# **Unit 02 Practice Problems**

## 2.1

Use this data set to draw a stem-and-leaf plot:

{ 0.9, 5.8, 6.6, 4.3, 8.8, 6.4, 3.2, 6.1, 2.0, 5.3, 7.2, 8.8, 8.6, 8.3, 1.5, 7.7, 4.1, 5.6, 8.1, 5.4, 7.0, 8.2, 7.4, 7.7, 7.0, 8.0, 7.6, 6.9, 3.6, 6.7, 4.6, 8.3, 8.4 }

#### 2.2

Twelve left-handed pitchers were randomly selected and their heights were rounded to the nearest tenth of an inch. Use the data to draw a stem-and-leaf plot:

{71.2, 71.7, 72.6, 72.6, 73.0, 73.5, 74.1, 74.9, 75.2, 75.3, 76.4, 76.6}

#### 2.3

Refer to Practice Problem 2.1. Describe the shape of the stem-and-leaf plot.

#### 2.4

Refer to Practice Problem 2.2. Describe the shape of the stem-and-leaf plot.

# 2.5

Refer to Practice Problem 1.1. The historian visits the 15 bells he randomly selected and measures their loudness in decibels (dB) when rung. The data is below. Draw a histogram and have the classes be [90,92), [92, 94), [94, 96), ... etc. Describe the shape of the histogram.

{ 90.2, 98.7, 92.4, 94.7, 97.6, 98.1, 98.3, 93.8, 99.9, 95.9, 96.2, 94.3, 99.3, 99.5, 97.9 }

#### 2.6

Use this data to draw a histogram and have the classes be [0, 5), [5, 10), [10, 15), ... etc. Describe the shape of the histogram.

{ 0.107, 0.741, 0.849, 1.590, 2.686, 3.432, 4.114, 4.253, 5.107, 5.754, 7.423, 9.031, 9.315, 9.413, 10.108, 12.041, 12.394, 13.085, 13.997, 15.108, 19.034, 22.222 }

Three data sets are described below. Choose exactly one of the following descriptions for each data set:

The mean is less than the median.

The mean is greater than the median.

The mean and median are approximately equal.

а

The owner of a car dealership is looking at a list of the mileages on all the cars available on the lot. Almost all of the cars are brand new, so almost all of the mileages are close to zero. However, just a few are used cars with mileages greater than 80,000 miles.

b

A doctor is looking at a histogram from a data set of blood pressure measurements. The histogram is symmetric and bell-shaped, with most observations (tallest rectangles) in the middle.

С

A professor is looking at a large data set of exam scores. Almost all of the scores are between 70-90 points. However, a small handful of students had extremely low scores in the neighborhood of 10-20 points.

## 2.8

Refer to Practice Problem 2.2. Find the sample mean and median among the heights.

#### 2.9

Refer to Practice Problem 2.5. Find the sample mean and median among the loudness measurements.

#### 2.10

In a data set with n = 13,642 values, where would you find the median? What if n = 173,209?

## 2.11

Find the sample variance of the data set { 0, 3, 3, 9, 10, 13, 18 } using the both the "regular" formula and "convenient" formula. Make a table and show your work. You should get the same answer with both formulas. Finally, find the standard deviation.

Find the sample variance of the data set { 5, 8, 9, 10, 17, 21, 24, 30 } using the both the "regular" formula and "convenient" formula. Make a table and show your work. You should get the same answer with both formulas. Finally, find the standard deviation.

#### 2.13

At a large high school exactly 100 juniors and 100 seniors are assembled in the auditorium. If you were to ask the students what grade they are in, the juniors would respond with "11" and the seniors would respond with "12". What is the sample mean of this data set? What is the approximate value of the standard deviation?

#### 2.14

The members of a book club calculate the sample mean and standard deviation among their ages.  $\overline{X} = 34.1$  and s = 7.2. What will be the sample mean and standard deviation among their ages exactly 5 years from today?

#### 2.15

Each slot machine in a Las Vegas casino contains a certain number of quarters. Suppose the sample mean and standard deviation among the numbers of quarters are  $\overline{X} = 512.4$  and s = 83.5. What is the sample mean and standard deviation among the dollar amounts of these quarters? (1 quarter = 0.25 dollars)

## 2.16

Create your own data sets, each with a sample size of n = 5, that satisfy the following descriptions. If no such data set could exist, write "<u>impossible</u>".

a

Both the mean and median equal ten, and the standard deviation is positive.

b The standard deviation is negative.

c Four of the five values are less than the mean.

The mean is negative, but the median is positive.

Refer to Practice Problem 2.2. Find the five number summary for the heights.

## 2.18

Refer to Practice Problem 2.5. Find the five number summary for the loudness measurements. Remember to order the data first.

#### 2.19

In a data set with n = 13,642 values, where would you find the three quartiles? What if n = 173,209?

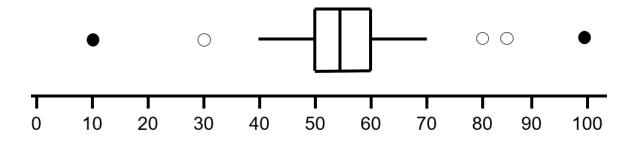
#### 2.20

Sixteen cigarette smokers made a New Years resolution to quit smoking. Ten weeks later they were asked how many cigarettes they smoked yesterday. The responses were: { 0, 0, 1, 2, 3, 3, 6, 7, 9, 10, 10, 10, 11, 13, 19, 26 }

Find the five number summary, IQR, the locations of the inner and outer fences, and then use these values to draw a boxplot.

# **2.21** Examine the following boxplot.

Find the value of the range, IQR, and the location of the fences.



# **Unit 03 Practice Problems**

## 3.1

A basket contains a red ball, a green ball, and a yellow ball. One ball will be drawn out of the basket at random. If the ball is red or green, the experiment ends. If the ball is yellow, then another ball will be drawn out of the basket at random (the yellow ball is not placed back into the basket). On the second draw, if the ball is red then the experiment ends. If the ball is green on the second draw, then a fair coin is flipped once. Draw a tree diagram and identify all the outcomes of the Sample Space and their probabilities.

#### 3.2

Redmond Elementary School has 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> grade classes. First, a class will be randomly selected and we will record if it is a class of 3<sup>rd</sup>, 4<sup>th</sup>, or 5<sup>th</sup> graders. Then, a student within the class will be selected at random and we will record if the student is a boy or girl. For example, one possible outcome in the Sample Space is 4G (4<sup>th</sup> grade class is selected, then a girl).

a Draw a tree diagram, then write out the Sample Space using set notation.

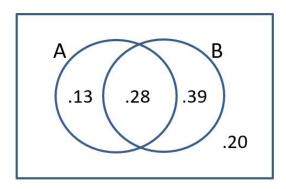
b Write the following events in set notation. For example: the event <u>a third grader</u> is selected = { 3B, 3G }.

A = the selected student is a girl

B = the selected student is a 3<sup>rd</sup> grader or a boy

C = the selected student is not in 4<sup>th</sup> grade

**3.3** Use this Venn Diagram to find the following probabilities. Also, shade the event on a picture of the diagram.



- P(A)
- P(B)
- $P(A \cap B)$
- $P(A \cup B)$
- $P(A \cap \overline{B})$
- $P(\overline{A} \cap B)$
- $P(\overline{A})$
- $P(\overline{B})$
- $P\left(\overline{A\cup B}\cup (A\cap B)\right)$
- $P((A \cap \overline{B}) \cup (\overline{A} \cap B))$
- $P(A \cup \overline{B})$
- $P(\overline{A} \cup B)$
- $P(\overline{A \cup B})$
- $P(\overline{A \cap B})$

If a male resident of a medical care facility is randomly selected, the probability he is over 80 years old = P(A) = 0.50. The probability he has prostate cancer = P(B) = 0.32. The probability he is both over 80 years old and has prostate cancer is 0.22.

a Draw a Venn Diagram with the given information.

b

For each event below, describe the man who is randomly selected if the event occurs.

 $A \cup B$ 

 $\overline{A} \cap B$ 

 $A \cup \overline{B}$ 

C

Find the following probabilities

 $P(A \cup B)$ 

 $P(\overline{A} \cap B)$ 

 $P(A \cup \overline{B})$ 

 $P(\overline{A} \cap \overline{B})$ 

 $P(\overline{A} \cup \overline{B})$ 

The instructor of a statistics course holds an optional review session at 8 am. If a student enrolled in the class is selected at random, the probability they attend the review session = P(R) = 0.60. The probability the student attends the review session and has not drank any coffee that morning =  $P(R \cap \overline{C}) = 0.06$ . The probability the student does not attend the review session and also has not drank any coffee =  $P(\overline{R} \cap \overline{C}) = 0.04$ .

a Draw a Venn Diagram with the given information.

b
Find the following probabilities and describe the student selected if the event occurs.

$$P(\overline{C})$$

$$P(R \cap \overline{R})$$

$$P(C \cup \overline{C})$$

$$P(\overline{R} \cap C)$$

$$P(\overline{R} \cap C)$$

$$P(\overline{R} \cup C \cup (R \cap C))$$

$$P(\overline{R} \cup C)$$

Fred randomly selects one ball at random out of a basket. The balls in his basket are blue and numbered 7, 8, and 9. Gina also selects one ball out of a basket. The balls her basket are red and numbered 5, 6, 7, 8, and 9. Fred and Gina will add the two numbers that appear.

```
Draw a table and find the following probabilities:
P(\text{sum} = 13)
P(\text{sum} = 16)
P(\text{sum} > 14)
P(\text{sum} \le 15 \cap \text{red} \le 7)
P(\text{sum} \le 15 \cup \text{red} \le 7)
P(\text{sum} = 14 \cap \text{red} = \text{blue})
P(\text{sum} = 14 \cup \text{red} = \text{blue})
b
What event(s) listed below are mutually exclusive with the event red = blue?
sum = 12
sum = 14
sum = 15
red < 7
red < 8
blue = 9
What event(s) listed below are mutually exclusive with the event <u>sum = 13</u>?
sum = 17
blue = 9
red = 6
red = 7
sum < 16
blue = 7
```

#### 3.7

Refer to Practice Problem 3.6. Find the following conditional probabilities:

```
P(\text{sum} = 16 \mid \text{red} = 8)

P(\text{blue} \ge 8 \mid \text{sum} = 16)

P(\text{sum} = 16 \mid \text{blue} \ge 8)

P(\text{sum} = 17 \mid \text{red} = 6)

P(\text{red} \le 7 \mid \text{sum} = 13)

P(\text{blue} = 7 \mid \text{sum is even})
```

Refer to Practice Problem 3.1.

а

Given that the green ball appears during the experiment, what is the probability the yellow ball appears during the experiment?

b

Given that the experiment ends after the first draw, what is the probability the green ball appears during the experiment?

C

Are the events  $A = \underline{\text{the first draw is yellow}}$  and  $B = \underline{\text{heads appears during the experiment}}$  independent? Prove your answer.

## 3.9

Refer to Practice Problem 3.5.

а

Given that a randomly selected student attends the review session, what is the probability they drank coffee? Compare  $P(C \mid R)$  to P(C). What do you notice? What does this tell you about the events C and R?

b

Based on your answer to the previous question, you should know in advance whether or not  $P(C \cap R)$  equals  $P(C) \cdot P(R)$ . You should also know if  $P(R \mid C)$  equals P(R). Confirm your answers.

С

In terms of the story problem, describe in words what it means for  $\mathcal{C}$  and  $\mathcal{R}$  to be independent.

An anthropologist is studying a population of natives in Papua New Guinea, all residents of an East Tribe or a West Tribe. If a native is selected at random, the probability they belong to the East Tribe is .60 and the probability they belong to the West Tribe is .40. If a member of the East Tribe is selected at random, the probability they have blood type O is .50, blood type A is .35, blood type B is .10, and blood type AB is .05. If a member of the West Tribe is selected at random, the probability they have blood type O is .35, blood type A is .20, blood type B is .25, and blood type AB is .20.

- a

  Draw a tree diagram with probabilities written on the branches. At the end of each branch, identify each outcome of the Sample Space and its probability.
- b
  What is the probability a randomly selected native has blood type A?
- Given that a randomly selected native has blood type AB, what is the probability they belong to the West Tribe?
- d Given that a randomly selected native does <u>not</u> have blood type O, what is the probability they belong to the East Tribe?

A company makes fluorescent light bulbs in three factories: Rockford, Sycamore, and Thompson. One bulb is selected at random.

70% of the bulbs are made in Rockford: P(R) = .70 20% of the bulbs are made in Sycamore: P(S) = .20 10% of the bulbs are made in Thompson: P(T) = .10

W = a bulb is still working after 10,000 hours of use

Rockford has a 90% success rate: P(W|R) = .90Sycamore has a 80% success rate: P(W|S) = .80Thompson has a 70% success rate: P(W|T) = .70

а

Draw a tree diagram with probabilities written on the branches. At the end of each branch, identify each outcome of the Sample Space and its probability.

b
What is the probability a randomly selected bulb is still working after 10,000 hours?

Given that a randomly selected bulb is not working after 10,000 hours, what is the probability it was made in Thompson?

d Given that a randomly selected bulb is working after 10,000 hours, what is the probability it was made in Rockford or Sycamore?

e
If the Rockford factory has the highest success rate, why does it produce the
most defective bulbs? Doesn't that seem weird? Explain.

An unfair coin has P(H) = .68

а

If the unfair coin is flipped 5 times, what is the probability of observing HTTHT?

b

If the unfair coin is flipped 7 times, what is the probability of observing tails at least once?

С

If the unfair coin is flipped 8 times, what is the probability of observing five heads and three tails in any order? Hint: the number of ways to arrange 5  $\,\mathrm{H}$ 's and 3  $\,\mathrm{T}$ 's equals 56.

# **3.13**The integers on an unfair six-sided die appear with the following probabilities:

P(one)	P(two)	P(three)	P(four)	P(five)	P(six)
.17	.27	.07	.24	.09	.16

а

If you roll the unfair die 4 times, what is the probability you will observe an even number on all 4 rolls?

h

If you roll the unfair die 5 times, what is the probability you will observe a five at least once?

c

If you roll the unfair die repeatedly, what is the probability you will observe a six for the first time on the  $5^{th}$  roll?

d

If you roll the unfair die 3 times, what is the probability you will observe a two on exactly 1 of the 3 rolls? <u>Hint</u>: notice this question does not require the two to appear on the first, second, or third roll.

Eight friends each write their name on a card and place it in a hat. Ashley, Bob, Carla, Danny, and Edith write their names on yellow cards. Frank, Greg, and Hannah write their names on pink cards.

а

If three cards will be drawn one at a time at random <u>with</u> replacement, what is the probability all three cards will be pink?

b

Find the answer to the previous question if the cards are drawn <u>without</u> replacement.

С

If four cards will be drawn one at a time at random <u>with</u> replacement, what is the probability Hannah's name will appear zero times?

d

Find the answer to the previous question if the cards are drawn without replacement.

е

If two cards will be drawn one at a time at random with replacement, what is the probability they will be different colors?

f

Find the answer to the previous question if the cards are drawn <u>without</u> replacement.

q

If four cards will be drawn one at a time at random with replacement, what is the probability that at least one persons name will appear more than once?

h

Find the answer to the previous question if the cards are drawn without replacement.

#### 3.15

Six high school students will choose a novel from the library to take home and read. Obviously, all the students must choose a different novel. The library has 894 novels. How many different arrangements are possible?

Twenty-five members of a hockey team must choose an integer between 1 and 99 to wear on the back of their hockey jersey. All the players must choose a different integer. How many different arrangements are possible?

#### 3.17

21 balls are in a basket: 10 are red, 6 are green, 3 are yellow, and 2 are orange. Exactly one of the red balls has a happy face drawn on it. It is known as the "Red Super Happy Ball". Four balls will be drawn out of the basket all at once at random without replacement. Find the probabilities of the following events:

a 2 balls are red, 1 is green, and 1 is orange.

b All 4 balls are red.

None of the balls are green.

d All of the balls are either red or green.

e At least one of the balls is yellow.

3 of the balls are red, and one of them is the Red Super Happy Ball.

g Two of the balls are either red or orange, the other two balls are either green or yellow, and the Red Super Happy Ball does not appear.

n
There are zero yellow balls or there are zero green balls.

Fact: 
$$\binom{15}{7}\binom{1}{1} + \binom{15}{8}\binom{1}{0} = \binom{16}{8}$$

а

Write a very short story explaining why it should make sense that  $\binom{15}{7}\binom{1}{1}+\binom{15}{8}\binom{1}{0}=\binom{16}{8}.$  Your example could be about balls in baskets, people serving on committees, or whatever you like.

b

Optional algebra challenge: prove the equality  $\binom{n-1}{r-1}\binom{1}{1}+\binom{n-1}{r}\binom{1}{0}=\binom{n}{r}$ 

#### 3.19

Draw a Venn Diagram with the given information.

$$P(A \cap B \cap C) = .07$$

$$P(A \cap B) = .09$$

$$P(A \cap C) = .17$$

$$P(B \cap C) = .10$$

$$P(A) = .41$$

$$P(B) = .20$$

$$P(C) = .36$$

а

Find the following probabilities:

$$P(A \cap \overline{B} \cap \overline{C})$$

$$P(A \cup B \cup C)$$

$$P(A \cup C)$$

$$P((A \cup C) \cap B)$$

$$P(C \cup (A \cap B))$$

$$P(\overline{(B \cap C)} \cap A)$$

b

Are  $A \cup C$  and B independent events? Prove your answer.

# **Unit 04 Practice Problems**

## 4.1

A basket contains 3 green and 2 yellow balls. One ball will be selected at random and then <u>not</u> replaced. Then a second ball will be randomly selected from the basket. X = # of green balls observed during the experiment.

а

Draw a tree diagram with probabilities written on the branches. At the end of each branch, identify each outcome of the Sample Space and its probability.

b

Write the pmf (probability mass function) of *X*.

#### 4.2

Refer to Practice Problem 3.17.

21 balls are in a basket: 10 are red, 6 are green, 3 are yellow, and 2 are orange. Four balls will be drawn out of the basket all at once at random without replacement. X = # of orange balls observed. Find the pmf (probability mass function) of X.

#### 4.3

Refer to Practice Problem 3.12.

An unfair coin has P(H) = .68. The coin will be flipped three times. X = # of heads. Find the pmf (probability mass function) of X.

## 4.4

A discrete random variable named *X* has the following pmf (probability mass function):

х	p(x)		
1	.60		
3	.20		
7	.10		
11	.10		

а

Find P(X > 6)

b

What is the probability two independent observations of *X* will both equal 1?

C

Find the population mean  $\mu$ , also known as E(X) = the expected value of X

d

Find the population variance of *X* with both the "regular" formula and "convenient" formula:

$$\sigma^2 = \sum (x - \mu)^2 \cdot p(x) = E(x^2) - \mu^2$$

Make a table and show your work. You should get the same answer with both formulas. Finally, find the standard deviation.

#### 4.5

Refer to Practice Problem 3.13.

If X = the outcome of one roll of the unfair die, find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ .

#### 4.6

A large group of people need to have their blood tested to determine whether or not they have Disease X. Instead of testing everyone individually, people are placed into groups of 10. The blood samples of the 10 people will be pooled and analyzed together. If the test is negative then we conclude 0 of the 10 people have the disease and no more testing is required. If the test is positive then we conclude at least one person in the group has the disease and all 10 people must then be individually tested, for a total of 11 tests. Let X = 1 the total number of tests needed for a group. What are the possible values of X? Find E(X) if the probability any person has the disease is p, then find E(X) if p = 0.02.

## 4.7

Refer to Practice Problems 3.13 and 4.5.

а

Suppose the integers { 1, 2, 3, 4, 5, 6 } were removed from the six faces of the unfair die and replaced with { 11, 12, 13, 14, 15, 16 }. Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$  and compare these values to what you found earlier.

b

Suppose the integers { 1, 2, 3, 4, 5, 6 } were removed from the six faces of the unfair die and replaced with { 10, 20, 30, 40, 50, 60 }. Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$  and compare these values to what you found earlier.

Refer to Practice Problem 4.3

An unfair coin has P(H) = .68. The coin will be flipped three times. X = # of heads. Is X a Binomial random variable? Explain why or why not. If it is, identify the values of the parameters n and p, then find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ .

## 4.9

Julian is shooting arrows at a target in the backyard. The probability he hits the bulls-eye is 0.35 on every attempt, and all of his attempts are independent of each other. X = # of successful bulls-eyes out of n = 30 attempts.

```
a
Find P(X = 9)
b
Find P(X = 12)
```

:

Find  $P(X \ge 16)$ . Express your answer using summation notation, then use a calculator or computer to find the answer.

d Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ 

# 4.10

An eight-sided die has eight faces with the integers { 1, 2, 3, 4, 5, 6, 7, 8 } equally likely to appear when the die is rolled.

a If the die is rolled five times, what is the probability an "8" appears exactly two times?

b
If the die is rolled six times, what is the probability an even number appears more than four times?

c If the die is rolled seven times, what is the probability a "6", "7", or "8" appears on exactly four of the seven rolls?

Jeremiah and Crystal are basketball players who like to compete against each other. They are about to attempt 10 free throw shots each. Jeremiah's probability of a successful free throw attempt is always .74. Crystal's probability of a successful free throw attempt is always .84. Jeremiah and Crystal are arguing about which event is more likely:

Jeremiah will have 8 or more successful free throws out of 10 attempts. Crystal will have 9 or more successful free throws out of 10 attempts. Who is correct?

#### 4.12

Refer to Practice Problem 4.9

Every day for a year, Julian shoots arrows at the target and writes down how many attempts were needed to hit the bulls-eye for the first time. At the end of the year, Julian has a data set with 365 observations.

a
Approximately what proportion of the 365 observations will be zeros? Ones?
Twos?

b

If Julian calculates  $\overline{X}$ ,  $s^2$ , and s for the data set, they will be approximately equal to what values?

#### 4.13

Jodie is very talented at throwing darts. The probability she successfully hits the bulls-eye equals .79. Assume that this probability of success never changes and that all of her throws are independent of each other. Jodie will begin throwing darts at the bulls-eye.

a
What is the probability Jodie's first success will occur on her third attempt?

b What is the probability Jodie's first success will <u>not</u> occur on her first or second attempt?

Refer to Practice Problem 4.11

Now Jeremiah and Crystal will each attempt free throws until they observe their first success. They are arguing about which event is more likely: Jeremiah's first success will occur on his first or second attempt. Crystal's first success will occur on her first attempt. Who is correct?

#### 4.15

Optional Challenge Problem:

An unusual vending machine works like this: if you insert \$1 into the machine, it will give you a playing card at random from the standard 52-card deck. Cards are dispensed "with replacement", so for example it is possible the vending machine could give you the Ace of Spades two times in a row. Suppose you are going to stand at this vending machine and insert \$1 bills until you have a complete deck of all 52 different cards. The random variable X = 0 the # of \$1 bills needed to acquire a complete deck.

Easy question: what are the possible values of *X*? Difficult question: what is the expected value of *X*?

#### 4.16

Refer to Practice Problem 4.10

a

What is the probability the second occurrence of a "7" will occur on the 5<sup>th</sup> roll of the eight-sided die?

b

What is the probability the fourth occurrence of an even number will occur on the 8<sup>th</sup> roll of the eight-sided die?

C

Suppose every time a 3, 4, or 5 is rolled, you win a prize. What is the probability the you will be awarded your third prize on the 7<sup>th</sup> roll of the eight-sided die?

Jeff is friends with Jodie (refer to Practice Problem 4.13) and he hits the bulls-eye with probability .40. Assume that this probability of success never changes and that all of his throws are independent of each other. Jeff will begin throwing darts at the bulls-eye.

а

What is the probability Jeff's fourth success will occur on his tenth attempt?

b

What is the probability Jeff will need to throw 4 or more darts in order to observe his second bulls-eye?

#### 4.18

Refer to Practice Problem 3.12.

An unfair coin has P(H) = .68. Suppose the coin will be flipped until heads is observed for the 100<sup>th</sup> time. X = # of flips needed. Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ 

#### 4.19

X = the # of parking tickets a campus parking officer writes on any particular day. X is a Poisson random variable with a mean of  $\lambda = 9.3$  tickets per day.

а

What is the probability the parking officer will write exactly 7 tickets tomorrow?

b

Every day for a year, the parking officer writes down how many tickets he wrote that day. At the end of the year, he has a data set with 365 observations. If he calculates  $\overline{X}$ ,  $s^2$ , and s for the data set, they will be approximately equal to what values?

Mrs. Hopkins enjoys watching hummingbirds approach her hummingbird feeder on a summer afternoon. X = the # of hummingbirds that approach the feeder per minute. Assume X is a Poisson random variable with a mean of  $\lambda = 4.7$  hummingbirds per minute.

а

What is the probability exactly 3 hummingbirds will approach her feeder within the next minute?

b

What is the probability more than 2 hummingbirds will approach her feeder within the next minute?

#### 4.21

An island in the South Pacific experiences frequent major earthquakes. X = the # of major earthquakes per 10-year period.  $X \sim POI(\lambda = 3.16)$ 

а

What is the probability there will be at least one earthquake within the next year? <u>Hint</u>: notice this question is asking about a one-year period, not a 10-year period.

b

What is the probability there will be fewer than five earthquakes within the next 15 years?

#### 4.22

Refer to Practice Problem 4.2.

What type of discrete random variable is X = # of orange balls observed? Identify the values of the parameter(s), then find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ .

A large classroom has 68 students: 19 are biology majors, 24 are chemistry majors, and 25 are math majors.

а

If ten students are randomly selected all at once and without replacement, what is the probability exactly 4 are math majors?

b

If three students are randomly selected all at once and without replacement, what is the probability they all have the same major?

#### 4.24

A basket contains 5 orange, 6 yellow, and 7 white balls. Identify each of the following random variables as Binomial, Geometric, Negative Binomial, or Hypergeometric. Also, identify the values of the parameter(s).

a Draw 5 balls all at once at random without replacement. A = # of white balls.

Draw 5 balls one at a time at random with replacement. B = # of white balls.

c Draw balls from the basket one at a time at random with replacement until you see the first yellow ball. C = # of draws needed.

d Draw balls from the basket one at a time at random with replacement until you see the third yellow ball. D = # of draws needed.

e Draw balls from the basket one at a time at random with replacement until you see a ball that isn't orange. E = # of draws needed.

f Draw 10 balls all at once at random without replacement. F = # of balls that aren't orange.

Refer to Practice Problem 4.4.

а

Write the moment generating function of X.

b

Use M(t) to find the first and second moments of X and confirm you get the same answers as Practice Problem 4.4.

С

What is the probability seven independent observations of X will add up to 33? Advice: use Mathematica.

# **Unit 05 Practice Problems**

## 5.1

A continuous random variable *X* has the following pdf:

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{2}{3} & 0 < x < 3\\ 0 & otherwise \end{cases}$$

а

Draw the pdf. Confirm the pdf integrates to 1. If you do not have calculus experience, use geometry.

b

Find P(1 < X < 3). If you do not have calculus experience, use geometry.

#### 5.2

A continuous random variable *X* has the following pdf:

$$f(x) = \begin{cases} -2x + 4 & 1 < x < 2 \\ 0 & otherwise \end{cases}$$

а

Draw the pdf. Confirm the pdf integrates to 1. If you do not have calculus experience, use geometry.

h

Find P(1 < X < 1.4). If you do not have calculus experience, use geometry.

## 5.3

A continuous random variable *X* has the following pdf:

$$f(x) = \begin{cases} \frac{1}{4}x^3 & 0 < x < 2\\ 0 & otherwise \end{cases}$$

а

Draw the pdf. Confirm the pdf integrates to 1. Calculus required.

b

Find P(0.9 < X < 1.2). Calculus required.

Refer to Practice Problem 5.1. Suppose a data set is made up of n=1000 independent observations of X.

а

If  $\overline{X}$ ,  $s^2$ , and s were calculated for the data set, they will be approximately equal to what values? Calculus required.

h

Approximately how many of the 1000 observations would you expect to be greater than 1?

#### 5.5

Refer to Practice Problem 5.2. Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ . Calculus required.

#### 5.6

Refer to Practice Problem 5.3. Find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ . Calculus required.

#### 5.7

X = circumference of a randomly selected baseball (inches)  $X \sim UNIF(9.0,9.1)$ 

а

What is the probability a randomly selected baseball has a circumference greater than 9.035 inches?

b

Find the population mean, variance, and standard deviation of *X*.

#### 5.8

An artist has a large collection of colored pencils. If one colored pencil is selected at random, its length is a Uniformly distributed random variable on the interval 6 inches to 7.2 inches. What is the probability a randomly selected colored pencil is between 6.3 and 6.7 inches long?

Emma is watching a severe thunderstorm. X = the waiting time (seconds) until the next lightning strike. X is Exponentially distributed with a mean waiting time of 5 seconds.

а

Since X is Exponentially distributed with a mean waiting time of 5 seconds, what is the value of  $\lambda$ ?

b

What is the variance and standard deviation among the waiting times?

С

What is the probability Emma will observe the next lightning strike within the next 7 seconds?

d

What is the probability Emma will not observe any lightning strikes within the next 10 seconds?

#### 5.10

A radioactive substance is rapidly emitting particles.  $X_{POI}$  = the # of particles emitted per second.  $X_{POI}$  is a Poisson random variable with a mean of  $\lambda = 2.5$  particles per second. Therefore, the waiting time until the next particle emission is Exponentially distributed.  $X_{EXP}$  is Exponential with a mean waiting time of  $\frac{1}{\lambda} = \frac{1}{2.5} = 0.4$  seconds.

а

What is the probability we will wait at least one second until the next particle is emitted?

h

What is the probability the next particle will be emitted sometime between 0.28 and 0.35 seconds from now?

С

Given there are no particle emissions in the next 0.20 seconds, what is the conditional probability there will be no particle emissions in the next 0.26 seconds?

d

Find the probability there will be no particle emissions for the next 0.06 seconds and compare it to your answer to the previous question. The Exponential random variable has a <u>Memoryless Property</u>:

$$P(X_{EXP} > t + t_0 \mid X_{EXP} > t_0) = P(X_{EXP} > t)$$

е

Do you drive a new car or an old car? Now that you know about the <u>Memoryless Property</u> of the Exponential random variable, do you believe the lifetimes of cars are Exponentially distributed? Why or why not? Discuss.

## 5.11

Write the pdf of  $N(\mu = 10, \sigma^2 = 4)$ .

## 5.12

Identify the values of  $\mu$  and  $\sigma^2$  of a Normal random variable with the following pdf:

$$f(x) = \begin{cases} \frac{1}{9\sqrt{2\pi}} e^{-\frac{(x+5)^2}{162}} & \text{if } -\infty < x < \infty \end{cases}$$

#### 5.13

X = weight of a randomly selected 12-year-old American boy  $X \sim N(\mu = 40 \text{ kg}, \sigma = 2.1 \text{ kg})$ 

а

Use the Empirical Rule to find an interval of weights where approximately 68% of the boys have a weight in the interval.

b

Use the Empirical Rule to find an interval of weights where approximately 95% of the boys have a weight in the interval.

С

Use the Empirical Rule to find an interval of weights where approximately 99.7% of the boys have a weight in the interval.

```
5.14
```

$$Z \sim N(\mu = 0, \sigma = 1)$$

Draw pictures and find each of the following probabilities:

a P(Z < -0.68)

b P(Z > -1.35)

c P(-1.24 < Z < 0.87)

d P(Z < 1.79)

e P(Z > 0.37)

f P(-2.05 < Z < 0)

# 5.15

$$Z \sim N(\mu = 0, \sigma = 1)$$

Draw pictures and find each of the following values:

а

Find the value of a if P(Z < a) = .9906

b

Find the value of *b* if P(Z > b) = .0708

C

Find the value of *c* if P(-c < Z < c) = .3830

d

Find the value of *d* if P(d < Z < 0.82) = .7271

e

Find the  $48^{th}$  percentile on the Z curve. What is the  $52^{nd}$  percentile?

f

Find the 89<sup>th</sup> percentile on the *Z* curve. What is the 11<sup>th</sup> percentile?

X = weight of a randomly selected 12-year-old American boy  $X{\sim}N(\mu=40~{\rm kg},\sigma=2.1~{\rm kg})$  Draw pictures.

a 
$$P(X < 38.53) = ?$$

С

What is the 91st percentile among the 12-year-old boy's weights?

#### 5.17

Engineers have designed a process to fill plastic milk jugs. X = amount of milk dispensed into a plastic jug (gallons)  $X \sim N(\mu = 1.02, \sigma = 0.01)$  Draw pictures.

а

If a milk jug is considered underfilled if it contains less than 1 gallon of milk, what proportion of milk jugs will be underfilled?

b

Milk jugs should not be overfilled because that would reduce profitability and possibly threaten the capacity of the milk jugs themselves. If a milk jug contains more than 1.0436 gallons, it is considered overfilled. What proportion of milk jugs will be overfilled?

С

61% of the milk jugs will contain more than k gallons of milk. What is the value of k?

Three QQ Plots were generated using three different data sets. Choose exactly one of the following descriptions for each data set:

The data set appears to be normally distributed.

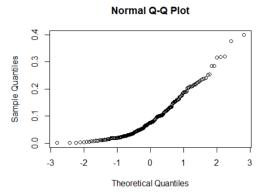
The data set appears to be heavy tailed.

The data set appears to be weak tailed.

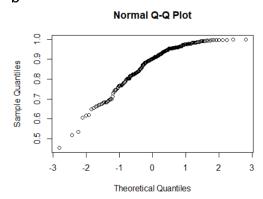
The data set appears to be right-skewed.

The data set appears to be left-skewed.

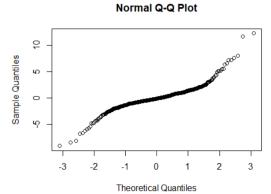








#### С



$$GAMMA(\alpha = 4, \beta = 3)$$

а

Write the pdf of  $GAMMA(\alpha = 4, \beta = 3)$ , simplifying it as much as possible.

b

If  $X \sim GAMMA(\alpha = 4, \beta = 3)$ , use a calculator or computer to find P(X > 10).

C

If  $X \sim GAMMA(\alpha = 4, \beta = 3)$ , find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ .

#### 5.20

Identify the values of  $\alpha$  and  $\beta$  of a Gamma random variable with the following pdf:

$$f(x) = \begin{cases} \frac{x^7 e^{-x}}{5040} & \text{if } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

## 5.21

$$X \sim BETA(\alpha = 1, \beta = 4)$$

а

Write the pdf of  $X \sim BETA(\alpha = 1, \beta = 4)$ , simplifying it as much as possible.

b

If  $X \sim BETA(\alpha = 1, \beta = 4)$ , find  $P\left(X > \frac{1}{2}\right)$ . This integral can be evaluated by hand using Integration by Substitution. It is also acceptable to use a calculator or computer.

С

If  $X \sim BETA(\alpha = 1, \beta = 4)$ , find  $\mu$ ,  $\sigma^2$ , and  $\sigma$ .

A continuous random variable X has the following moment generating function. Identify the name of the random variable and the value of its parameter(s).

$$M(t) = \frac{e^{67t} - e^{22t}}{45t}$$

# 5.23

A continuous random variable X has the following moment generating function. Identify the name of the random variable and the value of its parameter(s).  $M(t) = (1-21t)^{-7}$ 

# 5.24

A continuous random variable *X* has a moment generating function of

$$M(t) = 4(t-2)^{-2} = \frac{4}{(t-2)^2}$$

Find E(X) and Var(X)

# **Unit 06 Practice Problems**

## 6.1

Refer to Practice Problem 5.16.

X = weight of a randomly selected 12-year-old American boy

$$X \sim N(\mu = 40 \text{ kg}, \sigma = 2.1 \text{ kg})$$

Instead of randomly selecting one boy, suppose more than one boy is randomly selected and the sample mean weight among them is calculated.

а

If n = 4, find  $P(\overline{X} > 41)$ . Draw a picture.

b

If n = 8, find  $P(\overline{X} > 41)$ . Draw a picture.

# 6.2

Refer to Practice Problem 5.17.

Engineers have designed a process to fill plastic milk jugs.

*X* = amount of milk dispensed into a plastic jug (gallons)

$$X \sim N(\mu = 1.02, \sigma = 0.01)$$

Instead of randomly selecting one milk jug, suppose more than one milk jug is randomly selected and the sample mean among them is calculated.

а

What is the probability the sample mean amount of milk among n=7 randomly selected milk jugs will be less than 1.01 gallons? Draw a picture.

b

What is the probability the sample mean amount of milk among n=3 randomly selected milk jugs will be between 1.015 and 1.022 gallons? Draw a picture.

Refer to Practice Problem 5.7.

X = circumference of a randomly selected baseball (inches) $X \sim UNIF(9.0,9.1)$ 

Instead of randomly selecting one baseball, suppose more than one baseball is randomly selected and the sample mean circumference among them is calculated.

а

If n = 30, describe the approximate distribution of the sample mean.

b

If n = 30, approximate  $(9.04 < \overline{X} < 9.06)$ . Draw a picture.

С

If you were to continue approximating  $P(9.04 < \overline{X} < 9.06)$  with larger and larger sample sizes, do you think the probability would increase or decrease?

#### 6.4

Refer to Practice Problem 5.19. Also refer to the Famous Continuous Random Variables page of the Super Handout for the population mean, variance, and standard deviation of the random variable named Gamma.

$$GAMMA(\alpha = 4, \beta = 3)$$

а

If n = 34, describe the approximate distribution of the sample mean.

b

If n = 34, approximate  $(11.8 < \overline{X} < 13.2)$ . Draw a picture.

# **Unit 07 Practice Problems**

## 7.1

If someone wanted to compute a 76.2% confidence interval for  $\mu$  when  $\sigma$  is known, what value should they use for  $Z_{\frac{\alpha}{2}}$  while using the formula  $\overline{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ ?

## 7.2

If someone wanted to compute a 87.4% confidence interval for  $\mu$  when  $\sigma$  is known, what value should they use for  $Z_{\frac{\alpha}{2}}$  while using the formula  $\overline{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ ?

## 7.3

A population of moose lives in a national park. A random sample of n=35 moose has a sample mean weight of  $\overline{X}=944$  pounds. Based on data from previous studies, the biologists assume  $\sigma$  is known to be 92 pounds.

a Compute a 90% confidence interval for  $\mu$  = the population mean weight among all the moose living in the national park. What assumptions, if any, are required?

b What if the error margin is considered too large and an error margin of 20 pounds is desired at 90% confidence? What sample size is necessary?

## 7.4

An automotive engineer is interested in the population of all vehicles that have an engine of size of 2.0 L or smaller, and is particularly interested in  $\mu$  = the population mean mpg (miles per gallon) among these vehicles. The sample mean among a random sample of n=12 vehicles is 36.1 mpg and  $\sigma$  is known to be 4.4.

a Compute the 95% CI for  $\mu$ . What assumptions, if any, are required?

b What if the error margin is considered too large and an error margin of 1.6 mpg is desired at 95% confidence? What sample size is necessary?

Draw pictures:

а

On a *T* curve with 19 degrees of freedom, find the area to the right of 2.093.

b

On a T curve with 8 degrees of freedom, find the area between -1.397 and 1.397.

c

How many degrees of freedom does a *T* curve have if the area to the left of 2.763 equals 0.995?

d

How many degrees of freedom does a T curve have if the area between -2.160 and 2.160 equals 0.95?

## 7.6

If someone wanted to compute a 98% confidence interval for  $\mu$  when  $\sigma$  is unknown and n=23, what value should they use for  $t_{\frac{\alpha}{2}}$  while using the formula

$$\overline{X} \pm t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$
?

## 7.7

If someone wanted to compute a 90% confidence interval for  $\mu$  when  $\sigma$  is unknown and n=13, what value should they use for  $t_{\frac{\alpha}{2}}$  while using the formula

$$\overline{X} \pm t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$
?

## 7.8

Nutrition scientists randomly select n=10 students at a university and record how many calories they consume per day on average. The data set has a sample mean of  $\overline{X}=2681$  and a sample standard deviation of s=241.  $\sigma$  is unknown. Compute a 95% confidence interval for  $\mu$  = the population mean daily calorie consumption among all students at the university. What assumptions, if any, are required?

A medical researcher is studying how much weight women gain during pregnancy.  $\mu$  = population mean weight gain during pregnancy. She randomly selects n=30 women who recently gave birth and recorded how much weight they gained during pregnancy. The sample mean is  $\overline{X}=27.6$  pounds and the sample standard deviation is s=7.93 pounds.  $\sigma$  is unknown. Compute a 90% confidence interval for  $\mu$ . What assumptions, if any, are required?

## 7.10

In a large shipment of oranges, some oranges have seeds and some are seedless. You are interested in p = the proportion of oranges that are seedless. You randomly select 100 oranges and cut them open. 81 are seedless.

a Compute a 99% confidence interval for the proportion of oranges in the entire shipment that are seedless.

b

What if the error margin is considered too large and an error margin of .03 is desired at 99% confidence? What sample size is necessary?

#### 7.11

Forty high school students who recently earned their driver's license were randomly selected and 31 said they always wear a seat belt while driving.

a Use this data to compute a 95% confidence interval for p = the proportion of all recently licensed high school drivers who always wear a seat belt.

b What if the confidence level had been 90% instead of 95%? Would the confidence interval have been wider or thinner?

What if 310 out of 400 high school students said they always wear a seat belt instead of 31 out of 40? Would the 95% confidence interval have been wider, thinner, or the same width as the one you computed in part a?

d What if the error margin is considered too large and an error margin of .09 is desired at 95% confidence? What sample size is necessary?

# **Unit 08 Practice Problems**

## 8.1

Phil is driving his car and is half way to Jake's house and realizes he may have forgotten the homemade blueberry pie he promised to bring. If he decides he forgot the pie, he will turn around and go back home to look for it. If he decides the pie is somewhere in his car, he will continue driving to Jake's house. Phil is faced with the following hypothesis test:

H<sub>0</sub>: Phil forgot the pie

Ha: Phil did not forget the pie

Describe each of the following situations as a Type 1 error, Type 2 error, or no error.

а

Phil goes back home to look for the pie, but he didn't forget it after all. It was in his car the whole time.

h

Phil goes back home, finds the pie, then brings it to Jake's house.

C

Phil continues driving to Jake's house and shows up without a pie.

Ч

Phil continues driving to Jake's house and shows up with a pie. He didn't forget it after all.

## 8.2

Jennifer gives you a coin and tells you it is either a fair coin with P(H) = .50 or an unfair coin with P(H) = .80. You will conduct the following hypothesis test:

```
H_0: P(H) = .50
H_a: P(H) = .80
```

You will flip the coin four times and reject H<sub>0</sub> iff you observe 3 or 4 heads.

```
a Find \alpha = P(\text{Type 1 error})
```

b

Find  $\beta = P(\text{Type 2 error})$  and the Power of the test.

The null hypothesis, alternative hypothesis, and significance level for three hypothesis tests are given below. All three are a one-sample hypothesis test for  $\mu$  with known  $\sigma$ . Draw pictures of the rejection regions.

a H<sub>0</sub>:  $\mu = 7$  H<sub>a</sub>:  $\mu \neq 7$   $\alpha = .0394$  b H<sub>0</sub>:  $\mu = 21.5$  H<sub>a</sub>:  $\mu > 21.5$   $\alpha = .0301$  c H<sub>0</sub>:  $\mu = 9.004$  H<sub>a</sub>:  $\mu < 9.004$   $\alpha = .0166$ 

## 8.4

According to a wildlife survey conducted in Antarctica ten years ago, the mean height among all adult Emperor penguins was 110 cm. A biologist named Sarah wants to know if the mean height is different now. According to new data collected last month, a random sample of 32 adult Emperor penguins had a sample mean height of 110.6 cm. The population standard deviation is known to be 4.5 cm. Conduct all steps of a hypothesis test with  $\alpha = .0784$ . What assumptions, if any, are required to do this hypothesis test?

## 8.5

Ceramic bars are designed to have a population mean fracture strength of 90 MPa, however engineers studying the ceramic bars suspect the population mean is actually less than 90 MPa. A random sample of 20 ceramic bars has a sample mean fracture strength of 85.32 MPa, and the population standard deviation is known to be 10.3. Conduct all steps of a hypothesis test with  $\alpha = .0465$ . What assumptions, if any, are required to do this hypothesis test?

A social scientist named Ashley reads that in 1960, the mean age among American men at first marriage was 25.3 years. She wants to know if the mean age is greater now. Ashley doesn't know  $\mu$  or  $\sigma$  and takes a random sample of 20 American men who got married for the first time last month and records how old they were on their wedding day. The sample mean is 29.1 years and the sample standard deviation is 6.15. Conduct all steps of a hypothesis test with  $\alpha = .025$ . What assumptions, if any, are required to do this hypothesis test?

## 8.7

Over the course of several weeks Mitchell has trained his lab rats to run through a maze. At the end of the maze, the rats are rewarded with breakfast. It has been established that his population of rats complete the maze in an average of 10.00 seconds. Mitchell wonders if the time would be significantly less if he had the rats run the maze during a different time of day. As an experiment, he randomly selects 17 of the rats and has them run the race immediately after he feeds them dinner. n=17,  $\overline{X}=9.52$ , s=1.4 ( $\sigma$  is unknown). Conduct all steps of a hypothesis test with  $\alpha=.05$ . What assumptions, if any, are required to do this hypothesis test?

## 8.8

The President of Big State University claims that 75% of all students at the university are registered to vote. You suspect the true proportion is less than 0.75 and take a random sample. 56 out of 80 randomly selected students are registered to vote. Conduct all steps of a hypothesis test with  $\alpha = .01$ .

## 8.9

Many years ago, the proportion of citizens of Japan who were over 65 years old was 0.16. Demographers would like to know if the proportion is greater now. A random sample of n=120 Japanese citizens yielded 30 who were over 65. Conduct all steps of a hypothesis test with  $\alpha=.025$ .

A random sample of size n=49 will be taken from  $Y \sim N(\mu, \sigma=14)$  where  $\mu$  is unknown. Daisy will do a one-sample Z-test.

$$H_0$$
:  $\mu = 61$   
 $H_a$ :  $\mu \neq 61$ 

а

If  $\alpha = .1212$ , draw a picture of the rejection region on the appropriate curve and identify the critical value(s).

b

If the actual value of  $\mu$  equals 62, find the probability of Type 2 error and the Power of the test.

## 8.11

A random sample of size n=16 will be taken from  $Y \sim N(\mu, \sigma=28)$  where  $\mu$  is unknown. Samantha will do a one-sample Z-test.

$$H_0$$
:  $\mu = 90$   
 $H_a$ :  $\mu < 90$ 

а

If  $\alpha = .0274$ , draw a picture of the rejection region on the appropriate curve and identify the critical value(s).

b

If the actual value of  $\mu$  equals 85, find the probability of Type 2 error and the Power of the test. If the sample size is increased, what will happen to these probabilities? Try again with n=196.

## **Unit 12 Practice Problems**

Ollie's Organic Ice Cream, Inc. has made millions of dollars creating high quality ice cream products at their facility and then shipping them all over the world. They have hired you (an engineer with statistics expertise) to create five control charts for various processes at their facility. The data is located in an Excel file available on Canvas.

#### 12.1

A machine at Ollie's factory adds a small amount of peanut bits to the top of each Ollie's Peanut Crunch Ice Cream Cone. Of the hundreds of cones produced per day, five are selected at random and the sample mean amount of peanut bits is measured (in grams). Workers are concerned that the machine has become erratic within the last week. You have been provided with data for the last 40 days. Use the first 20 samples as the base period and draw an  $\overline{X}$  and  $s^2$  chart chart with the data. What assumptions about the data are you required to make? Does this process appear to be in control? Comment.

#### 12.2

Ollie's stores its ice cream products in a large industrial freezer as it awaits shipment. The temperature of the freezer must be closely monitored. If the freezer runs at too warm of a temperature, the ice cream will melt. If the freezer runs at too cold of a temperature, then Ollie's will spend too much on energy costs. The temperature (in °C) is recorded once an hour. Data from the last 100 hours has been provided. Use the first 25 observations as the base period and draw an *X* chart with the data. What assumptions about the data are you required to make? Does this process appear to be in control? Comment.

#### 12.3

Ollie's uses hundreds of eggs per day for its ice cream recipes and raises its own free-range egg-laying chickens on site. Conveyor belts and other machinery carefully transport 2000 eggs per day from one building at the facility to another. Inevitably, some of the eggs arrive cracked or smashed and are unusable. Ollie's wants to monitor the number of defective eggs and if it gets out of control, they will consider redesigning the egg delivery process. Data from the last 80 days has been provided. Use the first 20 observations as the base period and draw an np chart with the data. What assumptions about the data are you required to make? Does this process appear to be in control? Comment.

Ollie's most popular ice cream flavor is Cheerful Cherry Explosion because it has a bountiful amount of fresh cherries in every half-gallon container. Ollie's randomly selects one container per day for inspection and counts the number of cherries inside. Data from the last 90 days has been provided. Use the first 30 observations as the base period and draw a c chart with the data. What assumptions about the data are you required to make? Does this process appear to be in control? Comment.

# **Unit 09 Practice Problems**

## 9.1

A nutrition scientist owns some chickens. She selects one chicken and randomly selects six of her eggs. She measures two variables on each egg: X = mass of egg (grams) and Y = amount of protein in the egg (grams).

Х	Υ
60	6.3
60	6.4
65	6.8
67	6.6
67	6.9
68	6.9

а

Draw the scatterplot by hand or have a computer do it for you.

b

Make a table and use it to find  $\sum x^2$ ,  $\sum y^2$ , and  $\sum xy$ , then compute  $SS_{xx}$ ,  $SS_{yy}$ ,  $SS_{xy}$ , and r.

C

Compute the slope and intercept of the least-squares regression line. For this particular chicken, what is the predicted average protein content among eggs with a mass of 63 grams? Draw the least-squares regression line on your scatterplot by hand or have a computer do it for you.

d

Compute SS<sub>total</sub>, SSR, and SSE. Find the coefficient of determination and report what it means regarding these two variables.

е

Complete all steps of a hypothesis test for the true slope asking whether or not it equals zero ( $\alpha=.05$ ). Also write a simple linear regression ANOVA table with the data.

An agricultural scientist studying apple trees notices a positive association between  $X = trunk \ circumference$  (inches) and  $Y = apple \ yield$  (pounds). He randomly selects 5 apple trees and records X and Y for each:

Х	Υ
39	60
40	58
40	60
42	68
44	64

а

Draw the scatterplot by hand or have a computer do it for you.

b

Make a table and use it to find  $\sum x^2$ ,  $\sum y^2$ , and  $\sum xy$ , then compute  $SS_{xx}$ ,  $SS_{yy}$ ,  $SS_{xy}$ , and r.

С

Compute the slope and intercept of the least-squares regression line. What is the predicted average apple yield among trees with a trunk circumference of 41 inches? Draw the least-squares regression line on your scatterplot by hand or have a computer do it for you.

d

Compute SS<sub>total</sub>, SSR, and SSE. Find the coefficient of determination and report what it means regarding these two variables.

е

Complete all steps of a hypothesis test for the true slope asking whether or not it equals zero ( $\alpha = .05$ ). Also write a simple linear regression ANOVA table with the data.

One hundred Australians were randomly selected and two variables were recorded: X = foot length (inches) and Y = shoe length (inches). The data is located in a file available on Canvas named footshoe.csv. Use R to complete a simple linear regression model.