$-1.96 \leq -0.4 \leq 1.96$

(So hypothesis is accepted)

23/10/18

Autocorrelation Test :-

Step 1 :- Define the Null Hypothesis.

$H_0 : \hat{\rho}_{im} = 0$ The numbers are independent.

$H_1 : \hat{\rho}_{im} \neq 0$ The numbers are dependent.

Step 2 :- Find the value of i, m, N where, m is known as "lag".

Step 3 :- Using the value of i, m, N, find out the value of M.

$$\boxed{i + (M+1)m \leq N}$$

Step 4 :- $$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \cdot R_{i+(k+1)m}\right] - 0.25$$

Step 5 :- Find out the standard Deviation

$$\boxed{\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}}$$

Step 6 :- $$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}}$$

Step 7 :- Determine the value of $-\frac{Z_\alpha}{2}$ and $+\frac{Z_\alpha}{2}$.

Step 8 :- If $-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2}$ then Null hypothesis is accepted.

Q Given random numbers are :- 0.16, 0.82, 0.63, 0.04, 0.16, 0.19, 0.16, 0.82, 0.63, 0.04, 0.16, 0.30, 0.22, 0.88, 0.48, 0.29, 0.56, 0.44, 0.05, 0.81, 0.38, 0.59, 0.37, 0.71, 0.43, 0.92, 0.45, 0.57, 0.99, 0.20, 0.14, 0.64, 0.50, 0.73, 0.15, 0.02, 0.49, 0.86, 0.24, 0.90, 0.74, 0.41, 0.09, 0.80, 0.42

given $N = 40$.

$\alpha = 0.05$

$Z_{\alpha/2} = Z_{\frac{0.05}{2}} = 1.96$.

Test no's at position 2nd, 7th, 12th are auto-correlated or not.

Step 2: $i = 2$.
$m = 5$
$N = 40$.

Step 3: $i + (M+1)m \leq N$.

$2 + (M+1)5 \leq 40$

$(M+1)5 \leq 38$

$M+1 \leq 7.6$

$M \leq 6.6$.

We will take floor value of M.

$\boxed{M = 6.}$

Step 4.
$$\hat{\rho}_{im} = \frac{1}{6+1}\left[\sum_{k=0}^{6} R_{2+5k} \cdot R_{7+5k}\right] - 0.25$$

$$= \frac{1}{7}\left[R_2 \cdot R_7 + R_7 \cdot R_{12} + R_{12} \cdot R_{17} + R_{17} \cdot R_{22} + R_{22} \cdot R_{27} + R_{27} \cdot R_{32}\right] - 0.25$$

$$= \frac{1}{7}\left[0.16 \times 0.30 + 0.30 \times 0.56 + 0.56 \times 0.59 + 0.59 \times 0.45 + 0.45 \times 0.64 + 0.64 \times 0.49 + 0.49 \times 0.41\right] - 0.25$$

$$= \frac{1}{7}\left[1.6144\right] - 0.25$$

$$= \underline{\underline{-0.0193}}$$

Step 5 –
$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

$$= \frac{\sqrt{85}}{84} = 0.10975.$$

Step 6.
$$Z_0 = \frac{-0.0193}{0.10973}$$

$$= \underline{\underline{-0.17586}}.$$

Step 7: $-1.96 \leq -0.17586 \leq 1.96$.

Null Hypothesis is (Accepted).

Q Random Numbers given are:-

0.54	0.26	0.72	0.13	0.69
0.55	0.35	0.53	0.39	0.76
0.17	0.51	0.37	0.80	0.02
0.40	0.85	0.47	0.85	0.34
0.75	0.21	0.76	0.33	0.95
0.67	0.46	0.97	0.34	0.74
0.37	0.60	0.37	0.78	0.43
0.40	0.75	0.36	0.64	0.35
0.73	0.18	1.00	0.22	0.78
0.62	0.13	0.62	0.49	0.90

① Runs Up and Runs Down.

Sign

1 +	+	−	+	+ 21
−	+ 7	− 12	+	− 22
2 +	+	+	+ 17	+ 23
3 +	− 8	+	− 18	+
4 −	+	+ 13	+	−
−	+ 9	−	+ 19	− 24
	+	− 14	−	−
			− 20	+ 25
			+	+
			+	

step 4 $\mu_a = \frac{2N-1}{3} = \frac{99}{3} = 33$

$$\sigma_a^2 = \frac{16N-29}{90} = \frac{800-29}{90}$$
$$= 8.567$$

step 5 $Z_0 = \frac{a-\mu_a}{\sigma_a}$

$$= \frac{25-33}{2.9269} = -2.733$$

$-Z_{\alpha/2} = -1.96 \not> -2.733$

Null Hypothesis is rejected.

② Autocorrelated

$N = 50$
$i = 5$
$m = 5$

$5 + (M+1)5 \le 50$

$\Rightarrow (M+1)5 \le 45$

$\Rightarrow M+1 \le 9$ $\boxed{M=8}$

$M \le 8$

Step 4 $\hat{\rho}_{im} = \frac{1}{9}\left[\sum_{k=0}^{8} R_{5+5k} \cdot R_{10+5k}\right] - 0.25$

$= \frac{1}{9}\left[R_5 \cdot R_{10} + R_{10} \cdot R_{15} + R_{15} R_{20} + R_{20} R_{25} + R_{25} R_{30} + R_{30} R_{35} + R_{35} R_{40} + R_{40} R_{45} + R_{45} R_{50}\right] - 0.25$

$= \frac{1}{9}\left[0.75 \times 0.62 + 0.21 \times 0.13 + 0.76 \times 0.62 + 0.62 \times 0.21 + 0.13 \times 0.76 + 0.62 \times 0.33 + 0.33 \times 0.49 + 0.49 \times 0.95 + 0.95 \times 0.90\right] - 0.25$

$= \frac{1}{9}\left[2.8932\right] - 0.25$

$= 0.0714$.

$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13 \times 8 + 7}}{12(8+1)}$

$= \frac{\sqrt{104 + 7}}{108}$

$= \frac{\sqrt{111}}{108} = 0.0975$.

$Z_0 = \frac{0.0714}{0.0975} = \cancel{0.6769}$ 0.73230.

Accepted.

24/10/18

Q1 $a = 98$, $c = 47$, $m = 100$, $X_0 = 27$

2) $X_0 = 117$, $a = 43$, $m = 1000$, $c = 1$

3) $a = 7$, $c = 29$, $m = 100$, $X_0 = 37$.

3) Generate 10 random no. using LCM method

$(X_0 a + c) \bmod m$.

$X_1 = (117 \times 43 + 1) \bmod 1000$
$= (5032) \bmod 1000$
$= 032$.

$X_2 = (32 \times 43 + 1) \bmod 1000$
$= 377$.

$X_3 = (377 \times 43 + 1) \bmod 1000$

$= \underline{212}$.

$X_4 = 117$.

$X_5 = 32$.

$\left.\begin{array}{l} X_0 = 117 \\ X_1 = 32 \\ X_2 = 377 \\ X_3 = 212 \end{array}\right\}$ Cycle length = 4 as after that it is repeating.

Q.4. $R_0 = 0.117$; $R_2 = 0.032$

$R_3 = 0.377$; $R_4 = 0.212$

[0-1]

$N = 4$; $n = 10$

0 - 0.10	- 1
0.11 - 0.20	- 1
0.21 - 0.30	- 1
0.31 - 0.40	- 1
0.41 - 0.50	- 0
0.51 - 0.60	- 0
0.61 - 0.70	- 0
0.71 - 0.80	- 0
0.81 - 0.90	- 0
0.91 - 1.00	- 0

$f_e = \frac{4}{10} = 0.4$

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
1	0.4	0.6	0.36	0.9
1	0.4	0.6	0.36	0.9
1	0.4	0.6	0.36	0.9
1	0.4	0.6	0.36	0.9
0	0.4	-0.4	0.16	0.4
0	0.4	-0.4	0.16	0.4
0	0.4	-0.4	0.16	0.4
0	0.4	-0.4	0.16	0.4
0	0.4	-0.4	0.16	0.4
0	0.4	-0.4	0.16	0.4
				6.0

3.6
2.4

$X^2 = 6.0$.

$X^2_{\alpha, v} = 3.841 \not> X^2$

$X^2_{0.05, 1}$.

(H_0 is rejected).

R_i	Sign
0.117	-
0.032	-
0.377	-
0.212	-

$b = 1$

$N = 4$

n_1 = above the mean

n_2 = below the mean

mean:

2

2

Runs Up and Down

R_i	sign	
0.117	−	1
0.032	+	2
0.377	−	3
0.212		

$a = 3$

$\mu_a = \frac{2N-1}{3}$

$\Rightarrow \frac{7}{3} = 2.33$

$\sigma_a^2 = \frac{16 \times 4 - 29}{90}$

$= \frac{\cancel{35}\ 7}{\cancel{90}\ 18} = 0.389$

$Z_0 = \frac{a - \mu_a}{\sigma_a}$

$= \frac{3 - 2.33}{\sqrt{0.389}} = \frac{0.67}{0.62}$

$= 1.080$

$-1.96 \le 1.080 \le 1.96\ (\checkmark)$

Independence hypothesis accepted

Q Mid square method

$n = 2$

Seed = 42

26/10/18

Q.

0.63	0.63	0.84	0.38	0.71	0.89
0.28	0.17	0.54	0.93	0.44	0.66
0.30	1.00	0.56	0.85	0.72	0.91
0.42	0.61	0.57	0.68	0.95	0.50
0.97	0.19	0.09	0.14	0.28	0.33
0.05	0.94	0.99	0.18	0.96	0.89
0.71	0.64	0.01	0.84	0.51	0.54
		0.10	0.19	0.50	0.73
		0.69			0.76
					0.62
					0.92

where $\alpha = 0.05$

and $Z_{\alpha/2} = 1.96$

Test whether the 5th, 10th, 15th numbers in the sequence are autocorrelated or not?

Sol

$N = 50$; $i = 5$; $m = 5$.

$i + (M+1)m \le N$.

$\Rightarrow 5 + (M+1)5 \le 50$

$\Rightarrow (M+1)5 \leq 45$

$\Rightarrow M+1 \leq 9$

$\Rightarrow M \leq 8$

$\boxed{M=8}$

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{K=0}^{M} R_{i+km} \cdot R_{i+(K+1)m}\right] - 0.25$$

$$= \frac{1}{9}\left[\sum_{K=0}^{8} R_{5+5K} \cdot R_{10+5k}\right] - 0.25$$

$$= \frac{1}{9}\left[R_5 \cdot R_{10} + R_{10} \cdot R_{15} + R_{15} \cdot R_{20} + R_{20} \cdot R_{25} + R_{25} \cdot R_{30} + R_{30} \cdot R_{35} + R_{35} R_{40} + R_{40} R_{45} + R_{45} R_{50}\right] - 0.25$$

$$= \frac{1}{9}\left[0.97 \times 1.00 + 1.00 \times 0.84 + 0.84 \times 0.99 + 0.99 \times 0.93 + 0.93 \times 0.84 + 0.84 \times 0.95 + 0.95 \times 0.89 + 0.89 \times 0.89 + 0.89 \times 0.92\right] - 0.25$$

$$= \frac{1}{9}\left[7.5979\right] - 0.25$$

$= 0.844 - 0.25 = \underline{0.594}$

$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

$$= \frac{\sqrt{13 \times 8 + 7}}{12(8+1)} = 0.0975$$

$$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}} = \frac{0.594}{0.0975} = 6.092$$

$$-Z_{\frac{\alpha}{2}} \leq Z_0 \leq Z_{\frac{\alpha}{2}}$$

$= -1.96 \leq 6.092 \not\leq 1.96$ (Not accepted)

2/11/18

→ System State :- It is defined as

$(L_Q(t), L_S(t))$, where :-

→ $L_Q(t)$ is the number of waiting customers.

→ $L_S(t)$ is the no:- of customers under service (0 or 1) at a time t.

→ Events

Arrival of customer (A)
Departure of customer (D).
Simulation stopping event (E).

→ Event notices :- (info about future events).

Defined as (event type, event time), where

- (A, t) represents the arrival event scheduled to occur at future time t
- (D, t) represents the departure of the future time t.
- (E, 60) is the stoppage event of simulation at future time 60

Activities

Inter-arrival time.
Service time.

Initial condition.

① Clock is set to 0.

② $L_Q(0) = 0$, $L_S(0) = 1$.

③ FEL contains both the departure and arrival events.

Cumulative statistics.

① Total server busy time (B)
② Maximum Queue length (Q).

a* : The generated inter arrival time.

S* : The generated service time.

Discrete Event System Simulation :-

Q Interarrival distribution of customers

Time b/w Arrivals	1	2	3	4	5
Prob.	0.2	0.2	0.2	0.2	0.2
CP	0.2	0.4	0.6	0.8	1.0
Intervals	0 – 19	20 – 39	40 – 59	60 – 79	80 – 99.

Service Distribution

Service Time.	2	3	4	5
Prob.	0.30	0.28	0.25	0.17
CP	0.3	0.58	0.83	1.00
Inter-arrival	0 – 29	30 – 57	58 – 82	83 – 99.

RN

IAT	78	56	99	79	55	84	39	2	4	8
ST	37	66	79	1	82	67	10	26	69	67

~~Interarrival~~ IAT

1	78
2	56
3	99
4	79
5	55
6	84
7	39
8	2
9	4

	RDA	IAT	AT	RDS	ST	STB	STE
	–	–	0	37	3	0	3
1	78	4	4	66	4	4	8
2	56	3	7	79	4	8	12
3	99	5	12	1	2	12	16
4	79	4	16	82	4	16	20
5	55	3	19	67	4	20	24
6	84	5	24	10	2	24	26
7	39	2	26	26	2	26	28
8	2	1	27	69	4	28	32
9	4	1	28	67	4	32	36

	WT	Time spend in system	Idle time
1	0	3	0
2	0	4	1
3	1	5	0
4	0	2	0
5	0	4	2
6	1	5	0
7	0	2	0
8	0	2	0
9	1	5	0
10	4	8	0

Clock(t)	System State $L_Q(t)$	System State $L_S(t)$	FEL	Comment	Cumulative Statistics B	Cumulative Statistics MQ
0	0	1	(D,3) (A,4) (E,36)	First A occurs (a*=4) Schedule next A (s*=3) Schedule next D	0	0
3	0	0	(A,4) (E,36)	First D occurs (D,3)	3	0
4	0	1	(A,7) (D,8) (E,36)	~~Second~~ A occurs (a*=3) Schedule next A (s*=4) Schedule next D	0	0
7	1	1	(D,8) (A,12) (E,36)	Third A occurs (a*=5) Schedule next A	3	1
8	0	1	(D,12) (A,12) (E,36)	Second D occurs (S*=4) Schedule next D	7	0
12	0	0	(A,12) (E,36)	Third D occurs	11	0
12	0	[illegible]	(D,14) (A,16) (E,36)	Fourth A occur ak=4 schedule next A sk=2 Sched next D	11	0

	IAT	ST	AT	STB	STE	WT	TS	Idle
1	0	4	0	0	4	0	4	0
2	8	1	8	8	9	0	1	4
3	6	4	14	14	18	0	4	5
4	1	3	15	18	21	3	6	0
5	8	2	23	23	25	0	2	2
6	3	4	26	26	30	0	4	1
7	8	5	34	34	39	0	5	4
8	7	4	41	41	45	0	4	2
9	2	5	43	45	50	2	7	0
10	3	3	46	50	53	4	7	0

Clock	System State		FEL	Comment	Cumulative Stats	
	LQ(t)	LS(t)			B	MQ
0	0	1	(D,4) (A,8) (E,60)	1st A occurs, Schedule next A & 1st D at a* = 8 & s* = 4	0	0
4	0	0	(A,8) (E,60)	1st D occurs	4	0
8	0	1	(D,9) (A,14) (E,60)	2nd A occurs. Schedule next a* = 6, s* = 1	4	0
9	0	0	(A,14) (E,60)	2nd D occurs	5	0
14	0	1	(A,15) (D,18) (E,60)	3rd A occurs. Schedule next a* = 1, s* = 4	5	0
15	1	1	(D,18) (A,23) (E,60)	4th A occurs, customer delayed, next a* = 8	5	1
18	0	1	(D,21) (A,23) (E,60)	3rd D occurs, Schedule next s* = 3	9	1
21	0	0	(A,23) (E,60)	4th D occurs	12	1