Ernst-Abbe-Hochschule Jena
University of Applied Sciences

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# Masters in Scientific Instrumentation

## Optical Instruments

## Preparatory Report

## on

# Principle and Application of the Pulfrich-Refractometer.

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**Submitted by:**


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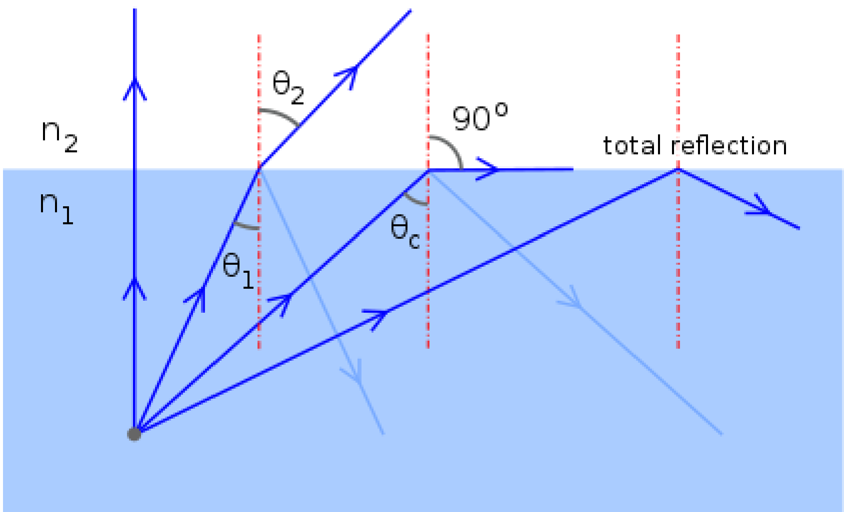
## Preparation

**3.1** Explain the basics of refraction of light including Total Internal Reflection. Calculate the critical angle of material with Brewster's angle of 60° in air.

Refraction Bending of light during transmission from one transparent substance into another. The bending is caused due to different speeds of light propagation in different mediums.

- The incident beam of light is refracted in such a way that it travels over the water's surface at a certain angle of incidence.
- The critical angle is the angle of incidence at which something happens. The refraction **angle is 90°**.
- The incident ray is reflected in the medium when the **angle of incidence is larger than the critical angle**. This event is referred to as total internal reflection.




$$\sin \emptyset\, 1 = \frac{N2}{N1}$$

$$\emptyset\ Critical = arc\ Sin\,\langle\frac{N2}{N1}\rangle$$

$$\emptyset = 0.0302$$

$$\text{Angle } \emptyset\ p = 60°$$

$$\emptyset p = arc\ tan\,\langle\frac{N2}{N1}\rangle \qquad \rightarrow \frac{N2}{N1} = 1.732$$

$$sin\emptyset b = \frac{N2}{\sqrt{N1^2 + N2^2}}$$

$$sin\emptyset c = \frac{N2}{N1}$$

$$\tan \emptyset b = \frac{1}{\sin \emptyset c}$$

$$sin\emptyset c = \frac{1}{\tan 60°}$$

$$= \frac{1}{\sqrt{3}}$$

$$\emptyset c = arc\ sin \langle \frac{1}{\sqrt{3}} \rangle$$

$$\boldsymbol{\emptyset c = 35°16}$$

3.2 Use Snell's law to calculate at which side of the prism the ray exits and its angle of refraction in the surrounding air.



3.2 A ray hits the prism as shown in fig. Use Snell's law to calculate at which side of the prism the ray exits and its angle of refraction in the surrounded air!

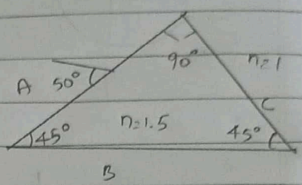
Applying Snell's law at left surface (A):

$\sin(40) = 1.5 \sin\theta_1$

$\sin\theta_1 = \dfrac{\sin 40}{1.5}$

$\theta_1 = \sin^{-1}(0.4285)$

$= 25.37^\circ$




This will give us angle of $(90^\circ - 25.372^\circ)$ at surface C,

angle $90 - 25.372 = 64.62^\circ$

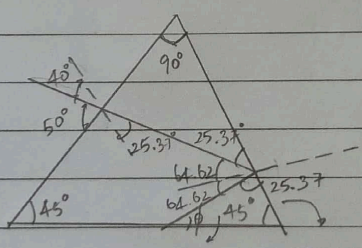
Applying Snell's law on surface C,

$1.5 \sin(64.62) = 1 \sin(\theta)$

$\sin\theta = 1.35$

This value is not acceptable. So, there is a possibility that total internal reflection will take place.

Thus, ray trace will look like;




angle $\phi = 180 - (45 + 25.37)$

$\phi = 109.62^\circ$

Actually the ray trace will be:

Angle with surface B's normal

$\theta_B = (\phi - 90)$

(let) $\theta_B = 19.62^\circ$

Applying Snell's law

$$1.5 \sin(19.62) = \sin(\theta_{out})$$

$$\sin(\theta_{out}) = 0.5036$$

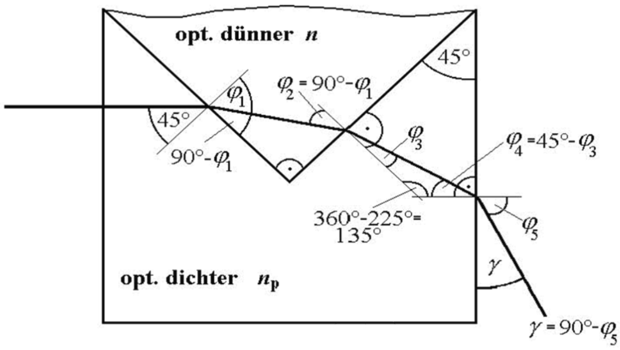
$$\boxed{\theta_{out} = 30.243^\circ}$$

3.3 Derive the formulas (1) and (2) in consideration of the ray trace and Snell’s law.

**Appendix: Derivation of the formula for the calculation of the refractive index using the Pulfrich refractometer with V-prism**

Relation between the refractive index of a fluid ($n$) and the angle of deflection ($\gamma$)

$n_p$ = refractive index of the V-prism (flint glass or crown glass)




$$\varphi_2 = 90° - \left(90° - 90° + \varphi_1\right) = 90° - \varphi_1$$

$$\frac{n}{n_p} = \frac{\sin 45°}{\sin\varphi_1} \quad \rightarrow \quad \sin\varphi_1 = \frac{1}{\sqrt{2}} \cdot \frac{n_p}{n}$$

$$\frac{n}{n_p} = \frac{\sin\varphi_3}{\sin\varphi_2} = \frac{\sin\varphi_3}{\cos\varphi_1} = \frac{\sin\varphi_3}{\sqrt{1-\sin^2\varphi_1}} \quad \rightarrow \quad \sin\varphi_3 = \frac{n}{n_p}\sqrt{1-\frac{n_p^2}{2n^2}}$$

$$\varphi_3 = 45° - \varphi_4$$

$$\frac{\sin\varphi_5}{\sin\varphi_4} = n_p \quad \rightarrow \quad \sin\varphi_4 = \frac{\sin\varphi_5}{n_p}$$

$$\varphi_5 = 90° - \gamma \quad \rightarrow \quad \sin\varphi_4 = \frac{\cos\gamma}{n_p}$$

Successive insertion yields the final equation for $n$:

$$\frac{n}{n_p}\sqrt{1-\frac{1}{2}\frac{n_p^2}{n^2}} = \frac{1}{2}\sqrt{2}\left(\sqrt{1-\frac{\cos^2\gamma}{n_p^2}} - \frac{\cos\gamma}{n_p}\right).$$

Rearranging and simplifying yields

$$n = \sqrt{n_p^2 - \cos\gamma\sqrt{n_p^2 - \cos^2\gamma}}$$

resp.

$$n = \sqrt{n_p^2 - \sin(90-\gamma)\cdot\sqrt{n_p^2 - \sin^2(90-\gamma)}} \quad .$$

3.4 Describe the working principle of refractometers by Abbe and Pulfrich. The V-shape in Fig. 1 is defined by an isosceles triangle with a 90° angle at the bottom. Specify the advantages and disadvantages of both methods.

Abbe Refractometer

In any encounter of light in an optical setup, the light can be diffracted, reflected, or absorbed when passing from one medium to another. Abbe Refractometer uses the reflection principle namely, total internal reflection. This works on the principle of the critical angle. The sample is put between two prisms: measurement and illumination prism. Light enters the sample from illuminating prism, gets refracted at an angle, then the position of the border between the bright and light area is measured. Experimentally, a 'shadow boundary' created at the sample's critical angle is used to determine the index. A filtered white light source is used to measure the index.

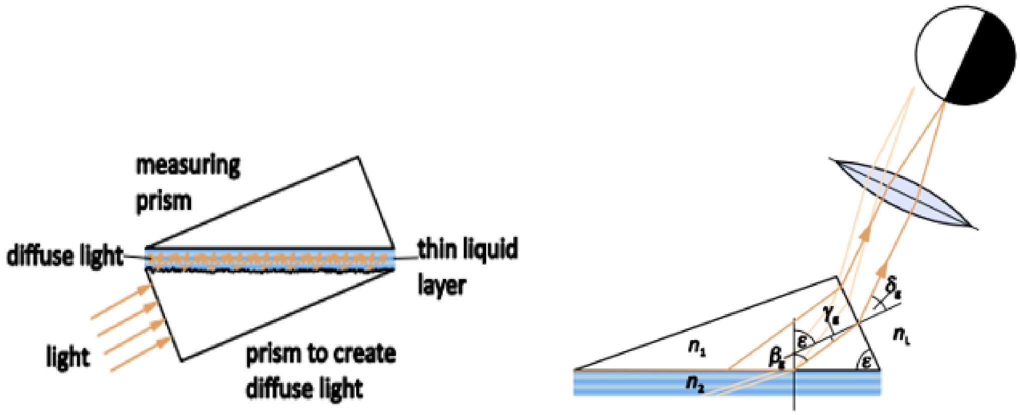



*Figure 1:* Ray diagram illustrating principle of Abbe Refractometer.

The operating principle of the Abbe refractometer is illustrated in Figure 1. The first figure shows two different prisms. The lower has a rough surface to create diffusely scattered light entering the liquid layer. The upper one is used to create a dark-bright boundary. The second Figure shows an incident ray at a glancing angle to the upper prism that is refracted under the critical angle $\beta_g$.

Let n1 and n2 denote the refractive indices of glass and of the liquid under study, respectively; ε is the refraction angle of the prism, and $\delta_g$ is the deflection angle. From Figure 1 the following relationships can be derived:

$$\sin \beta_g = n_2 / n_1 ,$$

$$\sin \delta_g = (n_1/n_L)\sin\gamma_g \approx n_1 \sin \gamma_g ,$$

$$\beta_g + \gamma_g = \varepsilon ,$$

where the refractive index of air is approximated by $n_L$=1.

Advantages:
1. Just a small sample is required.
2. Prism can be temperature controlled.
3. Abbe’s refractometers can be interfaced with computers, printers, and data processing equipment to further analyse readings.

Disadvantages:
1. Difficult to analyse materials with lower RI than that of a prism.
2. One must bring a possibly hazardous material close to the eyes to read hand-held refractometers.

## Pulfrich Refractometer

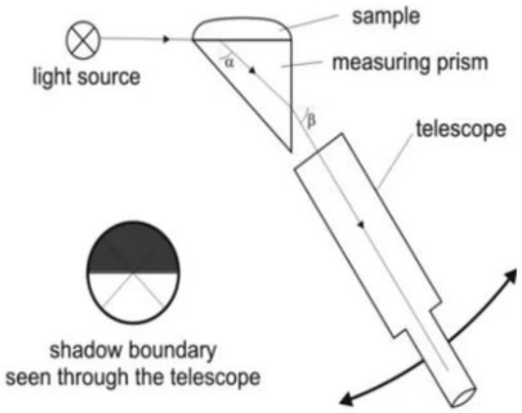
For Pulfrich Refractometer, Total Internal Reflection and Deflection are used. Thus, it gives higher intensity to the measurements. In this setup, a beam of monochromatic light is directed almost horizontally through the substance so that it meets the prism face at a grazing incidence angle. Thus it facilitates total internal reflection.

With the V-prisms used in this experiment not the critical angle for total reflection, but the deflection of the light is measured. The light beam passing through the collimator is incident on the V-prism and is refracted at the oblique surfaces that hold the fluid; finally, the light beam leaves the V-prism under an angle γ. This angle γ is measured with a telescope observing the collimator slit. γ is given by:

$$\sin(90°-\gamma)=\frac{1}{\sqrt{2}}\left(\sqrt{1{,}5\,n_1^2-n_\lambda^2}-\sqrt{n_\lambda^2-0{,}5\,n_1^2}\right)$$

alternatively, the wavelength-dependent refractive index $n_\lambda$ is given by:

$$n_\lambda=\sqrt{n_1^2-\sin(90°-\gamma)\cdot\sqrt{n_1^2-\sin^2(90°-\gamma)}}$$




Advantage:

1. Simple in operation.

Disadvantage:

1. Temperature Correction: Refractive index not determined in temperature 20° or if 20° cooling water is not used to circulate in the instrument.
2. Optical excitation (conjugated double bond): the difference between observed and calculated values of atomic and structural constants.

a) collimated light if using simple refraction and
b) converging light if using total internal reflection?

a) Collimated light has light rays that are parallel. The light spreads slowly as it travels. It is also related to collinear, as all the rays are lined up with each other.

As light rays travel from a high refractive index medium to a low refractive index medium, the medium acts as a convergent lens, converging the light rays. And after a certain angle called the critical angle, the light rays are totally reflected inside the high refractive index medium which is called total internal reflection.

3.6 Give reasons for the limitation of the measuring range if using different measuring methods with Pulfrich-Refractometer.

- Limited by the refractive index range of the measurement prism and sample.
- Accuracy may decrease at extreme angles of incidence or high refractive indices.
- Variation in sample thickness can affect measurement precision.
- Non-linear relationship between refractive index and measurement angle.
- Instrument design and alignment may introduce errors at certain measurement ranges.

3.7 Which kind of spectra do you know and how useful are they for the applied measuring methods?

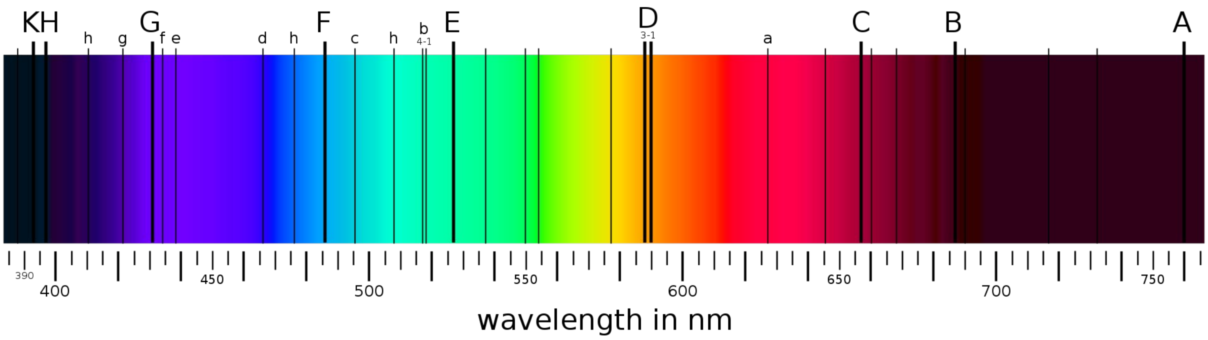
| **First Law**: Continuous Spectrum | **Second Law**: Emission Spectrum *Brightline* | **Third Law**: Absorption Spectrum *Dark line* |
|---|---|---|
| Hot bodies radiate a continuous spectrum | Hot gases under lower pressure emit energy in certain regions of the spectrum | Cooler gases absorb some of the the energy radiating from the hot body. |

| **Table 1. Optical Spectroscopy Methods** | | |
|---|---|---|
| **Method** | **Measured Quantity** | **Examples** |
| Absorption | Absorbance or the ratio of transmitted to incident radiant power, $A = -\log(\Phi/\Phi_o)$ | Atomic absorption, UV-VIS molecular absorption, IR absorption |
| Emission | Radiant power of emission, $\Phi_E$ | ICP and DCP emission, spark emission, laser-induced breakdown emission, flame emission, DC arc emission |
| Luminescence | Radiant power of luminescence $\Phi_L$ | Molecular fluorescence and phosphorescence, chemi- and bio-luminescence, atomic fluorescence |
| Scattering | Radiant power of scattering, $\Phi_s$ | Raman scattering, Mie scattering, turbidity |

3.8 List the wavelength and the color of the following spectral lines: e-line, F'-line, C'-line.

Spectral lines and their wavelengths and colors:
- e-line: Wavelength = 546.07 nm, Color = Green
- F'-line: Wavelength = 486.13 nm, Color = Blue
- C'-line: Wavelength = 656.28 nm, Color = Red

| Spectral | Wavelength (nm) | Color |
|---|---|---|
| A-line | 760 - 770 | Red |
| B-line | 680 - 700 | Reddish-Orange |
| C-line | 650 - 660 | Red |
| D-line | 580 - 590 | Yellow |
| e-line | 520 - 570 | Green |
| F-line | 490 - 500 | Cyan |
| F'-line | 460 - 470 | Blue |
| G-line | 430 - 440 | Indigo |
| h-line | 410 - 420 | Violet |
| H-line | 396 - 400 | Violet |

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