Introduction to Statistical Modeling

Case Western Reserve University, Spring 2026

Unit 07-01: Confidence Interval-Population Mean

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Example 1

Let us consider the following situation. Suppose the sample mean $\bar{y}$ is normally distributed, either because the population is normal or because the sample size is large ($n \geq 30$). Then

$$\bar{Y} \sim N\left(\mu,\ \frac{\sigma}{\sqrt{n}}\right).$$

Assume that the population standard deviation σ is known.

(a) Construct a 95% confidence interval for μ.

(b) Using the result from part (a), can we generalize our answer to construct a $(1 - \alpha)100\%$ confidence interval for μ when σ is known?

Exercise 1

Confidence Interval when σ is known: Group Work

We are interested in μ, the population mean weight among all CWRU students. Suppose $n = 30$, $\bar{y} = 150$ pounds, and $\sigma = 25$ (known). Construct a 95% confidence interval for μ. *Note:* You do NOT need to calculate exact values.

Exercise 2

True/False Game: T-P-S

Now that we have constructed a 95% confidence interval for the population mean weight among all CWRU students, what does it actually mean?

Make sure you synchronize with your group members and work through the following interpretations. Discuss whether each statement is **True** or **False**, and be ready to share your reasoning during our "Think–Pair–Share activity"!

(a) We sampled 95% of all CWRU students.

(b) We know that $141.05 < \mu < 158.95$.

(c) Based on our sample, values of μ between 141.05 and 158.95 are plausible for the population mean weight of CWRU students.

(d) $P(141.05 < \mu < 158.95) = 0.95$.

(e) We are 95% confident that $141.05 < \mu < 158.95$.

(f) Using this method, about 95% of intervals constructed from random samples would capture the true population mean μ in the long run.

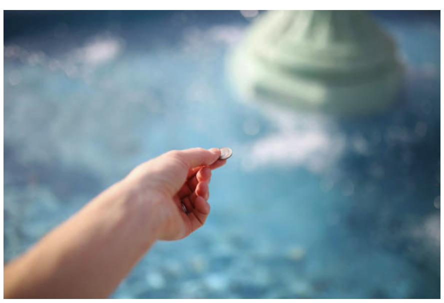
Example 3

Analogy: Horseshoes and Confidence Intervals

In the horseshoe analogy, the thrower cannot see or hear the pin, just as a sample has no direct knowledge of the true population mean μ.
Each horseshoe represents a confidence interval, and the pin represents the true parameter μ.

Example 4

Analogy: Coin Toss and Confidence Intervals



Imagine standing near Wade Lagoon by the Cleveland Museum of Art and tossing a special coin from a bridge toward the water!

Exercise 3

Understanding Confidence Interval: Group Work

(a) Consider the confidence interval formula for μ when σ is known:

$$\bar{Y} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

Without performing calculations, discuss what would make the interval **wider** or **thinner**.

(b) Would a 100% confidence interval for μ make sense? If we tried to construct a 100% confidence interval, what would happen to the critical value and the width of the interval? Explain why such an interval may or may not be useful in practice.

Example 5

Margin of Error & Sample Size

Recall the margin of error (ME) is

$$\text{ME} \; = \; z_{\alpha/2} \, \frac{\sigma}{\sqrt{n}}.$$

(a) If we want the margin of error to equal some small number B, what sample size is necessary?

$$z_{\alpha/2} \, \frac{\sigma}{\sqrt{n}} = B.$$

Solving for n:

$$\frac{\sigma}{\sqrt{n}} = \frac{B}{z_{\alpha/2}} \quad \Longrightarrow \quad \sqrt{n} = \frac{z_{\alpha/2}\sigma}{B} \quad \Longrightarrow \quad n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2.$$

(b) Following **exercise 1**, 95% confidence interval for μ was 150 ± 8.95, when $\sigma = 25$ (known).

If we want the margin of error to equal 3 pounds with 95% confidence, what should be the required sample size of CWRU students?

Example 6

Other Confidence Levels

Using the same setup as in **Example 1**, how would we construct confidence intervals for μ (when σ is known) with different confidence levels, such as 90% and 99%?

Exercise 4

Confidence Interval of μ when σ is unknown: Group Work

Suppose μ = population mean cholesterol level among 70-year-old American men. Let $n = 19$ (small n, so we must assume the population is normal), $\bar{y} = 44.2$, σ is unknown, and $s = 3.8$.

(a) Find a 90% confidence interval for μ. *Note:* You do NOT need to calculate exact values.

(b) Write down an appropriate interpretation of your finding in part (a) in context.

Exercise 5

Reverse-engineer: Group Work

Suppose $n = 16$, σ is unknown, and an 80% confidence interval for μ was $(48, 54)$. What were $\bar{y}$ and s?