Probability and Statistics

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January 21, 2026

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Content: Probability and Statistics

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1. Recall on set Theory
2. **Counting Techniques:** Permutation and combination.
3. **Probability:** Basic concepts and definition of probability, Conditional probability.
4. **Probability distributions:** Discrete distributions(e.g. binomial and Poisson distributions) and Continuous distribution (e.g. Normal Distribution).
5. Sampling Techniques

Counting Techniques

Permutations and Combinations

Introduction:

- Combinations and permutations are used in both statistics and probability, and they in turn, involve operations with factorial notation.
- Therefore combinations, permutations, and factorial notation are discussed in this section.
- This section aims at teaching a method of approach to certain problems involving arrangements and selections.
- This section helps us to count all elements of finite set; for counting all elements of finite set **the injection, surjection and Cartesian product** play important role.

Cartesian product

Consider A and B as the finite sets. A Cartesian product between A and B is defined as:

$$A \times B = \{(a, b) : \; a \in A \, and \, b \in B\}$$

In general,

$$\begin{aligned} A_1 \times A_2 \times \ldots \times A_n &= \{(a_1, a_, \ldots, a_n) : \; a_i \in A_i \, for \, i = 1, 2, \ldots, n \\ \#(A \times B) &= \#A.\#B \\ \#(A_1 \times A_2 \times \ldots \times A_n) &= \prod \#A_i \end{aligned}$$

Example

How many words of three alphabet can you form using the alphabet letters such that the first and third are consonants and the second is vowel.

Solution

Consider $\# C = 20$ and $\# V = 6$.

$$\#(C \times V \times C) = 20 \times 6 \times 20 = 2400 \, words$$

The union of 2 sets

Consider A and B as the finite sets.

$$\begin{aligned}
A \cup B &= \{x \, such \, that \, x \in A \, or \, x \in B\} \\
\#(A \cup B) &= \#A + \#B - \#(A \cap B) \\
\#(A \cup B) &= \#A + \#B \, if \, A \cap B = \emptyset \\
\#\emptyset &= 0 \\
\#U &= n
\end{aligned}$$

Arrangement with repetition

Definition: If we take p elements in n elements of E one after another with repetition means that before taking the second object in E we must put the first taken into E; we continue this process until we take p elements is called **arrangement with repetition.**

Notation:

$$\#\left(E^p\right) = \left(\# E\right)^p = n^p$$

Example

How many numbers can you write composed by 3 digits numbers different to zero?

Solution

$$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \ \#E = 9$$

We obtain $9^3 = 729$ numbers.

Arrangement without repetition

Definition: If we take r elements in n elements of E one after another without repetition means that taking one after the other in E; we continue this process until we take r elements is called **arrangement without repetition.**

Notation:

$$nPr = \frac{n!}{(n-r)!}$$

Example 1

How many permutations of six objects taken two a time can be made?

Answer:

$$6P2 = \frac{6!}{(6-2)!} = \frac{6 \times 5 \times 4!}{4!} = 30$$

Example 2

How many four-digit can be formed from the digits 2, 3, 4, 5, 6, and 7 without repetition?

Answer:

$$6P4 = \frac{6!}{(6-4)!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$$

Particular cases

Permutation (bijection)

We call permutation of finite set E of $\#E = n$, all arrangement at n elements of E.

$$nPn = n!$$

Example

How many possibilities can you arrange 4 cars into a parking of 4 places?

Answer:

$$4P4 = 4! = 4 \times 3 \times 2 \times 1 = 24\, possibilities$$

Permutation with repetition

The number of arrangements of n items, where there are k groups of like items of size $r_1, r_2, r_3, \ldots, r_k$, respectively, is given by

$$\frac{n!}{r_1!.r_2!.r_3! \ldots r_k!}$$

Example

How many different arrangements of the letters in the word ROOM can be made?

Answer:

$$n = 4,\ r_1 = 1,\ r_2 = 2,\ r_3 = 1$$

$$\frac{4!}{1!2!1!} = \frac{4!}{2!} = 12\, Arrangement$$

Combinations

- A combination is defined as a possible selection of a certain number of objects taken from a group without regard to the order.
- Consider E non-empty finite set of cardinal n and r is a natural number such that
$$0 \leq r \leq n.$$
We call combination at r elements of E all part of r elements of E.
$$\binom{n}{r} = nCr = \frac{n!}{r!(n-r)!}$$

Example

How many possible 14 students can form a committee of 6 students?
Each committee is a party of 6 students of the set of 14 students
Answer:

$$\binom{14}{6} \quad = \quad nCr = \frac{14!}{6!(14-6)!} = 3003$$

Exercises

1. How many 3 digit PINs can be formed from digits 0 to 9 if repetition is allowed?
2. How many 5 letter **codes** can be made from the English alphabet if letters can not repeat?
3. A password is 6 characters long, using 26 letters and 10 digits, no repetition. How many passwords?
4. How many ways are there to select three candidates from eight equally qualified recent graduates for openings in accounting form?
5. In how many ways can seven graduate students be assigned to one triple and two double hotel rooms during a conference?

Exercise 1: Password Security

A company requires 6 character passwords using only uppercase letters (A–Z) and digits (0–9).

- How many distinct passwords are possible?
- If an attacker tests 1 million passwords/second, what is the **maximum** time (in hours) to guarantee success?

Relevance: Key space analysis in cybersecurity

Exercise 2: Two-Factor Authentication (2FA)

A 2FA system requires:

- A 4 digit PIN (0–9, repetition allowed)
- Selection of **2 out of 5** security questions (order doesn't matter)

How many distinct authentication combinations are possible?

Relevance: Multi-factor authentication design

Exercise 3: Software Configuration Testing

An app has:

- Theme: Light, Dark, Auto (3)
- Language: EN, ES, FR, DE (4)
- Notifications: On, Off, Quiet (3)

1. How many total configurations?
2. How many test cases to cover **all pairs** of any two settings?

Relevance: Combinatorial software testing

Exercise 4: Simplified MAC Addresses

A 12-bit address must start with `101`. The remaining 9 bits can be any binary digits.

- How many unique devices can be addressed?

Relevance: Network addressing and namespace design

Exercise 5: Custom Access Roles

A role is any subset of 5 permissions: Read, Write, Delete, Share, Admin.

- How many distinct custom roles can be defined?

Relevance: Role-Based Access Control (RBAC)

Exercise 6: API Tokens

An 8-character token uses:

- Chars 1–2: uppercase letters (A–Z)
- Chars 3–4: digits (0–9)
- Chars 5–8: uppercase letters **or** digits

Repetition allowed. How many unique tokens?

Relevance: Secure token/key generation

Bonus Challenge: 2-Way Test Suite

Login form settings:

- Username format: Email, Phone, Username (3)
- Password policy: Weak, Strong (2)
- Captcha: Enabled, Disabled (2)

What is the **minimum** number of test cases to cover **every pair** of values at least once?

Relevance: Combinatorial interaction testing (e.g., NASA, Microsoft)