# Decision Trees: Discussion

Machine Learning


Some slides from Tom Mitchell, Dan Roth and others

# This lecture: Learning Decision Trees

1. **Representation**: What are decision trees?

2. **Algorithm**: Learning decision trees
   - The ID3 algorithm: A greedy heuristic

3. Some extensions

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# Tips and Tricks

1. Decision tree variants
2. Handling examples with missing feature values
3. Non-Boolean features
4. Avoiding ***overfitting***

# 1. Variants of information gain

Information gain is defined using entropy to measure the disorder/impurity of the labels.

There are other ways to measure disorder. Eg: MajorityError, Gini Index

Example: *MajorityError* computes:

> "*Suppose the tree was not grown below this node and the most frequent label were chosen, what would be the error?*"

> Suppose at some node, there are 15 + and 5 − examples. What is the *MajorityError*?

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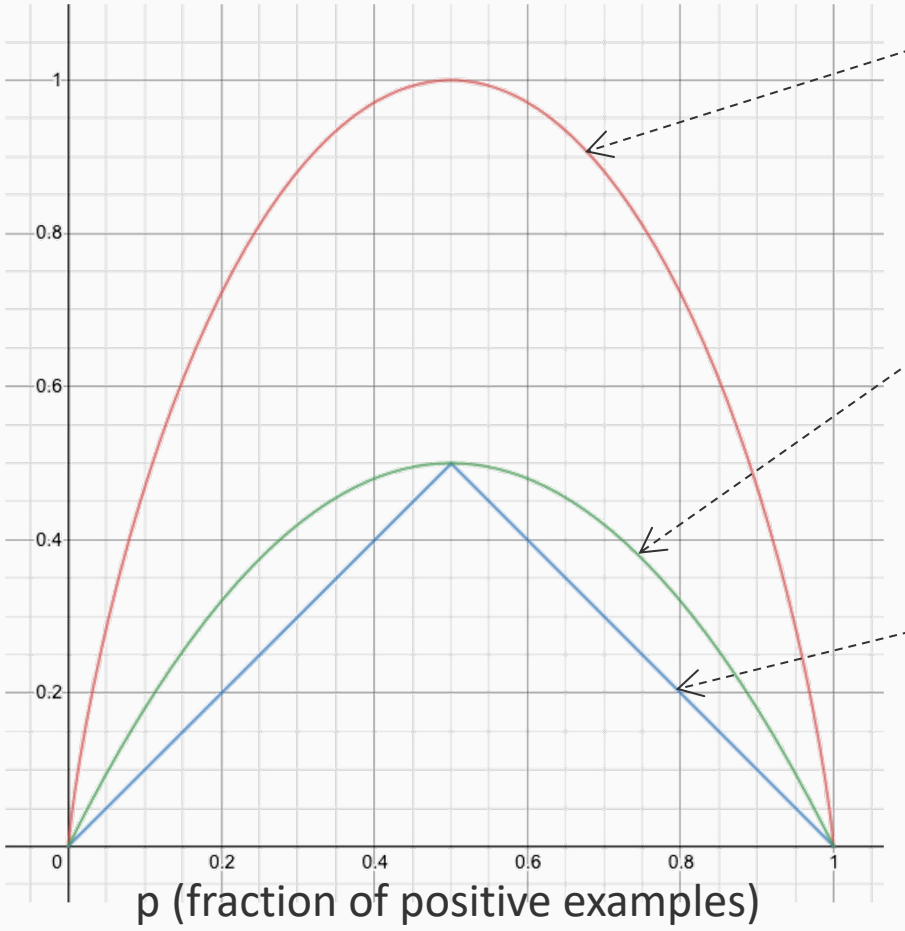
>
> Answer: ¼

Works like entropy

# 1. Variants of information gain

Let $p$ denote the fraction of positive examples. Then $1 - p$ is the fraction of negative examples.




Entropy:

$$-(p \log_2 p + (1-p) \log_2 (1-p))$$

Gini Index:

$$1 - (p^2 + (1-p)^2)$$

MajorityError:

$$\min(p, 1-p)$$

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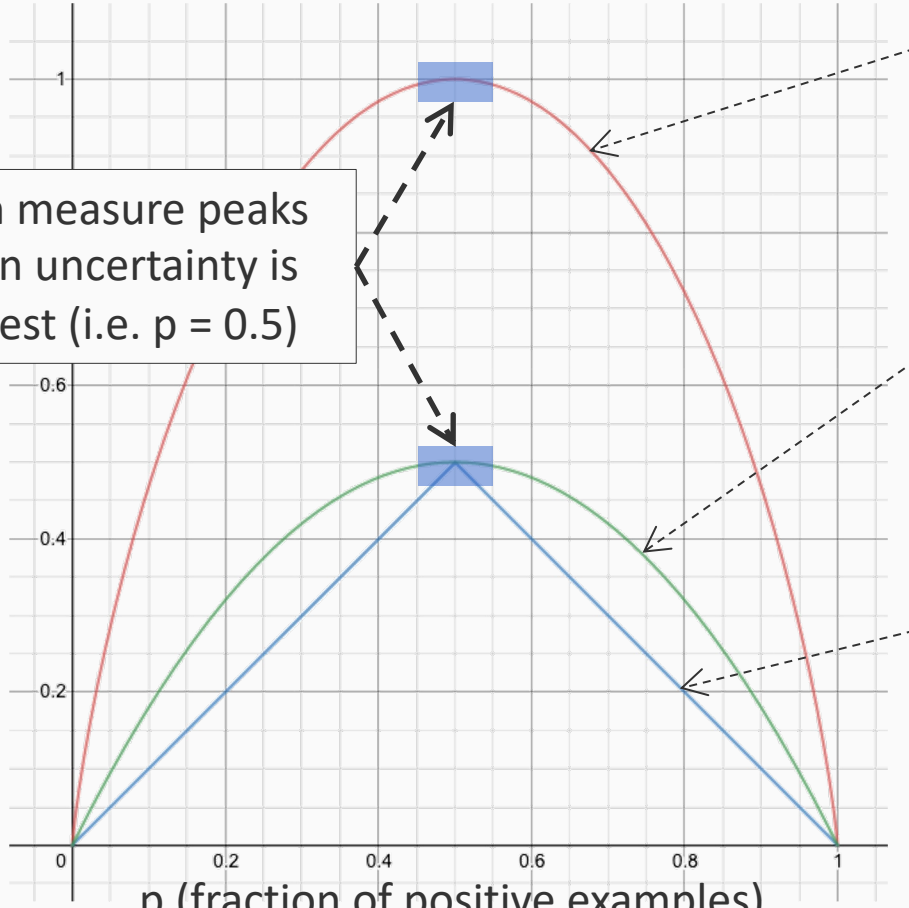


Each measure peaks when uncertainty is highest (i.e. p = 0.5)

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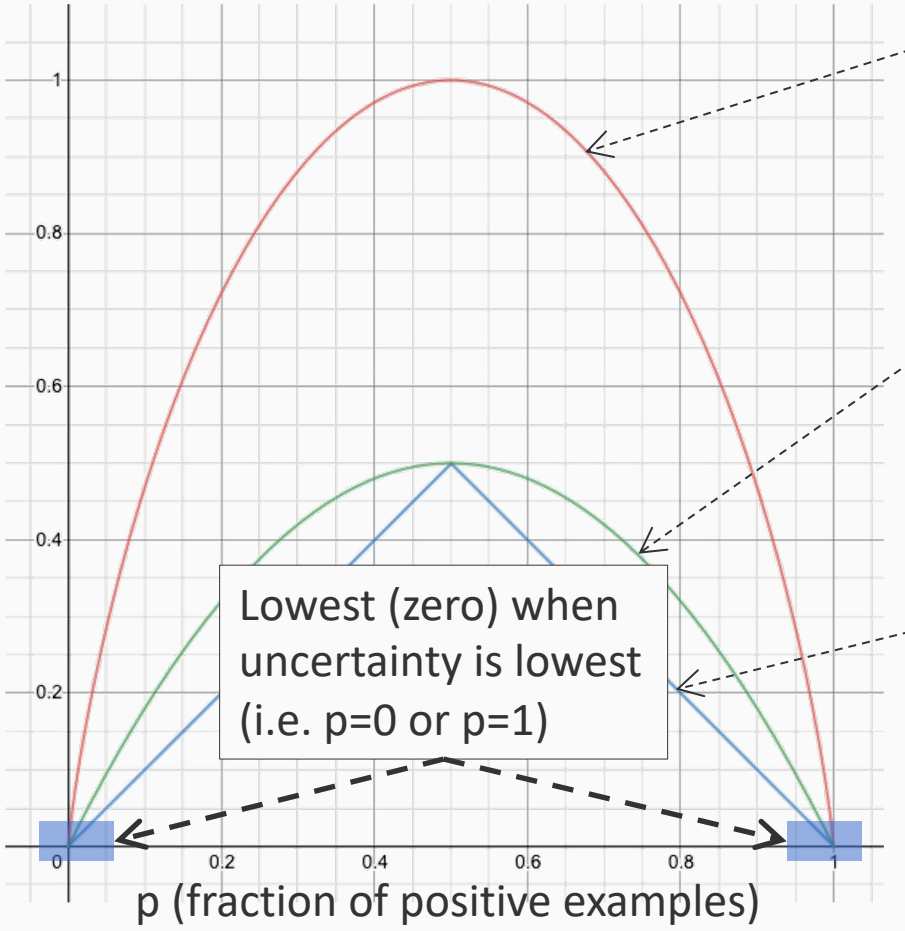


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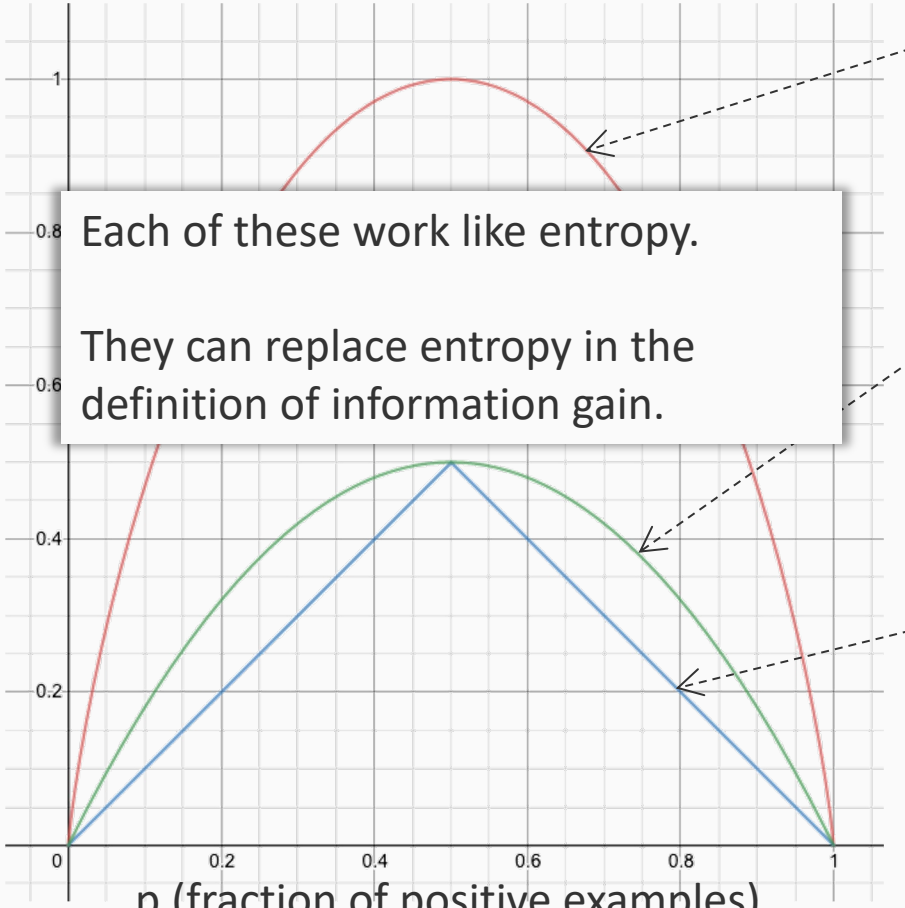


Each of these work like entropy.

They can replace entropy in the definition of information gain.

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# 2. Missing feature values

Suppose an example is missing the value of an attribute. What can we do at training time?

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 8 | Sunny | Mild | ??? | Weak | No |
| 9 | Sunny | Cool | High | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |

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Suppose an example is missing the value of an attribute. What can we do at training time?

Different methods to "Complete the example":

- Using the most common value of the attribute in the data
- Using the most common value of the attribute among all examples with the same output
- Using fractional counts of the attribute values
  - Eg: Outlook={5/14 Sunny, 4/14 Overcast, 5/15 Rain}
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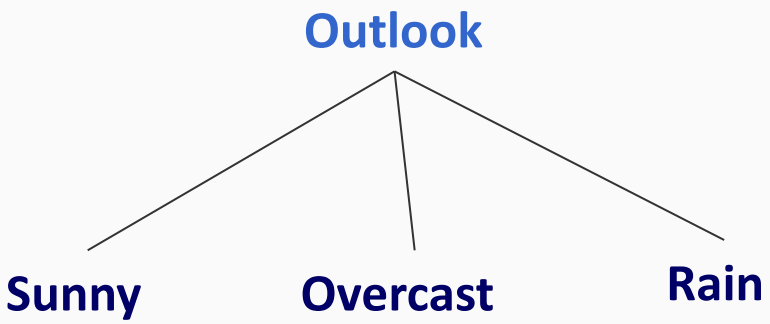
At test time? Use the same method

# 3. Non-Boolean features

- If the features can take multiple values
  - We have seen one edge per value (i.e a multi-way split)



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Convert Outlook=Sunny → {
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  - Or, perhaps group values into disjoint sets
- For numeric features, use thresholds or ranges to get Boolean/discrete alternatives

# 4. *Overfitting*

# The “First Bit” function

- A Boolean function with n inputs
- Simply returns the value of the first input, all others irrelevant

| $X_0$ | $X_1$ | Y |
|---|---|---|
| F | F | F |
| F | T | F |
| T | F | T |
| T | T | T |

$Y = X_0$

$X_1$ is irrelvant

What is the decision tree for this function?

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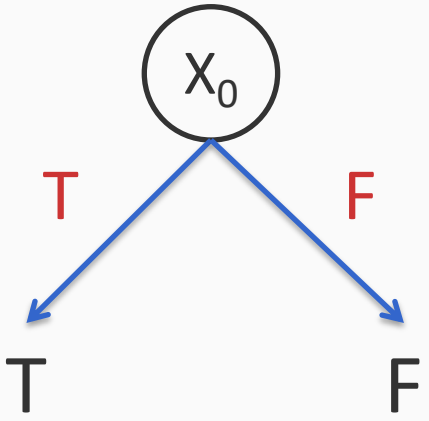

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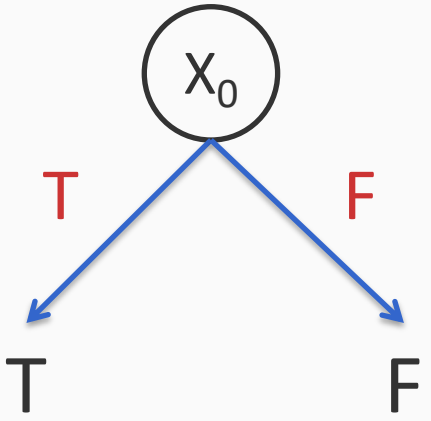
**Exercise**: Convince yourself that ID3 will generate this tree

What is the decision tree for this function?



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Suppose we have all $2^n$ examples for training. What will the error be on any future examples?

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Suppose we have all $2^n$ examples for training. What will the error be on any future examples?

Zero! Because we have seen every possible input!

And the decision tree can represent the function and ID3 will build a consistent tree

# Noisy data

What if the data is noisy? And we have all $2^n$ examples.

| $X_0$ | $X_1$ | $X_2$ | Y |
|---|---|---|---|
| F | F | F | F |
| F | F | T | F |
| F | T | F | F |
| F | T | T | F |
| T | F | F | T |
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Train and test sets are no longer identical.

Both have noise, possibly different

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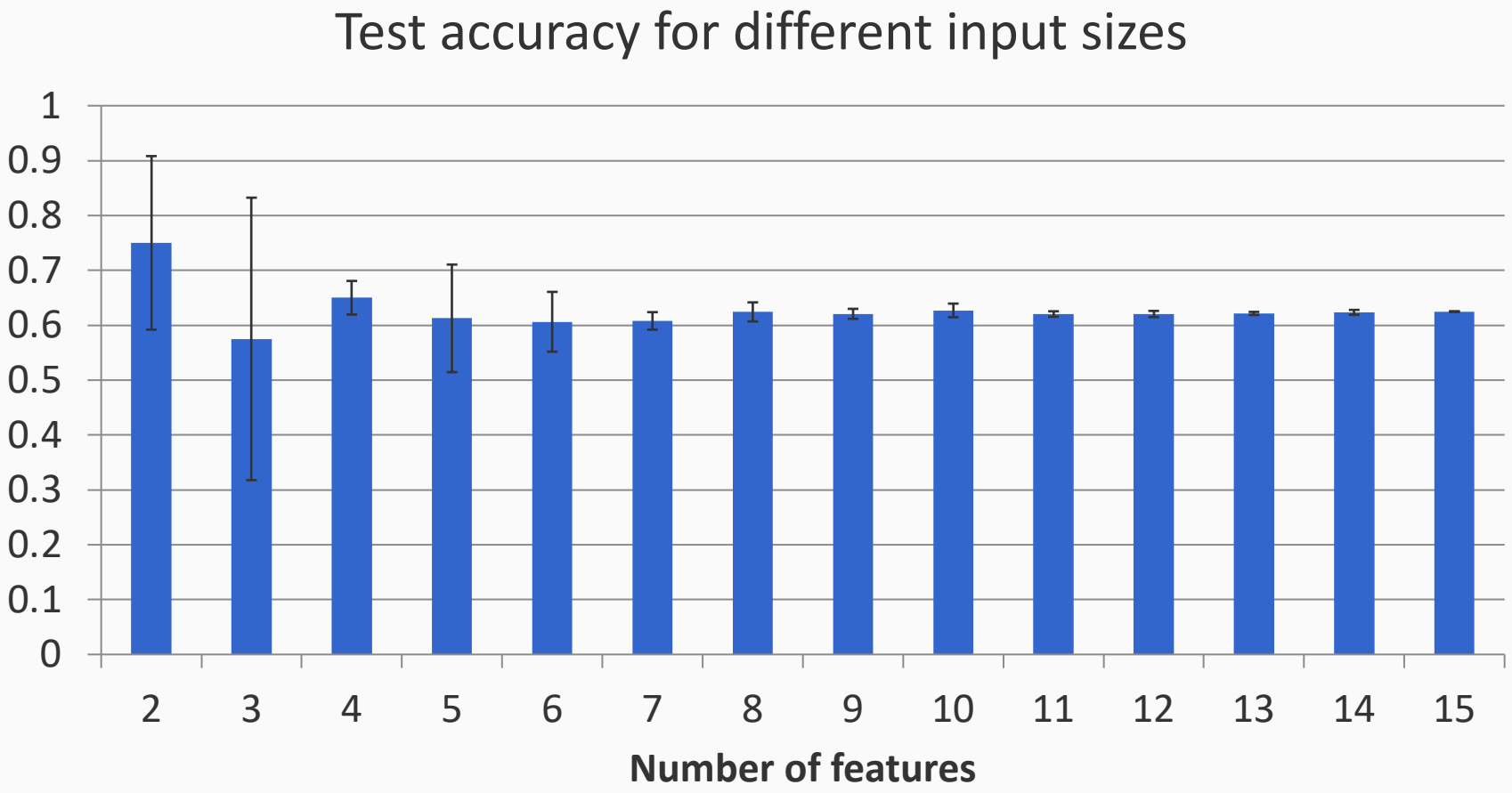
# E.g: Output corrupted with probability 0.25

The data is noisy. And we have all $2^n$ examples.




The error bars are generated by running the same experiment multiple times for the same setting

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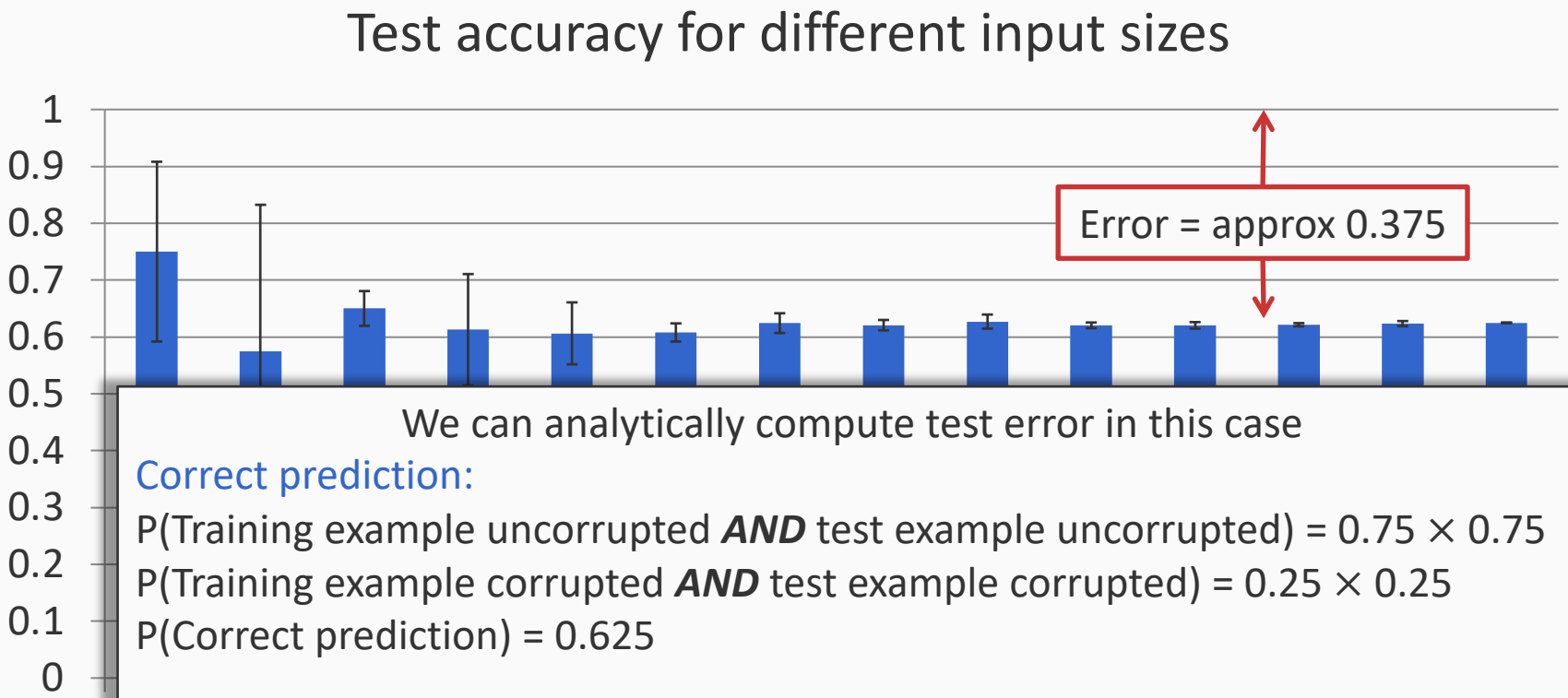


We can analytically compute test error in this case

Correct prediction:

P(Training example uncorrupted ***AND*** test example uncorrupted) = 0.75 × 0.75

P(Training example corrupted ***AND*** test example corrupted) = 0.25 × 0.25

P(Correct prediction) = 0.625

Incorrect prediction:

P(Training example uncorrupted ***AND*** test example corrupted) = 0.75 × 0.25

P(Training example corrupted and ***AND*** example uncorrupted) = 0.25 × 0.75

P(incorrect prediction) = 0.375

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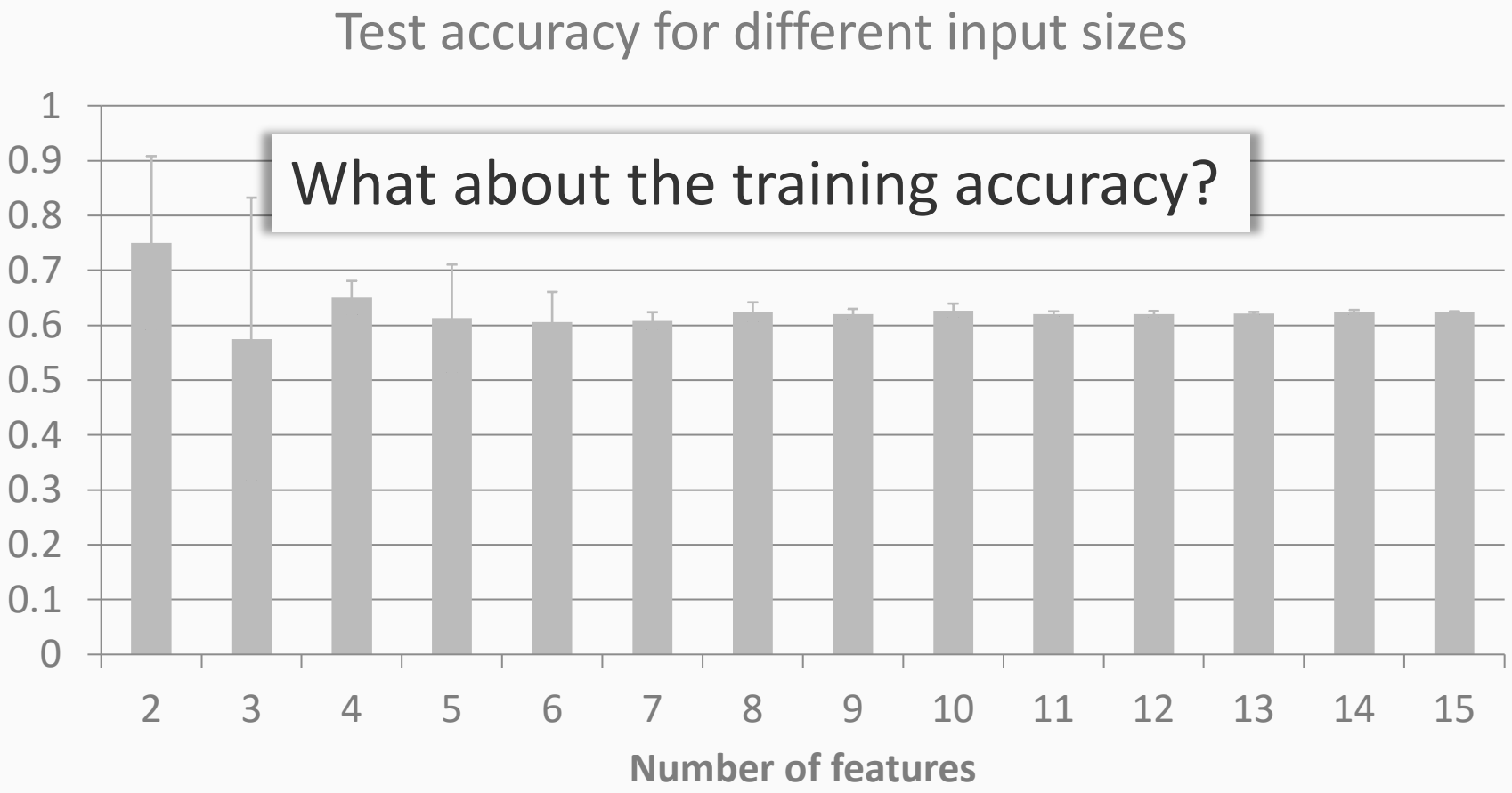

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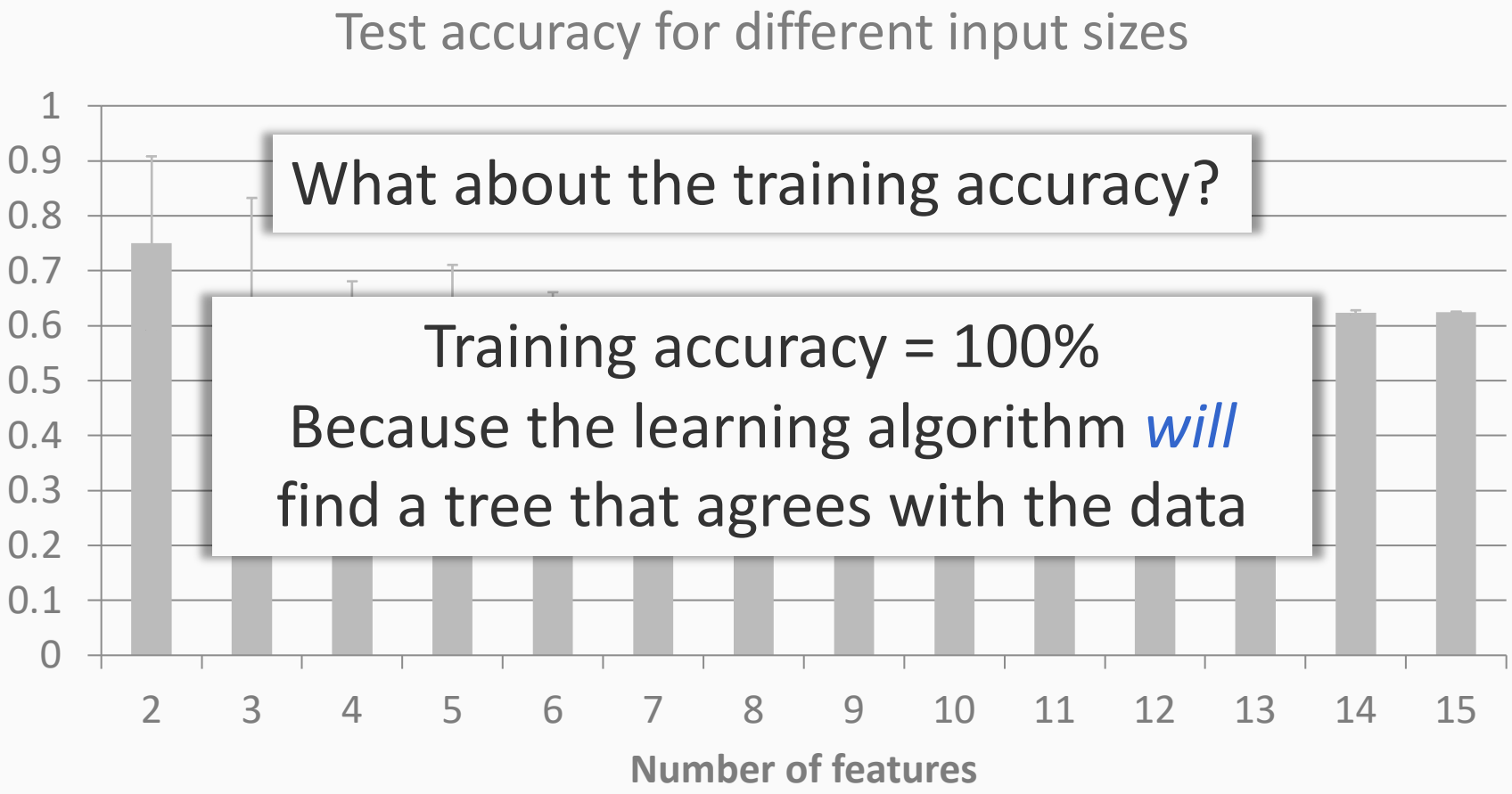

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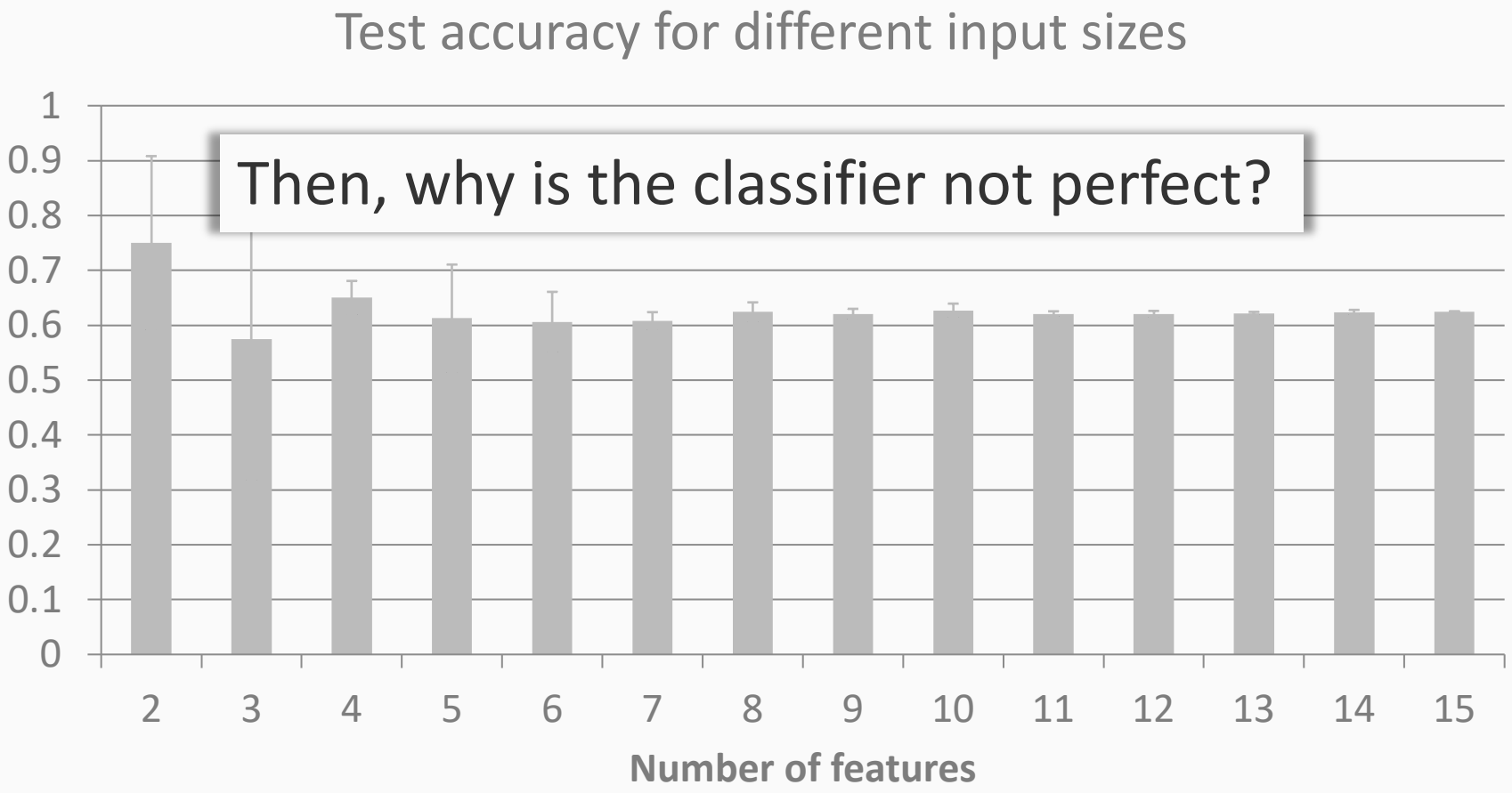

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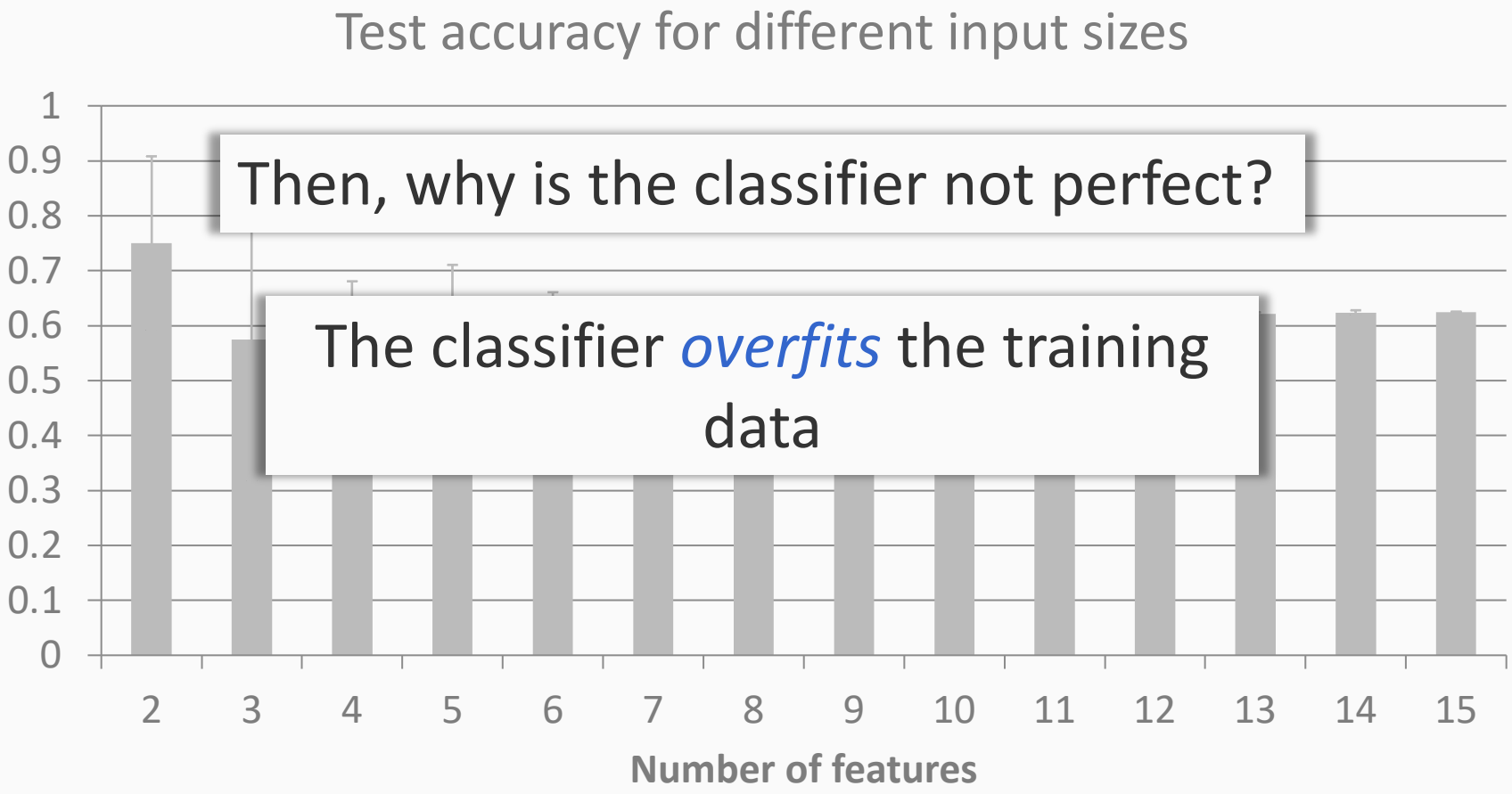

# Overfitting

- The learning algorithm finds a hypothesis that fits the noise in the data
  - Irrelevant attributes or noisy examples influence the choice of the hypothesis
- May lead to poor performance on future examples

# Overfitting: One definition

- Data comes from a probability distribution $D(X, Y)$
- We are using a hypothesis space $H$
- Errors:
  - Training error for hypothesis $h \in H$: $\text{error}_{\text{train}}(h)$
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- A hypothesis $h$ ***overfits*** the training data if there is another hypothesis $h'$ such that
  1. $h$ has lower training error than the competing hypothesis $h'$ but,
  2. $h'$ generalizes better than $h$.

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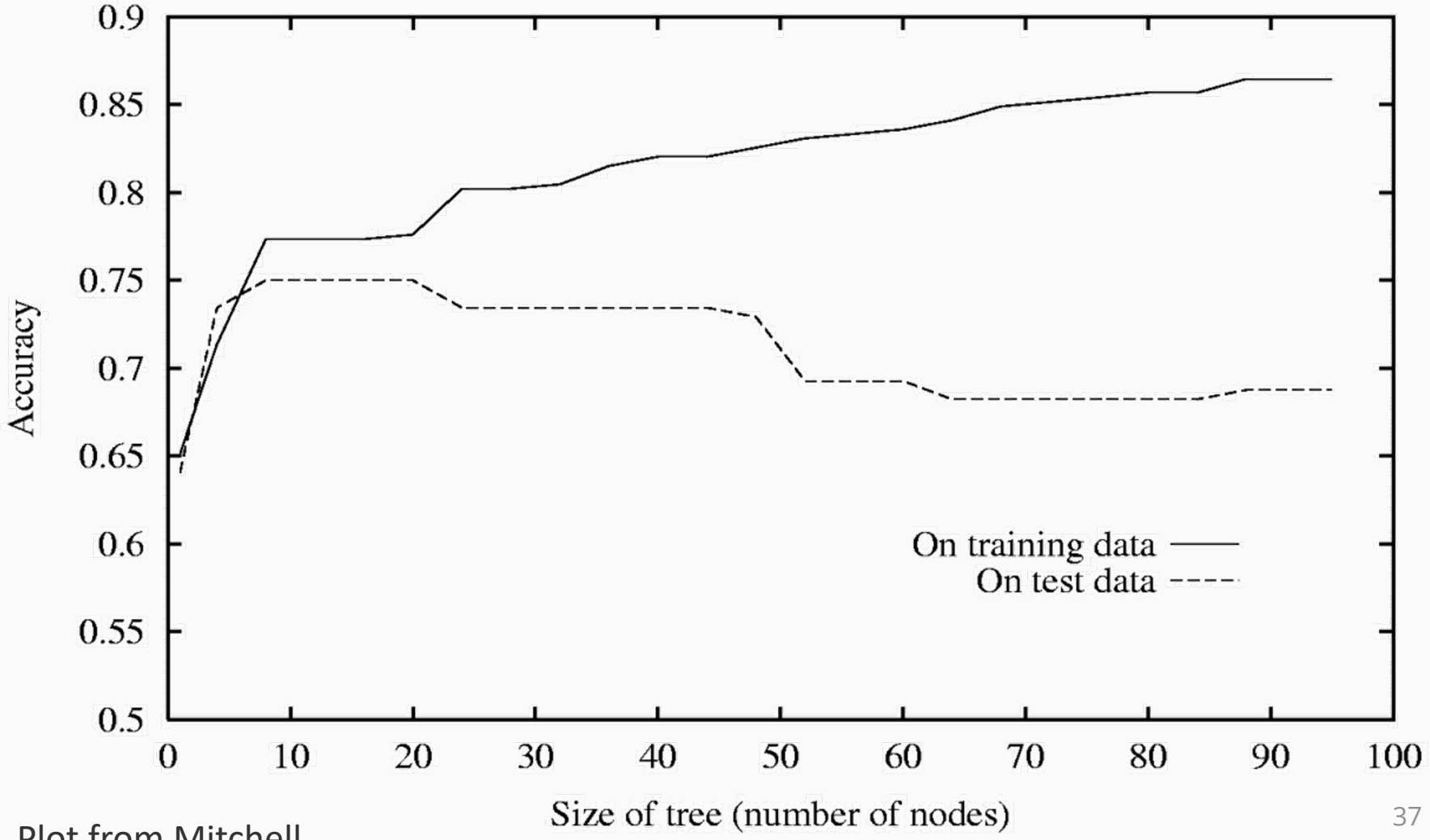
# Decision trees *will* overfit




Plot from Mitchell

# Avoiding overfitting with decision trees

## Occam's Razor

Favor simpler (in this case, shorter) hypotheses

Why? Fewer shorter trees, less likely to fit better by coincidence

- Some approaches:
  1. Fix the depth of the tree
     - ***Decision stump*** = a decision tree with only one level
     - Typically will not be very good by itself
     - But, we will revisit decision stumps later (short decision trees can make very good features for a second layer of learning)

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- Some approaches:
  2. Optimize on a ***held-out set*** (also called ***development set*** or ***validation set***) while growing the trees
     - Split your data into two parts –training set and held-out set
     - Grow your tree on the training split and check the performance on the held-out set after every new node is added
     - If growing the tree hurts validation set performance, stop growing

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- Some approaches:
  3. Grow full tree and then prune as a post-processing step in one of several ways:
     1. Use a validation set for pruning from bottom up greedily
     2. Convert the tree into a set of rules (one rule per path from root to leaf) and prune each rule independently

# Summary: Decision trees

- A popular machine learning tool
  - Prediction is easy
  - If we have Boolean features and binary classification, decision trees can represent any Boolean function
- Greedy heuristics for learning
  - ID3 algorithm (using information gain)
  - Robust implementations of some variants (eg. C4.5 algorithm) exist
- Can be used for regression too
- Decision trees are prone to overfitting unless you take care to avoid it