18/9/18.

The simulation of real system can be exact replicas of the real system, provided the randomness of events in the real system can be modelled into simulation models.

Random numbers are necessary to model this randomness of the real world into a simulation model.

- Most of the prog. languages & the simulation langs. have the feature of random number generation.

Digital random no:- random no:- generated in computer.

Properties of random numbers.

① Uniformity. (given range :- we have to generate b/w that range)

② Independence (random no. generation doesn't depend on any value)

A sequence of random no:- $R_1, R_2 \ldots$ must have two important statistical properties:-

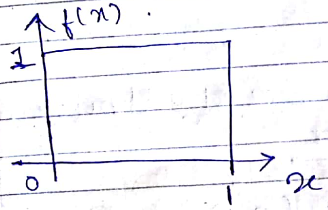
① Uniformity. (They are equally probable everywhere)

② Independence.

→ (The current value of RNs have no relation with the prev. value)

If we have random numbers uniformly distributed b/w 0,1 $= U(0,1)$ then pdf of this is

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$



$$E(x) = \int_0^1 x \cdot dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}.$$

Expected

$$\text{Variance } V(x) = \int_0^1 x^2\, dx - [E(x)]^2$$

$$= \left[\frac{x^3}{3}\right]_0^1 - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$$

Each random number R_i must be an independent sample drawn from a continuous distribution b/w 0 and 1 and pdf is given by

Same as previous

$E(x) =$ "

$V(x) =$ "

This is for uniformity & independence together.

Some consequences of Uniformity & Independence:

If the interval $(0,1)$ is divided into n equally divided class intervals then according to the independence or uniformity properties of R.N.

1. The probability of obtaining a value in a specific interval is not dependent on the previous value.
2. In a uniform distribution for a total of m observation taken, the expected number of observation in each interval is $\frac{m}{n}$.

As a part of simulation, such random numbers can be generated with the help of digital computer. Various methods or routines that can be used to generate numbers are available but it is important that they satisfy the following requirements:-

1. The routine should be fast.
2. The routine should be portable to different computers & should be flexible to be used in diff. prog. languages.
3. The routine should have long cycle. The cycle length represents the length of the random number sequence before previous number begin to repeat in an earlier order.
4. The random number should be replicable
5. The generated random numbers should closely approximate the ideal statiscal property of uniformity & independence.

20/9/18.

Random Numbers.

Truely Generated Random Numbers

(Tossing of a coin, in which we know what will be the sample space, but not the exact outcome when we are tossing)

Pseudo Random Number.

(Computerized random numbers i.e., generated by computer with help of some algorithm).

Mean will be either very low or high and same for Variance.

(So there is some error in Pseudo Random numbers)

Pseudo Random Numbers are generated by computers using an algorithm and seed.

* Seed: - It is the number from where we have to start, to generate pseudo random no:-

They are not strictly random, as if you start with same seed and same algorithm you will get the same number.

Eg: 23 56 81 75 23 56 67

seed

repetition in case of Pseudo R.N. but not in truly generated Random Numbers.

Technique to generate Pseudo Random Numbers:-

① Linear Congruential Method (LCM)

$$\boxed{X_{i+1} = (a \cdot X_i + c) \bmod m.} \quad (1)$$

LCM produces a sequence of integers say $X_1, X_2, X_3, \ldots$ between 0 to $m-1$ and is described by the above expression.

If $m = 100$ then no:- will be generated between 0 & 99.

a → multiplier
c → increment
m → mod.

LCM / Residue method

1. Additive congruential
2. Multiplicative congruential.
3. Mixed congruential

If in eq① put c = 0.

$$\boxed{X_{i+1} = (aX_i) \bmod m}$$ Multiplicative Congruential

If in eq① put a = 1

$$\boxed{X_{i+1} = (X_i + c) \bmod m}$$ additive congruential.

mixed congruential is same as eq①.

Q $X_0 = 21$.
$a = 13$.
$c = 43$.
$m = 100$.

$X_1 = (13 \times 21 + 43) \bmod 100$.
$= (273 + 43) \bmod 100$.
$= (316) \bmod 100$.
$= 16$.

$X_2 = (16 \times 13 + 43) \bmod 100$
$= (208 + 43) \bmod 100$.
$= (251) \bmod 100$
$= 51$.

$X_3 = (51 \times 13 + 43) \bmod 100$.
$= (663 + 43) \bmod 100$
$= (706) \bmod 100$
$= 06$.

$X_4 = (6 \times 13 + 43) \bmod 100$.
$= (121) \bmod 100$
$= 21$ (same as seed)

$X_5 = 16$; $X_6 = 51$; $X_7 = 06$.

The length of cycle (P) = 4.

Q $X_0 = 1$ | 2 | 3 | 4.

$a = 21$
$m = 2^6$
$c = 0$

	$X_0 = 1$	2	3	4
X_1	= (21) mod 64 = 21	(42) mod 64 = 42	(63) mod 64 = 63	84 mod 64 = 20
X_2	= 57	= 50	= 43	= 36
X_3	= 25	= 26	= 7	52
X_4	= 49	= 34	35	= 4
X_5	(1029) mod 64 = 5	= 10	19	P = 4
X_6	= (105) mod 64 = 41	$X_6 = 18$	15	
X_7	= 861 mod 64 = 29	$X_7 = 58$	59	
X_8	= 33	$X_8 = 2$	23	
		$\boxed{P = 8}$	35	
			31	
			11	
			39	
			51	
			47	
			27	
			55	
			3	
			P = 16	

$X_{i+1} = (a \cdot X_i)$ mod m

$\boxed{X_0 = 1}$

Put $i = 1$ $X_1 = 21 * 1$

$X_0 = 1$
$X_9 = 53$
$X_{10} = 25$
$X_{11} = 13$
17
$12 = 37$
$13 = 9$
$14 = 61$
$15 = 61$
$16 = 1$

$\boxed{P = 16}$

The best seed value is 1 and 3 as it has the highest length of the cycle = 16.

$$\boxed{R_i = \frac{X_i}{m}}$$

where R → random number.

Q $X_0 = 27$
$a = 17$
$C = 43$
$m = 100$

$X_1 = (27 \times 17 + 43)$ mod 100
$= (502)$ mod 100
$= 2$. ($R_1 = 0.02$)

$X_2 = (2 \times 17 + 43)$ mod 100
$= 77$ mod 100 $= 77$ ($R_2 = 0.77$)

$X_3 = (77 \times 17 + 43) \bmod 100$
$= (1352) \bmod 100$
$= 52$. $R_3 = 0.52$

$X_4 = (52 \times 17 + 43) \bmod 100$.
$= (927) \bmod 100$
$= 27$. same as seed
$R_4 = 0.27$.

so, $\boxed{P = 4}$

25/9/18:

Q Using the LCM Generator to generate 3 2-digit random integers.

$X_0 = 27$ $a = 8$ $c = 47$ $m = 100$.

$X_1 = (8 \times 27 + 47) \bmod (100)$
$= (263) \bmod 100$.
$= 63$.

$\{ X_{i+1} = (aX_i + c) \bmod m \}$

$X_2 = (63 \times 8 + 47) \bmod 100$.
$= (551) \bmod 100$
$= 51$.

$X_3 = (51 \times 8 + 47) \bmod 100$.
$= (455) \bmod 100$.
$= 55$.

Q Use the multiplicative method to generate the 4 3 digit random numbers assuming $X_0 = 117$; $a = 43$; $m = 1000$.

$X_1 = (117 \times 43) \bmod 1000$
$= (5031) \bmod 1000$
$= 031$.

$X_2 = (31 \times 43) \bmod 1000$
$= (1333) \bmod 1000$
$= 333$.

$X_3 = (333 \times 43) \bmod 1000$
$= 319$

$X_4 = 717$
$X_5 = 831$.

Mid/Middle Square Method:-

It was invented by John Von Neumann and was described at a conference in 1949. In mathematics, the mid square method is a method of generating pseudo random numbers. In practice it is not a good method, since its cycle is usually very short.

Steps:-

1. Starting with n digit numbers.
2. Squaring it
3. For 8 digit numbers remove 2 lower and higher order digit.
4. Taking n digits in the middle as the next number.
5. Repeat from step no:- 2.

Eg 1) $n = 4$. $X_0 = 5673$.

2) $(5673)^2 = 32182929$.

3) $X_1 = 1829$.

odd

Cycle - 2.
1. 1829
2. $(1829)^2 = 03345241$.
3. 3452.

Cycle 3.
1. 3452
2. $(3452)^2 = 11916304$.
3. 9163.

Q $n = 2$
$X_0 = 42$.

I. 1) $(42)^2 = 1764$.
2) 76.

II. 1) $(76)^2 = 5776$.
2) 77.

III. 1) $(77)^2 = 5929$.
2) 92.

IV. 1) $(92)^2 = 8464$.
2) 46.

V. 1) $(46)^2 = 2116$.
2) 11.

VI. 1) $(11)^2 = 121$
2) 12.

VII. $(12)^2 = 144$.
14.

VIII. $(14)^2 = 196$.
19.

IX. $(19)^2 = 361$
36.

X. $(36)^2 = 1296$.

XI. $(29)^2 = 841$

XII. $(84)^2 = 6056$

Application of random numbers.
Difference b/w truly random no and pseudo rn

XIII $(05)^2 = 0025$

XIV $(02)^2 = 0004$

XV $0^2 = 0$.

This method had a drawback that the cycle is short here as compared to the LCM method.

Problem in generation of pseudo random numbers.

1. The generated RN may not be uniformly distributed.
2. The generated RN might be discrete value instead of continuous value.
3. The mean of the generated RNs might be too high or too low.
4. The variance of the generated nos might be too high or too low.

Application of random numbers.

1. Gambling. 2. Statistical sampling. 3. Computer simulation. 4. Cryptography
5. Completely randomized design.

True vs pseudo-random numbers.

Pseudo random numbers are generated by computers using an algorithm and a seed. They are not strictly random as, if you start with the same seed and same algorithm, you will get the same numbers.

A truly random number, on the other hand, is completely unpredictable. I know of no way to generate a series of truly random numbers via computer; the usual ways are flipping coins, throwing darts, rolling dice & other physical processes.

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Random Number Streams:

$$\boxed{X_{i+n} = (a^n \bmod m) X_i \bmod m}$$

This directly helps us to find the value of any random number at nth position (which was previously generated by LCM method).

$n = 5$ $a = 19$ $X_0 = 63$.
$X_5 = ?$ $m = 100$

$X_5 = ((19)^5 \bmod 100)\ 63 \bmod 100$

$= (2476099 \bmod 100) *$

$= (99) * 63 \bmod 100$

$= (6137) \bmod 100$.

$= \underline{37}$.

LCM

$X_0 = 63$.

$X_1 = (19 \times 63) \bmod 100$.

$= (1197) \bmod 100$

$= 97$.

$X_2 = (97 \times 19) \bmod 100$.

$= (1843) \bmod 100$.

$= 43$.

$X_3 = (43 \times 19) \bmod 100$

$= (617) \bmod 100$

$= 17$.

$X_4 = (17 \times 19) \bmod 100$

$= 323 \bmod 100$

$= 23$.

$X_5 = (23 \times 19) \bmod 100$

$= 437 \bmod 100 = \underline{37 (V)}$.

Uniformity Test.

① Kolmogorov Smirnov Test. (K-S Test).

② Chi-Square Test.

K-S Test:

Steps in this test ~~are~~ are follow:-

① Define the hypothesis for Uniformity.

$H_0 : R_i \sim U[0,1]$
$H_1 : R_i \not\sim U[0,1]$
} two hypothesis.

② Arrange the data in increasing order.

$R_1 \leq R_2 \leq R_3 \leq \cdots \leq R_n$.

③ Compute D^+ and D^-

$$D^+ = \max \left\{ \frac{i}{N} - R_i \right\}$$

$$D^- = \max \left\{ R_i - \frac{(i-1)}{N} \right\}$$

④ $D = \max\{D^+, D^-\}$

⑤ Determine the critical value D_α for specified level of significance α.
generally, 'α' = 0·05

⑥ If $D > D_\alpha$
H_0 is Rejected.

Q The sequence of numbers 0·63, 0·49, 0·24, 0·57 and 0·71, 0·89 has been generated. Use K-S test with $\alpha = 0·05$ to determine if the hypothesis that the numbers are uniformly distributed on the interval [0, 1] can be rejected or not.

Step 1.

$H_0: R_i \sim U[0\ 1]$

$H_1: R_i \nsim U[0, 1]$

Step 2.

$0·24 \le 0·49 \le 0·57 \le 0·63 \le 0·71 \le 0·89$.

0·166

Step 3.

$D^+ = \max\left\{\frac{i}{N} - R_i\right\}$

$D^- = \max\left\{R_i - \frac{(i-1)}{N}\right\}$.

i	i/N	R_i	$i/N - R_i$	$R_i - \frac{(i-1)}{N}$
1	1/6	0·24	0·17 − 0·24 = −0·07	0·24.
2	1/3	0·49	0·33 − 0·49 = −	0·49 − 0·17 = 0·32
3	1/2	0·57	0·50 − 0·57 −	0·57 − 0·33 = 0·24
4	2/3	0·63	0·67 − 0·63 = 0·04	0·63 − 0·50 = 0·13
5	5/6	0·71	0·83 − 0·71 = 0·12	0·71 − 0·67 = 0·04
6	1	0·89	1 − 0·89 = 0·11	0·89 − 0·83 = 0·06

$D^+ = \max\{0·04, 0·12, 0·11\} = 0·12$.

$D^- = \max\{0·24, 0·32, 0·24, 0·13, 0·04, 0·06\} = 0·32$

Step 4.

$D = \max(D^+, D^-)$

$= 0·32$.

Step 5

$D_\alpha = D_{0·05} = 0·521$.

(Always be given in the ques)

Step 6. If $D > D_\alpha$.

$= 0.32 > 0.521$. (X).

So H_0 is not rejected

i.e, H_0 is accepted

and Random numbers are uniformly distributed.

Q. To perform a test for uniformity using the K-S test with the level of significance $\alpha = 0.05$ for the following numbers generated.

0.53 0.02 0.97 0.78 0.26.

Step 3.

i	i/N	R_i	$i/N - R_i$	$R_i - \frac{(i-1)}{N}$
1	0.2	0.02	0.18	= 0.02
2	0.4	0.26	0.14	= 0.06
3	0.6	0.53	0.07	= 0.13
4	0.8	0.78	0.02	= 0.18
5	1	0.97	0.03	= 0.17.

Step 4 $D^+ = 0.18$

$D^- = 0.18$.

Step 4 $D = 0.18$.

Step 5 $D_\alpha = 0.521$.

Step 6 $D_\alpha > D$. (✓)

H_0 is accepted

Random numbers are uniformly distributed.

Q $X_0 = 4$; $a = 21$; $c = 0$; $m = 64$

LCM

$X_0 = 4$. $= ~~0.04~~$

$X_1 = (4 \times 21) \bmod 64 = 20$. $= ~~0.20~~$

$X_2 = (20 \times 21) \bmod 64 = 36$. $= ~~0.36~~$

$X_3 = (36 \times 21) \bmod 64 = 52$. $= ~~0.52~~$

$X_4 = (52 \times 21) \bmod 64 = \underline{04}$.

Random numbers are 0.04 0.20 0.36 0.52

Step 3

i	i/N	R_i	$i/N - R_i$	$R_i - \frac{(i-1)}{N}$
1	0.25	0.04	0.21	0.04.
2	0.5.	0.20	0.30	—
3	0.75	0.36	0.39	—
4	1	0.52	0.48	—

$D^+ = 0.48$, $D^- = 0.04$.

~~$D = 0.48$.~~

~~$D_\alpha = 0.521$.~~

~~$D_\alpha > D$ (✓).~~

~~H_0 is accepted.~~

~~Random numbers are uniformly distributed.~~

$X_0 = \frac{\cancel{4}^{1}}{\cancel{64}_{16}} = 0.0625$ | $X_1 = \frac{\cancel{20}\ \cancel{10}\ 5}{\cancel{64}\ \cancel{32}\ 16} = 0.3125$ | $X_2 = \frac{\cancel{36}^{9}}{\cancel{64}_{16}} = 0.5625$

$X_3 = \frac{\cancel{52}^{13}}{\cancel{64}_{16}} = 0.8125$.

i	R_i	i/N	$\frac{i}{N} - R_i$	$R_i - \frac{(i-1)}{N}$
1	0.0625	0.25	0.1875	0.0625
2	0.3125	0.50	0.1875	—
3	0.5625	0.75	0.1875	—
4	0.8125	1.00	0.1875	—

$D^+ = 0.1875$.

$D^- = 0.0625$

$D = 0.1875$; $D_\alpha = 0.521$.

$D_\alpha > D$

H_0 is accepted.

So Random numbers are uniformly distributed.

28/9/18

Tutorial

Q1. Generate a sequence of 5 random numbers using mixed congruential method where $X_0 = 27$, $a = 8$, $c = 47$, $m = 100$

$X_1 = (aX_0 + c) \bmod m$

$= (8 \times 27 + 47) \bmod 100$

$= (216 + 47) \bmod 100$

$= 263 \bmod 100 = 63$.

$X_2 = (63 \times 8 + 47) \bmod 100 = 551 \bmod 100$

$= (504 + 47) \bmod 100 = 51$.

$X_3 = (51 \times 8 + 47) \bmod 100 = 455 \bmod 100 = 55$

$X_4 = (55 \times 8 + 47) \bmod 100 = 87$.

$X_5 = (87 \times 8 + 47) \bmod 100$

$= (696 + 47) \bmod 100 = 643 \bmod 100 = 43$.

Q2 Using K-S method determine the following numbers are uniformly distributed or not?

0·06, 0·09, 0·10, 0·11, 0·35, 0·38, 0·41, 0·46, 0·49, 0·53, 0·54, 0·61, 0·67, 0·72, 0·75, 0·78, 0·79, 0·85, 0·92, 0·95.

	R_i	i/N	$i/N - R_i$	$R_i - (i-1)/N$
1	0·06	0·05	—	0·06
2	0·09	0·1	0·01	0·04
3	0·10	0·15	0·05	0
4	0·11	0·2	0·09	—
5	0·35	0·25	—	0·15
6	0·38	0·3	—	0·13
7	0·41	0·35	—	0·11
8	0·46	0·4	—	0·11
9	0·49	0·45	—	0·09
10	0·53	0·5	—	0·08
11	0·54	0·55	0·01	0·04
12	0·61	0·6	—	0·06
13	0·67	0·65	—	0·07
14	0·72	0·7	—	0·07
15	0·75	0·75	0	0·05
16	0·78	0·8	0·02	0·03
17	0·79	0·85	0·06	—
18	0·85	0·9	0·05	0.
19	0·92	0·95	0·03	0·02
20	0·95	1·00	0·05	0

$D^+ = 0·09$
$D^- = 0·15$

$D = 0·15$.
$D_\alpha = 0·521$.

$D_\alpha > D$ (Thus H_0 is accepted).
So, it is uniformly distributed.

Probability distribution
- Discrete Probability distribution
- Continuous Probability Distribution

Discrete Probability distribution
- Bernoulli
- Binomial
- Negative Binomial
- Geometric Distribution
- Poisson Distr.

Bernoulli:
p = success
1 − p = q = failure
$E(x) = p$
$V(x) = pq$

Binomial:
$p(x) = {}^nC_x\, p^x q^{n-x}$
$E(x) = np$
$V(x) = npq$.

Negative binomial.

$$p(x) = {}^{n-1}_{x-1}C\, p^{x} q^{n-x}.$$

$$E(x) = \frac{x}{p}.$$

$$V(x) = \frac{xq}{p^2}.$$

Geometric distribution

$$p(x) = q^{x-1} p.$$

$$E(x) = 1/p.$$

$$V(x) = \frac{q}{p^2}.$$

Poisson distribution.

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$E(x) = V(x) = \lambda.$$

Q Ram is a high school basketball player. He is a 70% free throw shooter i.e, his probability of making a free throw is 0.70. What is the probability that Ram makes his 3rd free throw on his 5th shoot.

What is the probability Ram makes his first free throw on his 5th shoot.

Sol.— $n = 5$ $x = 3$. $p = 0.7$ $q = 0.3$

(i) $p(x) = {}^{n-1}_{x-1}C\, p^{x} q^{n-x}$ (Negative Binomial)

$= {}^{4}_{2}C\, p^{3} q^{2}$.

$= \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times (0.7)^3 \times (0.3)^2$

$= \frac{3 \times \cancel{4}^{2}}{\cancel{2}} \times 0.343 \times 0.09$.

$= 6 \times 0.343 \times 0.09$

$= 0.18522$

$= \underline{0.18522}$.

$E(x) = \frac{3}{\cancel{0.18522}\ 0.7} = \cancel{(\ldots)}\ 4.285$

$$V(x) = \frac{xq}{p^2} = \frac{3 \times 0.3}{0.49} = \frac{0.9}{0.49} = 1.$$

(Geometric

(ii) $p(x) = (0.3)^4 \cdot (0.7)$

$= 0.0081 \times 0.7$

$= \underline{0.00567}$

Q. A man was able to complete 3 files a day on an average. Find the probability that he can complete 5 files in the next day.

$\lambda = 3.$

$x = 5.$

$$p(x) = \frac{e^{-3} \cdot (3)^5}{5!}$$

$$= \frac{12.0982}{5!}$$

$= \underline{0.10}$

9/10/18.

- Random No. Test
  - Uniformity Test
    - K-S Test.
    - Chi-Square Test.
  - Independence Test
    - Runs Test.
      - Runs Up and Runs Down
      - Runs Above And Runs Below.

Chi-Square Test.

Step 1:- Define hypothesis for testing uniformity.

$H_0 : R_i \sim U[0,1]$

$H_1 : R_i \nsim U[0,1]$

Step 2:- Calculate the expected frequency "fe".

Step 3:- Calculate χ^2 (chi-square).

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

f_o = observed frequency

f_e = expected frequency

And then calculate the degree of freedom represented by V

For binomial distribution $V = n-1$

" Poisson " $V = n-2$.

" Normal " $V = n-3$.

For tabular form :- $\boxed{}_{m \times n}$

$V = (m-1)(n-1)$.

and $f_e = \dfrac{N}{n}$.

Step 4:- See the value of $X^2_{\alpha,V}$ from the table and the value calculated in step③.

Step 5:- If calculated value of $X^2 < X^2_{\alpha,V}$ then the null hypothesis Ho is accepted.

Everytime interval won't be given. Sometimes we have to give interval to the random nub.

Q.

0 – 0.10	0.10 – 0.20	0.21 – 0.30	0.31 – 0.40
0.05	0.19	0.21	0.34
0.06	0.14	0.21	0.31
0.04	0.11	0.22	0.34
0.06	0.14	0.22	0.35
0.09	0.14	0.30	0.34
	0.20		
	0.12		
5	7	5	5

0.41 – 0.50	0.51 – 0.60	0.61 – 0.70	0.71 – 0.80
0.49	0.51	0.66	0.76
0.49	0.51	0.65	0.75
0.47	0.55	0.65	0.76
0.40	0.51	0.64	
0.44	0.58	0.66	
		0.70	
5	5	6	3

0.81 – 0.90	0.91 – 1.00
0.84	0.92
0.87	0.99
0.90	0.99
0.83	0.98
	0.98
4	5

given

$X^2_{\alpha,V} = 16.9$

$\alpha = 0.05$ (row)

$V = 90$ (column)

$N = 50$

$n = 10$.

$f_e = \dfrac{N}{n} = \dfrac{5\cancel{0}}{1\cancel{0}} = 5$

Step 3

interval	fo	fe	fo-fe	$(fo-fe)^2$	$(fo-fe)^2/fe$
0-0.10	5	5	0	0	0
0.11-0.20	7	5	2	4	0.8
0.21-0.30	5	5	0	0	0
0.31-0.40	5	5	0	0	0
0.41-0.50	5	5	0	0	0
0.51-0.60	5	5	0	0	0
0.61-0.70	6	5	1	1	0.2
0.71-0.80	3	5	-2	4	0.8
0.81-0.90	4	5	-1	1	0.2
0.91-1.00	5	5	0	0	0
					2.0

$$\boxed{\chi^2 = 2.0}$$

$\chi^2_{\alpha, v} > \chi^2$ (Ho is accepted).
(✓)

Q. A sample survey of public opinion answer to the question
1. Do you drink?
2. Are u in favour of local option on sale of liquor?

		Q-1 Yes	No	Total
Q-2	Yes	56	31	87
	No	18	6	24
	Total	74	37	111

Test the hypothesis that the local option on the sale of liquor is not dependent on others drink.

Step 1 - Consider null hypothesis
Ho : the sale of the liquor is not dependent on others's drink.

Step 2 - $fo_{11} = 56$ $\quad fo_{21} = 18$
$fo_{12} = 31$ $\quad fo_{22} = 6$.

$fo_{11} = 56$
$fo_{12} = 31$
$fo_{21} = 18$
$fo_{22} = 6$

$fe_{11} \left(\frac{N_1 \times N_2}{N}\right) = \frac{87 \times 74}{111} = 58$

$fe_{12} = \frac{87 \times 37}{111} = 29$.

$fe_{21} = \frac{24 \times 74}{111} = 16$

$fe_{22} = \frac{24 \times 37}{111} = 8$.

	fe	fo-fe	$(fo-fe)^2$	$(fo-fe)^2/fe$
$fo_{11} = 56$	58	-2	4	0.07
$fo_{12} = 31$	29	-2	4	0.14
$fo_{21} = 18$	16	2	4	0.25
$fo_{22} = 6$	8	-2	4	0.5.
				$\boxed{0.96}$

$\chi^2_{0.05, 1} = 3.841 > \chi^2$ (Accepted).

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Q A dice is rolled 120 times with the following results

No. of turned up	1	2	3	4	5	6
Frequency	30	25	18	10	22	15

Test the hypothesis that dice is unbiased.

Step 1 Ho = dias is unbiased.

Step 2 $N = 120$

$n = 6$ $f_e = \frac{120}{6} = 20$

	f_o	f_e	$f_e - f_o$	$(f_e - f_o)^2$	$(f_e - f_o)^2 / f_e$
1	30	20	-10	1.00	5
2	25	20	-5	25	1.25
3	18	20	+2	4	0.2 $\frac{1}{5}$
4	10	20	+10	1.00	5
5	22	20	-~~0~~2	4	0.2
6	15	20	5	25	1.25

$\chi^2 = 12.9$

$\chi^2_{0.05,5} = 11.7$

$\chi^2_{\alpha,v} < \chi^2$ (Hypothesis rejected)

10/10/18

Test for Independence.

Runs Test → Runs Up and Runs Down.

Step 1 :- Define the Null Hypothesis for Independence.

H_0 : $R_i \sim$ Independent.

H_1 : $R_i \nsim$ Independent.

Step 2 :- Write down the sequence of runs up and runs down.

Step 3 :- Count the total number of runs present in the sequence.

Total no. of runs → a

Step 4 :- Compute mean and variance of a

$$\mu_a = \frac{2N-1}{3}$$

$$\sigma_a^2 = \frac{16N-29}{90}$$

Step 5. Compute standard normal distribution denoted by Z_0

where $Z_0 = \frac{a - \mu_a}{\sigma_a}$.

Step 6. Determine the critical value $Z_{\alpha/2}$ and $-Z_{\alpha/2}$

Step 7. If $-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2}$

then the Null hypothesis is accepted.

Q Test the following numbers by runs up and runs down method.

$\alpha = 0.05$

$Z_{\alpha/2} = Z_{0.025} = 1.96$.

Given random numbers :-

0.12, 0.01, 0.23, 0.28, 0.89, 0.31, 0.64, 0.28, 0.33, 0.93.

Step 2 :-

Ri	sign	count no
0.12	−	1
0.01	+	
0.23	+	2
0.28	+	
0.89	−	3
0.31	+	4
0.64	−	5
0.28	+	
0.33	+	6
0.93		

→ so $a = 6$

Step 4 $\mu_a = \frac{2N-1}{3} = \frac{19}{3} = 6.333$

$\sigma_a^2 = \frac{160-29}{90} = \frac{131}{90} = 1.455$

Step 5 $Z_0 = \frac{6 - \frac{19}{3}}{\sqrt{\frac{131}{90}}}$

$= \frac{6 - 6.33}{\sqrt{1.455}} = \frac{-0.33}{1.21}$

$= -0.27$

$-1.96 < 0.27 < 1.96$.

(Hypothesis accepted)

Runs above and below.

Step 1 – Define the Null Hypothesis for Independence

H_0 : Ri ~ Independent.

H_1 : Ri ≁ Independent.

* If our random no:- in points/decimal

form, we will subtract it from
+0.495 and if random no:- is 40 to ... etc we will subtract it from 49.5.

Step2 Write down the sequence of runs above & runs below.

Step3 Count the no:- of observation above mean n_1 and no:- of observation below the mean n_2.

Step4 :- Count the total number of runs denoted by b

Step5 Compute mean and variance of b.

$$\mu_b = \frac{2n_1n_2}{N} + \frac{1}{2}$$

$$\sigma_b^2 = \frac{2n_1n_2(2n_1n_2 - N)}{N^2(N-1)}$$

$$Z_0 = \frac{b - \mu_b}{\sigma_b}$$

Step6 Determine the critical value $Z_{\alpha/2}$ and $-Z_{\alpha/2}$

Step7 If $-Z_{\alpha/2} \le Z_0 \le Z_{\alpha/2}$, then null hypothesis is accepted.

Q ~~Same question~~

Ri	sign	count
0.11	−	
0.23	−	1
0.45	−	
0.08	−	
0.11	−	
0.50	+	2
0.09	−	3
0.60	+	4
0.80	+	

$b = 4.$

$N = 9$

$n_1 = 3$ (all +ve)

$n_2 = 6$ (all −ve).

$$\mu_b = \frac{2 \times 3 \times 6}{9} + \frac{1}{2}$$

$$= 4 + \frac{1}{2} = \frac{9}{2}$$

$$= 4.5.$$

$$\sigma_b^2 = \frac{2 \times 3 \times 6(36 - 9)}{81(8)}$$

$$= \frac{27 \times 3 \times 6}{81 \times 4} = 1.5.$$

$$Z_0 = \frac{4 - 4.5}{\sqrt{1.5}} = \frac{-0.5}{\sqrt{1.5}}$$

$$= \frac{-0.5}{1.22} = -0.$$

$$-1.96 \leq -0.4 \leq 1.96$$

(So hypothesis is accepted)

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