ST 260: Statistical Data Analysis

University of Alabama, Spring 2026

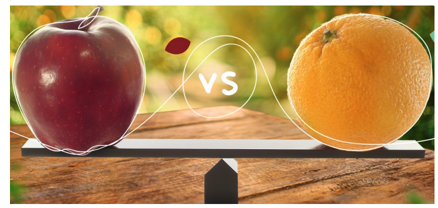
Introduction to Continuous Distributions (Part 3): The Standard Normal Distribution

Instructor: Md Mutasim Billah

Standard Normal Distribution

EXAM 1 scores – Apple vs Orange!

You					
Friend					



"Standardizing" an observation (finding its z-score) is measuring how many standard deviations above or below the mean it is. A positive z-score value means the value is above the mean; a negative z-score means it is below.

Properties of the Standard Normal Variable

- $Z \sim N(0, 1)$ has mean $\mu = 0$ and standard deviation $\sigma = 1$
- The standard normal curve is **symmetric** around 0
- **Area** under the curve equals **1**
- Standardizing any normal variable $X \sim N(\mu, \sigma)$:

$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad X = \mu + \sigma Z$$

- **Empirical Rule** (68–95–99.7)

Example 1

Finding probabilities given z-scores

Suppose $Z \sim N(0,1)$ (standard normal distribution). Compute the following probabilities (to 4 decimals).

(a) $P(Z \leq -0.77) = P(Z < -0.77)$

> Recall, for *any* continuous distribution, the probability at a single point is always zero (no width to allow area). As the standard normal distribution is continuous, so $P(Z = -0.77) = 0$.
>
> Which implies,
>
> $$P(Z \leq -0.77) = P(Z < -0.77) + P(Z = -0.77) = P(Z < -0.77) + 0 = P(Z < -0.77).$$

(b) $P(Z \geq -0.77)$

(c) $P(-1.05 \leq Z \leq 2.41)$

Example 2

Transforming Normal Distribution into a Standard Normal Distribution

Let X be the mass (in grams) of a randomly selected newborn baby, where

$$X \sim N(\mu = 3400,\ \sigma = 500).$$

(a) What is the probability that a randomly selected baby weighs less than 3800 grams?

(b) What is the probability that a randomly selected baby weighs more than 4170 grams?

(c) What proportion of babies weigh between 3800 and 4170 grams?

Example 4

Using z-scores to Compare How Unusual Values Are

The price of a cheeseburger in town follows a normal distribution with mean price \$9 and standard deviation \$1.5. A local restaurant decides to charge \$12 for a cheeseburger.

The price of a Honda Civic in town follows a normal distribution with mean price \$25,000 and standard deviation \$4,000. A local dealership decides to charge \$30,000 for a Honda Civic.

(a) Compute the standardized value (z-score) for each price.

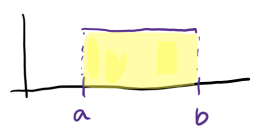
(b) Which price "most significantly" increased from the mean?

Example 5

Summary of Continuous Distributions

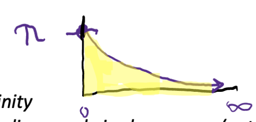
1. _(Continuous) Uniform Distribution:_

- **Picture**:
- **Parameters**: ***min*** *(a) and* ***max*** *(b)*
- **Domain** *(possible outcomes): from* ***min*** *to* ***max***
- **Properties**: *symmetric distribution, mean = median (midpoint), no mode*



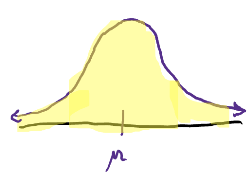
2. _Exponential Distribution:_

- **Picture** *(Shape):*
- **Parameters**: *"lambda", λ*
- **Domain** *(possible outcomes): zero to positive infinity*
- **Properties**: *right-skewed distribution, mean > median, mode is always zero (outcome under highpoint on graph)*



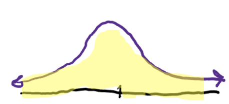
3. _Normal Distribution:_

- **Picture** *(Shape):*
- **Parameters**: *"mean", μ and "standard deviation", σ*
- **Domain** *(possible outcomes): negative infinity to positive infinity*
- **Properties**: *symmetric distribution, mean = median = mode*



4. _Normal Distribution:_

- **Picture** *(Shape):*
- **Parameters**: *"mean", $\mu = 0$ and "standard deviation", $\sigma = 1$*
- **Domain** *(possible outcomes): negative infinity to positive infinity*
- **Properties**: *symmetric distribution, mean = median = mode = 0*



Exercise

Think-Pair-Share!

(A) Suppose $Z \sim N(0, 1)$ (standard normal distribution). What is the value of $P(-1 \leq Z \leq 1)$?

   (a) 0.1587
   (b) 0.3174
   (c) 0.6826
   (d) 0.8413

(B) The heights of adult men in the United States are approximately normally distributed with a mean of 70 inches and a standard deviation of 3 inches.

   Is the following statement **TRUE** or **FALSE**?

   "The proportion of the population between 67 and 73 inches is approximately 0.6826."

   (a) TRUE
   (b) FALSE

(C) Daily high temperatures in a certain city follow a normal distribution with mean $\mu = 72°$F and standard deviation $\sigma = 8°$F. One day, the standardized temperature was 0.5 standard deviations below the average. What was the actual temperature on that day?

   (A) 68°F
   (B) 72°F
   (C) 76°F
   (D) 64°F