MATH 8.214 Marking guide

Q1. (6 marks)

a) Circumcenter: $\cap$ of 3 $\perp$ bisector lines of a triangle. 1

• It is the center of the circumcircle (circle passing thru the 3 vertices of the Δ) 1

b) Box product: $\langle \vec{u}, \vec{v}, \vec{w} \rangle = \langle [\vec{u}, \vec{v}], \vec{w} \rangle$ 1

• $|\langle \vec{u}, \vec{v}, \vec{w} \rangle| = V_{\square\, \vec{u}, \vec{v}, \vec{w}}$ 1

c) Cross product: • $[\vec{u}, \vec{v}] \perp \Pi_{\vec{u}, \vec{v}}$;

• $\det(\vec{u}, \vec{v}, [\vec{u}, \vec{v}]) > 0$; 1

• $\|[\vec{u}, \vec{v}]\| = \|\vec{u}\| \|\vec{v}\| \sin(\vec{u}, \vec{v}) = A_{\square\, \vec{u}, \vec{v}}$ 1

Q2. (9 marks). $A(6, \frac{\pi}{3})$; $B(8, 4\frac{\pi}{3})$

a) $d_{AB} = \|\vec{AB}\| = \sqrt{6^2 + 8^2 - 2 \times 6 \times 8 \cos(4\frac{\pi}{3} - \frac{\pi}{3})}$ LU 1

/3 $= \sqrt{100 + 96}$ LU $= \underline{14}$ LU 1

$d(A(r_1, \theta_1), B(r_2, \theta_2)) = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)}$ LU 1

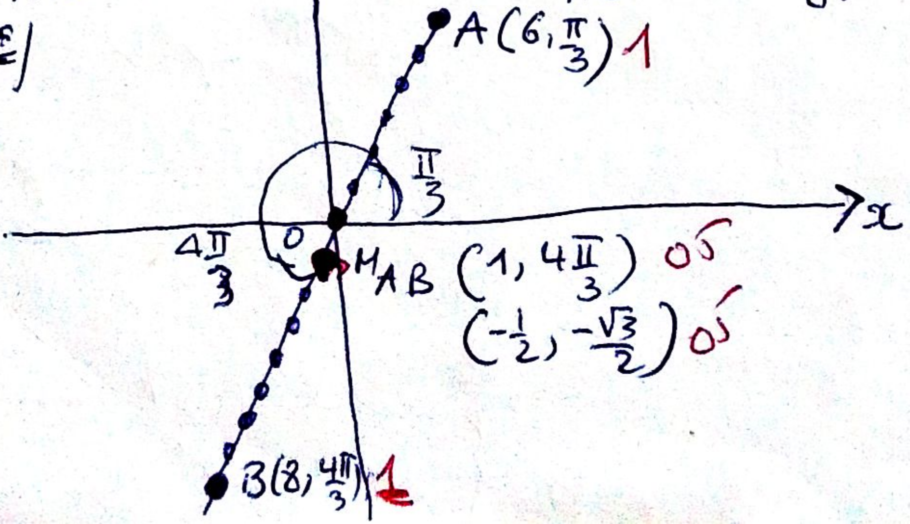
b) $A(6, \frac{\pi}{3}) \to A(6\cos\frac{\pi}{3}, 6\sin\frac{\pi}{3}) = A(3, 3\sqrt{3})$ 0.5

/3 $B(8, 4\frac{\pi}{3}) \to B(8\cos 4\frac{\pi}{3}, 6\sin 4\frac{\pi}{3}) = B(-4, -4\sqrt{3})$ 0.5

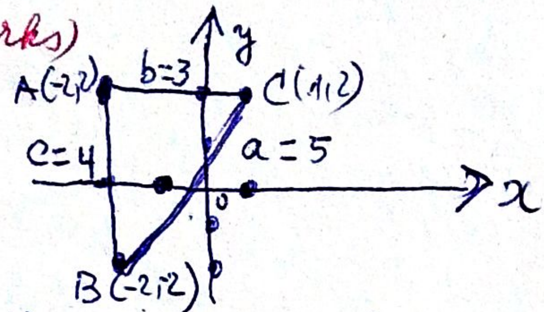
• $M_{AB}\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = M_{AB}\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 1 Cartesian

Mid-pt of AB $= M_{AB}(1, 4\frac{\pi}{3})$ 1 Polar

c) /3



Q_3 (15 marks)



a) m_a passes thru the mid-pt of BC $\to A'\left(-\frac{1}{2},0\right)$

/5 $m_a: \vec{v} = \vec{OA} + \lambda \vec{AA'} \to \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} \frac{3}{2} \\ -2 \end{bmatrix}$

$\frac{x+2}{\frac{3}{2}} = \frac{y-2}{-2} \to \left(m_a: 4x+3y+2=0 ;\ m_a: y = -\frac{4}{3}x - \frac{2}{3} \right)$ 2

$h_a \perp BC: \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 4 \end{bmatrix} \to BC: \frac{x+2}{3} = \frac{y+2}{4}$

$BC: 4x-3y+2=0 \to BC: y = \frac{4}{3}x + \frac{2}{3}$

h_a passes thru $A(-2,2)$; the slope of h_a is $-\frac{3}{4}$

$h_a: y = -\frac{3}{4}x + m \to 2 = -\frac{3}{4}(-2) + \cancel{1} + m \Rightarrow m = \frac{1}{2}$

$h_a: \left(y = -\frac{3}{4}x + \frac{1}{2}\right)$ or $\left(h_a: 3x+4y\cancel{+2}=0\right)$ 2

$\cos(m_a, h_a) = \frac{4\cdot 3 + 3\cdot 4}{\sqrt{4^2+3^2}\sqrt{3^2+4^2}} = \frac{24}{25}$ 1

or

$\sin(m_a, h_a) = \frac{7}{25}$ or $\tan(m_a, h_a) = \frac{7}{24}$

b) $I\left(\frac{a x_A + b x_B + c x_C}{a+b+c}, \frac{a y_A + b y_B + c y_C}{a+b+c}\right) = I\left(\frac{-10-6+4}{12}, \frac{10-6+8}{12}\right)$

/5 $\left(= I(-1, 1)\right)$ 1

$r = \frac{2S_{ABC}}{p} = \frac{2\times 6}{12} = 1$ 1

$\left(\gamma: (x+1)^2 + (y-1)^2 = 1\right)$ 1 canonical eq. of incircle

$A_{\gamma-\Delta} = A_\Delta - A_\gamma = (6 - \pi \cdot 1^2)$ sq U $= \underline{(6-\pi)}$ squ 1 1

incircle triangle

c) $G\left(\frac{x_A+x_B+x_C}{3}, \frac{y_A+y_B+y_C}{3}\right) = \left(\frac{-2-2+1}{3}, \frac{2-2+2}{3}\right)$

/5 $\left(G\left(-1, \frac{2}{3}\right)\right)$ 1

Circumcenter

$\vec{HG} = 2\vec{GO} \to$ 1 Since $H(-2,2)$; let $O(O_x, O_y)$

$\left(-1+2, \frac{2}{3}-1\right) = 2\left(O_x+1, O_y - \frac{2}{3}\right) \to \left(O\left(-\frac{1}{2}, 0\right)\right)$ 1

$\left(GO: \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{2}{3} \end{bmatrix} + \lambda \begin{bmatrix} \frac{1}{2} \\ -\frac{2}{3} \end{bmatrix}\right)$ 1 — Parametric eq.

$\left(\|\vec{GO}\| = \sqrt{\frac{1}{4}+\frac{4}{9}} = \underline{\frac{5}{6}\ LU}\right)$ 1

Length of GO