Math 348 - Red Queen Model Project

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1 Introduction

Evolutionary biology often focuses on the mechanisms that drive species diversification, the process by which new species arise and existing ones go extinct. In this context where organisms manage to persist and diversify in a constantly changing environment, the Red Queen hypothesis suggests that species must continuously adapt and evolve not just to gain a competitive advantage but simply to survive in an ever-changing environment [2]. A critical factor influencing diversification is the mode of reproduction. Asexual species reproduce rapidly and efficiently, while sexual species, although slower, introduce genetic variation that can enhance adaptability [1].This project explores three diversification models using the birth-and-death process: (1) asexual-only diversification, (2) separate asexual and sexual diversification with no interaction, and (3) a model allowing species to switch between reproductive modes. Both deterministic (expected value ODEs) and stochastic (Gillespie simulation) methods are used to analyze how reproductive strategies influence species richness and evolutionary stability.

2 Model and Methods

Model I: Asexual-Only Diversification

The first model considers a population of exclusively asexual species. Each species can reproduce by branching into a new species at rate λ_a and go extinct at rate μ_a. This is modeled using a birth-and-death process. Assuming that the model starts with a single species at time $t = 0$, the expected number of species at time t is the following.

$$\mathbb{E}[N(t)] = \begin{cases} \exp((\lambda_a - \mu_a)t), & \text{if } \lambda_a \neq \mu_a \\ 1, & \text{if } \lambda_a = \mu_a \end{cases}$$

This equation shows three possible regimes: supercritical ($\lambda_a > \mu_a$), where species grow exponentially over time due to rapid diversification; critical ($\lambda_a = \mu_a$), where the expected count is constant, but individual trajectories may still be extinct; and subcritical ($\lambda_a < \mu_a$), where extinction dominates and the expected number of species decreases exponentially. This model serves as a platform to understand the diversification dynamics that reproduces rapidly but lacks genetic variability. It demonstrates the trade-off between short-term growth and long-term evolutionary risk [1].

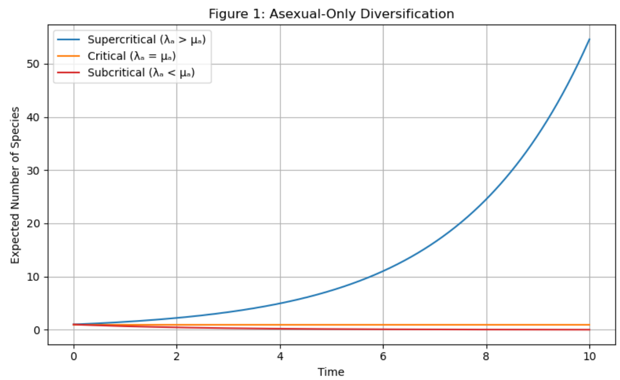

Figure 1: Expected number of asexual species over time under supercritical, critical, and subcritical birth-death regimes.

Model II: Independent Asexual and Sexual Diversification

In the second model, the project discusses two distinct reproductive types: asexual and sexual. Each type follows its own independent birth-and-death process. Asexual species diversify by creating new species at a higher rate (λ_a) and go extinct at a higher rate (μ_a), while sexual species undergo speciation more slowly (λ_s) but are more evolutionarily stable due to lower extinction (μ_s). The expected number of species is:

$$\mathbb{E}[N_a(t)] = \exp((\lambda_a - \mu_a)t), \quad \mathbb{E}[N_s(t)] = \exp((\lambda_s - \mu_s)t)$$

Model III: Switching Between Reproductive Modes

In the third model, the project illustrates the reproductive switching which allows species to transition between asexual and sexual strategies. Asexual species may evolve into sexual species at a switching rate $\sigma_{a\to s}$, while sexual species may revert to asexuality at rate $\sigma_{s\to a}$. This model captures the evolutionary flexibility that aligns with the Red Queen hypothesis—organisms must continually adapt, not just compete.

Each species type still undergoes speciation and extinction governed by its respective birth-death process. The deterministic dynamics of the population are modeled using the following coupled differential equations:

$$\frac{dN_a}{dt} = (\lambda_a - \mu_a)N_a - \sigma_{a\to s}N_a + \sigma_{s\to a}N_s$$
$$\frac{dN_s}{dt} = (\lambda_s - \mu_s)N_s + \sigma_{a\to s}N_a - \sigma_{s\to a}N_s$$

Here, $N_a(t)$ and $N_s(t)$ represent the number of asexual and sexual species at time t, respectively. The system is initialized with $N_a(0) = 1$ and $N_s(0) = 0$, and solved numerically to understand how populations evolve over time. To clarify how each event affects the population, a complete list of all stochastic transitions and their rates is provided below:

To explore the average trends, the model is numerically solved the above ODEs for parameters: $\lambda_a = 0.7$, $\mu_a = 0.5$, $\lambda_s = 0.4$, $\mu_s = 0.2$, $\sigma_{a\to s} = 0.1$, and $\sigma_{s\to a} = 0.05$. The solution

Table 1: Event types in the switching diversification model. Each event modifies the asexual (N_a) or sexual (N_s) species counts at the indicated rates.

Event	**Type Affected**	**Species Count Change**	**Rate**
Asexual Speciation	Asexual	$N_a \to N_a + 1$	$\lambda_a \cdot N_a$
Asexual Extinction	Asexual	$N_a \to N_a - 1$	$\mu_a \cdot N_a$
Switch to Sexual	Asexual $\to$ Sexual	$N_a \to N_a - 1$, $N_s \to N_s + 1$	$\sigma_{a\to s} \cdot N_a$
Sexual Speciation	Sexual	$N_s \to N_s + 1$	$\lambda_s \cdot N_s$
Sexual Extinction	Sexual	$N_s \to N_s - 1$	$\mu_s \cdot N_s$
Switch to Asexual	Sexual $\to$ Asexual	$N_s \to N_s - 1$, $N_a \to N_a + 1$	$\sigma_{s\to a} \cdot N_s$

shows that while asexual species dominate initially due to faster reproduction, sexual species gain in abundance over time due to their lower extinction rate and continuous input from switching events.

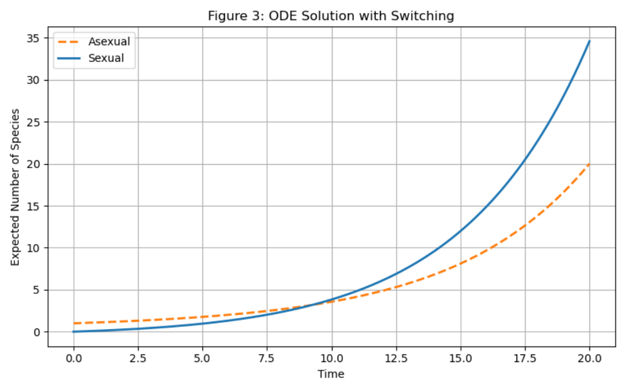

Figure 2: ODE solution for species switching between asexual and sexual reproduction. Asexual species initially dominate, but sexual species become more abundant over time due to evolutionary stability and switching.

3 Results

Deterministic Analysis

In Model I, the expected number of asexual species grows exponentially when the speciation rate exceeds the extinction rate ($\lambda_a > \mu_a$). However, in the critical ($\lambda_a = \mu_a$) and subcritical ($\lambda_a < \mu_a$) regimes, growth halts or declines, respectively. This highlights the fragile nature of asexual-only diversification under environmental or evolutionary pressures.Model II considers two distinct lineages: sexual and asexual, evolving independently. While asexual species initially outpace sexual species in terms of population growth—owing to their faster reproduction—the model reveals that both lineages grow exponentially at similar rates if net diversification rates are equal.Model III introduces switching between reproductive modes. Deterministic solutions from the coupled ODE system reveal a two-phase behavior: asexual species dominate in the early stages due to their rapid growth, but switching gradually redirects lineages toward the sexual state. Over time, sexual species become more abundant due to their lower extinction rate. The switching mechanism stabilizes the system and promotes evolutionary robustness.

Stochastic Simulations

To account for randomness in evolutionary dynamics, the project simulated Model III using the Gillespie algorithm. Each simulation run starts with one asexual species and proceeds through stochastic events including speciation, extinction, and switching. The results exhibit considerable variation across trajectories: in some runs, early extinction eliminates the lineage entirely; in others, the population stabilizes with a predominance of sexual species. Despite this variability, a consistent trend emerges—switching increases the likelihood of long-term survival by allowing lineages to transition into the more stable reproductive state.These stochastic outcomes reinforce the deterministic findings: while asexual reproduction offers a short-term advantage in diversification, the capacity to switch to sexual reproduction provides long-term evolutionary stability, aligning with the Red Queen hypothesis.

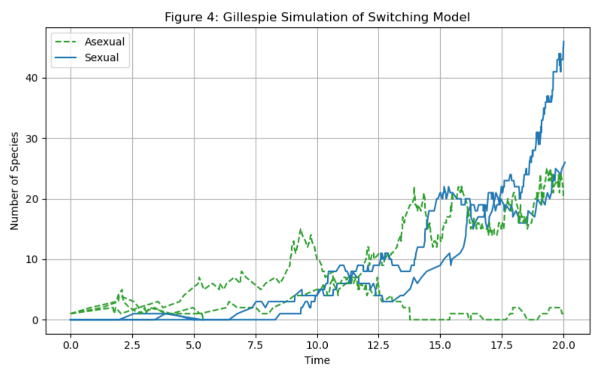

Figure 3: Stochastic simulation of switching model using Gillespie algorithm across multiple runs.

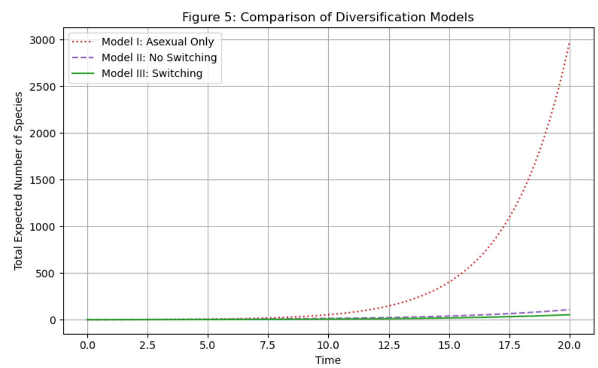

Figure 4: Comparison of total species richness across three models: asexual only, dual-type without switching, and switching model.

4 Conclusion

This project compares three models of species diversification to explore the role of reproductive strategy in evolutionary dynamics. The asexual-only model shows how fast diversification can be undermined by high extinction. The independent dual-type model reveals trade-offs between speed and stability. Finally, the switching model provides a compelling explanation for the coexistence and dominance of sexual reproduction in nature. By combining deterministic and stochastic techniques, it is found that flexibility in reproduction—alongside randomness—is key to long-term survival, aligning with the Red Queen's principle: evolve constantly or perish[2].

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References

[1] Chaparro-Pedraza, P.C., Roth, G., & Melián, C.J. (2024). Ecological diversification in sexual and asexual lineages. *Scientific Reports*, 14, 30369. https://doi.org/10.1038/s41598-024-81770-8.

[2] Solé, R. (2022). Revisiting Leigh Van Valen's "A New Evolutionary Law" (1973). *Biological Theory*, 17, 120–125. https://doi.org/10.1007/s13752-021-00391-w.