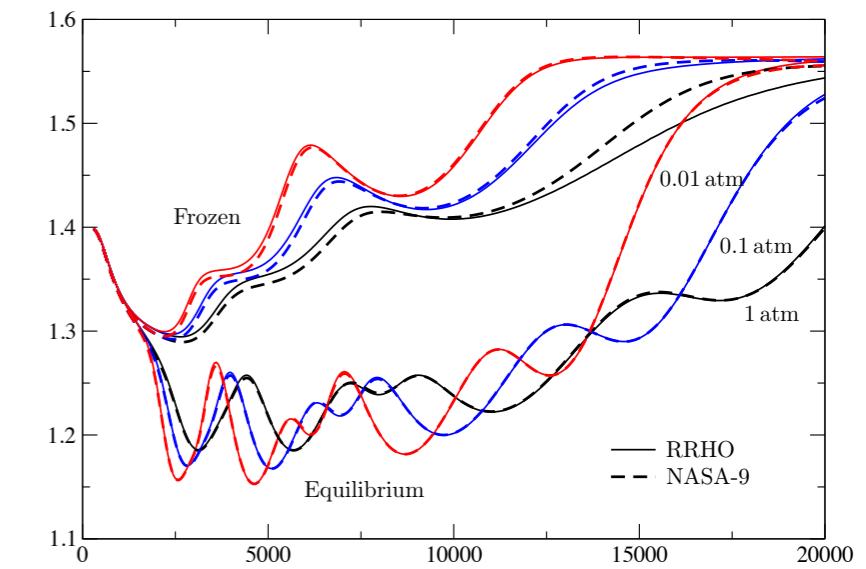
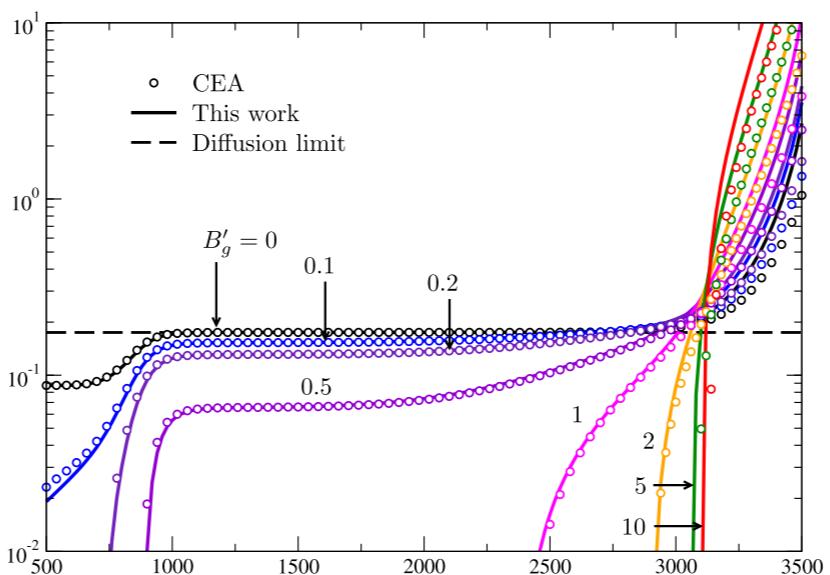
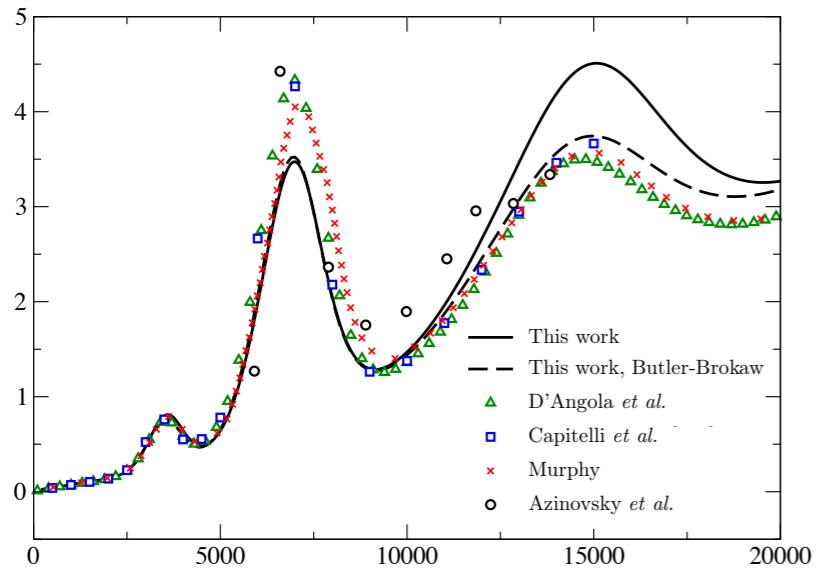


M⁺⁺ Mutation

Multicomponent Thermodynamic And Transport properties for IONized gases in C++



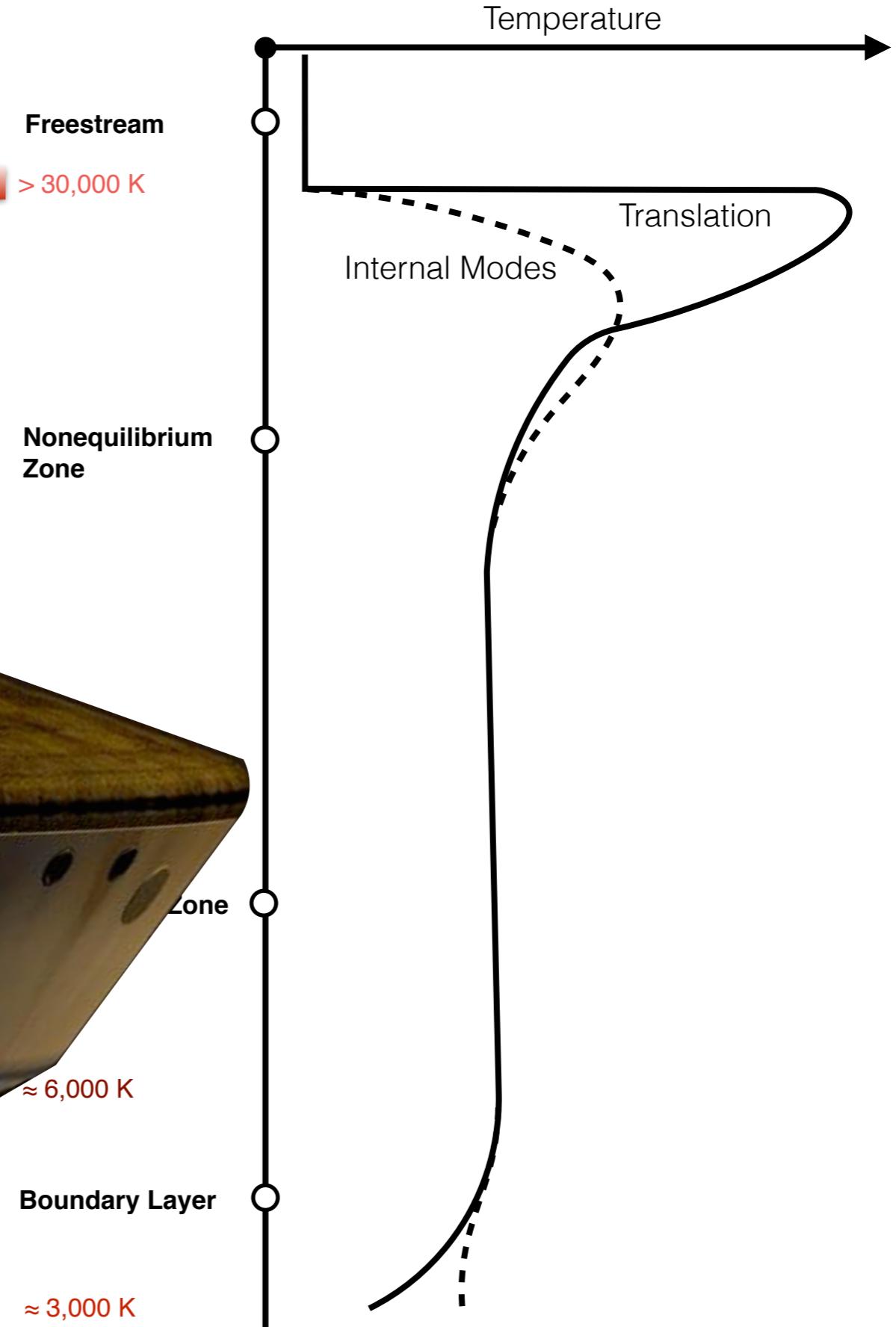
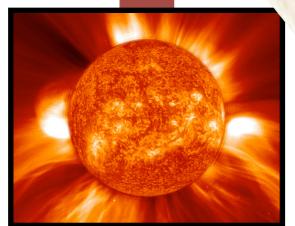
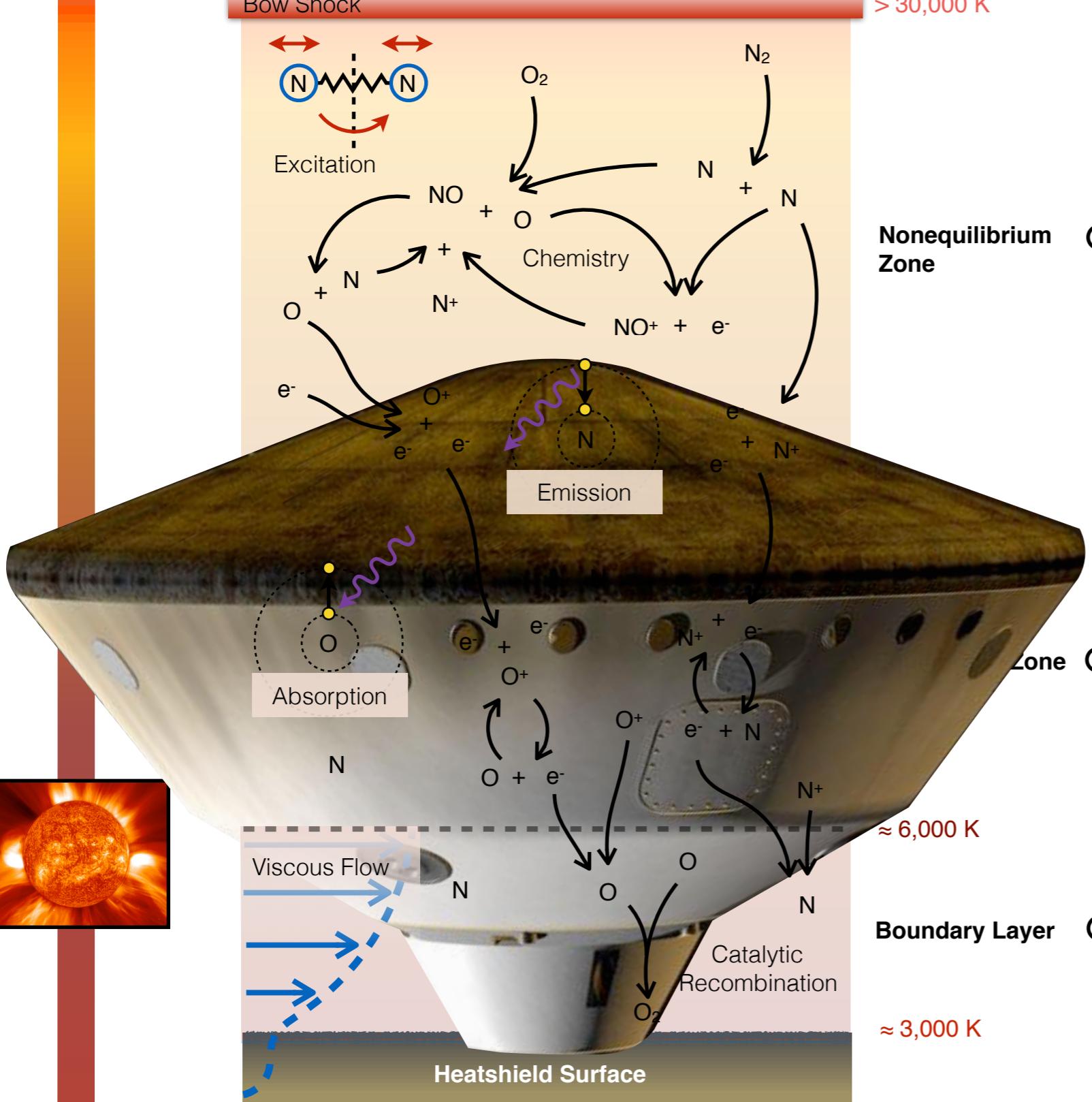
Presented by J.B. Scoggins, Georgios Bellas-Chatzigeorgis



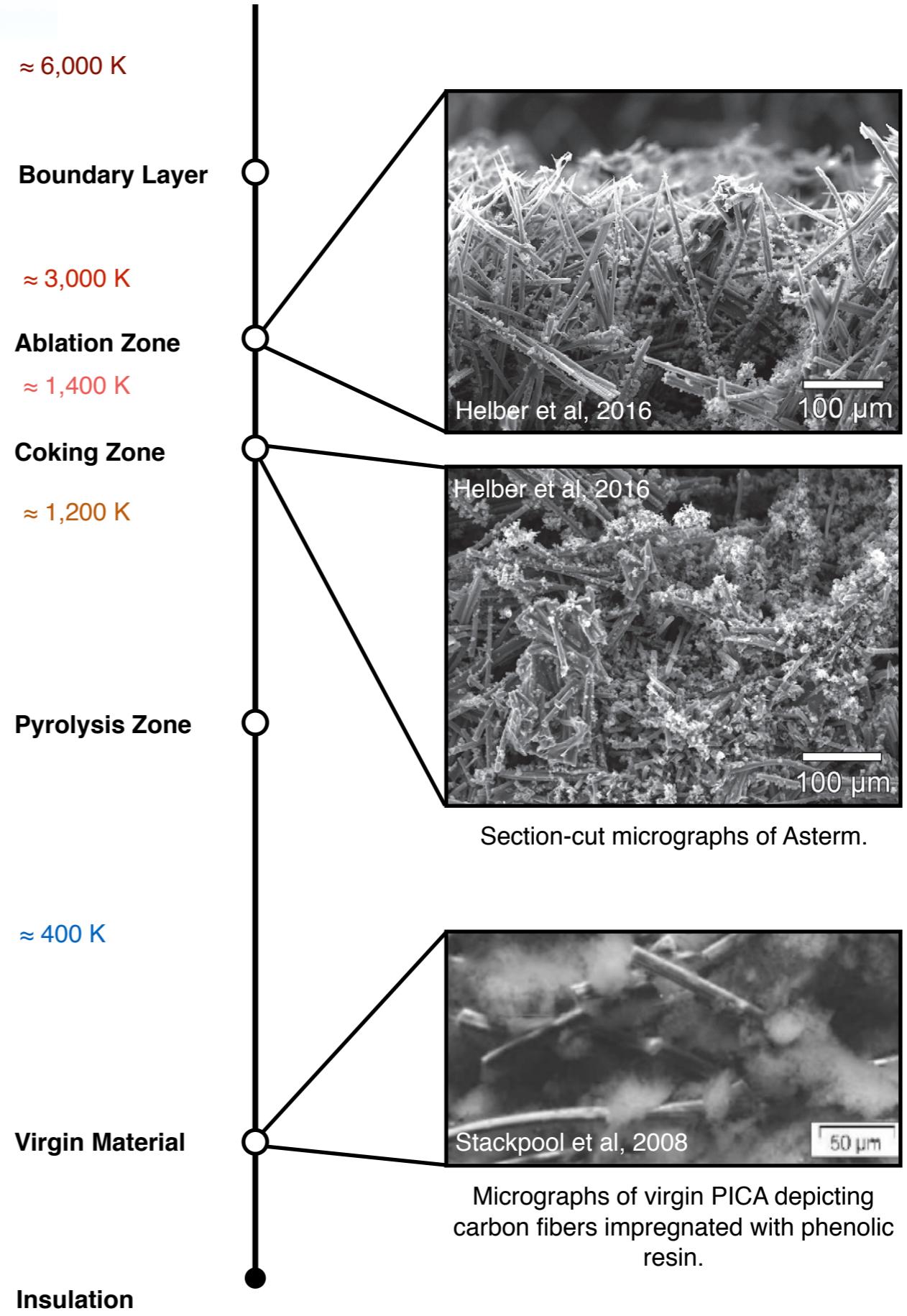
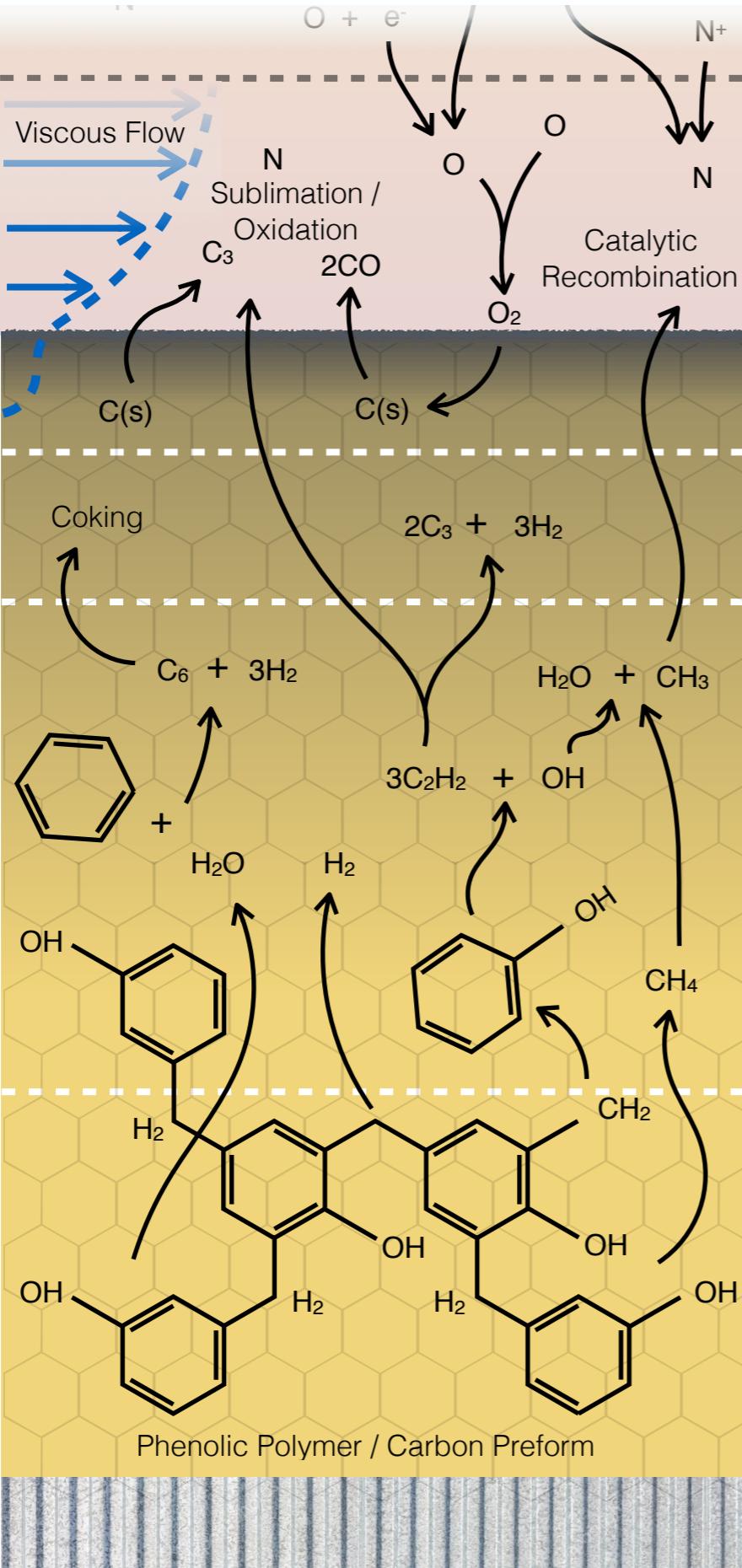
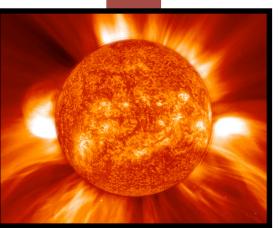
An improvisational feel...



Atmospheric entry physics (Earth entries)

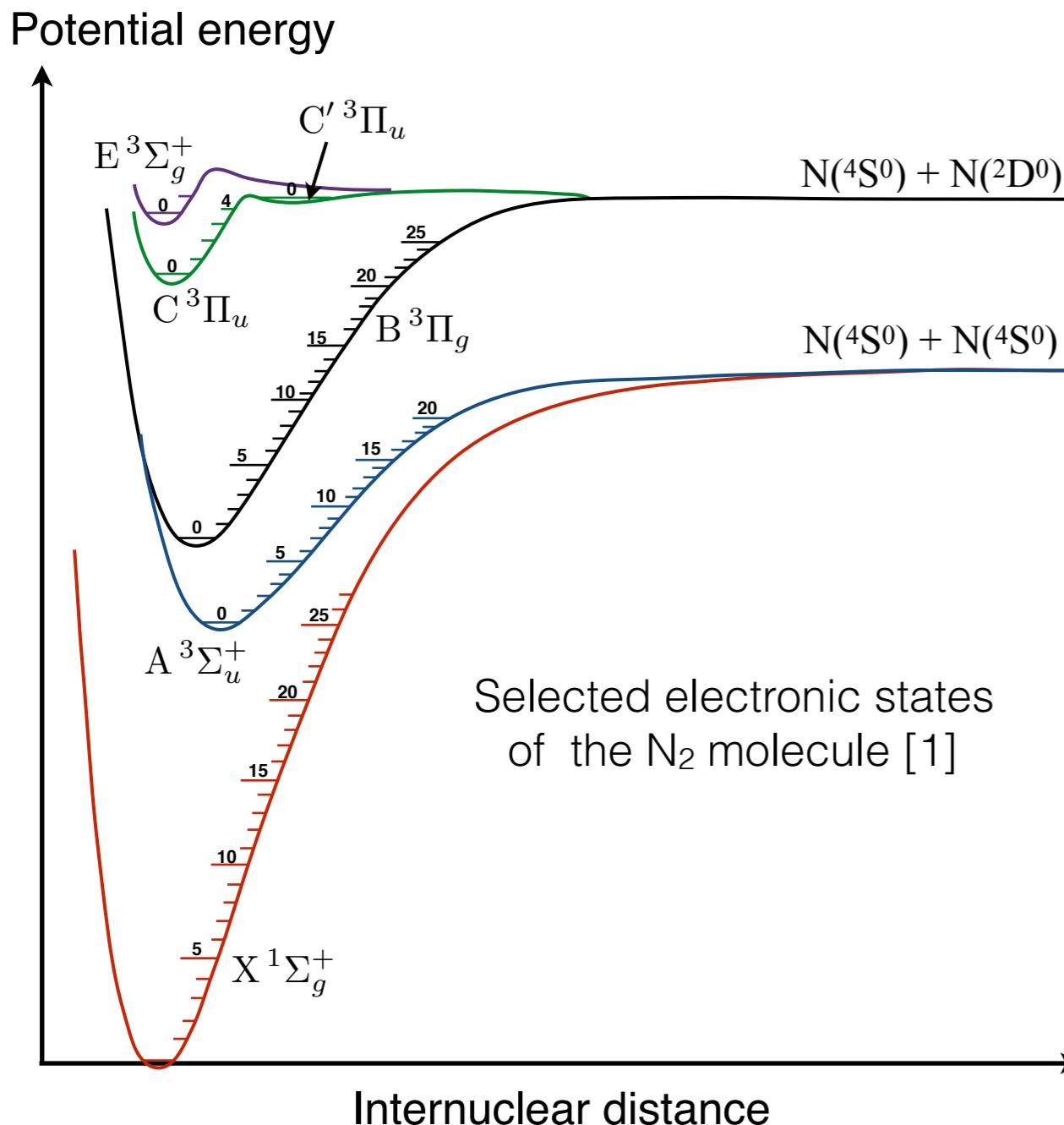


Thermal decomposition of the TPS



Energy partitioning regimes in hypersonics

Quantum mechanics dictates that atoms and molecule are permitted only **discrete energy levels**



Thermal Equilibrium

$$\tau_{\text{kin}} \ll \tau_{\text{flow}}$$



Boltzmann Distribution

$$n_j^l = \frac{n_j}{Q_j^{\text{int}}(T)} a_j^l \exp \left(-\frac{E_j^l}{k_B T} \right)$$

Multitemperature

State-specific

Energy binning

State-to-state

Nonequilibrium

$$\tau_{\text{kin}} \approx \tau_{\text{flow}}$$

Motivation

Governing equations for hypersonic flows

Detailed balance yields the following relationships for the conservation of mass, momentum, and energy

Pseudo-species mass conservation

$$\frac{\partial}{\partial t}(\rho_k) + \nabla \cdot (\rho_k \mathbf{u}) + \nabla \cdot (\rho_k \mathbf{V}_k) = \dot{\omega}_k + \dot{\phi}_k, \quad \forall k \in \mathcal{S}^*$$

Mass continuity

$$\frac{\partial}{\partial t}(\rho) + \nabla \cdot (\rho \mathbf{u}) = 0$$

Momentum conservation

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \nabla \cdot \bar{\Pi} + nq \mathbf{E}' + \mathbf{j} \times \mathbf{B}$$

Free-electron energy conservation

$$\frac{\partial}{\partial t}(\rho_e e_e) + \nabla \cdot (\rho_e e_e \mathbf{u}) = -p_e \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q}_e + \Omega_e + \mathbf{j}_e \cdot \mathbf{E}' + \mathcal{P}_e$$

Internal energy conservation of mode m

$$\frac{\partial}{\partial t}(\rho e^m) + \nabla \cdot (\rho e^m \mathbf{u}) = -\nabla \cdot \mathbf{q}^m + \Omega^m + \mathcal{P}_m$$

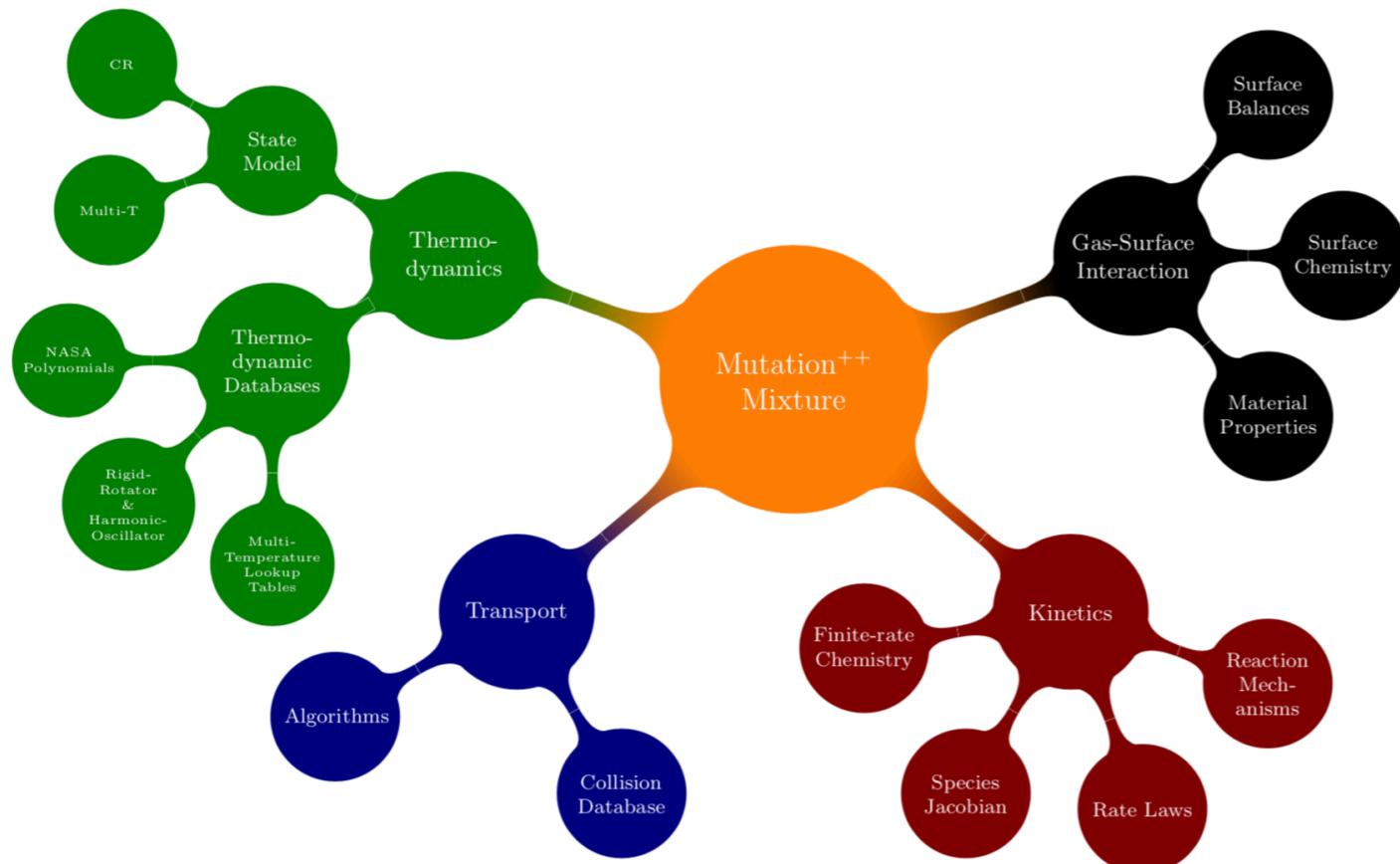
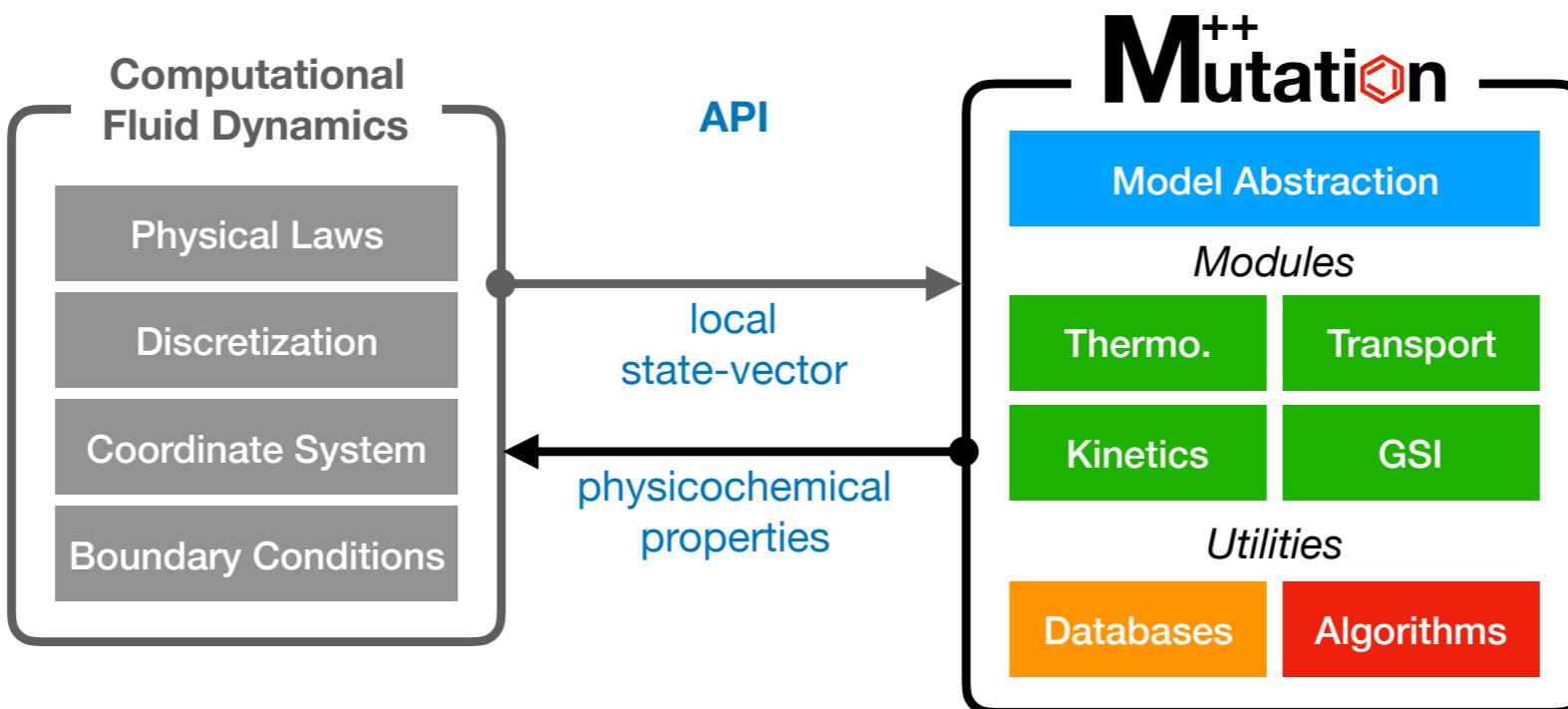
Total energy conservation

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho H \mathbf{u}) = \nabla \cdot (\bar{\Pi} \mathbf{u}) - \nabla \cdot \mathbf{q} + \mathcal{P} + \mathbf{j} \cdot \mathbf{E}'$$

Requires knowledge of:

- Thermodynamic and transport properties
- Chemical kinetics
- Energy exchange mechanisms
- Radiative source terms

Development of Mutation++

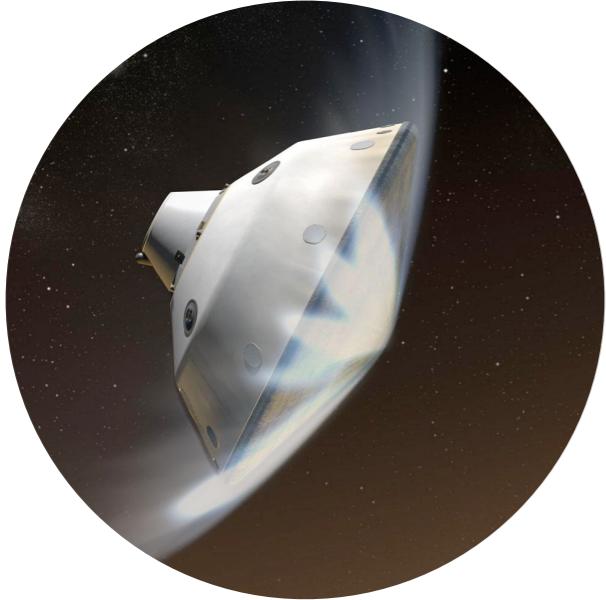


Objectives:

- Accurate property evaluation
- Efficient
- Extensible
- Interface to CFD
- Self documenting DBs
- Open source community

Design allows for broad application domain

Atmospheric entry



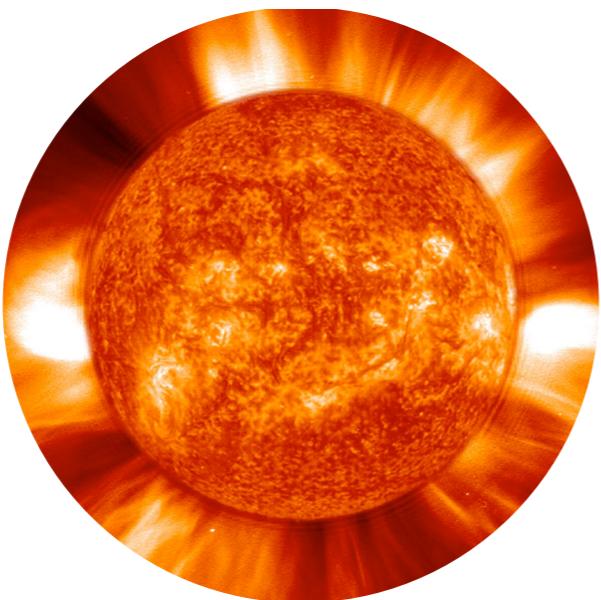
Magnetized plasmas



Meteor phenomena



Solar physics



Biomass pyrolysis



Flexible coupling strategies: “on-the-fly” or through tables

Let's get started...

Project Homepage

<https://github.com/mutationpp/Mutationpp>



Exercise

- 1) *From the homepage, navigate to the installation instructions.*
- 2) *Download and install the library.*

Test the library installation

Exercise

Run the tests and verify that the library is correctly installed.

Locating and modifying data

Types and locations of database files.

| Type | Data Directory |
|----------------------|-----------------|
| Mixture definitions | data/mixtures |
| Thermodynamic data | data/thermo |
| Transport data | data/transport |
| Reaction mechanisms | data/mechanisms |
| Energy transfer data | data/transfer |

Search order for database files.

Local Directory Data Directory

Exercise

- 1) Locate the “air5” mixture
- 2) Locate the reaction mechanism named in the mixture file.

Built-in Tool: checkmix

Loads a mixture and presents elements, species, and reactions in the mixture.

- ✓ Check if a mixture is found.
- ✓ Check exact location of the mixture file being loaded.
- ✓ Check if a mixture and corresponding support database files are formatted correctly.
- ✓ Check that no data is missing.
- ✓ Check which order the species and reactions are ordered internally.

Usage 1:

```
checkmix mixture
```

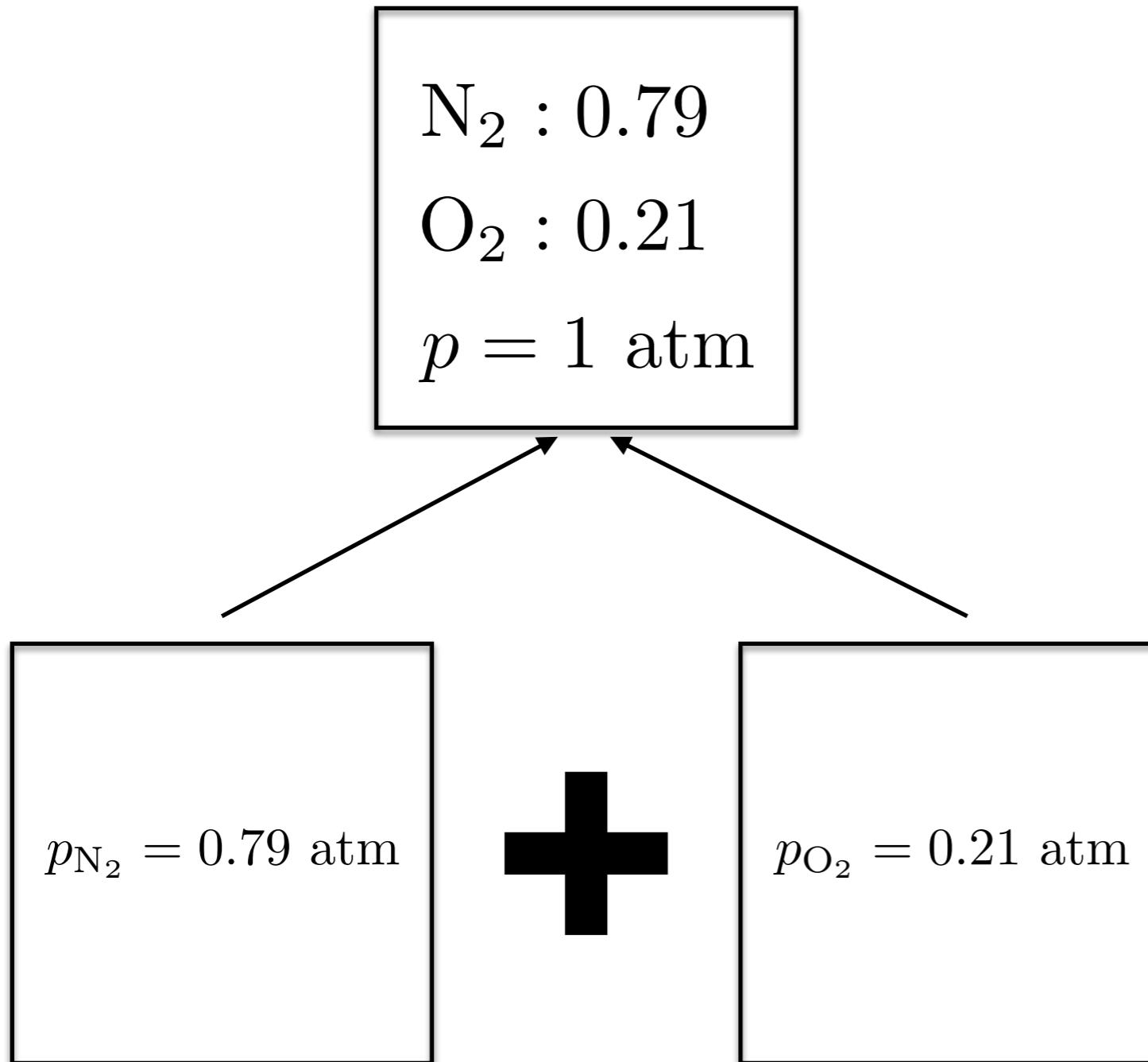
Usage 2:

```
checkmix [NASA-7 | NASA-9 | RRHO] species_list
```

Exercise

- 1) Run *checkmix* for “air5”.
- 2) Try the second usage with the same species as “air5”.

We are dealing with Mixtures



Perfect Gas Law

$$p = \rho RT$$

Dalton's Law

$$p = \sum_i^{n_s} p_i$$

Describing Chemical Mixtures

Mass Fractions

$$y_i = \frac{\rho_i}{\rho}$$



Mass of species i
Mass of mixture

$$\sum_i^{n_s} y_i = 1$$

Mole Fractions

$$x_i = \frac{n_i}{n}$$



Moles of species i
Moles of mixture

$$\sum_i^{n_s} x_i = 1$$

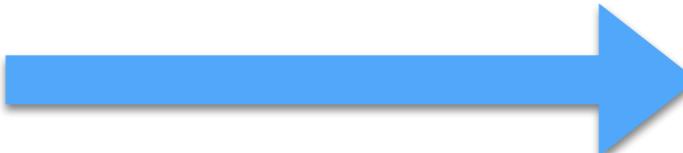
Elemental Mole Fraction

$$x_e = \frac{n_e}{n_{e,tot}}$$



Elements e
Total Elements

Species Properties

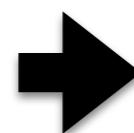


Mixture Properties

Describing Chemical Mixtures

Mass Fractions

$$y_i = \frac{\rho_i}{\rho}$$



Mass of species i
Mass of mixture

$$\sum_i^{n_s} y_i = 1$$

Mole Fractions

$$x_i = \frac{n_i}{n}$$



Moles of species i
Moles of mixture

$$\sum_i^{n_s} x_i = 1$$

Elemental Mole Fraction

$$x_e = \frac{n_e}{n_{e,tot}}$$



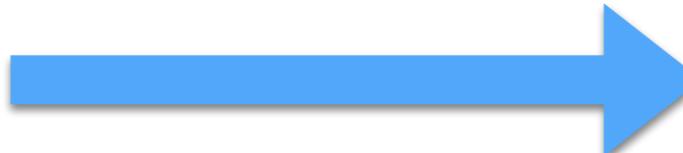
Elements e
Total Elements

Example:

$$x_{CO_2} = 0.8 \quad x_{CN} = 0.2$$

$$n_{e,tot} = 0.8 \cdot 1 + 0.8 \cdot 2 \quad CO_2 \\ + 0.2 \cdot 1 + 0.2 \cdot 1 \quad CN$$

Species Properties



Mixture Properties

Species Properties

NASA-9 Polynomials

- Computing properties based on polynomial fits

Enthalpy:

$$\frac{h_i}{R_i T} = -a_{0i} T^{-2} + a_{1i} \frac{\ln T}{T} + a_{2i} + \frac{a_{3i}}{2} T + \frac{a_{4i}}{3} T^2 + \frac{a_{5i}}{4} T^3 + \frac{a_{6i}}{5} T^4 + \frac{a_{7i}}{T}$$

Entropy:

$$\frac{s_i}{R_i} = -\frac{a_{0i}}{2} T^{-2} - a_{1i} T^{-1} + a_{2i} \ln T + a_{3i} T + \frac{a_{4i}}{2} T^2 + \frac{a_{5i}}{3} T^3 + \frac{a_{6i}}{4} T^4 + a_{8i}$$

Gibbs Free Energy (or any other thermodynamic property):

$$\frac{g_i}{R_i T} = \frac{s_i}{R_i T} - \frac{h_i}{T}$$

Rigid Rotator-Harmonic Oscillator

- Computing properties with analytical formulas (statistical thermodynamics)

e.g.: $e_i^{\text{trans}} = \frac{3}{2} R_i T$

Species to Mixture Properties

Energy and Enthalpy

$$e = \sum_{i=1}^{n_s} y_i e_i \quad \text{and} \quad h = \sum_{i=1}^{n_s} y_i h_i$$

Entropy

$$s = \sum_{i=1}^{n_s} y_i s_i - \frac{n k_B}{\rho} \sum_i x_i \ln x_i$$

Entropy of Mixing

Here all properties are in **kg**. When the properties are in **mole**, one should multiply with mole fractions!

Constrained multiphase equilibrium

A mixture is at equilibrium when the system Gibbs free energy is minimized, under mass balance constraints.

$$\tilde{G} \equiv \frac{G}{R_u T} = \sum_{m \in \mathcal{P}} \sum_{j \in \mathcal{S}_m} N_j (\tilde{g}_j + \ln N_j - \ln \bar{N}_m)$$

$\bar{N}_m = \sum_{j \in \mathcal{S}_m} N_j, \quad \forall m \in \mathcal{P}$
 $\sum_{j \in \mathcal{S}} B_{ji}^e N_j = c_i^e \quad \forall i \in \mathcal{E}$

yields small non linear system
of equations to be solved

Can use the Lagrange multiplier technique to solve constrained minimization problem.

| <u>$L = \tilde{G} - \sum_{i \in \mathcal{C}} \lambda_i \left(\sum_{j \in \mathcal{S}} B_{ji}^e N_j - c_i \right)$</u> | | | | | | <u>general linear columns constraints</u> | |
|---|--|---|--------------------------------------|---|--------------------------------------|---|---|
| Species | $\sum_{j \in \mathcal{S}} N_j \left(\tilde{g}_j + \ln N_j - \ln \sum_{k \in \mathcal{S}_{p_j}} N_k \right)$ | $\sum_{i \in \mathcal{C}} \lambda_i \left(\sum_{j \in \mathcal{S}} B_{ji}^e N_j - c_i \right)$ | | | | | |
| C | 1 | 0 | $\sum_{k \in \mathcal{S}_{p_j}} N_k$ | 1 | $\sum_{i \in \mathcal{C}} \lambda_i$ | $\sum_{j \in \mathcal{S}} B_{ji}^e N_j$ | 0 |
| CO | 1 | 1 | 1 | 1 | 1 | 0 | |
| CO ₂ | 1 | 2 | 1 | 1 | 1 | 0 | |
| $\frac{\partial L}{\partial N_j}$ | $\frac{\partial \tilde{G}}{\partial N_j}$ | $-\sum_{i \in \mathcal{C}} \lambda_i B_{ji}^e$ | 0 | 1 | 1 | 0 | |
| | | | 0 | 1 | 0 | 1 | |

necessary conditions
for minimization

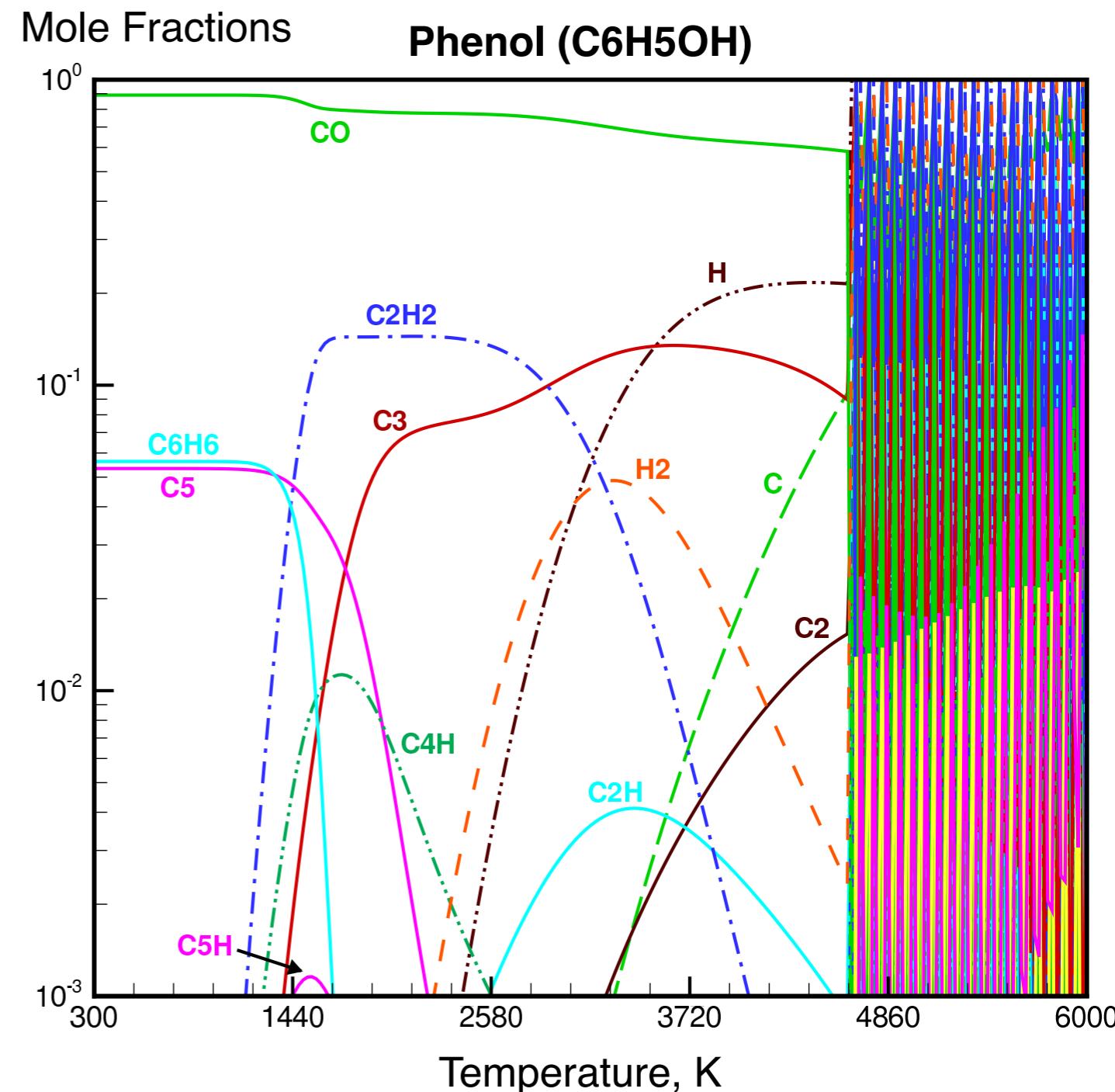
$= \tilde{g}_j + \ln N_j - \ln \sum_{k \in \mathcal{S}_{p_j}} N_k - \sum_{i \in \mathcal{C}} \lambda_i B_{ji}^e = 0, \quad \forall j \in \mathcal{S}.$

Multiphase Gibbs function continuation

Minimization of Gibbs energy under constraints

Solution of nonlinear system for phase moles and constraint potentials

- Newton iterations may fail to converge
- Pope [1] introduced Gibbs function continuation (GFC) method for one phase
- Have extended Popes formulation to multiple phases (MPGFC) in [2]
- Proof of guaranteed convergence when constraints are valid
- Showed that Pope convergence proof is a special case of the general proof



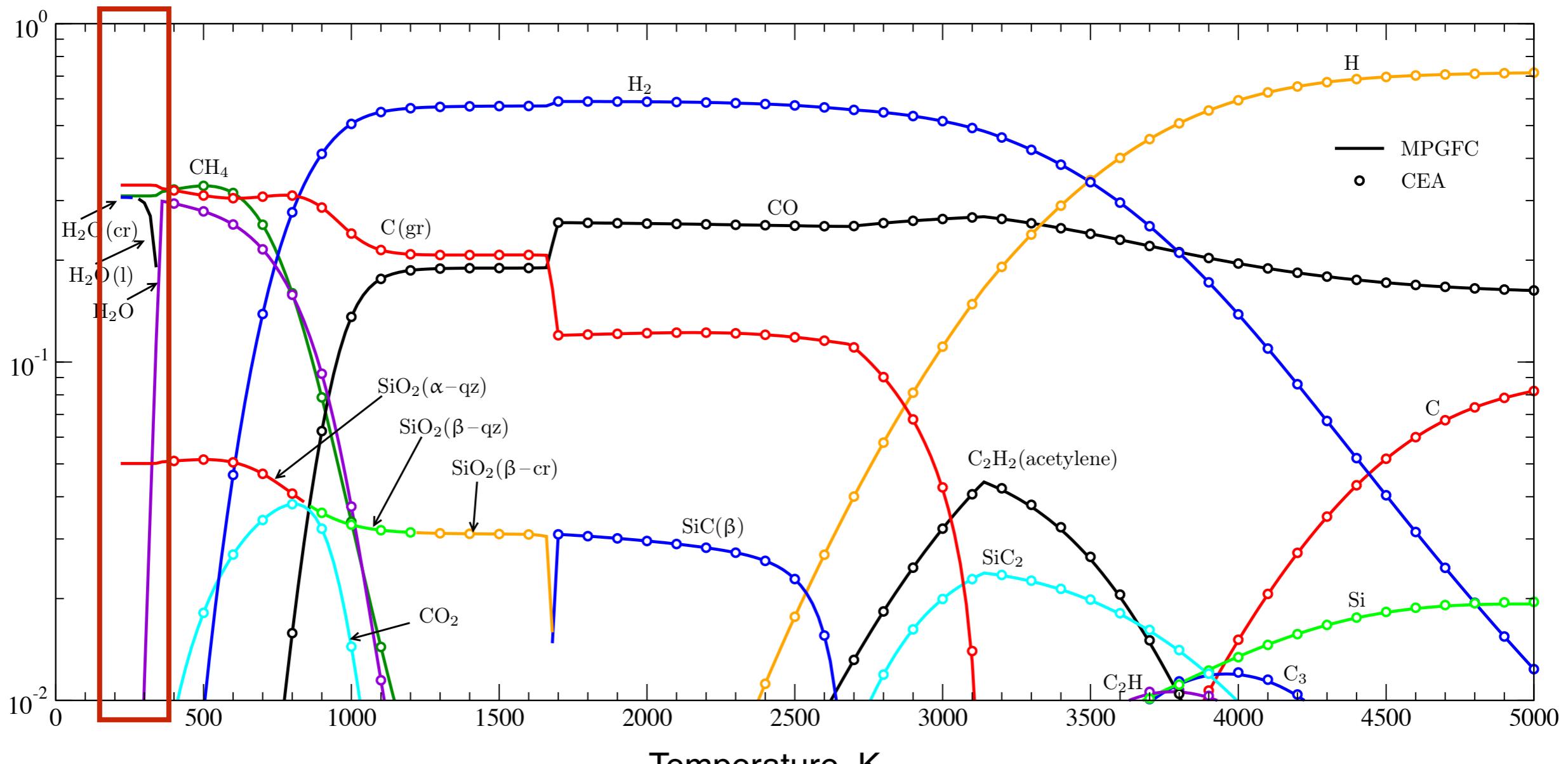
[1] Pope. *Combustion and Flame*, 139(3):222–226, 2004.

[2] Scoggins, Magin. *Combustion and Flame*, 162(12):4514–4522, 2015.

Multiphase Gibbs function continuation

Equilibrium mixture of 90% phenol (C_6H_5OH) with 10% silicon

Global mole fractions [1]



NASA CEA solver fails
to converge at low
temperatures

Built-in Tool: mppequil

Generates a table of equilibrium properties for a given mixture and conditions.

Usage:

```
mppequil [options] mixture
```

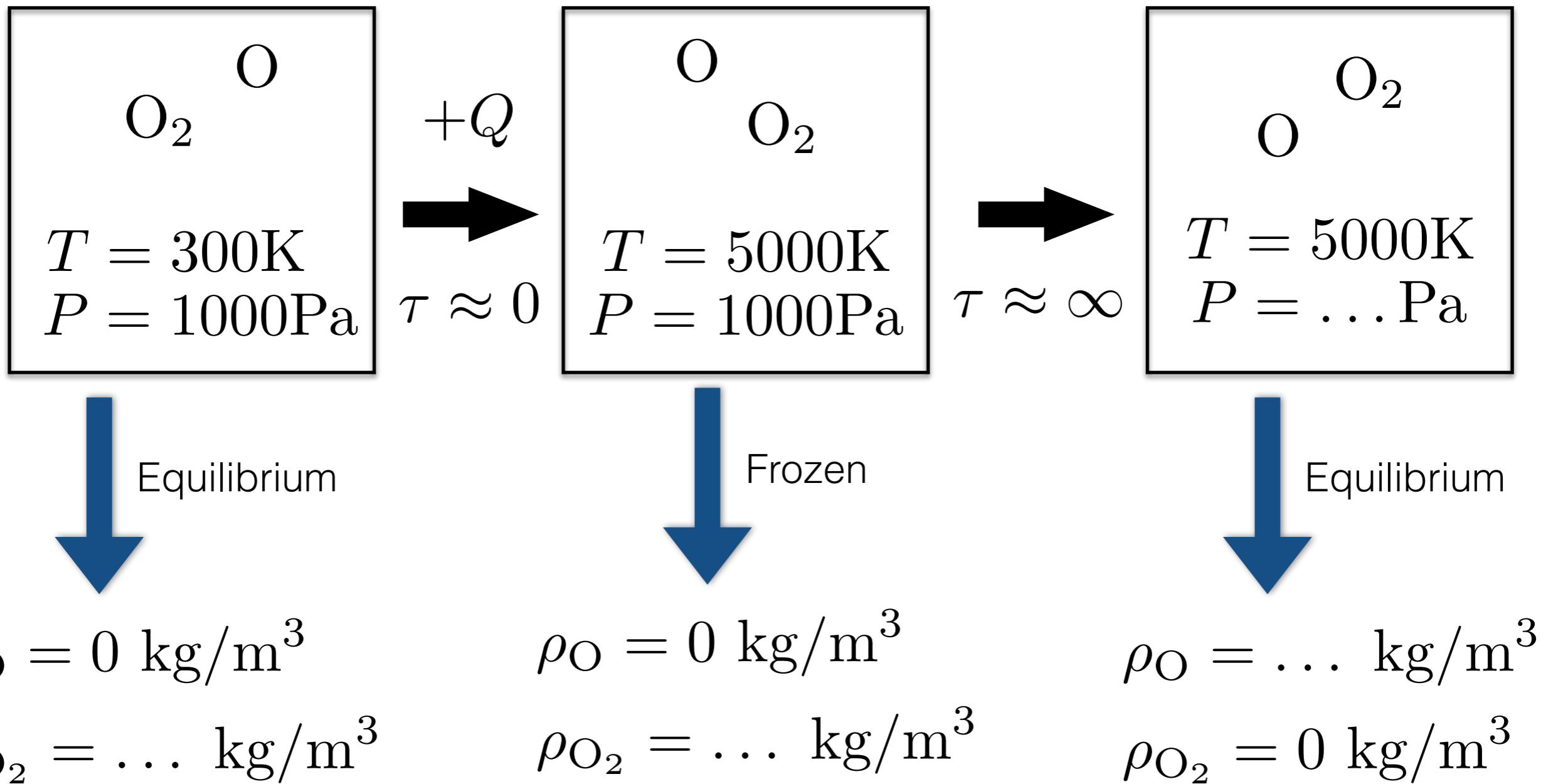
Help:

```
mppequil -h
```

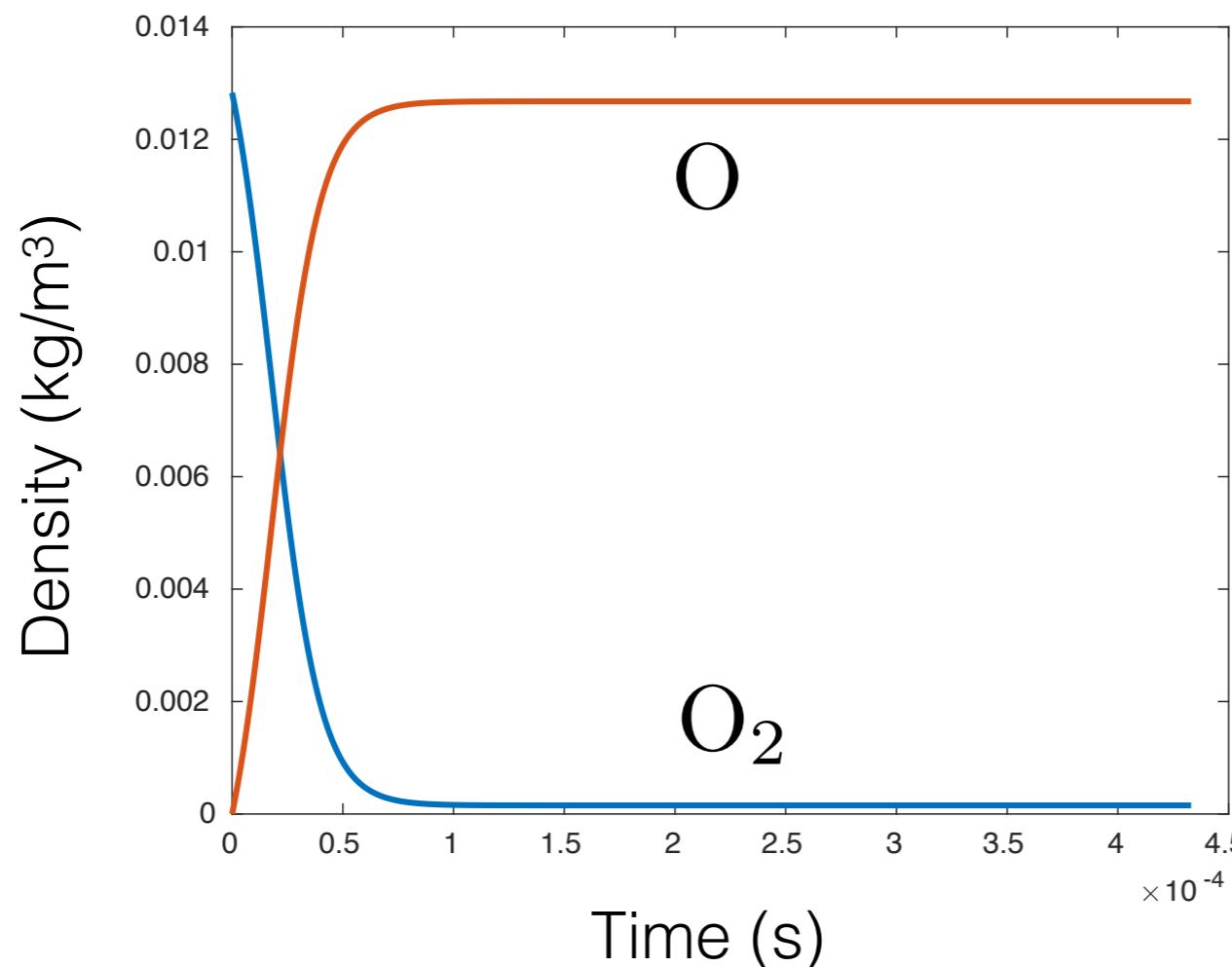
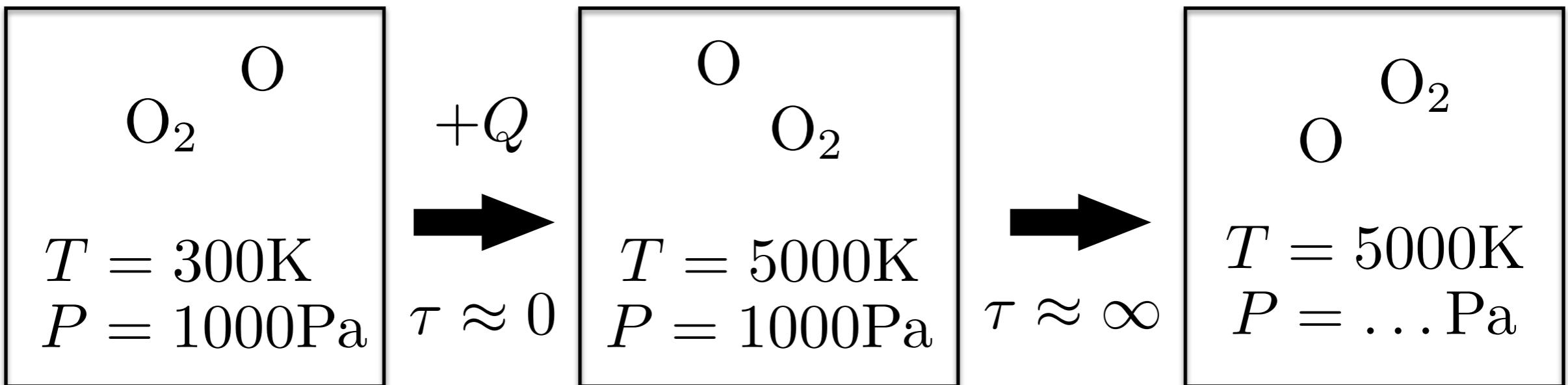
Exercise

- 1) *Check out the help for mppequil.*
- 2) *Compute the equilibrium mole fractions for 5-species air from 500K to 8000K at 1atm.*

Reaction occur at finite rate till equilibrium

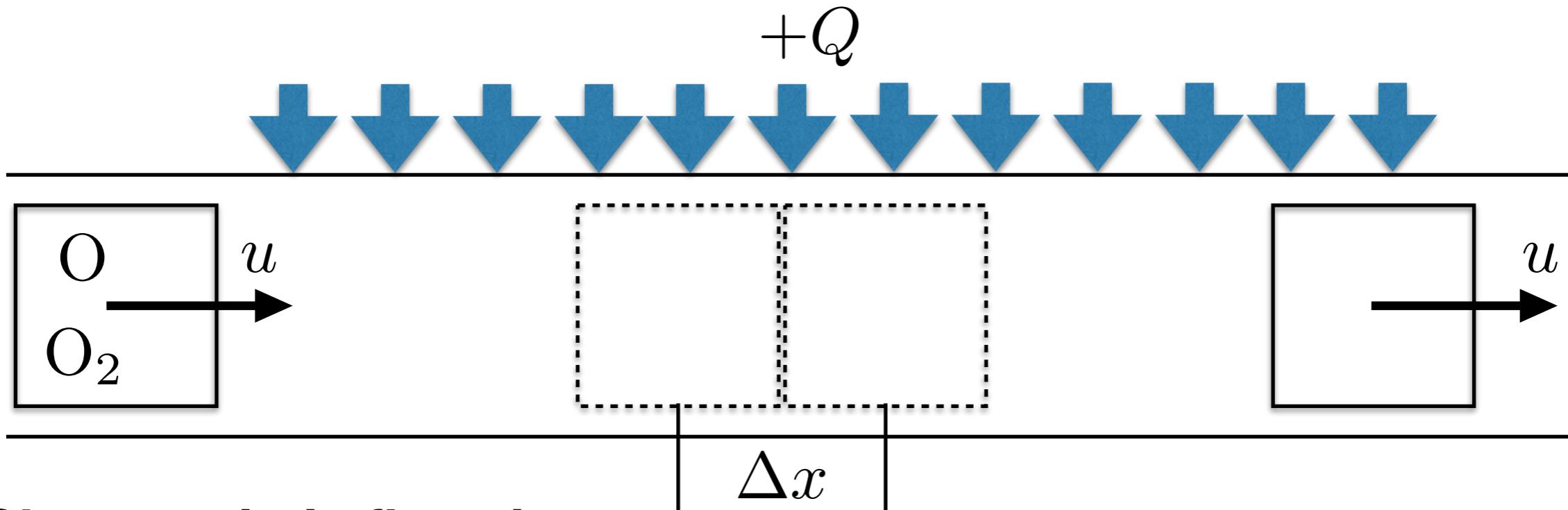


Reaction occur at finite rate till equilibrium



Finite Rate Chemistry
 $\text{O} + \text{O} \longrightarrow \text{O}_2$

When is finite-rate chemistry necessary?



Characteristic flow time

$$\tau_{\text{flow}} = \frac{\Delta x}{u}$$

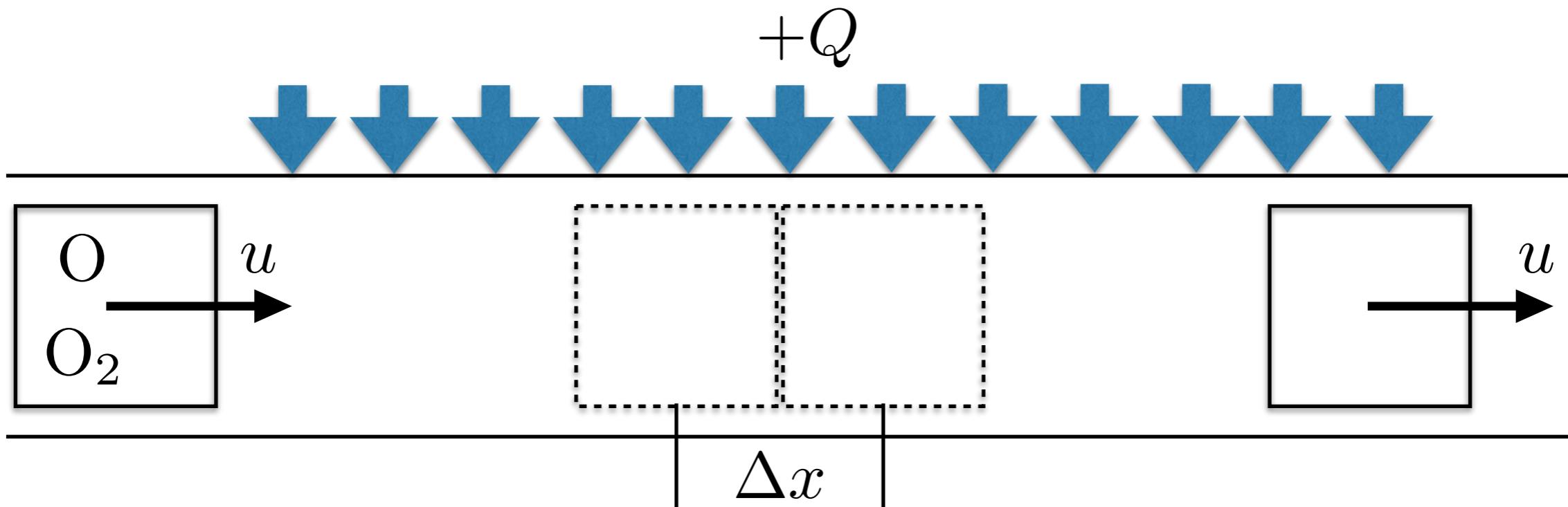
Damköhler Number:

$$Da = \frac{\tau_{\text{flow}}}{\tau_{\text{chem}}}$$

| | |
|------------|-------------|
| $Da \ll 1$ | Frozen |
| $Da \gg 1$ | Equilibrium |

Real flows are neither frozen nor in equilibrium

When is finite-rate chemistry necessary?



Frozen/Equilibrium:

$$\frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0$$

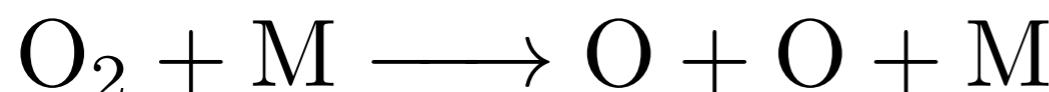
Finite Rate Chemistry

$$\frac{\partial \rho_i}{\partial t} + \nabla (\rho_i u + j_{\text{diff}}) = \dot{\omega}_i$$

Chemical Production: $\dot{\omega}_i \sim \frac{d[O]}{dt}$

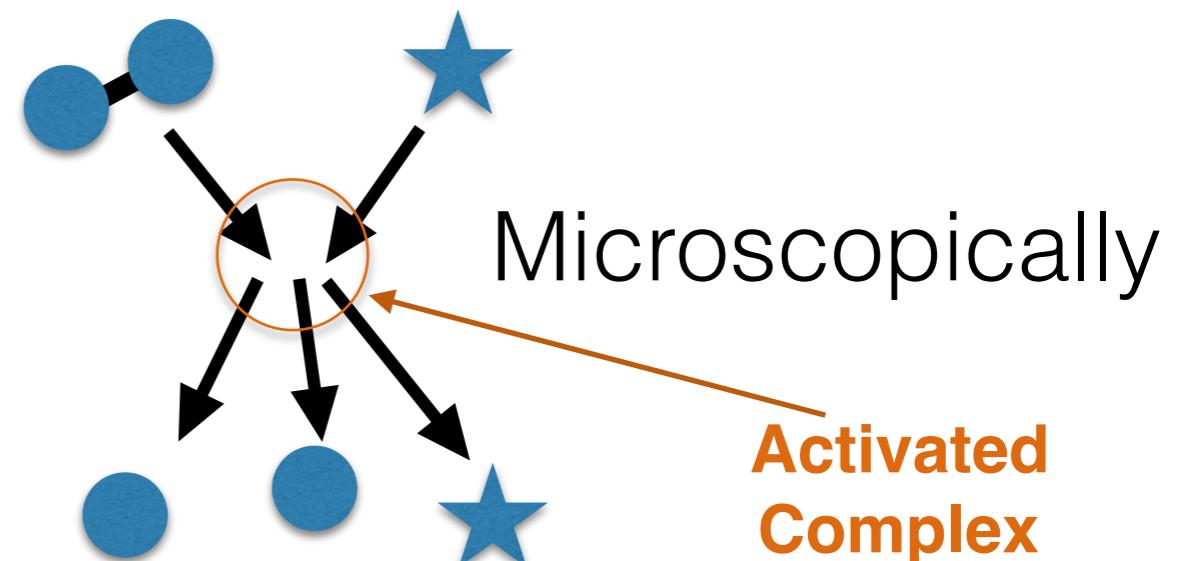
Chemical Reactions and their rate

Oxygen Dissociation



Law of Mass Action

$$r = k_f[\text{O}_2][\text{M}] \quad \text{Reaction Rate}$$



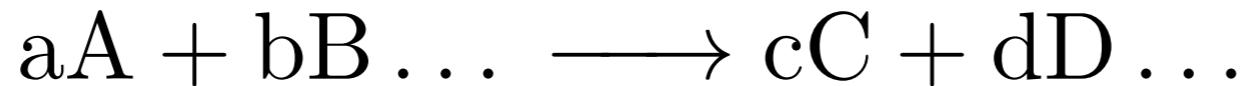
Chemical Production Rate of Species

$$\frac{d\text{O}_2}{dt} = -k_f[\text{O}_2][\text{M}]$$

Stoichiometry

$$\frac{d\text{O}}{dt} = 2k_f[\text{O}_2][\text{M}]$$

Generic Chemical Reaction



Reactants

$$\frac{d[A]}{dt} = -ak_f [A]^a[B]^b$$

$$\frac{d[B]}{dt} = -bk_f [A]^a[B]^b$$

Products

$$\frac{d[D]}{dt} = dk_f [A]^a[B]^b$$

$$\frac{d[C]}{dt} = ck_f [A]^a[B]^b$$

To go from **moles** to **kg** we multiply by the **molar mass**

Reaction Rate Coefficient

Arrhenius Law:

$$k_f = C \exp \frac{-E_a}{RT}$$

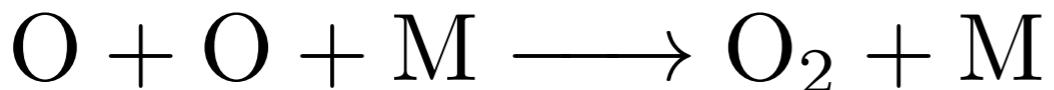
Activation Energy
Kinetic Energy of Atoms
Pre-exponential

The diagram shows the Arrhenius equation $k_f = C \exp \frac{-E_a}{RT}$. Three components are highlighted with orange circles and arrows pointing to them from text labels: 'Pre-exponential' points to the constant C , 'Activation Energy' points to the term $-E_a$, and 'Kinetic Energy of Atoms' points to the term RT .

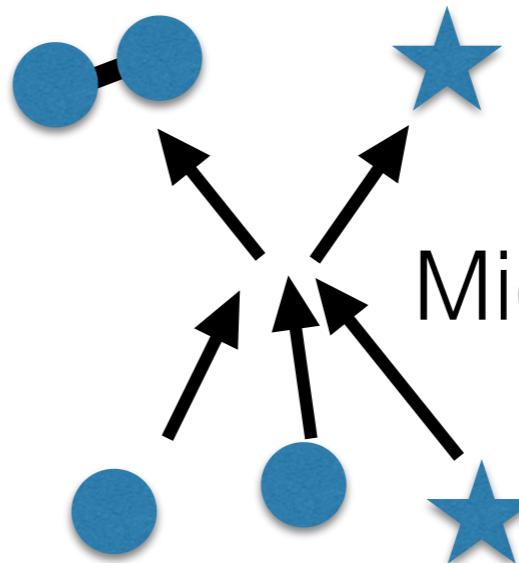
Generalized Arrhenius Law: $k_f = CT^b \exp \frac{-E_a}{RT}$

Determined Experimentally or from Quantum Chemistry

Backward Reactions (micro-reversibility)



$$r_b = k_b[\text{O}]^2[\text{M}]$$



Forward and backward rates are interdependent!

At Equilibrium:

$$r_f = r_b$$

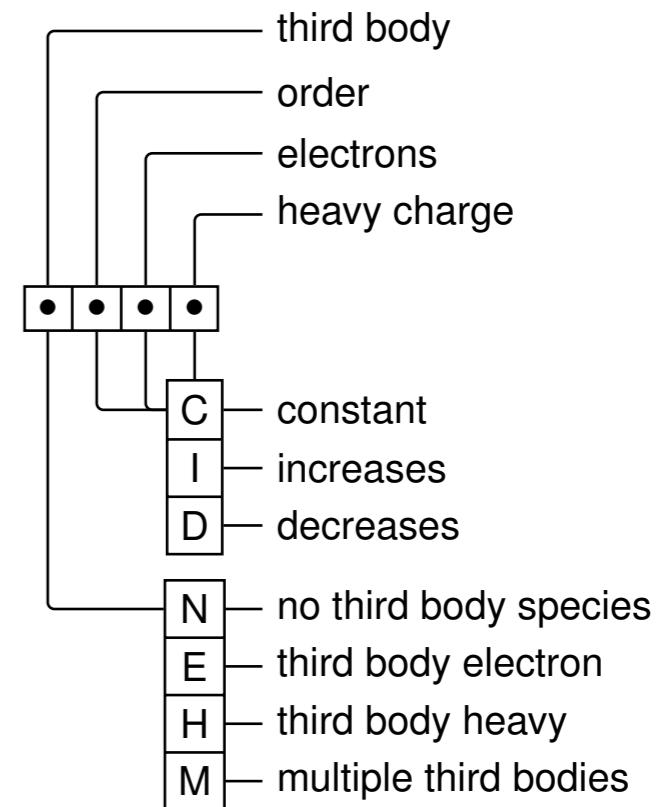
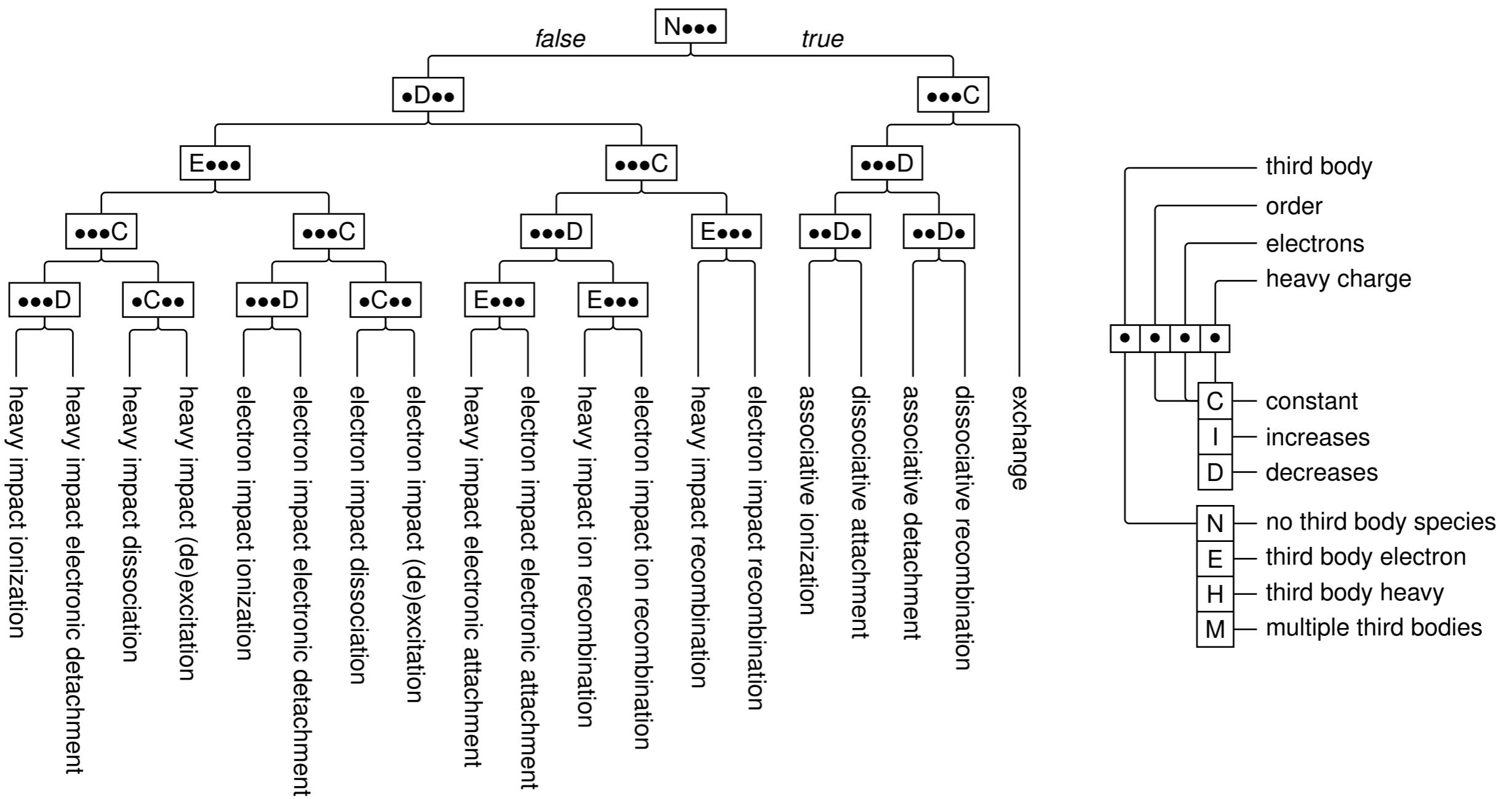
or:

$$\frac{k_f(T)}{k_b(T)} = \frac{[\text{O}_2]}{[\text{O}]^2}$$

Equilibrium Constant: $K_C(T)$

Provided rigorously
from thermodynamics

Automatic reaction classification



What happens when I have multiple reactions?

```
<!-- Park Air-5 Reaction Mechanism -->
<mechanism name="air5_park">

  <arrhenius_units A="mol,cm,s,K" E="kcal,mol,K" />

  <!-- 1 -->
  <reaction formula="N2+M=2N+M">
    <arrhenius A="3.0E+22" n="-1.6" T="113200.0" />
    <M>N2:0.2333, N0:0.2333, O2:0.2333</M>
  </reaction>

  <!-- 2 -->
  <reaction formula="O2+M=2O+M">
    <arrhenius A="1.0E+22" n="-1.5" T="59360.0" />
    <M>N2:0.2, N0:0.2, O2:0.2</M>
  </reaction>

  <!-- 3 -->
  <reaction formula="N0+M=N+O+M">
    <arrhenius A="5.0E15" n="+0.0" T="75500.0" />
    <M>NO:20.0, N:22.0, O:22.0</M>
  </reaction>

  <!-- 4 -->
  <reaction formula="N2+O=N0+N">
    <arrhenius A="5.69E+12" n="+0.42" T="42938.0" />
  </reaction>

  <!-- 5 -->
  <reaction formula="O2+N=N0+O">
    <arrhenius A="2.49E+09" n="+1.18" T="4005.5" />
  </reaction>
</mechanism>
```

What happens when I have multiple reactions?

```
<!-- Park Air-5 Reaction Mechanism -->
<mechanism name="air5_park">

<arrhenius_units A="mol,cm,s,K" E="kcal,mol,K" />

<!-- 1 -->
<reaction formula="N2+M=2N+M">
    <arrhenius A="3.0E+22" n="-1.6" T="113200.0" />
    <M>N2:0.2333, NO:0.2333, O2:0.2333</M>
</reaction>      Pre-exp      b      Activation
                                         Temperature
                                         Ea/R

<!-- 2 -->
<reaction formula="O2+M=2O+M">
    <arrhenius A="1.0E+22" n="-1.5" T="59360.0" />
    <M>N2:0.2, NO:0.2, O2:0.2</M>
</reaction>

<!-- 3 -->
<reaction formula="NO+M=N+O+M">
    <arrhenius A="5.0E15" n="+0.0" T="75500.0" />
    <M>NO:20.0, N:22.0, O:22.0</M>
</reaction>

Third Body Efficiency
<!-- 4 -->
<reaction formula="N2+O=NO+N">
    <arrhenius A="5.69E+12" n="+0.42" T="42938.0" />
</reaction>

<!-- 5 -->
<reaction formula="O2+N=NO+O">
    <arrhenius A="2.49E+09" n="+1.18" T="4005.5" />
</reaction>
</mechanism>
```

What happens when I have multiple reactions?

```
<!-- Park Air-5 Reaction Mechanism -->
<mechanism name="air5_park">

<arrhenius_units A="mol,cm,s,K" E="kcal,mol,K" />

<!-- 1 -->
<reaction formula="N2+M=2N+M">
    <arrhenius A="3.0E+22" n="-1.6" T="113200.0" />
    <M>N2:0.2333, N0:0.2333, O2:0.2333</M>
</reaction>

<!-- 2 -->
<reaction formula="O2+M=2O+M">
    <arrhenius A="1.0E+22" n="-1.5" T="59360.0" />
    <M>N2:0.2, NO:0.2, O2:0.2</M>
</reaction>

<!-- 3 -->
<reaction formula="N0+M=N+O+M">
    <arrhenius A="5.0E15" n="+0.0" T="75500.0" />
    <M>NO:20.0, N:22.0, O:22.0</M>
</reaction>

<!-- 4 -->
<reaction formula="N2+O=N0+N">
    <arrhenius A="5.69E+12" n="+0.42" T="42938.0" />
</reaction>

<!-- 5 -->
<reaction formula="O2+N=N0+O">
    <arrhenius A="2.49E+09" n="+1.18" T="4005.5" />
</reaction>
</mechanism>
```

Example:

$$\frac{d[O]}{dt} = \begin{array}{l} +2k_{f,2}[O_2][M] \\ +k_{f,3}[NO][M] \\ -k_{f,4}[N_2][O] \\ +k_{f,5}[O_2][N] \end{array}$$

$$\begin{array}{l} -2k_{b,2}[O]^2[M] \\ -k_{b,3}[N][O][M] \\ +k_{b,4}[NO][N] \\ -k_{b,5}[NO][O] \end{array}$$

Forward

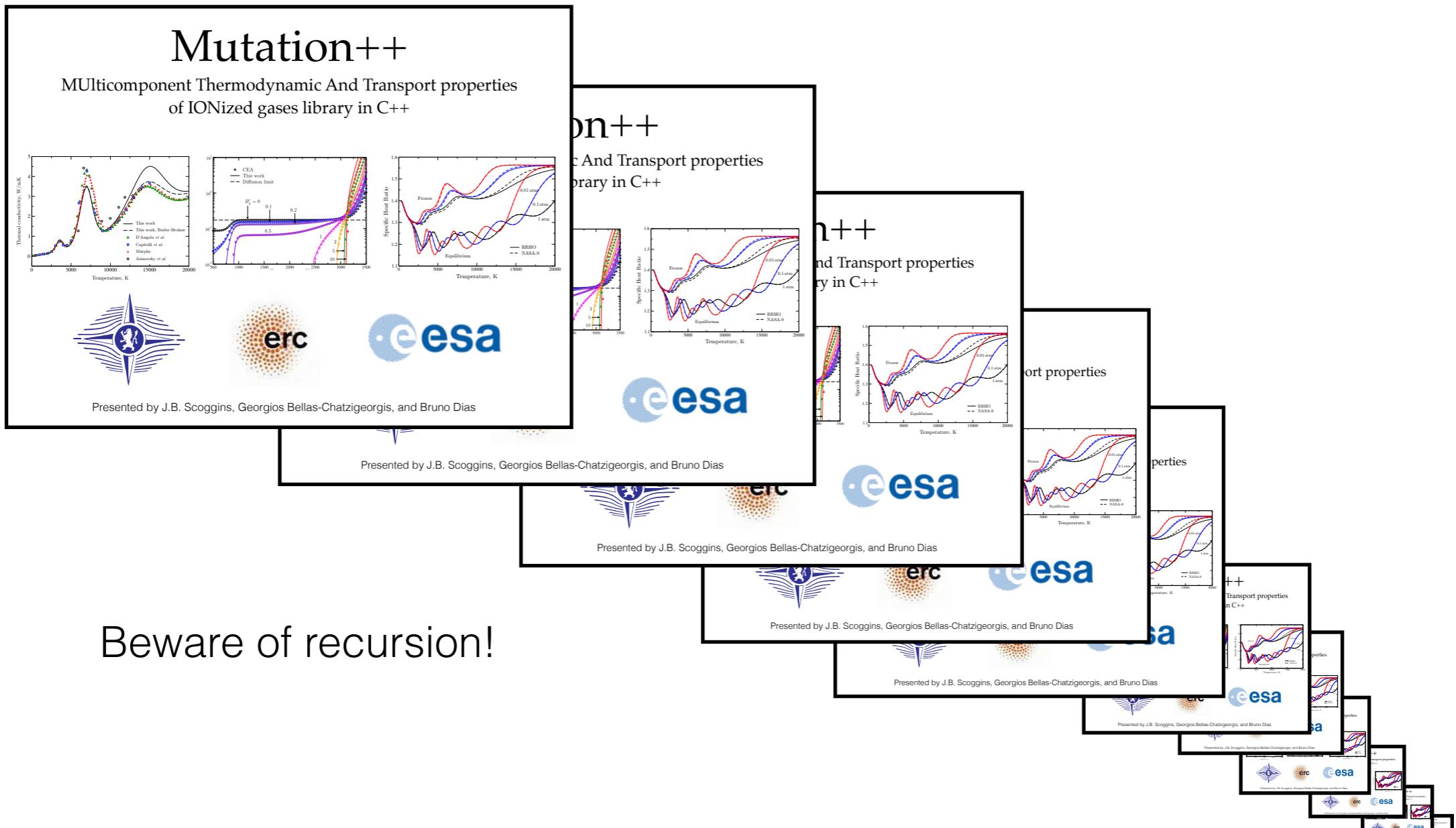
Lunch Time!



Before we begin...

Please download the training materials, including the example programs and this presentation:

<https://tinyurl.com/yxk467g4>



Hello World!

Hello World! Mixture

Printing species in a mixture

hello.cpp

```
#include "mutation++.h"
#include <iostream>

int main(int argc, char* argv[])
{
    // Load air5 mixture
    Mutation::Mixture mix("air_5");

    // Print the species
    std::cout << "Species: ";
    for (auto& s : mix.species())
        std::cout << s.name() << ' ';
    std::cout << '\n';

    return 0;
}
```

hello.f90

```
program main
    use mutationpp
    implicit none
    character(len=10) :: mixture
    character(len=12) :: species, state_model
    integer :: j, ns

    mixture      = "air_5"
    state_model = "ChemNonEq1T"

    ! Initialize the mutation++ library
    call mpp_initialize(mixture, state_model)
    ns = mpp_nspecies()

    write(*,'(A12)',advance='no') "Species: "
    do j = 1,ns
        call mpp_species_name(j, species)
        write(*,'(A12)',advance='no') species
    end do
    write(*,*)

    ! Clean up the memory in the mutation++ library
    call mpp_destroy()

end program main
```

Using M++ in your own projects

Setting up dependencies

Makefile for C++ code.

```
# Change this path to your Mutation++ install path
MPP_INSTALL_DIR = $(MPP_DIRECTORY)/install

CXX_FLAGS = -I $(MPP_INSTALL_DIR)/include \
            -I $(MPP_INSTALL_DIR)/include/mutation++ \
            -I $(MPP_DIRECTORY)/thirdparty/eigen \
            -O3 -std=c++11

# use .dylib for Mac, .so for Unix
UNAME_S := $(shell uname -s)
ifeq ($(UNAME_S),Linux)
    CXX      = g++
    CXX_LIBS = $(MPP_INSTALL_DIR)/lib/libmutation++.so
endif
ifeq ($(UNAME_S),Darwin)
    CXX      = c++
    CXX_LIBS = $(MPP_INSTALL_DIR)/lib/libmutation++.dylib
endif

hello : hello.o
    $(CXX) -o $@ $(CXX_FLAGS) $^ $(CXX_LIBS)

%.o : %.cpp
    $(CXX) -c $(CXX_FLAGS) $<
```

Makefile for Fortran code.

```
# Change this path to your Mutation++ install path
MPP_INSTALL = $(MPP_DIRECTORY)/install

FC_FLAGS = -J$(MPP_INSTALL)/include/
           -O3 -g -fdefault-real-8

# use .dylib for Mac, .so for Unix
UNAME_S := $(shell uname -s)
ifeq ($(UNAME_S),Linux)
    FC      = gfortran
    FC_LIBS = $(MPP_INSTALL)/lib/libmutation++_fortran.so
endif
ifeq ($(UNAME_S),Darwin)
    FC      = gfortran
    FC_LIBS = $(MPP_INSTALL)/lib/libmutation++_fortran.dylib
endif

hello : hello.o
    $(FC) -o $@ $(FC_FLAGS) $^ $(FC_LIBS)

%.o : %.f90
    $(FC) -c $(FC_FLAGS) $<
```

Exercise

Compile and run the `hello.cpp` or `hello.f90` programs.

Creating a Mixture from scratch

Understanding the MixtureOptions class

hello.cpp

```
#include "mutation++.h"
#include <iostream>

int main(int argc, char* argv[])
{
    // Load air5 mixture
    Mutation::Mixture mix("air_5");

    // Print the species
    std::cout << "Species: ";
    for (auto& s : mix.species())
        std::cout << s.name() << ' ';
    std::cout << '\n';

    return 0;
}
```

mixture_options.cpp

```
#include "mutation++.h"
#include <iostream>

int main(int argc, char* argv[])
{
    // Setup mixture options
    Mutation::MixtureOptions options;
    options.setSpeciesDescriptor("N O NO N2 O2");
    options.setThermodynamicDatabase("RRHO");

    // Create the mixture
    Mutation::Mixture mix(options);

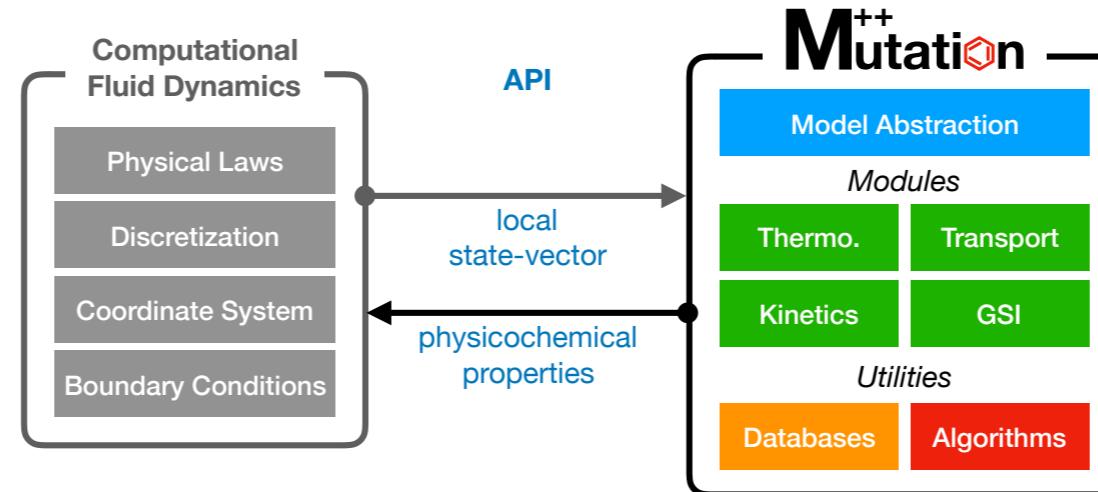
    // Print the species
    std::cout << "Species: ";
    for (auto& s : mix.species())
        std::cout << s.name() << ' ';
    std::cout << '\n';

    return 0;
}
```

Exercise

- 1) Modify hello.cpp to use MixtureOptions as above.
- 2) Try using more generic species descriptor to load all species with N and O (including ions).

Setting the state of the mixture



```
mix.setState(const double* const vec1, const double* const vec2, int vars = 0);
```

| State Model | vars | vec1 | vec2 |
|--------------|------|------------------------|-------------------------------------|
| Equil | 0 | mixture density | mixture energy density |
| | 1 | static pressure | temperature |
| | 2 | element mole fractions | pressure, temperature |
| ChemNonEq1T | 0 | species densities | mixture energy density |
| | 1 | species densities | temperature |
| | 2 | species mass fractions | pressure, temperature |
| ChemNonEqTTv | 0 | species densities | total and vibronic energy densities |
| | 1 | species densities | trans. and vibronic temperatures |

*Natural variables for CFD are always the default (0) variable set.

Setting the state of the mixture

Example equilibrium code

set_state_equil.cpp

```
#include "mutation++.h"
#include <iostream>

int main(int argc, char* argv[])
{
    // Create the mixture
    Mutation::MixtureOptions options("air_5");
    options.setStateModel("Equil");

    Mutation::Mixture mix(options);
    mix.addComposition("N2:0.79, O2:0.21", true);

    double P = Mutation::ONEATM;
    double T = 7000.0;
    mix.setState(&P, &T, 1);

    for (int i = 0; i < mix.nSpecies(); ++i) {
        std::cout << std::setw(10) << mix.speciesName(i);
        std::cout << std::setw(15) << mix.X()[i] << '\n';
    }

    return 0;
}
```

Exercise

Try above code with a pure nitrogen mixture.

Setting the state of the mixture

Example non-equilibrium code

set_state_noneq.cpp

```
int main(int argc, char* argv[])
{
    // Same setup as before...

    // Equilibrate the solution to T and P
    double P = Mutation::ONEATM; double T = 7000.0;
    mix.equilibrate(T, P);

    // Print mole fractions
    std::cout << "-- Before changing temperature --\n";
    std::cout << "Mole fractions:\n";
    printMoleFractions(mix);

    // Print the species production rates
    std::cout << "Production rates:\n";
    printProductionRates(mix);

    // Change the temperature but leave the densities the same
    std::vector<double> rho(mix.nSpecies());
    mix.densities(rho.data());
    T = 1000.0;
    mix.setState(rho.data(), &T, 1);

    // Print mole fractions
    std::cout << "-- After changing temperature --\n";
    std::cout << "Mole fractions:\n";
    printMoleFractions(mix);

    // Print the species production rates
    std::cout << "Production rates:\n";
    printProductionRates(mix);

    return 0;
}
```

Output

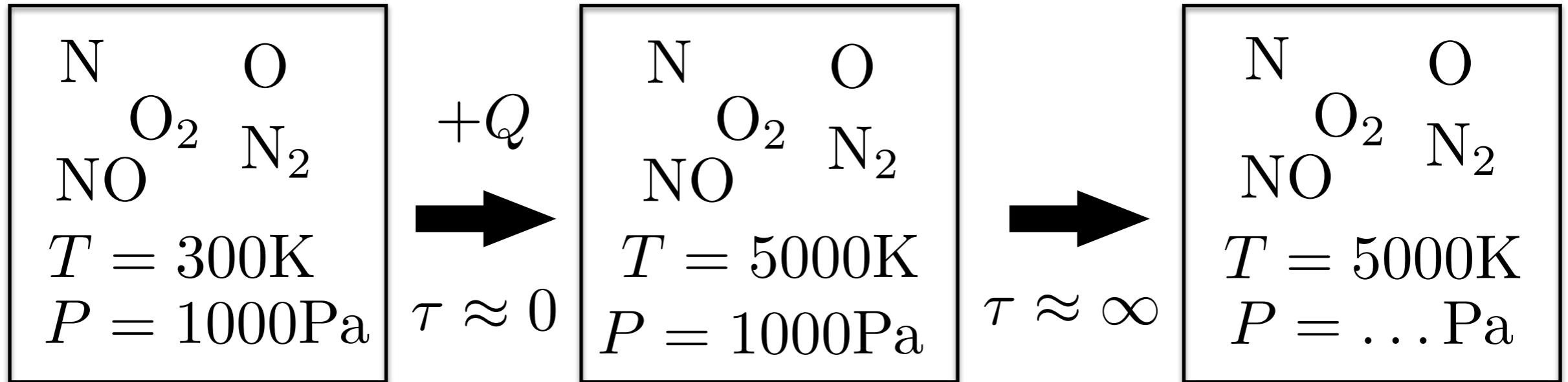
```
-- Before changing temperature --
Mole fractions:
    N      0.499208
    O      0.258275
    NO     0.00258515
    N2     0.239898
    O2     3.42487e-05
Production rates:
    N      4.125e-13
    O      -1.66924e-13
    NO     -2.81131e-13
    N2     -2.81269e-13
    O2     3.16824e-13
-- After changing temperature --
Mole fractions:
    N      0.499208
    O      0.258275
    NO     0.00258515
    N2     0.239898
    O2     3.42487e-05
Production rates:
    N      -1036.07
    O      -331.057
    NO     320.88
    N2     886.286
    O2     159.962
```

Atoms are recombining at the lower temperature.



Now for a more interesting example

Implementing a 0D reactor at constant volume and temperature



$$\frac{d\rho_i}{dt} = \dot{\omega}_i \quad \xrightarrow{\text{Explicit}} \quad \rho_i^{n+1} = \rho_i^n + \Delta t \cdot \dot{\omega}_i^n$$

Instead: Runge-Kutta 4

$$\rho_i^{n+1} = \rho_i^n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t^{n+1} = t^n + \Delta t$$

$$k_1 = \Delta t \cdot \dot{\omega}_i(\rho^n)$$

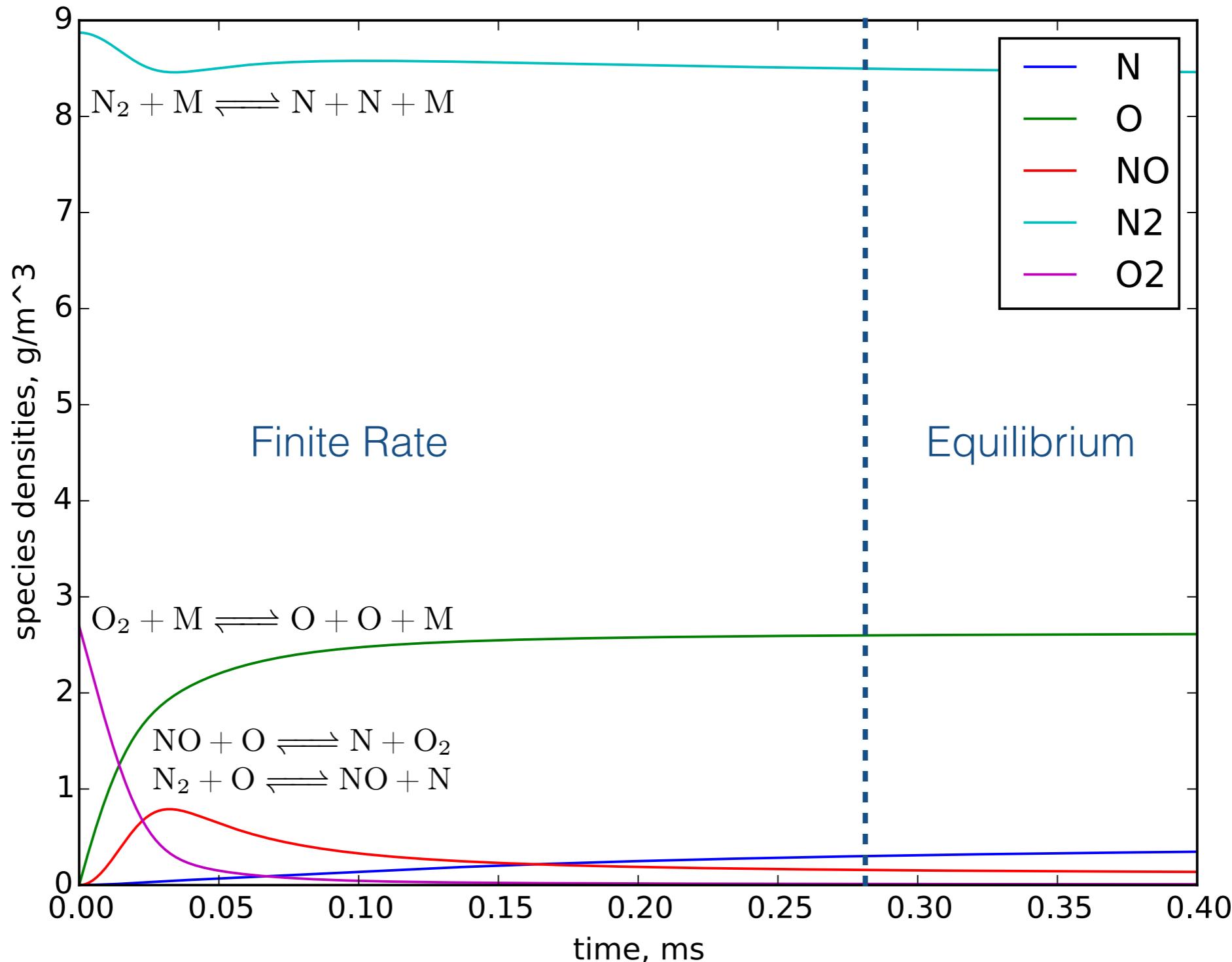
$$k_2 = \Delta t \cdot \dot{\omega}_i(\rho^n + \frac{k_1}{2})$$

$$k_3 = \Delta t \cdot \dot{\omega}_i(\rho^n + \frac{k_2}{2})$$

$$k_4 = \Delta t \cdot \dot{\omega}_i(\rho^n + k_3)$$

Results (cvisoRK4)

Implementing a 0D reactor at constant volume and temperature



What else is available in the library?

Building the API documentation

What you will need: Doxygen (<http://www.stack.nl/~dimitri/doxygen/index.html>)

1. Change to `mutation++/build` directory
2. Run `ccmake ..`
3. Turn `BUILD_DOCUMENTATION` to ON
4. Type '`c`' '`c`' '`g`'
5. Run `make docs`
6. Open `mutation++/docs/html/index.html` in your favorite browser

Happy Coding!