

## Unit 06 Practice Problems

### 6.1

a

$$P(\bar{X} > 41) = P\left(Z > \frac{41 - 40}{2.1/\sqrt{4}}\right) = P(Z > 0.95) = 1 - .8289 = .1711$$

b

$$P(\bar{X} > 41) = P\left(Z > \frac{41 - 40}{2.1/\sqrt{8}}\right) = P(Z > 1.35) = 1 - .9115 = .0885$$

### 6.2

$$P(\bar{X} < 1.01) = P\left(Z < \frac{1.01 - 1.02}{0.01/\sqrt{7}}\right) = P(Z < -2.65) = .004$$

b

$$P(1.015 < \bar{X} < 1.022) = P\left(\frac{1.015 - 1.02}{0.01/\sqrt{3}} < Z < \frac{1.022 - 1.02}{0.01/\sqrt{3}}\right) =$$
$$P(-0.87 < Z < 0.35) = .6368 - .1922 = .4446$$

### 6.3

a

From Problem 5.7 we know:

$$\mu = E(X) = \frac{\theta_1 + \theta_2}{2} = \frac{9.0 + 9.1}{2} = 9.05$$

$$\sigma^2 = Var(X) = \frac{(9.1 - 9.0)^2}{12} = \frac{1}{1200}$$

$$\sigma = SD(X) = \frac{\theta_2 - \theta_1}{\sqrt{12}} = \frac{9.1 - 9.0}{\sqrt{12}} = \frac{1}{\sqrt{1200}}$$

According to Central Limit Theorem Part 2, since  $n \geq 30$

$$\bar{X} \approx N\left(\mu = 9.05, \frac{\sigma}{\sqrt{n}} = \frac{\left(\frac{1}{\sqrt{1200}}\right)}{\sqrt{30}} \approx 0.00527\right)$$

b

$$P(9.04 < \bar{X} < 9.06) = P\left(\frac{9.04 - 9.05}{0.00527} < Z < \frac{9.06 - 9.05}{0.00527}\right) =$$

$$P(-1.90 < Z < 1.90) = .9713 - .0287 = .9426$$

c

The interval (9.04,9.06) is symmetric around  $\mu = 9.05$  and the  $\bar{X}$  curve gets taller and thinner as  $n$  increases. The taller and thinner  $\bar{X}$  curves will have more of their area/probability in the interval (9.04,9.06). Therefore the probability will increase with larger and larger values of  $n$ . For example if  $n = 40$  then

$$P(9.04 < \bar{X} < 9.06) = P\left(\frac{9.04 - 9.05}{\left[\frac{\left(\frac{1}{\sqrt{1200}}\right)}{\sqrt{40}}\right]} < Z < \frac{9.06 - 9.05}{\left[\frac{\left(\frac{1}{\sqrt{1200}}\right)}{\sqrt{40}}\right]}\right) =$$

$$P(-2.19 < Z < 2.19) = .9857 - .0143 = .9714$$

## 6.4

a

From Problem 5.19 we know:

$$\mu = \alpha\beta = 4(3) = 12$$

$$\sigma^2 = \alpha\beta^2 = 4(3^2) = 36$$

$$\sigma = \sqrt{36} = 6$$

According to Central Limit Theorem Part 2, since  $n \geq 30$

$$\bar{X} \approx N\left(\mu = 12, \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{34}}\right)$$

b

$$P(11.9 < \bar{X} < 13.2) = P\left(\frac{11.9 - 12}{6/\sqrt{34}} < Z < \frac{13.2 - 12}{6/\sqrt{34}}\right) =$$

$$P(-0.19 < Z < 1.17) = .8790 - .4247 = .4543$$