Classical Probability

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Probability notion

Introduction and terminology

- **Probability, (probability theory),** branch of mathematics that deals with measuring or determining quantitatively the likelihood that an event or experiment will have a particular outcome.
- It studies random experiment.
- Probability is based on the study of permutations and combinations and is the necessary foundation for statistics.

Elementary Probability Theory

Meaning of Probability

Assume that an experiment can be repeated many times, with each repetition called a **trial**, and assume that one or more outcomes can result from each trial, then the probability of a given outcome is the number of times that outcome occurs divided by the total number of trials. If the outcome is sure to occur, it has a probability of 1; if an outcome can not occur, its probability is 0.

Example

- The probability of flipping a fair coin and getting tails is 0.50, or 50%. If a coin is flipped 10 times, there is no guarantee, that exactly 5 tails will be observed, the proportion of tails can range from 0 to 1.

a) Random experiment

In the study of probability, any process of observations is referred to as an experiment.

- A **random experiment** is a situation involving chance or probability that leads to well-defined results called outcomes.
- An experiment is called **random experiment** if its outcomes cannot be predicted.
- Then, the results of an observation are called outcomes of the experiment.
- So an outcome is the result of a single trial of an experiment (H: head; T: tail)

Examples

Example

To throw a die , to roll a coin , to draw a card from a deck (set of cards), or selecting a diamond , a spade , a club , or a heart [Throwing an ace, a king, a jack, a queen of diamonds, spades, clubs, or hearts.

b) Sample Space

- The set of all possible outcomes of a random experiment is called the Sample Space (**universal set**) and it is denoted by S.
- An element in is called a **sample point.**
- An **event** is one or more outcomes of a probability experiment.
- **Complement of an event:** all the elements in the sample space except the given elements.
- **Mutually exclusive events or Disjoint events** : two events which cannot happen at the same time.

Examples

Example of rolling a die

- A single 6-sided die is rolled. The possible outcomes of this experiment are 1, 2,3,4,5, and 6.
- $\Omega = U = S$ is the set of all possible outcomes.
- A single outcome of this experiment is rolling a 1, a 2, a 3,. . .
- Rolling an even number (2, 4, or 6) is an event.
- Rolling an odd number (1, 3, or 5) is also an event.

Examples of random experiments:

- The roll (throw) of a die: $\Omega = U = S = \{1, 2, 3, 4, 5, 6\}$.
- The toss of a coin: $\Omega = U = S = \{H, T\}$.

Exercise

Exercise 1:

Find the sample space for the experiment of tossing a coin

1. Once
2. Twice

Solution

1. There are two possible outcomes, head or tail. Thus $S = \{H, T\}$
2. There are four possible outcomes : pairs of heads and tails: $S = \{HH, HT, TH, TT\}$

Tree Diagrams

- Tree diagram is a graphical way of listing all the possible outcomes.
- The outcomes are listed in an orderly fashion, so listing all of the possible outcomes is easier than just trying to make sure that you have them all listed.
- It is called a tree diagram because of the way it looks.
- The first event appears on the left, and then each sequential event is represented as branches off of the first event.
- The tree diagram to the right would show the possible ways of flipping two coins.
- The final outcomes are obtained by following each branch to its conclusion:
- They are from top to bottom: $\{HH, HT, TH, TT\}$

Exercise 2:

1. Find the sample space for the experiment of roll a die
   1. Once
   2. Twice

Solution

1. $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow \#S = 6$
2. There are 36 possible outcomes:$S = \{1, 2, 3, 4, 5, 6\} * \{1, 2, 3, 4, 5, 6\} \Rightarrow \#S = 36$

Sample Space: Two Dice

All 36 possible outcomes:

	1	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Probability

Measuring probabilities using a calibration experiment

- Probabilities are generally hard to measure. It is easy to measure probabilities of events that are extremely rare or events that are extremely likely to occur.
- But consider your probability for the event "There will be a white Christmas this year".
- You can remember years in the past where there was snow on the ground on Christmas.
- Also you can recall past years with no snow on the ground.
- So the probability of this event is greater than 0 and less than 1. But how do you obtain the exact probability?

Definition

If the possibility space S has a finite number of sample points, then, we denote this number by $n(s)$. Consider an event A which is a subset of S, then $n(A) \leq n(S)$.

Tossing a die. **Outcomes:**

- The possible outcomes of this experiment is $S = \{1, 2, 3, 4, 5, 6\}$.
- Let A_1 be the event of obtaining an even number: $A_1 = \{2, 4, 6\}, \quad \Rightarrow \#A_1 = 3.$
- Let A_2 be the event of obtaining an odd number :$A_2 = \{1, 3, 5\}, \quad \#A_2 = 3.$

Therefore, the probability is the measure of how likely an event is.

Exercise 3

Tossing a die

1. Let A be the event that the appeared number is even. $A = \{2, 4, 6\}$.
2. Let B be the event that the occurring number is odd. $B = \{1, 3, 5\}$.
3. Let C be the event that the occurring number is less than 4. $C = \{1, 2, 3\}$.

Questions

1. What is the probability of each outcome?
2. What is the probability of rolling an even number? An odd number?

The Classical Definition of Probability

For an event A_i, its probability $p(A_i)$ is given by:

$$
\begin{aligned}
p(A_i) = \frac{n(A_i)}{n(S)} &= \frac{\text{number of possible outcomes of event } A_i}{\text{number of total outcomes}} \\
&= \frac{\text{the number of ways event } A \text{ can occur}}{\text{the total number of possible outcomes}}
\end{aligned}
$$

This assumes all outcomes are **equally likely**.

Example: Rolling a Fair Die

Let's say we roll a fair 6-sided die.
Define:

- Event A_1: "Roll an even number" $\rightarrow$ outcomes: $\{2, 4, 6\} \rightarrow 3$ outcomes
- Event A_2: "Roll an odd number" $\rightarrow$ outcomes: $\{1, 3, 5\} \rightarrow 3$ outcomes
- Total outcomes: 6

Then:

$$p(A_1) = \frac{3}{6} = \frac{1}{2}, \quad p(A_2) = \frac{3}{6} = \frac{1}{2}$$

Comparing Probabilities

We can compare how likely events are:

If $p(A) > p(B)$

Then event A is **more likely** to occur than event B.

If $p(A) = p(B)$

Then events A and B are **equally likely** to occur.

In our die example:

$$p(A_1) = p(A_2) = \frac{1}{2} \Rightarrow \text{Even and odd numbers are equally likely.}$$

- The probability of each outcome is always the same $\frac{1}{6}$. The outcomes are equally likely to occur.
  Equally likely events: events which have the same probability of occurring.

$$\begin{aligned} P(1) &= \frac{Numberofwaystorolla1}{Totalnumberofsides} = \frac{1}{6} \\ P(2) &= \frac{1}{6}, \ldots \end{aligned}$$

- Classical probability uses the sample space to determine the numerical probability that an event will happen. It is also called theoretical probability.

Axioms of Probability

1. $P(S) = 1$
2. $A \subset S$, $0 \leq P(A) \leq 1$. All probabilities are between 0 and 1 inclusive.
3. $A, B \subset S \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If $A \cap B = \emptyset$. A and B are mutually exclusive events in S. Then $P(A \cup B) = P(A) + P(B)$.
5. The probability of an event which cannot occur (impossible event) is 0.
6. The probability of an event which is not in the sample space is zero.
7. The probability of an event which must occur is 1.
8. If A' is the event of not occurring the event, then $P(A') = 1 - P(A)$.

Exercises

1. 1. A box contains 6 red balls, 5 green, 8 blue and 3 yellow. A single ball is chosen at random from the box. What is the probability of choosing
    - a red ball?
    - a green ball?
    - a yellow ball?
    - a red or green ball?
2. The outcomes are not equally likely to occur. You are more likely to choose a blue ball than any other color. You are least likely to choose a yellow ball.
3. Choose a number at random from 1 to 5. What is the probability of each outcome? What is the probability that the number chosen
    - is even?
    - is odd ?
    - is prime?