FINAL EXAM TOTAL MARKS 40

Q1(10 marks)

Given vertices A (1,1,1), B(0,1,1), C(1,0,1), D(1,1,0)

a) find area and angle of B of face BCD (3 marks)

b) write canonical equation and write length of altitude dropped from A. (4 marks)

c) find the volume of tetrahedron (3 marks)

Q2(10 marks)

a)Given function f(x,y): $\sqrt{9-x^2-y^2}$

i)find and sketch domf of given function(2 marks)

ii) find and sketch th range of function (2 marks)

iii) sketch the graph of a given function(1 marks)

b) find partial derivative of following function (5 marks)

f(x,y): $x^2+y^2+\frac{2x}{\sqrt{x^2+y^2}}\ where\ x=rcos\emptyset, y=rsin\emptyset$

find $\frac{\partial^2 f}{\partial r\partial\emptyset}, \frac{\partial^2 f}{\partial\emptyset\partial\emptyset}, \frac{\partial^2 f}{\partial r\partial r},$

Q3 (10 marks)

a) find the tanget plane and normal line to the surface at P(2,-1,2)

S: $x^2+y^2+z^2-9=0\ (5\ marks)$

b) examine possible extrema of the function

$x^2+y^2-2x+2y=0$

Q4)(10 marks)

a) $\iint xydxdy \text{ where } D: y1 = x, y2 = x^2 \text{ (3 marks)}$

b) $\iint \frac{y}{\sqrt{x^2+y^2}} \text{ where } x^2 + y^2 \leq 1, y \geq 0$ (3 marks)

$$c) \iiint \frac{x}{z^2} sin2ydxdydz \text{ where } D \equiv \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq \pi \\ \frac{1}{2} \leq z \leq 1 \end{cases} \quad (4\ marks)$$

Joseph Taylor's Answer

Q1) a) BC =(1,-1,0), BD(1,0,-1)

Area $= \frac{1}{2}\begin{vmatrix} i & -j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$=-i-j-k normal vector = (-1,-1,-1), then length is $\sqrt{3}$

Area = $\frac{\sqrt{3}}{2}$

$$\text{Angle of B is CosQ} = \frac{\text{BC. BD}}{||BC||\ ||BD||}$$

$$\text{BC. BD} = 1 + 0 + 0 = 1$$
$$||BC|| = \sqrt{2}, ||BD|| = \sqrt{2}$$

$$Q = arccos(\frac{1}{2})$$

$$b)\ canonical\ equation$$

$\begin{vmatrix} x-1 & y-0 & k-1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$=x-1-y-k+1=0, normal vector(1,-1,-1) then for length of altitude, I just said that is cros product BC and BD, then it's normal vector b the one I use to get length and still root3 is answer I got.

Volume I got 1/6 after doing determinat of AB,AC,AD

Q2)domf x$x^2 + y^2 \leq 9$

And Is howed circle with radius 3

ii) range [0,3]

to sketch the range, I don't know, I just showd graph with numberline not exceed 3 iii)sketching graph of function I tried again the circle

And later I did shape seems to be cone.

b) $f(r,Q)r^2 + \frac{2rcosQ}{r} = r^2 + 2cosQ$

$$\frac{\partial f}{\partial r} = 2r, \frac{\partial f}{\partial \emptyset} = -2sinQ, \frac{\partial^2 f}{\partial r \partial r} = 2,$$

$$\frac{\partial^2 f}{\partial r \partial \emptyset} = 0, \frac{\partial^2 f}{\partial \emptyset \partial \emptyset} = -2cosQ,$$

Q3) a)S: $x^2 + y^2 + z^2 - 9 = 0$

$$\frac{\partial f}{\partial x} = 2x, at\ px = 2, = 2.2 = 4$$

$$\frac{\partial f}{\partial y} = 2y, at\ py = -1 = 2.-1 = -2$$

$$\frac{\partial f}{\partial z} = 2z, at\ pz = 2 = 2.2 = 4$$

Ts=$Ts = 4(x-2) - 2(y+1) + 4(z-2) = 4x - 8 - 2y - 2 + 4z - 8 = 2x - y + 2z - 9 = 0$

$$Ns = \frac{x-2}{4} - \frac{y-1}{2} + \frac{z-2}{4}$$

b)
$x^2 + y^2 - 2x + 2y = 0$

$$\frac{\partial f}{\partial x} = 2x - 2, \frac{\partial f}{\partial y} = 2y + 2$$

Critica points

$$2x - 2 = 0, 2y + 2$$

X=1, y=-1

Second derivative $\frac{\partial f}{\partial xx} = 2, \frac{\partial f}{\partial xy} = 0, \frac{\partial f}{\partial yy} = 2$
Hessian detarmant=$\frac{\partial f}{\partial xx} . \frac{\partial f}{\partial yy}) - \left(\frac{\partial f}{\partial xy} . \frac{\partial f}{\partial xy}\right) = 2.2) - 0 = 4 > 0\ and\ \frac{\partial f}{\partial xx} > 0, it's\ local\ minmum$

Q4) a) $\iint xydxdy \; where \; D: y1 = x, y2 = x^2 \; (3 \; marks)$

$$x = x^2, x - x^2 = 0, x(1-x) = 0, x = 1 \; and \; x = 0, 0 \leq x \leq 1$$

$$xydydx = x\frac{y^2}{2}$$

$$\iint \frac{1}{2}\iint x[x^4 - x^2] = \frac{x^5}{5} - \frac{x^3}{3} = \frac{1}{2}\left[\frac{x^6}{6} - \frac{x^4}{4}\right] = \frac{1}{2}\left[\frac{1}{6} - \frac{1}{4}\right] = -1/12$$

b) $\iint \frac{y}{\sqrt{x^2+y^2}} \; where \; x^2 + y^2 \leq 1, y \geq 0$ (3 marks)

$$\frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r} = sinQ$$

$x^2 + y^2 \leq 1$, 0<=r<=1

$$y \geq 0, 0 \leq Q \leq \pi$$

integral= $\iint sinQrdrdQ$

Instegral w.r.t.r is $\frac{r^2}{2} sinQ = which \; give \; us \; this \; if \; we \; replace \; boundaries$

$$\frac{1}{2} integral \; sinQ = \frac{1}{2}[0c0sQ] with \; boundaries \; of \; teta, -cospi - cos0$$
$$= -(-1) - 1 \; here \; it \; complicated \; me, I \; found \; zero$$
$$but \; again \; I \; didn't use \; that \; negative, then \; I \; had \; two \; answers, 0 \; and - 1$$

$$c) \iiint \frac{x}{z^2} sin2ydxdydz \; where \; D \equiv \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq \pi \\ \frac{1}{2} \leq z \leq 1 \end{cases} \quad (4 \; marks)$$

Here what I did, I don’t know if exist!

I took that z^2 up but with power -2, and integral became

$$\iiint xz^{-2} sin2ydxdy$$

And now each part on it’s own, I think for final answer I found -3/8

But integral of z, I found $\frac{z^{-1}}{-1}$ $then \; I \; turns \; them \; into - z^{-1} \; and \; I \; replace \; boundaries \; for \; z$

And for sin2Q, let u =2y du=2dy, dy=du/2 then replaced and found -1/2cosu and put boundaries 1/2 [-cospi-cos0