Premises: "If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on", "If the sailing race is held, then the trophy will be awarded" and "The trophy was not awarded".
Conclusion: "It rained"

Find whether the argument is valid or not

Let R = it rains,
F = it is foggy,
S = the sailing race will be held,
D = the lifesaving demonstration will go on
T = trophy will be awarded.

P1: (~R or ~F) --> S ^ D
P2: S --> T
C: R

1.S --> T
2.~T

Using Modus Tollens on 1 and 2:
3. ~S

Using Addition on 3
4. ~S or ~D

Using de-morgans on 4:
5. ~S or ~D = ~(S^D)

6.(~R or ~F) --> S ^ D

Using Modus Tollens on 6 and 5
7. ~(~R or ~F)

Using de morgans on 7:
8. R^F

Using simplification on 8:
9: R

So Valid

Limitations of Propositional Logic

Propositional logic:

- ❖ the world is described in terms of elementary propositions and their logical combinations
- ❖ an elementary proposition, also known as a basic proposition or atomic proposition, is a fundamental statement in propositional logic that cannot be further divided or decomposed

Elementary statements:

- ❑ Typically refer to objects, their properties and relations.
- ❑ But these are not explicitly represented in the propositional logic
- ❑ For Example:

  Hasan is a GUB student

  Hasan

  - object

  A GUB student

  - a property

Objects and properties are hidden in the statement, it is not possible to reason about them

➢ For propositional logic, statements that must be repeated for many objects

Example:

If Hasan is a CSE GUB graduate then Hasan has passed CSE105

Translation

Hasan is a CSE GUB graduate → Hasan has passed CSE105

Solution:

❖ make statements with variables

If x is a CSE GUB graduate then x has passed CSE105

x a CSE GUB graduate → x has passed CSE105

- Propositional logic fails for Statements that define the property of the group of objects

- Example:

Some of the CSE graduates graduate with honors.

- Example 2:

p = Everyone enrolled in the university, has lived in a dormitory

q = Karim has never lived in a dormitory

r = SO, Karim is not enrolled in the university

Predicate Logic

Remedies the limitations of the propositional logic

- Explicitly models objects and their properties
- Allows to make statements with variables and quantify them

Basic building blocks of the predicate logic:

- Constant –models a specific object

Examples: “Hasan”, “Khulna”, “7”

- Variable – represents object of specific type
 (defined by the *universe of discourse)*

Examples: x, y
(universe of discourse can be people, students, numbers)

- Predicate - Represents properties of objects

Examples: Red(car23), student(x), married(John,Ann)

Predicates :

- ❑statements involving variables which are neither true nor false until or unless values of the variables are specified

For example:

X is an animal.

- ❖This is neither true nor false . So, not propositions
- ❖Here, x is a subject.
- ❖**Subject is something we are discussing about**
- ❖'is an animal' is a predicate
- ❖**Predicate refers to a property that subject of the statement can have**

Assigning value to Predicate

Predicate - Represents properties of objects
A predicate P(x) assigns a value true or false to each x depending on whether the property holds or not for x.

x is a prime number (universe of discourse is integers)

What are the truth values of:
- P(2) T
- P(3) T
- P(4) F
- P(5) T

All statements P(2), P(3), P(4), P(5) are propositions.

Is P(x) a proposition? No. Many substitutions are possible.

So in predicate logic, a statement can be divided into 2 parts:

- ❑ **Subject**
- ❑ **Predicate**

Shorthand Notation:

For example, x is greater than 5 can be represented by G(x)

- ✓ where G() denotes the predicate ‘is greater than 5
- ✓ x denotes the subject or variable
- ❑ After assigning value of x, G(x) becomes a proposition that means can have truth value
- ❑ For example:
- ❖ G(6) = 6 is greater than 5 (true)
- ❖ G(3) = 3 is greater than 5 (false)

Q(x,y) denotes "x is a current player of team y"

So

- ❖ Q(Neymar, Brazil) : Neymar is a current player of team Brazil (True)
- ❖ Q(Sakib Al Hasan, Bangladesh) : Sakib Al Hasan is a current player of team Bangladesh (True)
- ❖ Q(Messi, Barcelona) : Messi is a current player of team Barcelona (False)
- ❖ Q(Tamim Iqbal, Bangladesh) : Tamim Iqbal is a current player of team Bangladesh (False)

Predicate is not proposition

Important:

• statement **P(x) is not a proposition** since there are many objects it can be applied to

This is the same as in propositional logic …

… But the difference is:

• predicate logic allows us to explicitly manipulate and substitute for the objects

• Predicate logic permits quantified sentences where variables are substituted for statements about the group of objects

Quantifiers:

- Words that refer to quantities like ‘some’ or ‘all’
- It indicates for how many elements a given predicate is true
- Quantifiers are used to express the quantities without giving an exact number

Types of Quantifiers:

- Universal Quantifiers
- Existential Quantifiers

Quantified Statements

Predicate logic lets us to make statements about groups of objects

• To do this we use special quantified expressions

Two types of quantified statements:

• **universal**

Example: ‘ all CSE GUB graduates have to pass CSE 105”

– the statement is true for all graduates

• **existential**

Example: ‘Some CSE GUB students graduate with honor.’

– the statement is true for some people

Universal Quantifiers

For example:

P(x): x+2 > x

P(1): 1+2 > 1 (true)

P(2): 2+2 > 1 (true)

P(3): 3+2 > 1 (true)

……..

- ❖ So, P(x) is true for all positive integer x
- ❖ It can be represented as ∀x P(x)
- ❖ ∀ means ‘for all’
- ❖ It is read as: for all positive integer x, P(x) is true

Universal Quantifier

Defn: The universal quantification of P(x) is the proposition:
"*P(x) is true for all values of x in the domain of discourse.*"

The notation $\forall x P(x)$ denotes the universal quantification of P(x), and is expressed as for every x, P(x).

Example:

• Let P(x) denote x > x - 1.
• What is the truth value of $\forall x P(x)$?

• Assume the universe of discourse of x is all real numbers.

• Answer: Since every number x is greater than itself minus 1.
Therefore, $\forall x P(x)$ is true.

Universally Quantified Statement

Predicate logic lets us make statements about groups of objects

Universally quantified statement

- CSE-major(x) $\rightarrow$ Student(x)

– Translation: “if x is a CSE-major then x is a student”

– Proposition: no.

- $\forall x \text{ CSE-major}(x) \rightarrow \text{Student}(x)$

– Translation: “(For all people it holds that) if a person is a CSE-major then s/he is a student.”

– Proposition: yes.

Existential Quantifiers

$Q(x)$: $x < 5$ [domain is set of positive integers]

$Q(1)$: $1 < 5$ (true)

$Q(2)$: $2 < 5$ (true)

$Q(3)$: $3 < 5$ (true)

$Q(4)$: $4 < 5$ (true)

$Q(5)$: $5 < 5$ (false)

$Q(6)$: $6 < 5$ (false)

$Q(7)$: $7 < 5$ (false)

$Q(8)$: $8 < 5$ (false)

$Q(9)$: $9 < 5$ (false)

....

- ❖ So, there exists some values of x for which Q(x) is true
- ❖ This can be written as: $\exists x\, Q(x)$
- ❖ $\exists$ is read as ‘There exists some values’

Existential Quantifier

Definition: The existential quantification of P(x) is the proposition
"*There exists an element in the domain (universe) of discourse such that P(x) is true.*"

The notation: $\exists x P(x)$

Example 1:

- Let T(x) denote x > 5 and x is from Real numbers.
- What is the truth value of $\exists x T(x)$?

- **Answer:**
- Since 10 > 5 is true. Therefore, it is true that $\exists x T(x)$

Quantified statements

Statements about groups of objects

Example:

- CSE-GUB-graduate (x) ∧ Honor-student(x)

– **Translation: “x is a CSE-GUB-graduate and x is an honor** student”

– **Proposition: no.**

- $\exists x \text{ CSE - NSU - graduate } (x) \wedge \text{Honor - student}(x)$

– **Translation:** “There is a person who is a CSE-NSU-graduate and who is also an honor student.”

– **Proposition: ? yes**

Universal quantifier –the property is satisfied by all members of the group

Existential quantifier – at least one member of the group satisfy the property