24/7/18.

Modelling is a process in which we make a model which includes construction + working of the model.

The operations performed on the model is simulation.

Data → Quantitative < Discrete, Continuous

Data → Qualitative

Disadv of simu

1. Initial cost.
2. Expertise.

Why we need Simulation ?

1. Safety.
2. Complex Problem.
3. Time effectiveness.
4. Prediction.
5. Cost reduction.
6. Analyze and verify theoretical models which may be too difficult to grasp (from pure conceptual level).
7. Performance optimization
8. Video games.
9. Scientific modelling of natural systems or human systems to gain insight into their functioning.
10. training of civilian and military Personnel.
11. Testing, training and education.
12. Safety engineering.
13. Social simulation.
14. Automobiles (Car racing simulator).
15. Biomechanics
16. Classroom of the future
17. Comm. satellite

27/7/18.

Definition of Modelling & Simulation

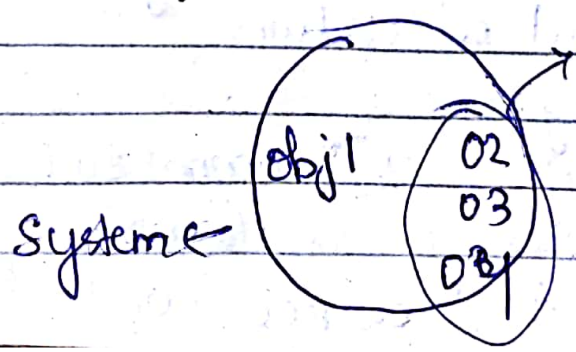
Modelling is the process of representing a model which includes its construction and working. This model is similar to a real system which helps the analyst to predict the effect of changes to the system.
It is an act of building a model.

Simulation is the process of using a model to study the performance of a system. Simulation of a system is the operation ~~which~~ of a model in terms of time or space which helps analyze the performance of an existing system.
It is an act of using a model for simulation

System.

- Components of the system
- System Environment
- System Model.

entity (combination of Objects is entity)



Object 1 depends on O2, O3 and O4.

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Any object which have some action to perform and is dependent on number of objects is called system.

Eg:, bank, library, college, etc.

University → large system.

Clg1 Clg2 Clg 3

If we combine few of these objects joined in some regular interactions or interdependence is become a large system.

1. Components of the system.

① Entity ② Attribute ③ Activity.
④ State of the system. ⑤ Event.

activity of student

College ← system.

coming time & going time

Students Classes Department } Entity.

attribute { hardworking Study

Entity → An entity is an object of interest in a system.
For e.g, in a factory system department orders, products and number of parts are entities.

Attributes → An attribute denotes the properties of an entity.

Activity → Any process causing changes in a system is known as activity.

State of the system → This mean a description of all the entities, attributes and activities as they exist at one point in time.

Event → It is defined as an occurence that may change the state of the system.

II System Environment:

The external components which interacts with the system and produce necessary changes are said to be system environment.

① Endogenous System.
② Exogenous System.

<u>Endogenous</u> → If the system is not affected by the environment is known as endogenous system.
Eg:- Classroom in the absency of students.

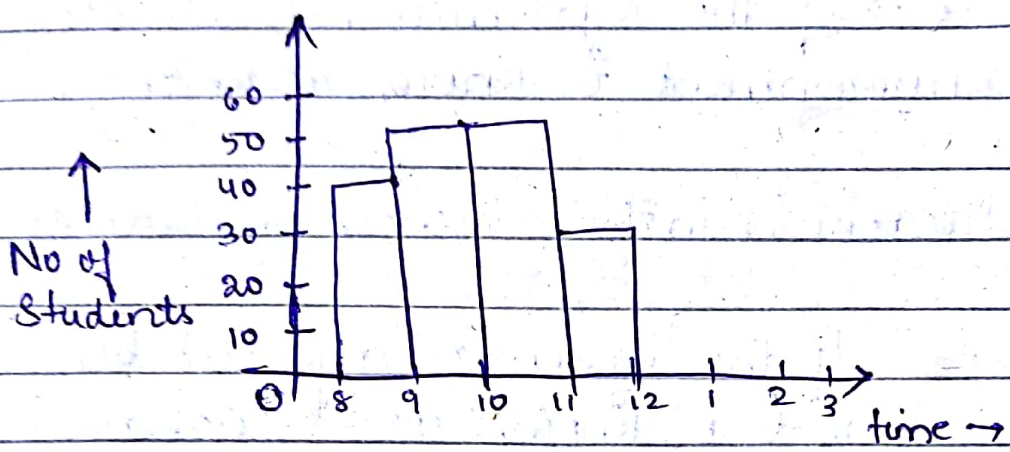
<u>Exogenous</u> → If the system is affected by the environment is known as exogenous system.
Eg:- Economic model of the country is affected by the world economic conditions.

→ <u>Open system and close system</u>:-

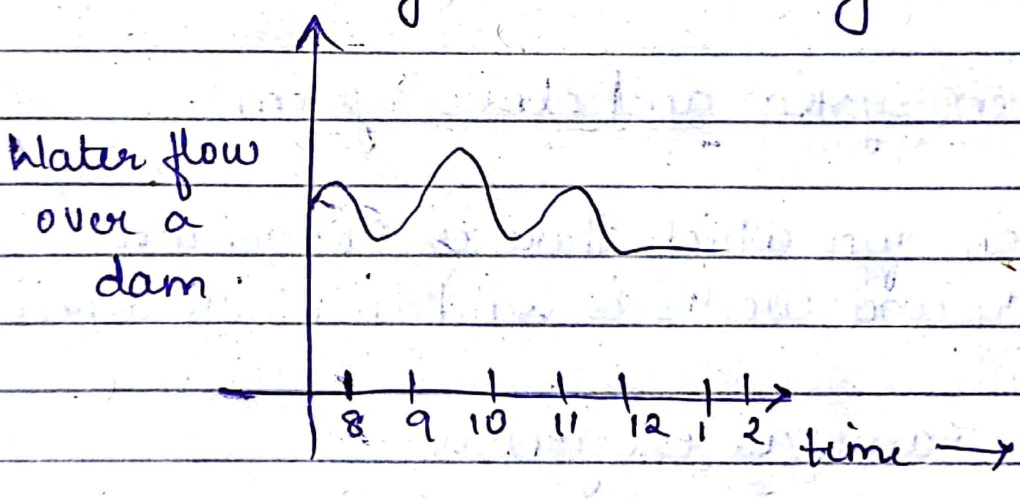
A system for which there is exogenous activity and event is said to be an open system.
Eg:- same as exogenous.

A system for which there is no exogenous activity and event is said to be close.
eg:- Water in an insulated flask.

→ **Discrete system:-** In this system the state variable change only at a discrete set of points in time.



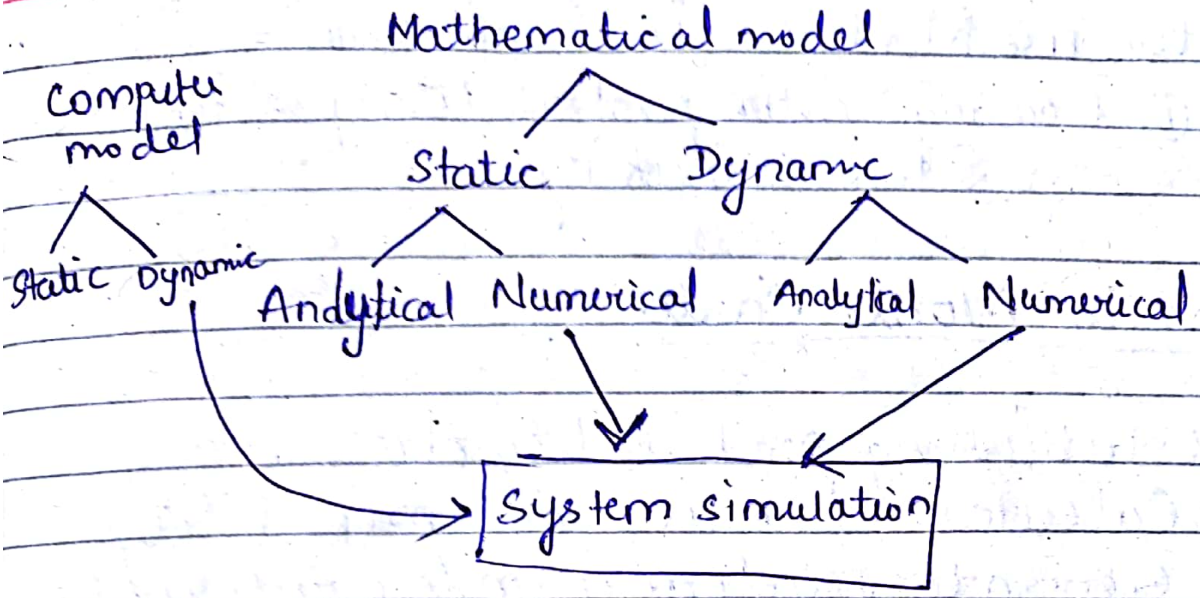
→ **Continuous system:-** In this system the state variable changes continuously over time.



(III) System model.

Model

- Physical model
  - Static
  - dynamic
- Mathematical model
  - Static
  - dynamic
- Computer model
  - Static
  - Dynamic



Eg:- Making of a house.

① Physically draw on paper.

② Draw mathematical equations.

③ Then using software (Autocad) we generate Computer model.

<u>Stochastic Variable</u>: or Random Variable

Eg:- Toss a Coin } Random experiment

$$\underbrace{X}_{\text{Random variables}} = \begin{cases} 0 & \text{Head} \\ 1 & \text{Tail} \end{cases}$$

<u>Stochastic modelling</u>:

It is used when we are making model for probability purpose problem.

Eg:- Monte Carlo Simulation Technique

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→ We use Monte Carlo Simulation :-
① Dealing with probability problems.
② In Decision making.

Steps in Monte Carlo

① Establishing probability distribution.
② Calculate cumulative probability.
③ Generate random number intervals.
④ generate random numbers.

Q Jhon is a dentist who schedules all his patients for 30 minutes.

Category	Time Req.	No. of patients	Pro
Filling	45 min	40	40/100
Crown	60 "	15	15/100
Cleaning	15 "	15	15/100
Extraction	45 "	10	10/100
Checkup	15 min	20	20/100
	Total =	100	

Calc: Find out the average waiting time for patients as well as the idleness of the doctor.
Simulate the dentist clinic for 4 hours.
Arrival time start at 8:00 am.

given - 40, 82, 11, 34, 25, 66, 17, 79
random numbers.

* If we have ~~[illegible]~~ two numbers after the decimal then random intervals ~~numbers~~ generate b/w 0 to 99;
and if 3 numbers after decimal then random intervals generate b/w 0 to 999.

* Total time = 4 hrs.
Per patient = 30 ~~patients~~ min.
Total patients = 8.

[illegible]b. of patient	C.P	Random Intervals
[illegible] = 0.4	0.40	0 - 39
= 0.15	0.55	40 - 54
= 0.15	0.70	55 - 69
= 0.1	0.80	70 - 79
= 0.2	1.00	80 - 99.

Now we will make the simulated table.

Patient	Scheduled ~~Arrival~~	Random Number	Category	Time
1	8:00	40	Crown	60min
2	8:30	82	Checkup	15min
3	9:00	11	Filling	45
4	9:30	34	Filling	45
5	10:00	25	Filling	45
6	10:30	66	Cleaning	15
7	11:00	17	Filling	45
8	11:30	79	Extraction	45

Patient	Arrival	Service Start	Service Duration	Service Ends	Waiting	[illegible]
1	8:00	08:00	60 min	9:00	0	[illegible]
2	8:30	9:00	15 min	9:15	30	0
3	9:00	9:15	45 min	10:00	15	0
4	9:30	10:00	45 min	10:45	30	0
5	10:00	10:45	45 min	11:30	45	0
6	10:30	11:30	15 min	11:45	60	0
7	11:00	11:45	45 min	12:30	45	[illegible]
8	11:30	12:30	45 min	1:15	60	0

Total waiting = 285.

Avg. waiting time $= \frac{285}{8}$

$= 35.625$ minutes

Idleness of doctor = 0 minutes.

Q

Daily demand of cake	Prob	Random no.
0	0.01	48, 78, 09,
15	0.15	51, 56, 77,
25	0.20	15, 14, 68 and
35	0.50	09.
45	0.12	
50	0.02	

Q ① Simulate demand for next 10 days.

② Stocic simulation if owner made ~~Simual~~ 35 cake/day.

Also find average daily demand on the basis of simulated data.

CP	Intervals
0.01	0
0.16	1 - 15
0.36	16 - 35
0.86	36 - 85
0.98	86 - 97
1.00	98 - 99

Simulated demand

		Stock
48	35	0
78	35	0
09	15	20
51	35	0
56	35	0
77	35	0
15	15	20
14	15	20
68	35	0
09	15	20

Total demand for 10 days = 270

Total stock = 80.

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If owner is making 35 cake per day
∴ Total stock = $35 \times 10 = 350$.

Average daily demand on basis of simulated data $= \frac{270}{10} = 27$ cakes/day.

Q The automobile company manufactures around 150 scooters.
The daily production varies from 146 to 154.

Prod./day	Prob.
146	0.04
147	0.09
148	0.12
149	0.14
150	0.11
151	0.10
152	0.20
153	0.12
154	0.08

The finished scooters are transported in a lorry accomo-dating 150 scooters.
Using the following random numbers
80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57.

Simulate:-
(i) Avg. no. of scooters waiting in the factory

② Avg no:- of empty space in the lorry?

Sol -

Prod./day	Prob.	CP	Random no:
146	0.04	0.04	0-3
147	0.09	0.13	4-12
148	0.12	0.25	13-24
149	0.14	0.39	25-38
150	0.11	0.50	39-49
151	0.10	0.60	50-59
152	0.20	0.80	60-79
153	0.12	0.92	80-91
154	0.08	1.00	92-99

① Radom	Prod/day	Scooters waiting	Space in lorry
80	153	3	0
81	153	3	0
76	152	2	0
75	152	2	0
64	152	2	0
43	150	0	0
18	148	0	2
26	149	0	1
10	147	0	3
12	147	0	3
65	152	2	0
68	152	2	0
69	152	2	0
61	152	2	0
57	151	1	0

Avg scooters waiting = $\frac{21}{15} = \frac{7}{5} = 1.4$

$\frac{9}{15} = \frac{3}{5} = 0.6$

Avg:- no:- of empty space in lorry

2/8/18

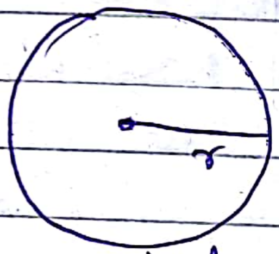
Stochastic model

Static — Dynamic

Static ↓ Monte Carlo.

→ Estimating the value of π using Monte Carlo.

Step 1.

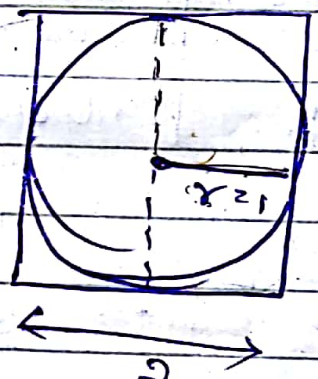


Area of circle = πr^2.

Assume $r = 1$

$$\boxed{A_c = \pi}$$

Step 2:

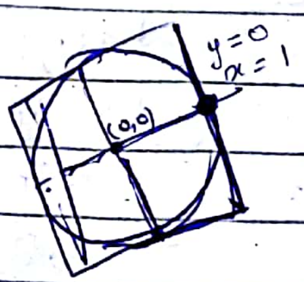


Area of square = 4

Step-3

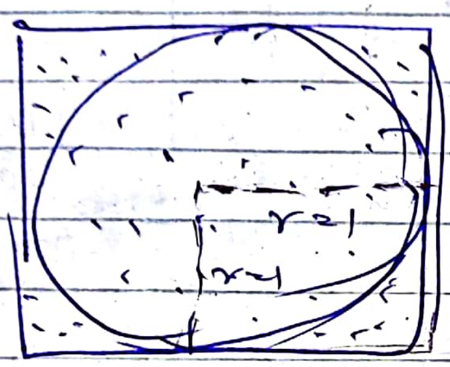
$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi}{4}$$

$$\boxed{\pi = \frac{4 * A_c}{A_s}}$$



Step-4:

Considering random numbers in fig in step2.

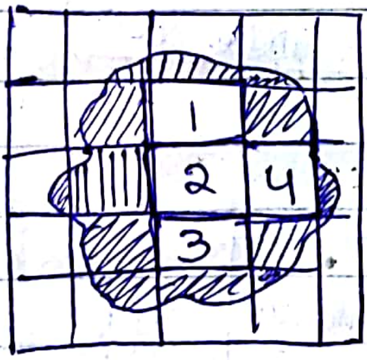


Area of circle ≈ No:- of random no:- in a circle.

Area of square ≈ No:- of random no:- in a square.

Now $\boxed{\pi = 4 * \frac{n_c}{n_s}}$

Estimating the area of irregular shape using Monte Carlo.

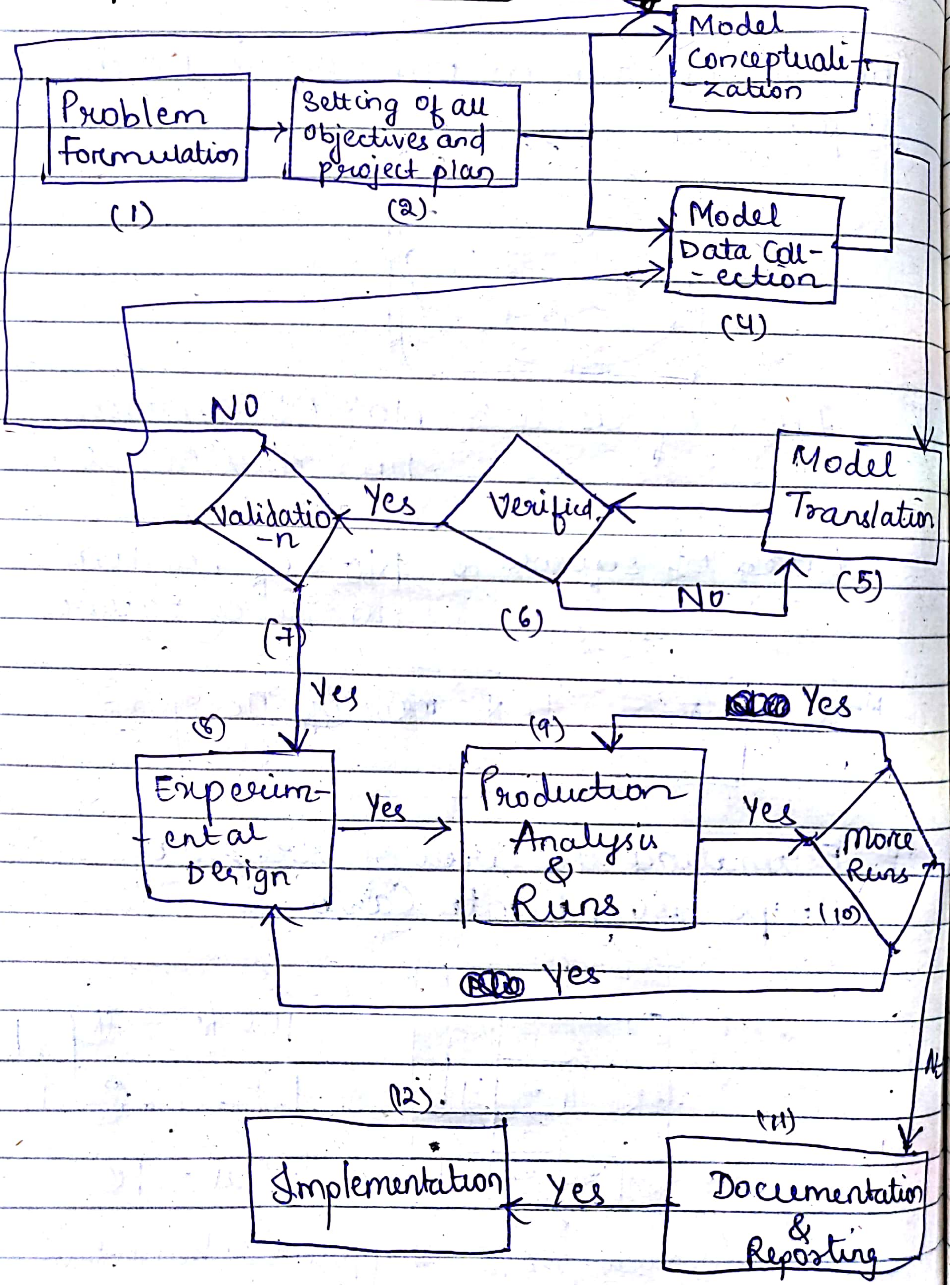


100% = 1, 1, 1, 1, 1, 1, 1, 1

Total - 10.

So our total area = 10.95 Suppose we have blocks 4 → 5, 10, 50, 30% = 95%.

Steps in simulation study :-



(1) & (2) → 1st phase.
(3) – (7) → 2nd phase.
(8) – (10) → 3rd phase.
(11) – (12) → 4th phase.

Advantages of Simulation.

1. Easy to Understand.
2. Easy to test.
3. Easy to upgrade.
4. Easy to identifying constraints.
5. Easy to visualize the problems.

Disadvantages:

1. ~~Initial cost~~ Simulation process is expensive
2. Time consuming.
3. It requires expert to understand.
4. Operations are performed on the system using random numbers, hence it is difficult to understand results.

Applications.

1. Defence.
2. ~~Air~~ Flight.
3. Aeronautics.

3/8/18

Implementation of estimation of π in program.

Suppose we have generated 10 random points.

Points (in one quad.)	x	y	if $x^2+y^2<1$ Print x or 0	if $x^2+y^2<1$ Print y or 0
1	0.5	0.9	0	0
2	0.9	0.3	0.9	0.3
3	0.3	0.2	0.3	0.2
4	0.9	0.8	0	0
5	0.6	0.5	0.6	0.5
6	0.8	0.7	0	0
7	0.6	0.4	0.6	0.4
8	0.3	0.5	0.3	0.5
9	0.2	0.6	0.2	0.6
10	0.3	0.8	0.3	0.8

$$\pi = \frac{\overset{2}{\cancel{4}} * 7}{\underset{5}{\cancel{10}}}$$

$$= \frac{14}{5} = 2.8$$

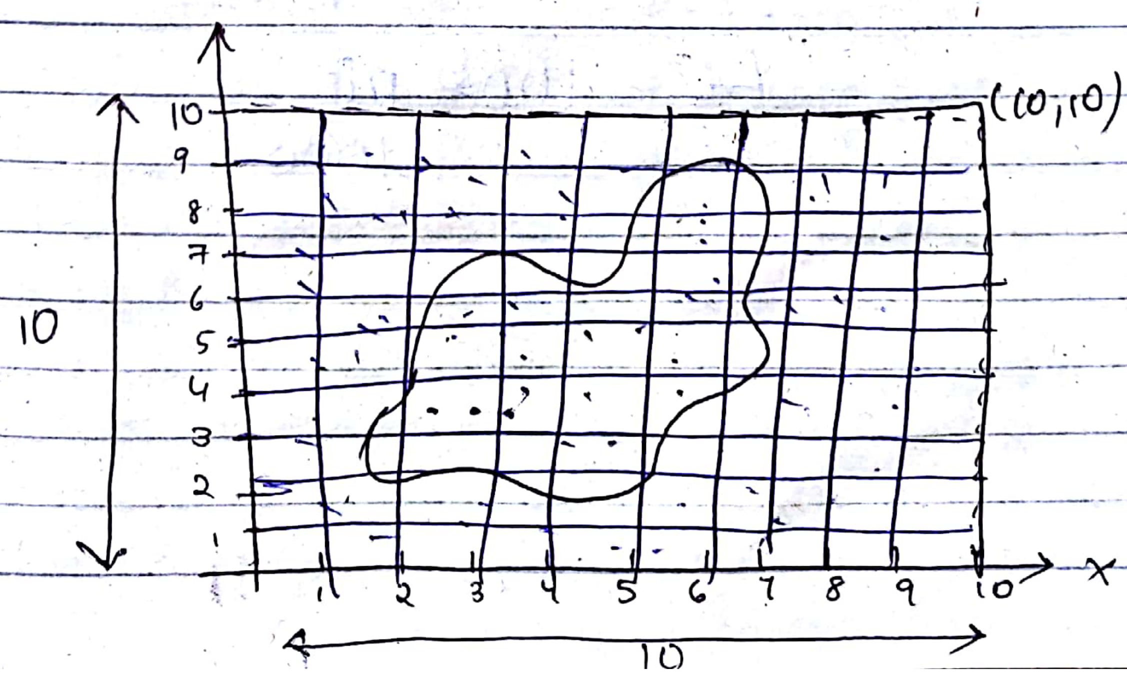
(This value is very less as we have taken very less number of random points)

we have done this for circle

we have done this for square.

if $x^2+y^2<1$ (1,0)	if $x^2+y^2<1$ Print(1,1)
0	1
1	1
1	1
0	1
1	1
0	1
1	1
1	1
1	1
1	1
7	10

Q



Sol- $\dfrac{\text{Area of irregular figure}}{\text{Area of square}} = \dfrac{n_i}{n_s}$.

$$\Rightarrow \frac{A_i}{A_s} = \frac{n_i}{n_s}.$$

$$\Rightarrow \boxed{A_i = \frac{n_i}{n_s} \times A_s.}$$

Here in this figure, $A_i = 100 * \dfrac{n_i}{n_s}$.

Suppose we have 1000 random points

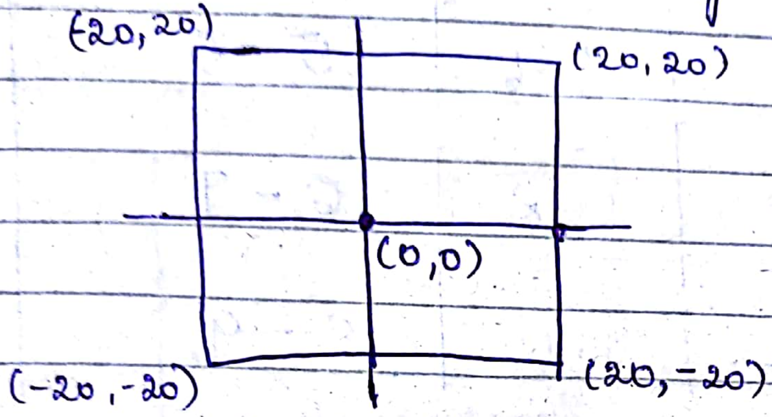
so $n_s = 1000$.

$$A_i = 100 * \frac{n_i}{1000}.$$

7/8/18

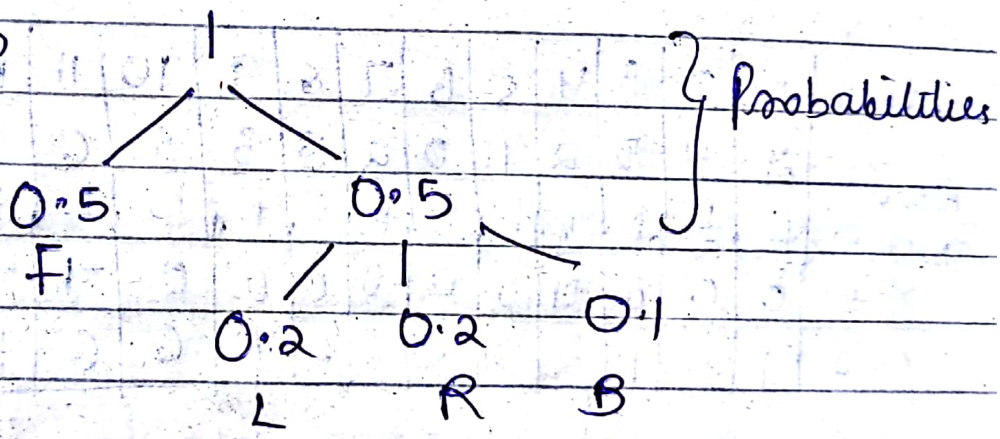
Random Walk Problem

→ Assume a 20X20 room. (eg.)



Calculate the next 20 steps.

given



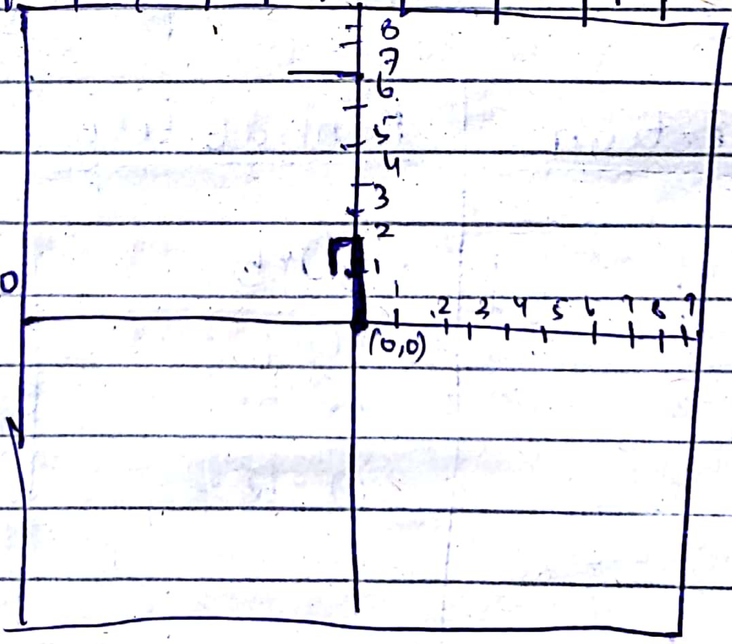
Direction	Probability
F	0.5
B	0.1
L	0.2
R	0.2

Applying Monte Carlo on this table/ situation

Direction	Prob	CP	RN Intervals
F	0.5	0.5	0 - 4
B	0.1	0.6	5 - 5
L	0.2	0.8	6 - 7
R	0.2	1.0	8 - 9

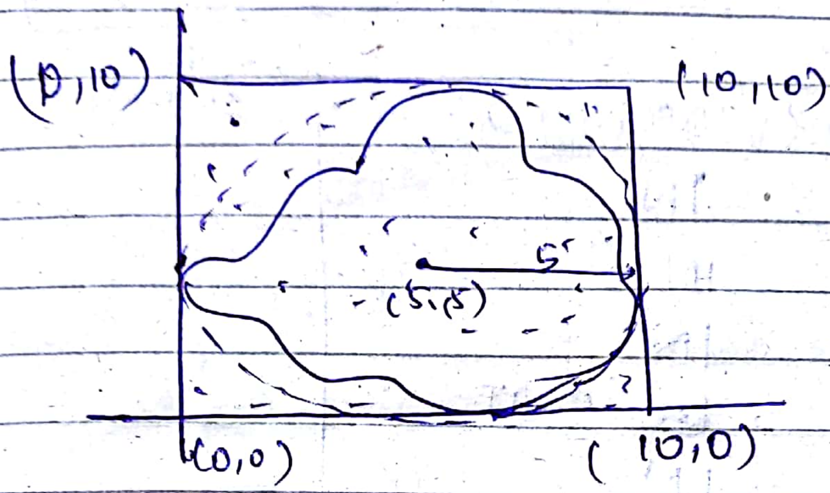
Steps	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
RNo	2	4	5	2	1	0	2	3	3	5	6	4	4	3	9	8
Direch.	F	F	B	F	F	F	F	F	F	B	L	F	F	F	R	R
x=0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	1
Y=0	1	2	1	2	3	4	5	6	7	6	6	7	8	9	9	9

* If (X,Y) is within room, then OUT=No else OUT=Yes.



OUT	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Random points in irregular surface problem (Continuation of previous problem).



17	18	19	20
7	6	4	5
L	L	F	B
0	-1	-1	-1
9	9	10	9

→ Consider a geometric figure that will cover maximum of the irregular figure.

→ Here we are considering a circle

$$x^2 + y^2 = r^2$$

$$(x-5)^2 + (y-5)^2 = 25$$

$$(y-5)^2 = 25 - (x-5)^2$$

(To consider points only inside the circle).

$$(y-5)^2 < 25 - (x-5)^2$$

| N | N |

→ After that we will generate random numbers b/w 0 to 10

$\Rightarrow y^2 + 25 - 10y < \cancel{25} - x^2 - \cancel{25} + 10x$

$\Rightarrow \boxed{y^2 < 10y + 10x - x^2 - 25}$

x	y	$y < \sqrt{10x + 10y - x^2 - 25}$ → If true IN/OUT
6	5	IN
7	4	IN
9	3	IN
9	8	OUT
6	6	IN

$60+30-36-25$ $\frac{25}{29}$ 11

$70+40-49-25$ $\frac{24}{36}$

$90+30-81-25$ $\frac{39}{25}$ 14

$90+80-81-25$ $\frac{26}{64}$

→ Calculate the number of IN; this will equal to ni (no:- of random points in irregular surface)

→ Put this in the formula and get the area of the irregular surface.

$A_I = 100 * \frac{ni}{ns}$ → Total random points

$= 100 * \frac{ni}{1000}$ For eg 1000

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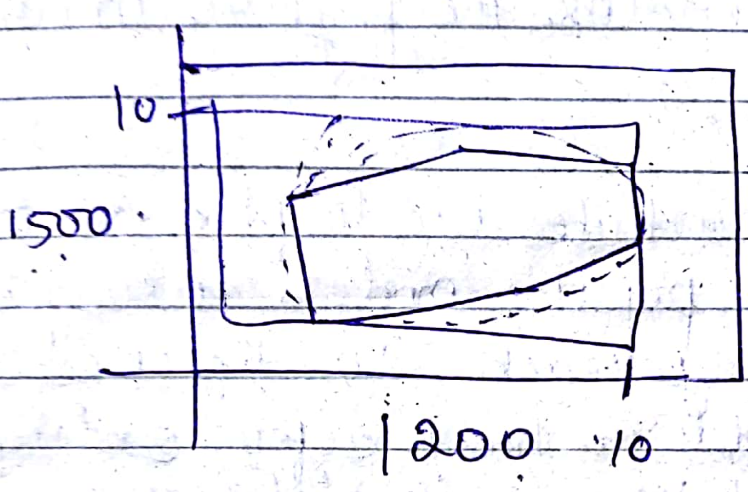
Here $n_i = 4$ (no:- of ins)

$$\boxed{A_I = \frac{100 \times 4}{1000} = 0.4}$$

Q:



$$\underline{A_i} = A_s * \frac{n_i}{n_s}$$

$$n_i = \frac{A_i \times n_s}{A_s}$$

$$= \frac{A_i \times 100}{1500 \times 1200}$$ (suppose $n_s = 100$)

(Here we can consider 1500 ~ 15, 1200 ~ 12).

Now we can calculate the no:- of random points using area's by dividing the surface in blocks.

Now, if we want to implement this,

then we will enclose the figure in nearest circle, calculate random points and then from this we can calculate area. (This can happen by first enclosing it in square and then enclose the figure in circle)

9/8/18

Q A garment retailer use to place an order for 27 most popular models as soon as the inventory reaches 20. He used to have 25 models on hand before ordering because the lead time on the delivery of the garment is 2 weeks. Simulate the sales of 15 wa

Sales per week	Probability	CP	Random IV
5	0.05	0.05	0 - 4
6	0.05	0.10	4 - 9
7	0.10	0.20	10 - 19
8	0.10	0.30	20 - 29
9	0.10	0.40	30 - 39
10	0.20	0.60	40 - 59
11	0.20	0.80	60 - 79
12	0.1	0.90	80 - 89
13	0.05	0.95	90 - 94
14	0.05	1.00	95 - 99

Q Calculate the average ending inventory.

Week	RN	Sales	Bel Inv	End Inv	Shortage	Placed Q
1	23	8	25	17	0	27
2	59	10	17	7	0	—
3	82	12	7	0	5	—
4	83	12	27	15	0	27
5	61	11	15	4	0	—
6	0	5	4	0	1	—
7	48	10	27	17	0	27
8	33	9	17	8	0	—
9	6	6	8	2	0	—
10	32	9	29	20	0	27
11	82	12	20	8	0	—
12	51	10	8	0	2	—
13	54	10	27	17	—	27
14	66	11	17	6	—	—
15	55	10	6	0	4	—
				121.	12	

Q1 Average ending inventory $= \frac{121}{15}$

$= 8.0666$

Q2 Number of shortage week = 4.

Simulation In Dynamic System:-

⇒ Queuing System:

Customer — Service provider

(Two most imp entities of Queuing system).

- Basic concepts

→ Queue.

→ Queuing Theory → when we are dealing queue, mathematically then it is being used.

Dynamic System

In this system, time plays an important role. The system state in dynamic system changes with time. Hence, in such system a clock known as Simulation clock is used to track simulated time.

Queuing System

This system consist of two entities :- Customer and server.

Customer:- It is the entity requesting

service or needing service. The number of customer and Service ~~no~~ required may be one or more.

Server – It is the entity providing the service. The number of server may be one or more.

Basic Concept:

① Queue :–
A line of people or vehicle waiting for their turn.

② Queuing theory:–
It is the mathematical study of waiting line or queue. This technique provides basis of decision making about the resources needed to provide a service.

③ Reason of Queue:–
① A Queue is formed when a no:- of customer are more than the service provider.
② The serving time to a customer is more than the arrival time of the customer.

* Application of Queing Theory :-

④ Reason to study Queue.

→ To allocate the resources.
→ To find out the cost of offering the service.

Components of Queuing System :-

① Calling Population or Customer.

On the basis of size :-

finite Population (Doctor will check only 5 patients).

Infinite Population. (Children coming in university to take admission but some are admitted & some not).

② Arrival.

On the basis of behaviour.

Balking

Reneging

Jockying

customer
Pattern (coming in which pattern)
- single
- Batch
- with appointment
- without appointment, etc.

→ Poison and Exponential are two methods for the calculation of arrival.

Tutorial-1

① $3 \rightarrow 6$
$5 \rightarrow 4$

$$P = \frac{(6+4)-1}{20} = \frac{9}{20}$$

② Total = 7.

$$\frac{{}^5C_2}{{}^7C_2} = \frac{\lfloor 5}{\lfloor 3} \times \frac{\lfloor 5}{\lfloor 7}$$

$$= \frac{5 \times \cancel{24} \cancel{4} 2}{{}_3\cancel{6} \times 7 \times \cancel{6}}$$

Ans ← $10/21$

(3) Two consecutive tails : $\frac{1}{4}$

(4) $$\frac{^{52}C_4 \times {^{4}C_1} \times {^{51}C_1} \times {^{4}C_1}}{^{52}C_2}$$

$$\frac{4}{\cancel{52}_{13}} \times \frac{\cancel{4}}{51_{13}} = \frac{4}{663}$$

(5) $$P(SSC \mid SSM) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{40}{60} = \frac{2}{3}$$

(6) $$\frac{4+1}{6} = \frac{5}{6}$$

(7) $$\cancel{\frac{^{4}C_2 \times {^{4}C_1}}{^{8}C_3}} = \frac{\lfloor 4}{\cancel{\lfloor 2 \times \lfloor 2}} \times 4$$

$$\frac{^{4}C_2 \times {^{4}C_1}}{^{8}C_3} \qquad \frac{\lfloor 8}{\lfloor 5 \times \lfloor 3}$$

$$= \frac{3}{7} \qquad = \frac{\lfloor 4 \times \lfloor 3}{6 \times 7 \times 8} = \frac{3}{7}$$

6 2
5 3
4 4

$\frac{Q}{36}$

$\frac{2}{36} + \frac{5}{36}$ $\quad$ $5/36$

(8) $P = \frac{2}{5}$ $\rightarrow$ Ans

$\boxed{(9)\ 1 - \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) = \frac{3}{4} \rightarrow \text{Ans}}$

~~(9) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} +$~~

~~$\left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{2}{3}\right) + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right)$~~

~~$+ \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right)$~~

~~$= \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{8}\ \frac{1}{12} + \frac{1}{24} + \frac{1}{24}$~~

~~$= \frac{6+3+2}{24} = \frac{11}{24} = \frac{1}{9} + \frac{1}{12} + \frac{1}{6}$~~

~~$= \frac{12+2+2+2}{24} = $~~

(10) $\frac{{}^{26}C_2 + {}^{4}C_2 - 11}{{}^{52}C_2}$

$= \frac{\frac{26 \times 25}{2} + 5}{\frac{52 \times 51}{2}}$

13
26
25
130
52
650

$= \frac{26 \times 25 + 10}{52 \times 51}$ $\quad$ $\boxed{= \frac{55}{221}}$ $\rightarrow$ Ans

26
25
130 650
52

$= \frac{660}{52 \times 51} = \frac{330}{26 \times 51} = \frac{110}{13 \times 51} = \frac{55}{17 \times 13} \rightarrow$ Ans

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(7) 2C + 1I

$$\uparrow$$

$$\frac{{}^4C_2}{8} \times \frac{{}^4C_1}{7} + \frac{{}^4C_1}{8} \times \frac{{}^4C_2}{7}$$

$$= \frac{\cancel{2} \times \cancel{4} \times 6}{\cancel{56}\ 7} = \frac{6}{7}$$

$$\begin{array}{ccccccc} C & & C & & I \\ \frac{\cancel{4}}{\cancel{8}\ \cancel{2}} & \times & \frac{\cancel{3}}{7} & \times & \frac{\cancel{4}\ \cancel{2}}{\cancel{6}} \end{array} + \begin{array}{ccccc} I & & C & & C \\ \frac{4}{8} & \times & \frac{4}{7} & \times & \frac{3}{6} \end{array} + \frac{4}{8}$$

$$= \frac{2 \times 1 \times 3}{2 \times 7} = \frac{3}{7} \rightarrow \underline{\text{Ans}}.$$

Binomial

$$\boxed{P(x) = {}^nC_x (p)^x (q)^{n-x}}$$

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10/8/18

I Calling population :-

Number of customers waiting for their turn → size of population

① Finite population

I C

$\frac{1}{7} \times \frac{3}{6}$

A limit ~~pop~~ on the customers for waiting in the queue.

② Infinite

No limit on the customer.

II Arrival

It determine the way customers enter in the system.

ⓐ Behaviour of arrivals:

① Balking :- Customer do not join the queue.

② Renging :- Customer wait for some time in the queue but leave before being served.

③ Jokeying → Customer moves from one queue to another queue, hoping to receive service.

ⓑ Pattern of arrival.

① The customer can arrive in batches or single (lone)
② The customer may arrive in scheduled time or unscheduled time.

Arrival time Calculation.

→ Poission Distribution.
→ Exponential Distribution.

Poission - Probabilities for the number of customers that may arrive in any specific interval of time.

Exponential - Probability for time gap between ~~any~~ two consecutive arrivals.

III Queue Discipline (Rules).

1. FCFS or FIFO.
2. LCFS
3. Random order.
4. Emergency.

IV Queuing Process.

→ It will tell us No. of Queue + length
→ It is divided into 2 types :-
 (a) Finite source Queue. (movie theatre)
 (b) Infinite source Queue. (railway)

Queuing process refers to the number of queues and their respective length.

Finite source Queue → Services are provided to limited number of customers.

Infinite source Queue → services are provided to all customers.

V Service Process.

Service Process ~~of~~ means process of man or machine to serve the customer.

Formulas.

① Arrival time.

(a) Arrival rate.

(b) Average arrival rate (λ)

Eg:- arrival rate = 6 min/per.

$1C \underline{\quad 6min \quad} 2C$

$6min \rightarrow 1 person.$

$1hr \rightarrow \frac{1}{6} \times 60 = 10 persons.$

↑

ARR.

② Service Time.

ⓐ Service rate.

ⓑ Average service rate (μ).

$1C \underline{\quad 10min \quad} 2C$

eg:- Avg service rate for 8 h ??
given service rate = 10 min.

$$\mu = \frac{8 \times 60}{10} = 48 persons.$$

③ Utilization rate (R) or ρ.

$$R = \frac{\lambda}{\mu} = \frac{ARR}{ASR}$$

~~given μ = 10~~ For a service provider (μ)

Capacity = 10

But ~~happened~~ came = 8 (λ)

$$R = \frac{8}{10} = 0.8$$

$$= 80\%$$

There are 3 possibilities in R :-

① If $\lambda > \mu$

$10 > 8$.

(i) infinite Queue. (because the leftover will form another queue).

(ii) Service Provider is busy.

(iii) Failure of service system.

② If $\lambda = \mu$.

① Customer comes and served.

② No queue will be there.

③ Server will be busy.

(3) If $\lambda < \mu$.

(1) No queue will be there.
(2) There is ideal time for server.

14/8/18

(4) Probability of 'n' customer in a system.

Model.

Single server | multi server.

$M|M|1$ ← no:- of servers

M (first) — arrival distribution

M (second) — service distribution

$\infty | \infty$.

∞ (first) — ∞ population

∞ (second) — ∞ queue length

Probability of n customer at a time t for a small event h.

$P_n(t+h) = P_{n-1}^{(t)}$ * probability of one arrival * no service

$+ P_{n+1}^{(t)}$ * probability of one service * no arrival

$+ \; p_n^{(t)} *$ probability of no arrival * no service.

$$p_n(t+h) = p_{n-1}^{(t)} * \lambda h * (1-\mu h)$$
$$+ \; p_{n+1}^{(t)} * \mu h * (1-\lambda h)$$
$$+ \; p_n^{(t)} * (1-\mu h) * (1-\lambda h)$$

$$= p_{n-1}(t)\,\lambda h - p_{n-1}(t)\,\lambda h\,\mu h$$
$$+ \; p_{n+1}(t)\,\mu h - p_{n+1}(t)\,\mu h\,\lambda h$$
$$+ \; p_n^{(t)} - p_n^{(t)}\mu h - p_n^{(t)}\lambda h + p_n\,\mu h\,\lambda h$$

We will now neglect the square terms.

$$= p_{n-1}(t)\,\lambda h + p_{n+1}(t)\,\mu h + p_n(t)$$
$$- \; p_n(t)\,\mu h - p_n(t)\,\lambda h.$$

$$= p_{n-1}(t)\,\lambda h + p_{n+1}(t)\,\mu h + p_n^{(t)}(1 - \mu h - \lambda h)$$

$$\Rightarrow P_n(t+h) - P_n(t) = h\big(P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)\big)$$

$$\Rightarrow \frac{P_n(t+h) - P_n(t)}{h} = P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)$$

$\Rightarrow$ Applying Steady state (Transient state $\rightarrow$ when the system starts and then it becomes constant which is the Steady state, where time remains constant)

So now our time is 0 and h is very small as it is a small event

So, $\cancel{\dfrac{P_n(t+h) - P_n(t)}{h}} = 0$

$$\Rightarrow 0 = \lambda P_{n-1} + \mu P_{n+1} - (\lambda+\mu)P_n$$

$$\boxed{\Rightarrow (\lambda+\mu)P_n = \lambda P_{n-1} + \mu P_{n+1}}$$

For $n=0$ (For 0 customers).

$\because$ There will be arrival but no service.

~~So the term~~ will be there.

So the term will not be there.

$$P_{n-1}(t) * \lambda h * \underbrace{(1-\mu h)}_{0}$$

$$= 0.$$

Now our eqn is

$$P_n(t+h) = P_{n+1}\,\mu h(1-\lambda h) +$$

$$P_n(t)\,(1-\lambda h) * \underbrace{(1-\mu h)}$$

(as there will be service provider)
So it will be 1

$$= P_{n+1}\,\mu h(1-\lambda h) + P_n(t)\,(1-\lambda h)$$

$$\Rightarrow \frac{P_n(t+h) - P_n(t)}{h} = P_{n+1}\,\mu - P_n(t)\,\lambda.$$

(neglecting h^2 term)

$\Rightarrow 0 = P_{n+1}\mu - P_n\lambda$

$\Rightarrow 0 = P_1\mu - P_0\lambda$

$\Rightarrow P_1\mu = P_0\lambda$

$\boxed{\Rightarrow P_1 = \frac{\lambda}{\mu} P_0}$ $\Rightarrow P_1 = R P_0$

Probability of zero customers

$P_0 + P_1 + P_2 + P_3 \cdots P_n = 1 \rightarrow$ (Probability of all customers)

$\Rightarrow P_0 + R P_0 + R^2 P_0 + R^3 P_0 \cdots R^n P_0 = 1$

$\Rightarrow P_0 (1 + R + R^2 \cdots R^n) = 1$

(when $n \to \infty$)

$\Rightarrow P_0 \left(\frac{1}{1-R}\right) = 1$

$\Rightarrow \boxed{P_0 = 1 - R}$ (Idle rate)

① $\boxed{P_0 = 1 - \frac{\lambda}{\mu}}$

(2) $P_1 = R P_0$

$= R(1-R)$

$= \frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)$

(3) $P_2 = R \cdot P_1 = R^2 P_0$

$= \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)\right)$

$= \left(\frac{\lambda}{\mu}\right)^2 \left(1-\frac{\lambda}{\mu}\right)$

(4) $P_n = R^n \cdot P_0$

$$\boxed{P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)}$$

<u>Q</u> Arrival time = 15 min.
ST = 25 min.

$\lambda = 15$
$\mu = 25$.

(1) AAR =

① 15 min → 1 person

3×60 min → $\frac{3 \times 60}{15} = 12$ persons $\lambda =$

② ASR

25 min → 1 person

60 min → $\frac{\cancel{60}^{12}}{\cancel{25}_{5}} \times 3 = 7.2$ $\mu =$

③ $p(7)$

$= \left(\frac{\lambda}{\mu}\right)^7 \left(1 - \frac{\lambda}{\mu}\right) = \left(\frac{\lambda}{\mu}\right)^7 \left(1 - \frac{\lambda}{\mu}\right)$

$= \left(\frac{15}{25}\right)^7 \left(1 - \frac{3}{5}\right) = \left(\frac{12}{7.2}\right)^7 \left(1 - \frac{12}{7.2}\right)$

$= \left(\frac{3}{5}\right)^7 \left(\frac{2}{5}\right)$ → This comes out to be -ve.

$=$ ~~1.536×10^{-5}~~ → Ans

So this is the case of the failure.

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Input
(λ, μ, c)

Calculate probabilities

Output

L_q - length of queue
L_s - length of system
W_q - waiting time of queue
W_s - waiting time of system
L'_s - length of system when no:- of customer is not known

16/8/18 . Average/Expected/Mean are all same.

→ L_q → Average ~~length~~ no:- of customers in the queue

→ L_s → Average no:- of customer in the system.

$$L_s = \frac{UR}{IR} = \frac{\text{Utilisation rate}}{\text{Idle rate}}$$

$$= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}$$

$$\boxed{L_s = \frac{\lambda}{\mu-\lambda}}$$

$$L_q = L_s - UR$$

$$= \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu} = \frac{\lambda\mu - \cancel{\lambda\mu} + \lambda^2}{\mu(\mu-\lambda)}.$$

$$\boxed{L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}}$$

→ W_q - waiting time in the queue - average.

$$= \frac{\text{Average no:- of customer in a queue}}{\text{Arrival rate}}$$

$$= \frac{L_q}{\lambda}$$

$$= \frac{\lambda^{\cancel{2}}}{\mu(\mu-\lambda)\cancel{\lambda}} = \boxed{\frac{\lambda}{\mu(\mu-\lambda)}}$$

→ W_s = average no:- of customers waiting in a system.

$$= \frac{L_s}{\lambda}$$

$$= \frac{\cancel{\lambda}}{(\mu-\lambda)\times\cancel{\lambda}} = \boxed{\frac{1}{\mu-\lambda}}$$

→ Expected length of non-empty queue Lq

$$Lq' = \frac{Lq}{p(n>1)}$$

$$= \frac{Lq}{1-(P_0+P_1)}$$

$$= \frac{Lq}{1-(1-R+R(1-R))}$$

$$= \frac{Lq}{\cancel{1}-\cancel{1}+\cancel{R}-\cancel{R}+R^2}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)\times\frac{\lambda^2}{\mu^2}}$$

$$Lq' = \boxed{\frac{\mu}{\mu-\lambda}}$$

Q. On the desk of an off. of a banking company, the arrivals of the customer follows poisson law on an average at every 10 minutes a customer arrive; ~~~~ service time = 6 min.

Calculate AAR and ASR.

① 1 hour

② 15 min.

③ 8 hours.

AAR

①

10 min — 1

60 min → $\frac{60}{10}$ = 6 person/hour

②

10 m → 1

15 m → $\frac{15}{10}$ = 1.5 person

③

8h → $\frac{8 \times 60}{10}$ = 48 person

ASR

① $\mu = \frac{60}{6}$ = 10 person

② $\mu = \frac{15}{6}$ = 2.5 person

③ $\mu = \frac{8 \times 60}{6}$ = 80 person.

Utilisation rate

① $R = \frac{6}{10} = 0.6$

② $R = \frac{1.5}{2.5} = \frac{3}{5} = 0.6$

③ $R = \frac{48}{80} = \frac{6}{10} = 0.6$

Idle rate

① $P_0 = 1 - 0.6 = 0.4$

② $P_0 = 1 - 0.6 = 0.4$

③ $P_0 = 1 - 0.6 = 0.4$

Q In a health clinic, the Average rate of arrival of patient is 12 patients/hr. On an average a doctor can serve patient at the rate of 1 patient every 4 minutes.

Assume the arrival of patient follows poisson law.

① Find the avg. no of patient in the wt. line and in clinic.

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② The aveg. wt time in the wt. line or in the queue and also the avg wt. time in the clinic.

<u>Sol</u> $\lambda = 12$

4 min → 1 patient

60 min → $\frac{60}{4} = 15$ patient

$\mu = 15$.

① $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\cancel{12} \times \cancel{12}}{\cancel{15}(\cancel{3})} = 3.2$

② $L_s = \frac{\lambda}{\mu-\lambda} = \frac{12}{3} = 4$.

③ $W_s = \frac{1}{\mu-\lambda} = \frac{1}{3}$ hr $= 20$ min

$= 0.33$ hr

④ $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\cancel{12}\,4}{15 \times \cancel{3}}{}^{\times 60} = 0.267$ hr.

$= 16$ min

17/8/18.

Tutorial.

Q.

Arrive → random 1 to 8 minutes apart.

Each inter-arrival time → has same probability so Each Prob. of AT = 1/8.

given

Assume → first customer arrive at 0.

Service time (min)	Probability	CP.	RI.
1	0.10	0.10	0-9
2	0.20	0.30	10-29
3	0.30	0.60	30-59
4	0.25	0.85	60-84
5	0.10	0.95	85-94
6	0.05	1.00	95-99

Time b/w arrivals (min)	Prob.	CP.	Random IV
1	0.125	0.125	100-124
2	0.125	0.250	125-249
3	0.125	0.375	250-374
4	0.125	0.500	375-499
5	0.125	0.625	500-624
6	0.125	0.750	625-749
7	0.125	0.875	750-874
8	0.125	1.000	875-999

2)

Customer	Random Number	Time b/w Arrival
1	—	—
2	913	8
3	727	6
4	115	1
5	948	8
6	309	3

Custome	Random	Service time
1	84	4
2	09	1
3	74	4
4	53	3
5	17	2
6	79	4

3)

ST+WT

Customer	Inter AT	AT	ST	Time service begin	WT Time	Time Service ends	Time Spent in System	Idle time of server
1	—	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
	26		18		3		21	12

1. Avg. wt time $= \frac{3}{6} = \frac{1}{2} = 0.5$

2. Prob. that a customer has to wait in the queue

$$= \frac{1}{6}$$

3. Fraction of idle time of server

$$= \frac{12}{30} = \frac{2}{5} = 0.4 \times 100$$
$$= 40\%$$

4. Avg. service time $= \frac{18}{6} = 3$

5. Avg. time b/w arrivals $= \frac{26}{5}$

$$= \frac{\text{Sum of time b/w arrivals}}{\text{No. of arrivals} - 1} = 5.2$$

6. Avg. wt time those who wait

$= \frac{\cancel{\text{Sum of time}}\ 1}{\cancel{\text{No. of}}} = \frac{3}{1} = 3$

7. Avg. time a customer spends in system $= \frac{21}{6} = \frac{7}{2} = 3.5$

Q $\frac{1}{\mu} = 30$ minutes.

$$\mu = \frac{1}{30} \text{ person/min}$$

$$= \frac{6\cancel{0}}{3\cancel{0}} = 2 \text{ person/hr.}$$

$$= 16 \text{ person per 8 hour.}$$

$\lambda =$ 10 per 8 hour.

$$R = \frac{\lambda}{\mu}$$

$$P_0 = 1 - R$$

Probability for idle server

$$= 1 - \frac{10}{16} = \frac{6}{16} = \frac{3}{8}$$

$$\text{Idle time} = \frac{3}{\cancel{8}} \times \cancel{8}$$

$$= 3 \text{ hours each day.}$$

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{10}{16-10}$$

$$= \frac{10}{6} = \frac{5}{3}$$

$$= 1.66 \approx 2 \text{ jobs.}$$

Q $\lambda = 4$ cars/hour.

$\mu = 8$ cars/hour

(b) $W_s = \frac{1}{\mu-\lambda} = \frac{1}{8-4} = \frac{1}{4}$ hr

(average time a car spends in the system)

(c) $L_s = \frac{\lambda}{\mu-\lambda} = \frac{4}{8-4} = \frac{4}{4} = 1$ car

(average no. of cars in the system).

(a) → $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{4}{8(8-4)}$

$= \frac{4}{8\times4} = \frac{1}{8}$ hr

(Average time that a car waits for water-wash to begin)

— X —

21/8/18

M:M:1 | N:∞] Kendall's Notation

M → Arrival rate

M → Service rate

1 → server

N → finite queue

∞ → ∞ population

S	C	n − c
server	Cust	

Suppose $n=5$. (i.e, the queue can contain on 5 customers)

As soon as the 6th customer comes, there will be balking as he cannot join the queue, Because the queue length is fixed to 5.

Probability of ~~number~~ zero/ no customer

$$p_1 = Rp_0 \qquad p_0 + p_1 + \cdots + p_n = 1$$

$$p_n = R^n p_0 \qquad p_0 + Rp_0 + R^2 p_0 + \cdots + R^n p_0 = 1$$

$$p_0 (1 + R + R^2 + \cdots + R^n) = 1$$

$$p_0 \left[\frac{1 - R^{n+1}}{1-R} \right] = 1$$

$$\Rightarrow \boxed{p_0 = \frac{1-R}{1-R^{n+1}}}$$

$$\therefore \boxed{p_n = \left(\frac{1-R}{1-R^{n+1}} \right) * R^n}$$

Performance measure in M:M:1:N model.

$$L_s = \sum_{n=0}^{N} n p_n.$$

i.e, $1 \cdot p(1) + 2 \cdot p(2) \dots$

$$L_s = \sum_{n=0}^{N} n R^n p_0$$

Put $R^n = R^{n-1} \cdot R$

$$L_s = \sum_{n=0}^{N} n R^{n-1} \cdot R p_0$$

$$= p_0 R \sum_{n=0}^{N} R^{n-1} \cdot n \qquad \left\{ \because \frac{dx^n}{dx} = nx^{n-1} \right.$$

$$= p_0 R \sum_{n=0}^{N} \frac{dR^n}{dR}$$

$$= p_0 R \frac{d}{dR} \sum_{n=0}^{N} R^n.$$

$$= p_0 R \frac{d}{dR} \left[\frac{1 - R^{n+1}}{1-R} \right]$$

$$= P_0 R \left\{ \frac{(-1)(1-R)(N+1)R^N - (-(1-R^{N+1}))}{(1-R)^2} \right\}$$

$$= P_0 R \left[\frac{(1-R^{N+1}) - (1-R)R^N(N+1)}{(1-R)^2} \right]$$

$$= P_0 R \left[\frac{1 - R^{N+1} - R^N(N+1) + R^{N+1}(N+1)}{(1-R)^2} \right]$$

$$= P_0 R \left[\frac{1 - \cancel{R^{N+1}} - (N+1)R^N + \cancel{R^{N+1}} + NR^{N+1}}{(1-R)^2} \right]$$

$$= P_0 R \left[\frac{1 + NR^{N+1} - (N+1)R^N}{(1-R)^2} \right]$$

$$= R \left[\frac{\cancel{(1-R)}}{(1-R^{N+1})} \right] \left[\frac{1 + NR^{N+1} - (N+1)R^N}{(1-R)^{\cancel{2}}} \right]$$

$$= R \left[\frac{1 + NR^{N+1} - (N+1)R^N}{(1-R^{N+1})(1-R)} \right]$$

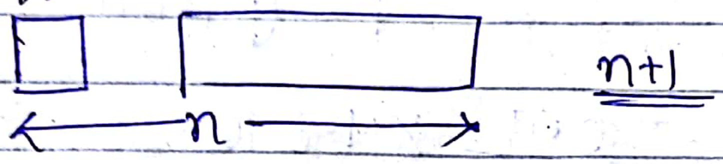
$$\boxed{p(n>0) = 1 - p(0)}$$

($p(n>0) \rightarrow$ probability that atleast 1 customer is getting service) = 1 - probability that there are no customers

$$L_q = L_s - (p(n>0))$$
$$= L_s - [1 - p(0)]$$

$$\boxed{L_q = L_s - 1 + p(0)}$$

Effective arrival rate



Customer that arrive but not join in the queue.

$$\boxed{\lambda_{eff} = \lambda(1 - p_n)}$$

$$\boxed{W_s = \frac{L_s}{\lambda_{eff}}}$$

$$\boxed{W_q = \frac{L_q}{\lambda_{eff}}}$$

Q $\lambda = 20$, $\mu = 8$; $N = 6$.

(i) Utilization rate.

$$UR = \frac{\lambda}{\mu} = \frac{20}{8} = \underline{\underline{1.11}}.$$

(ii) $P(6\text{ customer}) = \frac{1-R}{1-R^{N+1}} \cdot R^N$

$$= \frac{1-1.11}{1-(1.11)^{6+1}} \cdot (1.11)^6$$

$$= \frac{-0.11}{-1.076} \times 1.870$$

$$= \underline{\underline{0.1912}} \longrightarrow \underline{\underline{\text{Ans}}}$$

(iii) $L_S = \frac{R}{1-R^{N+1}} \left[\frac{1+N \cdot R^{N+1} - (N+1)R^N}{(1-R)} \right]$

$R = 1.11$
$N = 6$

$$L_S = \frac{1.111}{-1.076} \left[\frac{1+6 \times (2.076) - 7(1.87)}{-0.11} \right]$$

$$= 9.39 \left[1 + 12.46 - 13.09 \right]$$

$$= 9.39 \times 0.37$$

$$= \underline{\underline{3.47}} \longrightarrow \underline{\underline{\text{Ans}}}$$

(iv) $\lambda_{eff} = \lambda(1 - p(n))$

$= 20(1 - 0.1912)$

$= 20 \times 0.808$

$= \underline{\underline{16.17}} \longrightarrow \text{Ans}$

(v) $L_q = L_s - [1 - p(0)]$

$= 3.47 - [1 - 0.102]$

$= 3.47 - 0.988$

$= \underline{\underline{2.482}} \longrightarrow \text{Ans}$

(vi) $W_s = \dfrac{L_s}{\lambda_e} = \dfrac{3.47}{16.17} = \underline{\underline{0.214}} \rightarrow \text{Ans}$

(vii) $W_q = \dfrac{L_q}{\lambda_e} = \dfrac{2.482}{16.17} = \underline{\underline{0.153}} \rightarrow \text{Ans}$

(viii) Ideal rate $= p(0) = \underline{\underline{0.102}} \longrightarrow \text{Ans}$