

Image subtraction → ① helps in moving object detection. (can identify whether image is moving or stable).

② Addition of background or subtraction of background.

Set operations on 2-D images.

① Union
② Intersection
③ A − B
④ A^c



Study about OR, AND, X-OR, X-NOR, etc.

1/2/19.

Neighbourhood Process (Spatial Filtering)

Two types of filtering technique

- Spatial Filtering
  - Linear } → smoothing
    1. Averaging
    2. Weight-averaging
  - Non-linear } → sharpening
    1. Median
    2. Min
    3. Max
    4. Midrange filter
- Frequency Domain filtering
  - → Low pass
  - → High pass

Mask / Template / Kernel / window

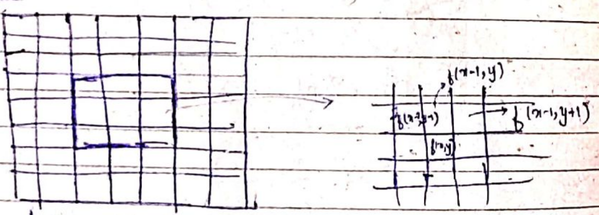


Image M×N

Window Size m×n
(2a+1) (2b+1)

when a, b is any non-negative number
→ odd (as we get mid value in odd)

If we keep on increasing the mask size the image will keep on blurring, and if sometimes we take large mask then our information will be lost.

→ y

x ↓		
w(-1,-1)	w(-1,0)	w(-1,1)
w(0,-1)	w(0,0)	w(0,1)
w(1,-1)	w(1,0)	w(1,1)

→ Mask coefficient.

Resultant image, $g(x,y)$:

$$\boxed{g(x,y) = \sum_{s=-a}^{s=a} \sum_{t=-b}^{t=b} w(s,t)\, f(x+s, y+t)}$$

→ Correlation

In correlation → given mask is applied as it is
In convolution → given mask is rotated 180° and then applied.

$$\boxed{g(x,y) = \sum_{s=-a}^{s=a} \sum_{t=-b}^{t=b} w(s,t)\, f(x-s, y-t)}$$ → Convolution

1	3	5
2	4	6
5	9	8

mask

→ In case of correlation mask will be same.

we will rotate 180° in case of convolution

8	9	5
6	4	2
5	3	1

The criteria for a mask is that sum of its elements should always be 1.

There are many ways to design a mask.

① 3X3 Mask.

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

② 5X5 Mask

$\frac{1}{25}$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

We are designing mask to identify the neighbourhood pixel.

$\frac{1}{5}$

0	1	0
1	1	1
0	1	0

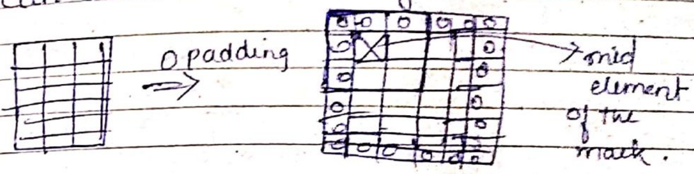
$\frac{1}{13}$

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

0 Padding → adding one extra level to an image having intensity value 0.

This is done to smooth the boundary.

Now, if we apply mask to the image, then we can smooth the boundary also.



If there is large deviation of intensity level and then if we do 0 padding to smooth the edges, then there will not be much change to the intensity level at the edge, thus the smoothing will not be that good. So here we do replicate.

In replicate we add the boundary same as the value of image edge intensity level.

1	200	60
50	0	20
100	100	0
150	200	300

replicate →

1	1	200	60	60
1	1	200	60	60
50	50	0	20	20
100	100	10	0	0
150	150	200	300	300
150	150	200	300	300

~~Sharpening~~ Non-linear filter is mainly used for salt and pepper noise (which is an additive noise).

5/8/17

1-D ① f(x) 0 0 0 1 0 0 0 0

w = 1 2 3 4 8

② 2D f(x,y)

0 0 0 0 0
0 0 0 0 0 w = 1 2 3
origin 0 0 1 0 0 4 5 6
0 0 0 0 0 7 8 9
0 0 0 0 0

Operations to be performed :-
① Convolution
② Correlation

① Correlation

Maximum padding we can do is m-1

m = 2a-1
n = 2b-1 } Mask order

For 1-D:

w = 2 3 4 6 8

No:- of columns = 5
Maximum padding = 5-1
= 4

0 0 0 0 | 0 0 0 1 0 0 0 0
① 1 2 3 4 | 8
| 0 0 0 1 . 0 0 0 0

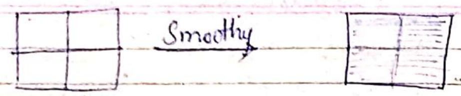
② 1 2 3 4 8
0 0 0 1 0 0 0 0

③ 1 2 3 4 8
0 0 0 1 0 0 0 0

④ 1 2 3 4 8
1 2 3 4 8

Finally $\boxed{0\ 0\ 0\ 8\ 4\ 3\ 2\ 1}$ → Ans

Here for each bit we do sum of product I cut multiply with each bit with the corresponding window value.
Then sum of all these values, gives the value for that pixel



Here if we want to do edge detection then it is not possible after applying Smoothing filter.

→ So for this we use sharpening filter (which is a high pass filter). ~~This~~ By using this we can detect the edge.

1	2	3	8
7	6	5	3
4	5	2	9
6	0	3	1

Mask $\frac{1}{9}$

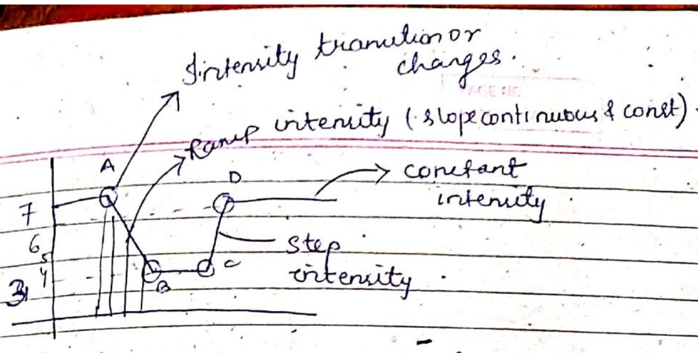
1	1	1
1	1	1
1	1	1

If we want to do smoothing using this filter

Then → $1\times1 + 2\times1 + 3\times1$
$+ 7\times1 + 6\times1 + 5\times1$
$+ 4\times1 + 5\times1 + 2\times1$

$= \frac{35}{9}$

So now centre pixel 6 is replaced with 35/9.



→ From A to B there is gradual change, but C to D there is drastic change.
Thus, we want to bifurcate these points.

→ If we will be able to bifurcate these points then we can detect the edges.

→ Generally smoothing filter acts like a low pass filter.

Derivatives

1st Derivative

→ Must be zero in area of constant intensity.
→ " " non zero at the onset of intensity ramp/step.
→ Must be non-zero along the ramps.

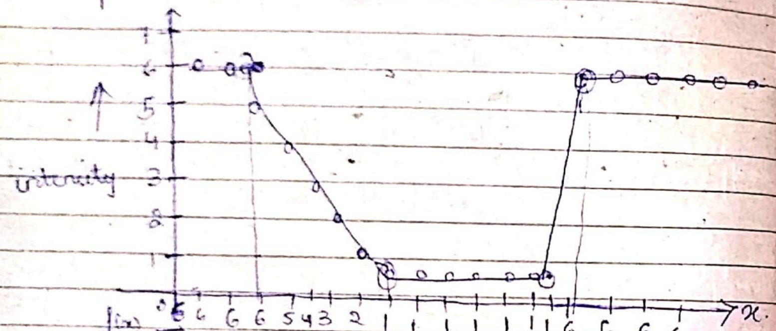
2nd Derivative

→ Must be zero in area of const. intensity

Highlight the points b/w which there is drastic change.

→ Must be non-zero at the onset and end of step or ramp.

→ Must be zero along the ramp of constant slope.



$f(x)$: 6 6 6 6 5 4 3 2 1 1 1 1 1 1 1 6 6 6 6

$\frac{\partial f}{\partial x}$: 0 0 -1 -1 -1 -1 -1 0 0 0 0 0 0 5 0 0 0 0

For 1-D $\frac{\partial^2 f}{\partial x^2}$: 0 0 -1 0 0 0 0 1 0 0 0 0 0 5 -5 0 0 0

1st ~~Oder~~ derivative.

$$\frac{\partial f}{\partial x} = f(x+1) - f(x).$$

Sum of total values of mask should be zero.

$f(y)$: $\begin{array}{|c|}\hline 0 \\ \hline -1 \\ \hline +1 \\ \hline\end{array}$ → y-axis

w_2 $\begin{array}{|c|c|c|}\hline 0 & -1 & 1 \\ \hline\end{array}$ → x-axis, $f(x)$

Sum = 0 − 1 + 1 = 0.

2nd derivative.

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

$\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline\end{array}$ → $f(x)$ → Total sum = 0

Laplacian.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2}.$$

For y-axis.

$\begin{array}{|c|}\hline 1 \\ \hline -2 \\ \hline 1 \\ \hline\end{array}$

In 2-D (along ~~with 1-D we will consider~~ y-axis we will consider following)

1st derivative.

$$\frac{\partial f}{\partial y} = f(y+1) - f(y).$$

2nd derivative

$$\frac{\partial^2 f}{\partial^2 y} = f(y+1) + f(y-1) - 2f(y).$$

Laplacian.

$$\nabla^2 f = \frac{\partial^2 f}{\partial y^2}.$$

In 2-D

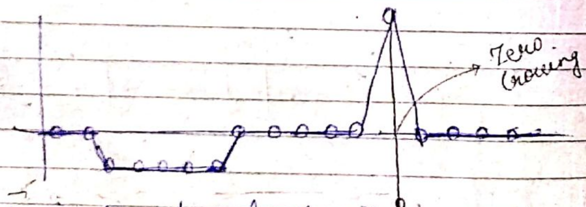
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ ~~Using~~ Summing $\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix}$

Total sum = 0

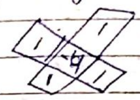
If we plot for the previous graph, the first derivative is



First derivative drawback, it is unable to bifurcate the start and end of ramp or

So we move on to second derivative step.

Considering diagonal mask,



Combining horizontal, vertical & diagonal mask we get,

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Total sum = 0

Eg:-

40	40	40	150	150
40	40	40	150	150
40	40	40	150	150
250	250	250	250	250
250	250	250	250	250

The whole image is divided into 3 regions

R1	R2
R3	

* Applying laplacian mask on that image,

1	1	1
1	-8	1
1	1	1

Applying zero padding on the image,

0	0	0
0	40	40
0	40	40

$\Rightarrow 40 \times (-8) + 40 \times 3$

$= 120 - 320$

$= -200$ → getting negative intensity

Applying replicate padding,

40	40	40
40	40	40
40	40	40

$\Rightarrow 40 \times (-8) + 40 \times (+8)$

$= \underline{\underline{0}}$

So its better to apply replicate padding in these cases.

→ After applying the mask, on the padded image, we would be able to detect the edges of the image.

* Boost filter (Unsharped masking).

↳ Subtracting the laplacian from image

$f(x,y) - \nabla^2 f(x,y)$.

This is known as <u>high boost filter</u>.

After padding image is →

40	40	40	40	150	150	150
40	40	40	40	150	150	150
40	40	40	40	150	150	150
40	40	40	40	150	150	150
250	250	250	250	250	250	250
250	250	250	250	250	250	250
250	250	250	250	250	250	250

<u>Mask</u>:

1	1	1
1	-8	1
1	1	1

0	0	330	-480	0
0	0	330	-480	0
-630	-630	-850	-80	-300
630	630	520	410	300
0	0	0	0	0

Image after applying mask-ing

Scanned by CamScanner

08/02/19

→ Zero crossing will only be there in 2nd derivative.

→ This is the mid point ~~of~~ between two intensity level.

Robert operator
Prewitt operator
Sobel operator

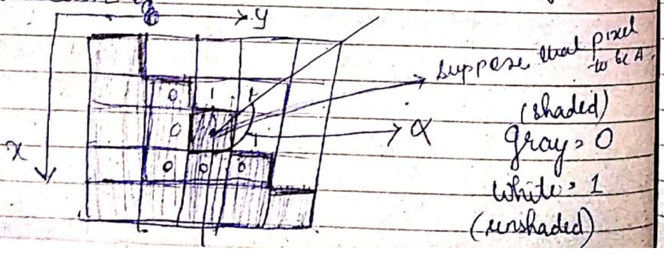
When we are talking about gradient, we are talking about 1st derivative.

$$\underset{\text{gradient}}{\nabla f} = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Steps in Edge Detection

1. Image smoothing for noise reduction.
2. Detection of edge points.
3. Edge localization.

→ Gradient basically tells us about the ~~magnitude~~ strength and direction ~~of~~



$$M(x,y) = \sqrt{g_x^2 + g_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{g_y}{g_x}\right)$$

For the pixel, A
we subtract top row − bottom row
(Here top means down row)

$$\Rightarrow 0-0 + 0-1 + 0-1$$
$$\Rightarrow -2.$$

$$\text{So, } \frac{\partial f}{\partial x} = g_x = -2$$

Similarly we will do for column
Right column − Left column

$$\Rightarrow 1-0 + 1-0 + 0-0$$
$$\Rightarrow \underline{\underline{2}}$$

$$g_y = \frac{\partial f}{\partial y} = 2$$

$$M(x,y) = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1) = -45^\circ$$

$$= 180 - 45 = 135^\circ$$

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x,y).$$

-1	1

$$\frac{\partial f}{\partial y} = f(x, y+1) - f(x,y).$$

-1
1

Robert Cross-gradient operator ⇒

-1	0
0	1

0	-1
1	0

Suppose an ~~image~~ mask.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Prewitt operator,

$$g_x = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

$$g_y = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$$

Mask can be like for x-direction

-1	-1	-1
0	0	0
1	1	1

For y-direction

-1	0	1
-1	0	1
-1	0	1

Sobel operator

$$g_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3).$$

$$g_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7).$$

Mask in x-direction, y-direction

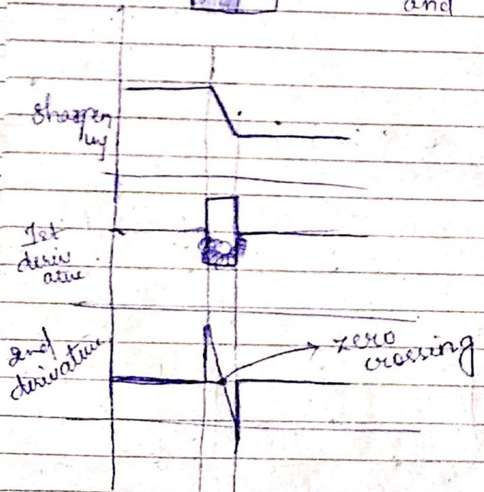
-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

⇒ As in sobel smoothing is there so sobel is good in performance then prewith
But if we have a light image, then doing smoothing ~~so~~ makes the image blur. So here sobel performs bad than prewith.
→ Prewith is easy than sobel to apply.

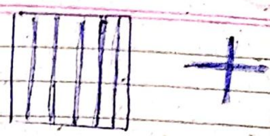
Q

Graph for sharpening filter, 1st derivative and 2nd derivative

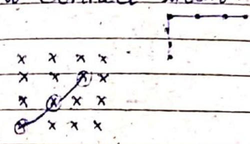


In 2nd derivative sign represent the darker or lighter region
(−) → darker
(+) → brighter side

Q Try it for

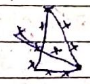


12/08/19
Now If we identify the boundary pixel then how to connect them.



We have three collinear pixel so now the question is how to attach them

If the direction of boundary and neighbour pixel is same then we can say they are connected



There is problem of discontinuity (thinking of the edge point) and similarity of edge pixel

Segmentation
Once all the pixel are encountered then there is segmentation of the image.

Segmentation types

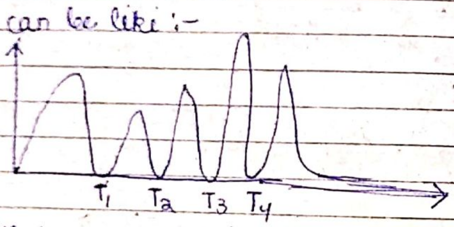
① Discontinuity

② Similarity (will discuss later).

Multi-level thresholding

Each part is of different intensity.

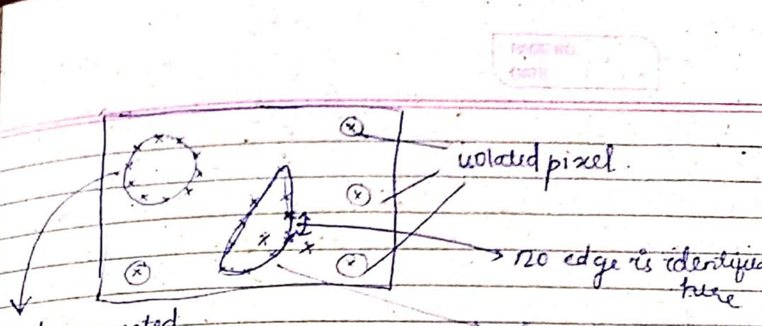
Histogram can be like :-



So we will have multiple / variable thresholding.

$$g(x,y) = \begin{cases} a & \text{if } f(x,y) \le T_1 \\ b & T_1 \le f(x,y) \le T_2 \\ c & T_2 < f(x,y) \le T_3 \\ d & T_3 < f(x,y) \le T_4 \\ e & \text{otherwise.} \end{cases}$$

Global and local thresholding can be applied only when different parts of image are clearly visible. i.e, totally different intensity but problem occurs when we have noise in the image



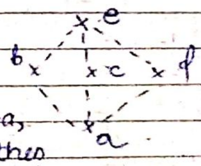
Interconnected pixels due to similar nature.

$$M(x,y) = \sqrt{g_x^2 + g_y^2}$$

$$\alpha = \tan^{-1}\left(\frac{g_y}{g_x}\right).$$

To identify this,

We find the difference of intensity b/w b-a, c-a, d-a, mod them (+ve value) and then if the diff. lies b/w threshold values then, it can be connected to a. Similarly b-e, c-e and d-e.



Complexity of this connectivity = $\frac{N(N-1)}{2}$

$O(n^2)$ is very high.

And comparison → $O(n-1)$ complexity

Total complexity = $O(n^3)$

Strength of all the pixels is found out and difference b/w two pixel is ~~found~~ is found out where the difference is less that pixels are

Connected.

May be we can consider the directionality difference b/w the angle is found out.

local connectivity linking:

$$|\nabla f(x,y) - \nabla f(x',y')| \leq T \rightarrow \text{strength}$$

$$\alpha = \tan^{-1}\left(\frac{g_y}{g_x}\right)$$

$$|\alpha(x,y) - \alpha(x',y)| \leq A \rightarrow \text{direction.}$$

Due to high complexity of this method, we move to the method given below.

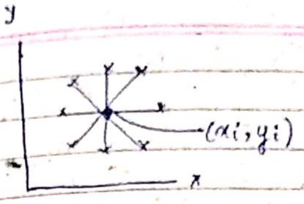
Global edge point linking method :-

→ Objective :- (1) To reduce the complexity.

Hough Transformation - Provide the global edge linking in case of discontinuity b/w two edge pixels.

14/02/19

(A)

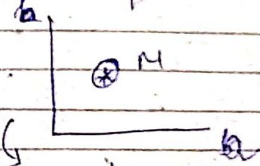


$y_i = ax_i + b.$

$b = -ax_i + y_i.$

Keeping a, b fixed and varying x_i and y_i the line in figure A will be converted into a point in a, b plane.

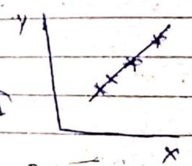
B.



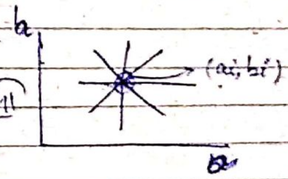
This point M in figure B, will be converted to line in figure A.

$b = -ax_i + b.$

I

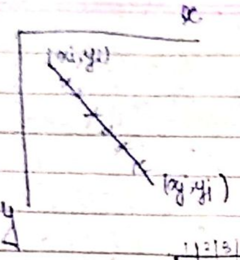


II

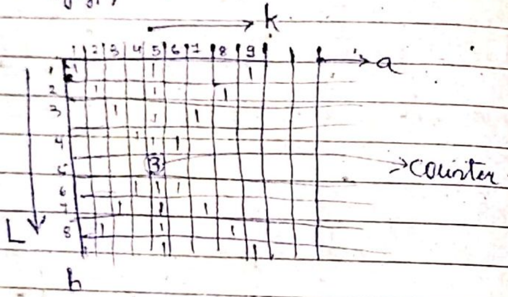


$y_i = ax_i + b.$ $\quad b = -ax_i + y_i$

Now all the points in this line in fig I will now will be passed through a fixed point (a_i, b_i).



(x_i, y_i) $b = -ax_i + y_i$



All the collinear points will pass through the same point.
The counter will show how many collinear points are in a line in x-y plane.

→ How much high the value of accumulator will go then, there will be that much high collinear points will be on a line.

→ We have a nature of line in the x-y axis as slope and intercept
(a) (b).

$a_{min} = 0$ $b_{min} = 0$ { Assuming values in only +ve quad. (i.e. absolute value)
$a_{max} = \infty$ $b_{max} = \infty$

We tell this only when we have counter or intersection point in the +ve quad of a, b.

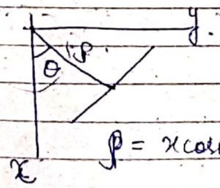
But if we are getting intersection point somewhere outside the +ve quad, then we can say,

$a_{min} = -\infty \Rightarrow b_{min} = -\infty$
$a_{max} = +\infty \Rightarrow b_{max} = \infty$

So, it is difficult to recognise all lines and all collinear points in a line.

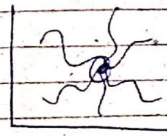
This infinite problem can be solved, when we consider angle instead of magnitude.

So, now



Now max. angle can be 360°

$\rho = x\cos\theta + y\sin\theta$



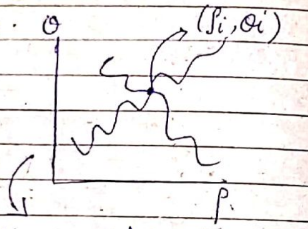
So now, our range is fixed between -1 and 1 and angle 360°
(so infinite range problem is overcomed).

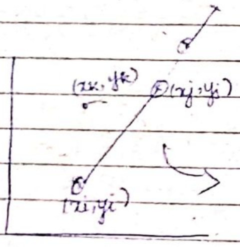
$y = ax + b$

(x_i, y_i) $\rho = x_i \cos\theta + y_i \sin\theta$ → ρ-θ plane

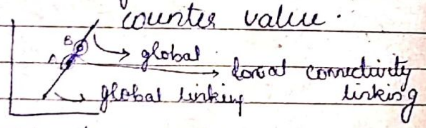
(x_j, y_j) $\rho = x_j \cos\theta + y_j \sin\theta$

xy plane.





Now on changing θ and ρ value, we will again get accumulator or counter value.



If these two points A and B are very similar or are very close then we can connect it directly according to local connectivity linking.

→ ~~Edge point find out~~
→ ~~Convert into~~

① First compute the gradient of image (to identify the edge pixels).

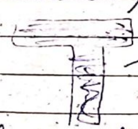
② Subdivide the pixels into ρ, θ plane. The no. of sinusoidal curves in ρ, θ plane corresponds to the no. of pixels in x, y plane. The accumulator value gives the no. of collinear points on a line.

→ Find global connectivity
→ Then find local connectivity linking to link the points (disjoint) that are very close or are very similar in nature.

③ To examine the count of ρ, θ accumulator for high pixel concentration.

④ Examine the continuity b/w pixels in a selected sets.

When we do not have very clear pixels, then hough transformation (global edge point linking method) works very well.



→ Hough transformation.

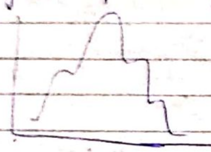
→ If we have clear or distinct edge then local edge linking method works well.

→ local edge linking works good.

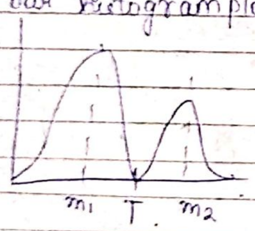
(TILL MIDSEM).

Similarity based segmentation is based on ~~freq~~ thresholding.

Histogram plot before thresholding is as:-



When we apply global thresholding ~~@~~ the we divide are basically in 2 intensity level.
So, now our histogram plot will be as:-



$$\boxed{T_{new} = \frac{1}{2}(m_1 + m_2)}$$

new threshold value.

$m_1, m_2 \rightarrow$ mean.

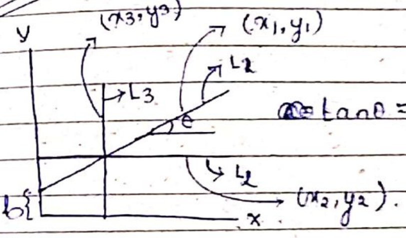
→ Global thresholding, we will continuously find our threshold till we find a constant threshold value.

Entropy of the particular pixel is -

$$\boxed{-p_i \log p_i}$$

There are varities of techniques for global thresholding.

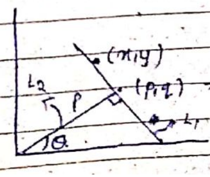
15/02/2019



$\tan\theta = \frac{b}{a}$

The slope of the vertical line is infinity (∞) so how will we draw this vertical line a-b plane (as our calculation of intercept will be difficult, as slope is ∞).

Now we will see in ρ, θ plane



Slope of L_1 is $\dfrac{y-\rho\sin\theta}{x-\rho\cos\theta}$

Slope of line $L_2 = \tan\theta$.

As L_1 & L_2 $\perp$ to each other,

Slope of $L_1 = -\dfrac{1}{\tan\theta}$

$$\Rightarrow -\frac{1}{\tan\theta} = \frac{y-\rho\sin\theta}{x-\rho\cos\theta}$$

$$\Rightarrow -\frac{\cos\theta}{\sin\theta} = \frac{y-\rho\sin\theta}{x-\rho\cos\theta}$$

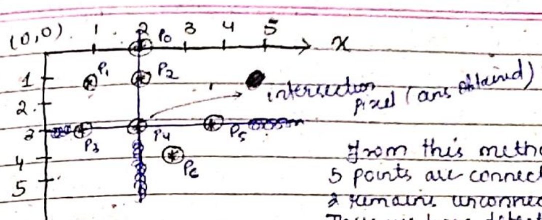
$$\Rightarrow -x\cos\theta + \rho\cos^2\theta = y\sin\theta - \rho\sin^2\theta$$

$$\Rightarrow \rho(\cos^2\theta + \sin^2\theta) = y\sin\theta + x\cos\theta$$

$$\Rightarrow \boxed{\rho = y\sin\theta + x\cos\theta}$$

So in ρ,θ transformation we are not having problem of ∞ slope and thus we are able to calculate ρ.

S



From this method, the 5 points are connected but 2 remains unconnected. Thus we have detected the edge.

$P_0 = (2,0)$ $\quad P_3 = (1,3)$ $\quad P_6 = (3,4)$

$P_1 = (1,1)$ $\quad P_4 = (2,3)$

$P_2 = (2,1)$ $\quad P_5 = (4,3)$

$\theta =$	$-45°$	$0°$	$45°$	$90°$
$P_0(2,0)$	$\sqrt{2}=1.4$	2	$\sqrt{2}=1.4$	0
$P_1(1,1)$	0	1	1.4	1
$P_2(2,1)$	0.7	2	2.12	1
$P_3=(1,3)$	-1.4	1	2.82	3
$P_4=(2,3)$	-0.7	2	3.53	3
$P_5=(4,3)$	0.7	4	4.94	3
$P_6=(3,4)$	-0.7	3	4.94	4

$r = x\cos\theta + y\sin\theta$

$\frac{2}{\sqrt{2}}$ $+0$ $\frac{1}{\sqrt{2}}$

$\Rightarrow$

	-1.4	-0.7	0	0.7	1	1.4	2	2.1	2.8	3	3.5	4	4.9
-45°	1	2	1	2	0	1	0	0	0	0	0	0	0
0°	0	0	0	0	2	0	3	0	0	1	0	1	0
45°	0	0	0	0	0	2	0	1	1	0	1	0	2
90°	1	0	1	0	2	0	0	0	0	3	0	1	0

Taking the highest accumulator value,

$(\rho, \theta) = (2, 0°)$ & $(3, 90°)$.

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$$2 = x\cos 0^\circ + y\sin 0^\circ$$

$$\Rightarrow \boxed{x = 2}$$

$$3 = x\cos 90^\circ + y\sin 90^\circ$$

$$\boxed{y = 3}$$

The intersection point where the lines will cut is (2, 3).

$$2 = x\cos 0^\circ + y\sin 0^\circ$$
$$\Rightarrow \boxed{x = 2}$$

$$3 = x\cos 90^\circ + y\sin 90^\circ$$
$$\boxed{y = 3}$$

The intersection point where the lines will cut is (2, 3).

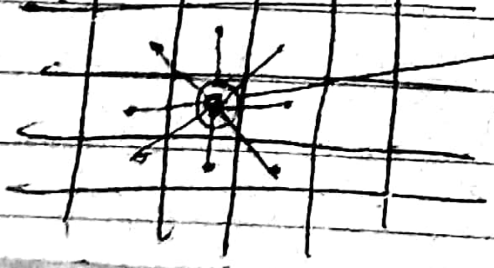
— x —

Similarity Based Segmentation (Region based)

Identifying the region by using
- identifying the region by using region growing technique
- Region split and merge

In region based we did not particularly identified edges of a region.

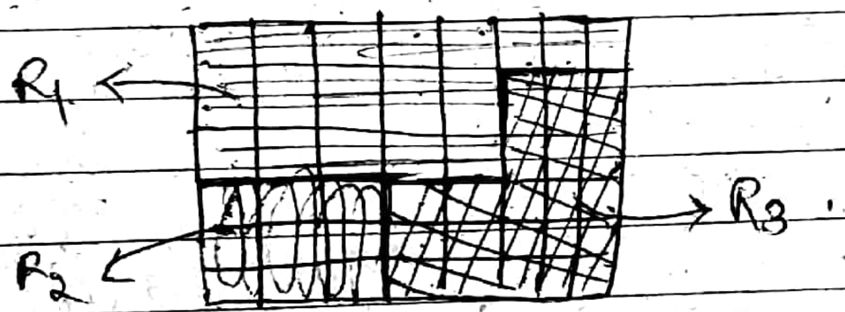
RGT



→ seed point.

We will match the neighbourhood pixel of the seed pixel and find similarity if they are similar then we can say that they are connected and then again take another seed point, and check its neighbourhood.

Step 1 - How to select seed point.
Step 2:- Define similarity function.



In an image any point can be seed point.

Let R represent the entire image region. We may view segmentation as a process that partitions R into n subregions $R_1, R_2 \ldots R_n$ such that:-

(a) $\bigcup_{i=1}^{n} R_i = R$. or $\sum_{i=1}^{n} R_i = R$.

(b) R_i is a connection region, $i = 1, 2, \ldots, n$

(c) $R_i \cap R_j = \phi$ for all i and j, $i \neq j$.

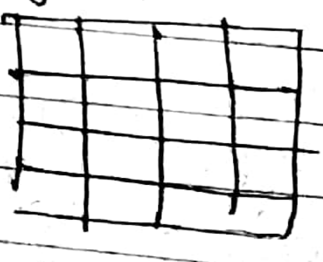
Scanned by CamScanner

(d) $P(R_i) = \text{TRUE}$ for $i = 1, 2, \cdots n$
↳ similarity property of a particular region is fixed i.e, all the pixel in a region have similar property.

(e) $P(R_i \cup R_j) = \text{False}$ for all $i \neq j$
↳ no similarity of properties.

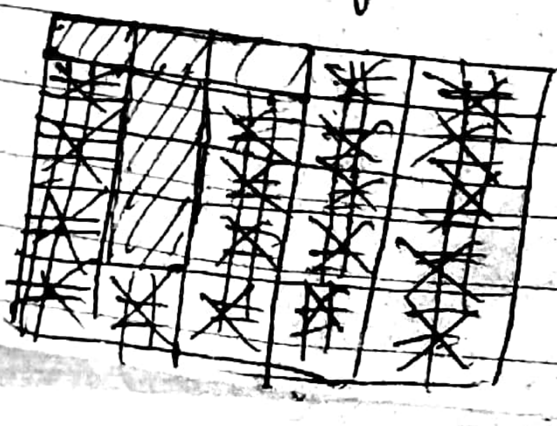
It is true only if the pixel are in boundaries and have similar properties or area have neighbour boundaries.

We will choose a seed point by checking the range of the gray level image i.e, for example



0 – 15.

Then we will try to take seed point of intensity 15 or close to 15 value, not in the mid.
And we are free to take multiple seed values.



→ Here we will see pattern feature specially texture feature and we match the similarity.

Here we can take seed value depending on the number of texture feature.

No:- of seed value = No:- of texture.
But if we have large number of texture then taking a random seed value might be difficult.
In that case we use region split and merge.

Example

→ seed value.

	1	2	3	4	5	6
1	1	2	1	2	5	6
2	0	1	0	1	5	4
3	1	2	1	0	7	6
4	3	7	1	1	4	5
5	1	3	5	6	6	5
6	1	0	7	6	6	5

Range = 0-7.
∴ 1st seed value = 5 (can be any 5, 6, 7 ..)

① seed value = 5.
seed pt = (2,5)

② $I_{max} = 7$; $I_{min} = 0$
Let threshold value $T = 3$.

③ If neighbour pixel intensity difference ≤ 3

then they are connected → It is the proper-ty of a particular region

1	2	1	2	5	6
0	1	0	1	5	4
1	2	1	0	7	6
3	7	1	1	4	5
1	3	5	6	6	5
1	0	7	6	6	5

Incase there is an isolated pixel in a region then that may be a noise etc.
In that it is better to use region split and merge

$(I_{max} - I_{min}) < 3$

In this we divide the region into 4 parts

If this, $(I_{max} - I_{min}) < 3$ property holds in a region then we consider that region otherwise further split that region

then they are connected → It is the property of a particular region

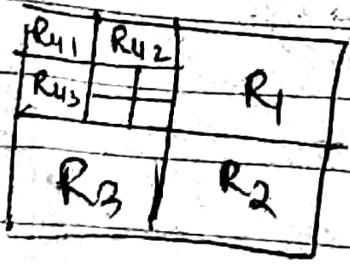
1	2	1	2	5	6
0	1	0	1	5	4
1	2	1	0	7	6
3	7	1	1	4	5
1	3	5	6	6	5
1	0	7	6	6	5

Incase there is an isolated pixel in a region then that may be a noise etc.
In that it is better to use region split and merge

$(I_{max} - I_{min}) < 3$

In this we divide the region into 4 parts.

If this, $(I_{max} - I_{min}) < 3$ property holds in a region then we consider that region otherwise further split that region.



Now in order to merge R_1 and R_2

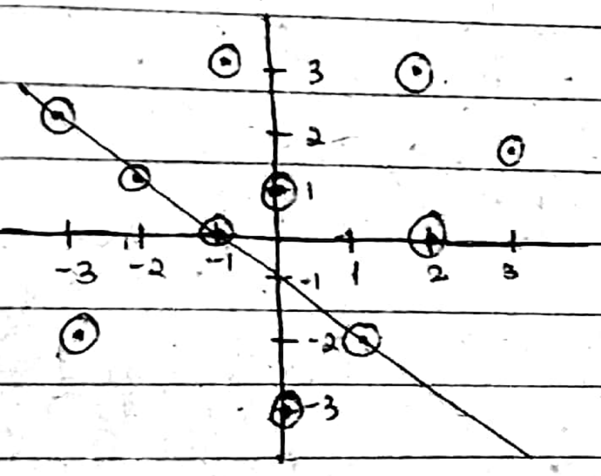
$I_{max\,R_1} - I_{min\,R_2} < 3$ and $\longrightarrow$ (1)

$I_{max\,R_2} - I_{min\,R_1} < 3 \longrightarrow$ (2)

If both (1) & (2) satisfies then we can merge R_1 and R_2.

21/02/19

Q



$P_1 = (-3,2)$ ✓

$P_2 = (-1,3)$

$P_3 = (-2,1)$ ✓

$P_4 = (-1,0)$ ✓

$P_5 = (0,1)$

$P_6 = (-3,-2)$

$P_7 = (0,-3)$

$P_8 = (1,-2)$ ✓

$P_9 = (2,0)$

$P_{10} = (2,3)$

$P_{11} = (3,2)$

Using Hough transformation, find the collinear points.

	-45°	0	45°	90°
P_1	-3.5	-3	-0.7	2
P_2	-2.8	-1	1.4	3
P_3	-2.1	-2	-0.7	1
P_4	-0.7	-1	-0.7	0
P_5	-0.7	0	0.7	1
P_6	-0.7	-3	-3.5	-2
P_7	2.1	0	-2.1	-3
P_8	2.1	1	-0.7	-2
P_9	1.4	2	1.4	0
P_{10}	-0.7	2	3.5	3
P_{11}	0.7	3	3.5	2
P_{12}				

	-3.5	-3	-2.8	-2.1	-2	-1	-0.7	0	0.7	1	1.4	2	2.1	3	[illegible]
-45°	1		1	1			4		1		1		2		
0		2			1	2		2		1		2		1	
45	1			1			4		1		2				2
90		1			2			2		2		2		2	

$$(\rho, \theta) = (-0.7, -45^\circ)$$

$$(-0.7, 45^\circ).$$

$$\Rightarrow -0.7 = x\cos(-45^\circ) + y\sin(45^\circ)$$

$$\rightarrow -0.7 = \frac{x+y}{\sqrt{2}}$$

$$\boxed{\Rightarrow x+y = -1}$$

$$\Rightarrow -0.7 = \frac{x+y}{\sqrt{2}}$$

$$\boxed{\Rightarrow x+y = -1}$$

So, we will see those point whose $x+y = -1$
so points we get are $(-3,2)$, $(-2,1)$, $(-1,0)$ and $(1,-2)$.

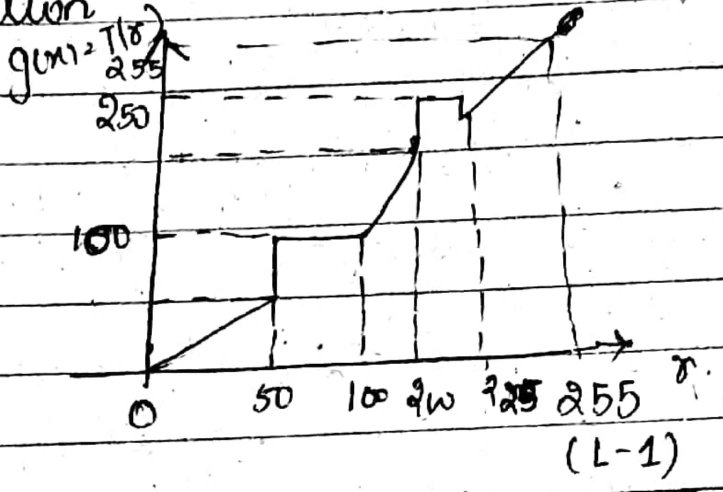
→ If instead of pixels, intensity level or image is given then which boundary pixels to consider.
Suppose, black & white image is given,

0	0	1
0	0	1
0	0	0
1	0	0

If no. of 1 is less, then consider pixel position of 1 or else 0.
If same then consider any one (1 or 0).

If image is a grey level image, then the range of intensity will be given or pixel position will be fixed.

Q. If for a given image, we have some function



Given image :

150	50	75	80	100	200

On this image, apply transformation function

$$
\begin{cases}
100, & 50 \leq r < 100 \\
250, & 200 \leq r < 225 \\
r, & 100 < r < 200 \\
r, & 0 < r \leq 50 \\
r, & r \geq 225
\end{cases}
$$

Now our transformed image is,

150	100	100	100	100	250