Introduction to Statistical Modeling: HW 06

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Submission Guidelines
Please follow these guidelines carefully when completing and submitting your homework. Clear and well-organized work helps ensure fair and accurate grading.

- Write your answers neatly and legibly.
- Circle or clearly mark your final answers so they can be easily found on the page.
- Do the problems in order (Problem 1 before Problem 2, etc.).
- Show all work for nontrivial calculations. Answers without supporting work may receive little or no credit.
- Do **not** write directly on the homework question pages. Use blank paper or a new page on your writing device to write your solutions.

Work that is excessively messy or difficult to follow may not receive full credit. Please take care to present your solutions clearly and logically.

Problem 6.1 - Standard Normal Probabilities

Find the following probabilities if $Z \sim N(\mu = 0, \sigma = 1)$. **You are required to draw a picture for every part.**

a) $P(Z < -0.68)$

b) $P(Z > 2)$

c) $P(0 < Z < 1.73)$

Find the value of z if...

d) $P(Z > z) = 0.5$

e) $P(Z < z_0) = 0.8643$

f) $P(-z < Z < z) = 0.90$

g) $P(-0.20 < Z < z) = 0.0673$

h) What is the 32nd percentile of $Z \sim N(0, 1)$?

i) What is the 68th percentile of $Z \sim N(0, 1)$?

Problem 6.2 - Weights of 11-Year-Old Boys

Biologists at CWRU are studying the weights of 11-year-old American boys. This population of weights is normally distributed with mean $\mu = 81$ pounds and standard deviation $\sigma = 9$ pounds. Let $Y =$ the weight of a randomly selected 11-year-old American boy.
What is the probability the weight of a randomly selected boy is...

a) Less than 91.44 pounds?

b) Greater than 85.86 pounds?

c) Between 74.61 pounds and 88.38 pounds?

d) The probability that the weight of a randomly selected boy is greater than k pounds is 0.0250. What is k?

e) What is the 78th percentile among the weights?

Problem 6.3 - Interest Rate Forecasts

One method of arriving at economic forecasts is to use a consensus approach. A forecast is obtained from each of a large number of analysts; the average of these individual forecasts is the consensus forecast. Suppose that the individual 1996 January prime interest–rate forecasts of all economic analysts are approximately normally distributed with mean 7% and standard deviation 2.6%.
If a single analyst is randomly selected from among this group, what is the probability that the analyst's forecast of the prime interest rate will

a) exceed 11%?

b) be less than 9%?

Hint: If we define $Y =$ interest-rate forecast, then what is the distribution of Y?

Problem 6.4 - Achievement Test Scores

Achievement test scores of all high school seniors in Ohio state have mean 60 and variance 64. A random sample of $n = 100$ students from one large high school in the Cleveland area had a mean score of 58.

a) Is there evidence to suggest that this high school is inferior? Find the necessary probability and comment.

b) If we had $n = 10$ students, could we answer part (a)? Explain.

Hint: Calculate the probability that the sample mean is at most 58 (given n).

Problem 6.5 - Degrees of Freedom

What are the degrees of freedom corresponding to each of the following probabilities? **You are required to draw a picture for every part.**

a) $P(T > 2.763) = 0.005$

b) $P(T < -2.650) = 0.01$

c) $P(-1.313 < T < 1.313) = 0.80$

Problem 6.6 - Comparing Variability

Each of the following data sets has a mean of 10. Without using a calculator, put the data sets in a list of increasing variance/standard deviation.

$$A : \{4, 6, 8, 10, 12, 14, 16\}$$
$$B : \{-2, 3, 4, 6, 10, 19, 30\}$$
$$C : \{9, 10, 10, 10, 10, 10, 11\}$$
$$D : \{9, 9, 9, 10, 11, 11, 11\}$$

Problem 6.7 - Sample Variance by Hand

Find the sample variance and sample standard deviation for the following data set by hand. Use both the regular and convenient formula.

$$\{-2, 3, 4, 6, 10, 19, 30\}$$

Problem 6.8 - Transformations of Data

The data set $\{0, 2, 3, 3, 4, 6\}$ has sample mean 3, sample variance 4, and sample standard deviation 2.

Find the sample mean, sample variance, and sample standard deviation of:

$$A : \{3000, 3002, 3003, 3003, 3004, 3006\}$$
$$B : \{0, 10, 15, 15, 20, 30\}$$
$$C : \{0, 0.2, 0.3, 0.3, 0.4, 0.6\}$$
$$D : \{-10, -12, -13, -13, -14, -16\}$$

AI

You're encouraged to AVOID the use of AI and to NEVER use it as your first approach to an exercise. Learning comes from you doing the puzzling, not from you producing a correct answer.

(a) Did you use AI for any part of these problems?

(b) If so, describe: where you used it (on which problems), how long you worked on the problems before turning to AI, and what prompts you used or typed into AI.

GOOD LUCK!