Problem Formulation for Multidisciplinary Optimization in Aerospace Engineering

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1. Introduction

In real engineering problems of aerospace systems, multiple disciplines have to work together to satisfy the need of high performance complex system design. Therefore, these systems need coupled simulations and design optimization to have better design in shorter time. This is why multidisciplinary design optimization(MDO) emerged within aerospace engineering field in the late 1970s. One of the first application of it was wing design where the disciplines of aerodynamics, structures and controls are strongly coupled. However, studies are not limited by aerospace field, it has been applied to many different real engineering systems in automotive engineering, çivil engineering etc. . In this study MDO of aerospace systems are mainly considered.

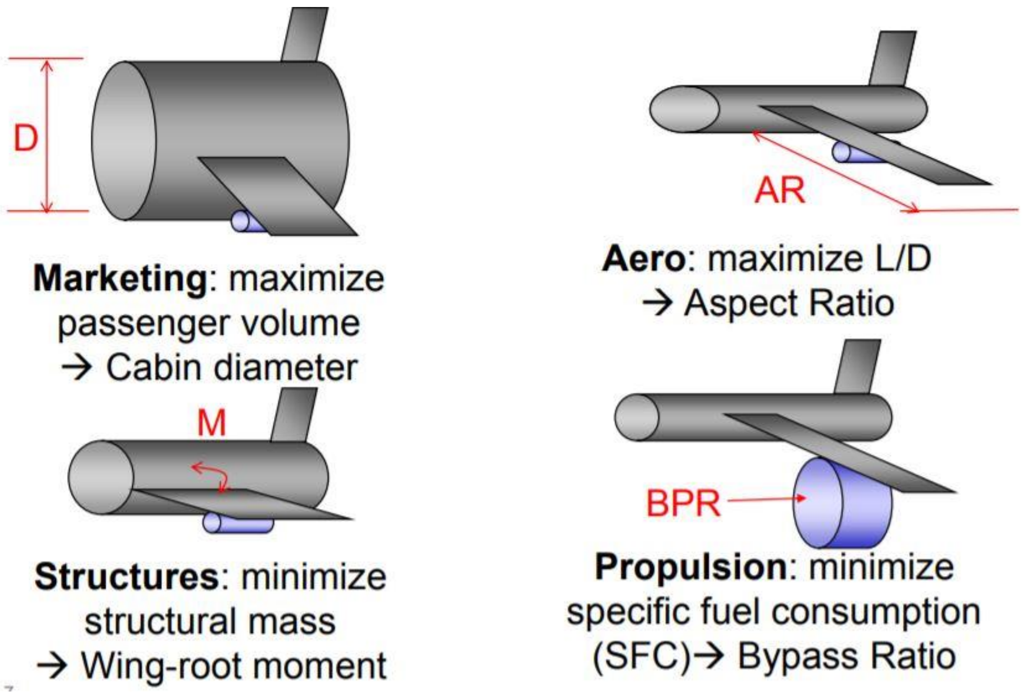

Figure 1: Discipline Level Optimization. Source: MIT

To understand the significance of MDO better, a simple schematic of airplane design optimization example is given in Figure 1. As it is seen, depending on the discipline, the design variable changes so the design loop takes long time to be completed since every discipline studies the design optimization individually which is known as discipline level optimization. MDO offers a system level optimization which means it includes these all disciplines in one analysis. This type of analysis called as multidisciplinary analysis which is explained in following section.

2. Multidisciplinary Design Analysis

In multidisciplinary design analysis different disciplines do the analysis individually but the output of each discipline analysis provides an input for other discipline. Explaining this by using a real case application of aeroelasticity would give a better understanding.

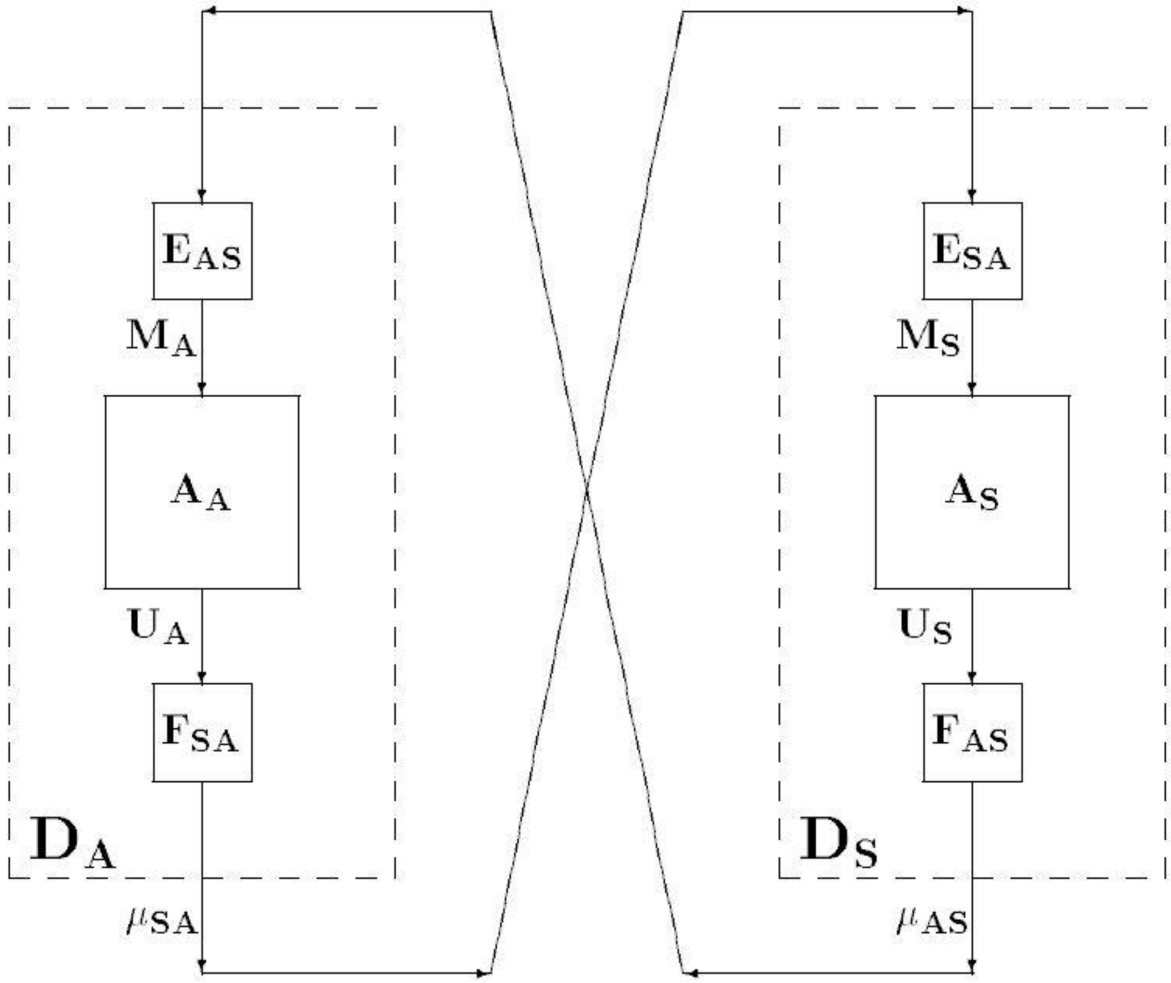

Figure 2: Multidisciplinary Analysis for an Aeroelastic System

In Figure 2 the system of aeroelastic optimization is given, two boxes with dashed lines represents two different disciplines which are aerodynamics(D_A) and structures(D_s). The necessary computations for these disciplines are done by individual analysis codes such as computational fluid dynamics(CFD) code for aerodynamics discipline(A_A) and finite element code (FEM) for structure discipline(A_s). M_A and M_s are the input variables of the analysis, aerodynamic and structure respectively. After analysis completed the results are given as coupling variables U_A and U_s which has the neccessary information for other discipline. For instance U_A is the result of aerodynamic analysis and it fits into interdisciplinary mapping F_{SA}. The information of the grid values are passed by μ_{SA} to other interdisciplinary mapping E_{SA}. As a result multidisciplinary analysis is achieved when feasibility is observed in each disciplines which has input by using interdisciplinary mappings.

3. Optimization

In aerospace design optimization problems, the objective and constraint functions are usually nonlinear. So general purpose nonlinear optimizer is required. Optimization algorithms are mainly categorized into as gradient based and gradient free algorithms. Gradient based method requires computing of the gradient to find the best direction for converging to the optimum value. However, only objective function and constraints are needed for gradient free algorithms. In Figure 3, for both gradient based and gradient free algorithms their dependency to number of design variables can be clearly seen. The computational cost of gradient-free optimizers (ALPSO, NSGA2) scale exponentially with number of design variables, while the gradient-based optimizers (SNOPT, SLSQP) scale linearly [Martins et al., 2016]. Therefore, for number of design variables greater than $\mathcal{O}$ (10^2) it is unfeasible to use gradient free algorithms which is the case for most of the aerospace applications. By considering this reality, gradient based algorithms are decided as more suitable. Howevere there are still some other issues for gradient based algortihms. Firstly, the assuption of continuous and differentiable function can be wrong for some discontinuities. However, in practice it is seen that this did not prevent the algorithm from converging minimumSecondly, the gradient based algorithms converges to only one local minimum. This problem is also investigated for practical applications and it is observed that this situation also can be ignored. [Martins et al. 2016].

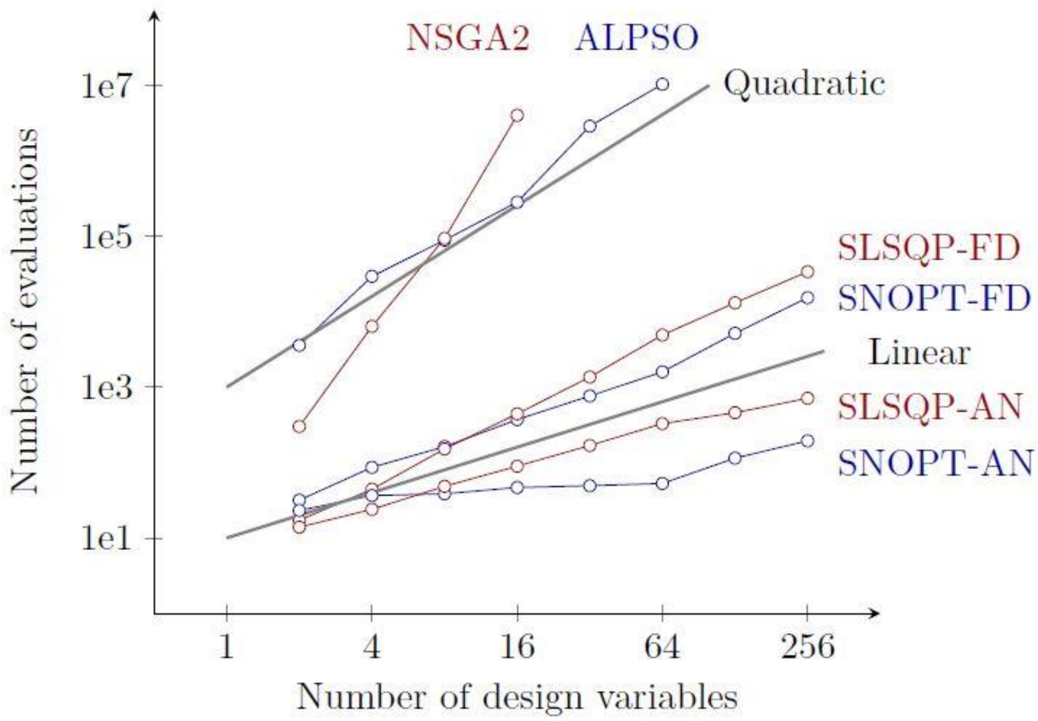

Figure 3 : Computational cost of minimizing a constrained multidimensional Rosenbrock function with respect to the number of design variables for different optimization algorithms[Martins et al. 2016]

The challenging part of gradient based optimization algorithms is to develop or choose a method for computing the gradients to have an accurate and efficient algorithm. Main methods for derivative calculations are: Finite Difference, Complex-Step, Direct Method and Adjoint Method. As it is indicated previously, number of design variables are high in aerospace applications so this plays a crucial role for choosing the better algorithm for derivative calculation as well. When the given methods are investigated, except Adjoint method the cost of other three methods are proportional to number of design variables. Therefore, using of adjoint method or developing a unified method are found as the best options to compute gradients.

4. Multidisciplinary Design Optimization Architectures

In a multidisciplinary design optimization problem, having an efficient interaction between different disciplines is significant for the performance of the MDO formulations. Therefore, several architectures are developed to formulate this interaction and the main formulations are "Multidisciplinary Feasible(MDF)", "Individual Discipline Feasible(IDF)" and "All-at-Once(AAO)" architectures. In this section, details of these methods are described by giving the general mathematical expressions and procedures.

4.1. All-at-Once Architecture

This is the most fundamental architecture of MDO since no single discipline analysis needs to be solved at each iteration. In this architecture every analysis variable is a design variable and every equation of disciplines is a optimization constraint. In other words, the formulation does not search feasibility for single discipline in each iteration before optimization convergence is obtained. In Figure 4, the general structure of AAO for a sample aeroelasticity problem is given. In this diagram, f and C_D represents the objective function and design constraints respectively; X_D, X_{U_A} , X_{U_S} are the optimization design variables, variables of aerodynamic discipline and variables of structure discipline respectively. Lastly, W_A and W_S gives the residuals for aerodynamic and structure disciplines. These residuals are sent to optimizer and optimizer does the necessary computation just in case both residuals are equal to zero since this is obligatory for a feasible analysis.

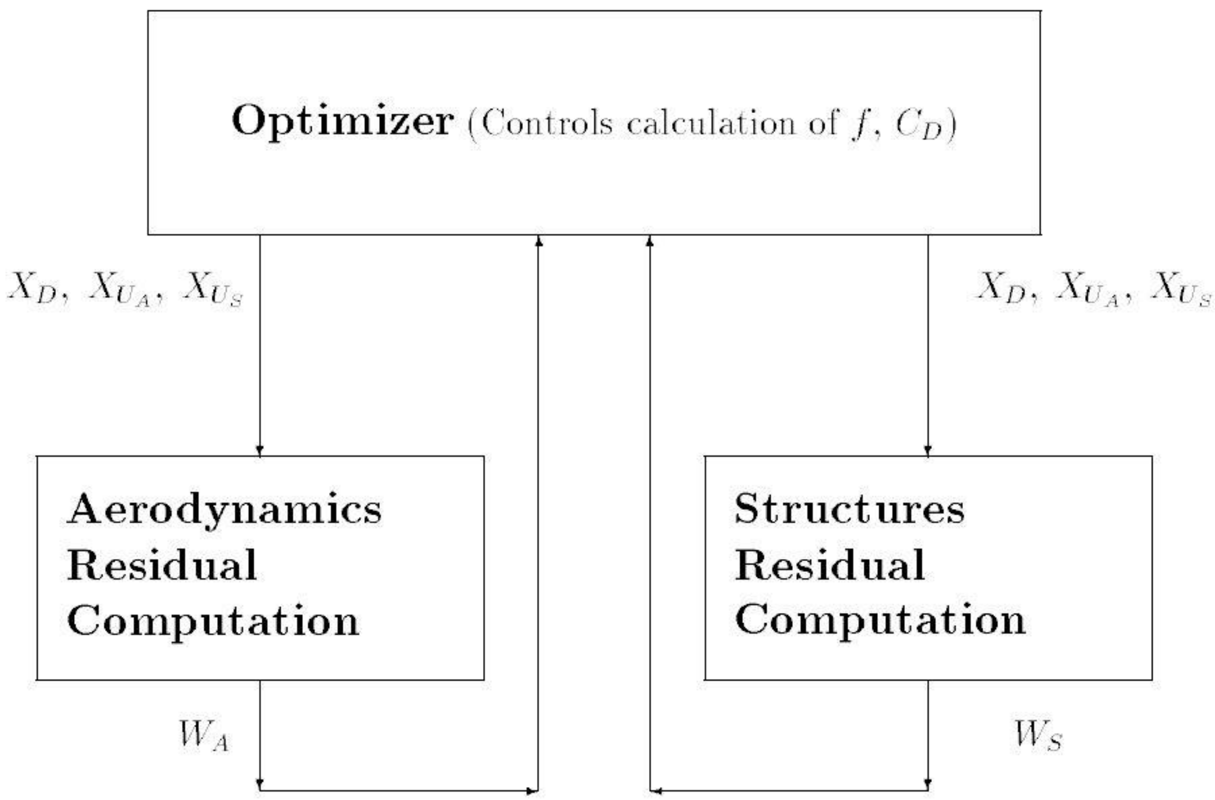

Figure 4 Diagram of AAO Architecture [1]

As it is clearly seen, in AAO architecture residual computation is done without solving discipline equations(structure and aerodynamic). Therefore, the residuals should be available for predetermined values of design variables(X_D, X_{U_A}, X_{U_S}). General formulation for this MDO architecture is given in equation

$$\begin{aligned} &\text{minimize} && \mathrm{f(X)} \\ &\text{with respect to} && \mathrm{X}=(X_D, X_{U_A}, X_{U_S}) \\ &\text{subject to} && C_D(X) \geq 0 \\ & && W_A(X_D, M_{AS}(X), X_{U_A})=0 \\ & && W_S(X_D, M_{SA}(X), X_{U_S})=0 \end{aligned}$$

Equation 1: AAO Formulation

This formulation does not seem as a good option for real engineering applications since the problem needs whole design variables, discipline variables and discipline analysis equations that makes the problem size high and in addition to this residiuals are not available or computed for many engineering designs to use them as inputs of optimizer.

4.2. Individual Discipline Feasible

In individual discipline feasible(IDF) architecture discipline equations are solved in each iterations unlikely AAO and the feasibility is obtained for each discipline seperately. However, multidisciplinary feasibility is expected to obtain in optimizer part of the diagram(Figure 5). General formulation for IDF can be expressed as:

$$\begin{aligned} &\text{minimize} && \mathrm{f}(X_D, U_A(X), U_S(X)) \\ &\text{with respect to} && \mathrm{X}=(X_D, X_{\mu_{AS}}, X_{\mu_{SA}}) \\ &\text{subject to} && C_D(X, U_A(X), U_S(X)) \geq 0 \\ & && C_{AS} \equiv X_{\mu_{SA}} - F_{AS}(X_D, U_S(X))=0 \\ & && C_{AS} \equiv X_{\mu_{AS}} - F_{SA}(X_D, U_A(X))=0 \end{aligned}$$

Equation 2: IDF Formulation

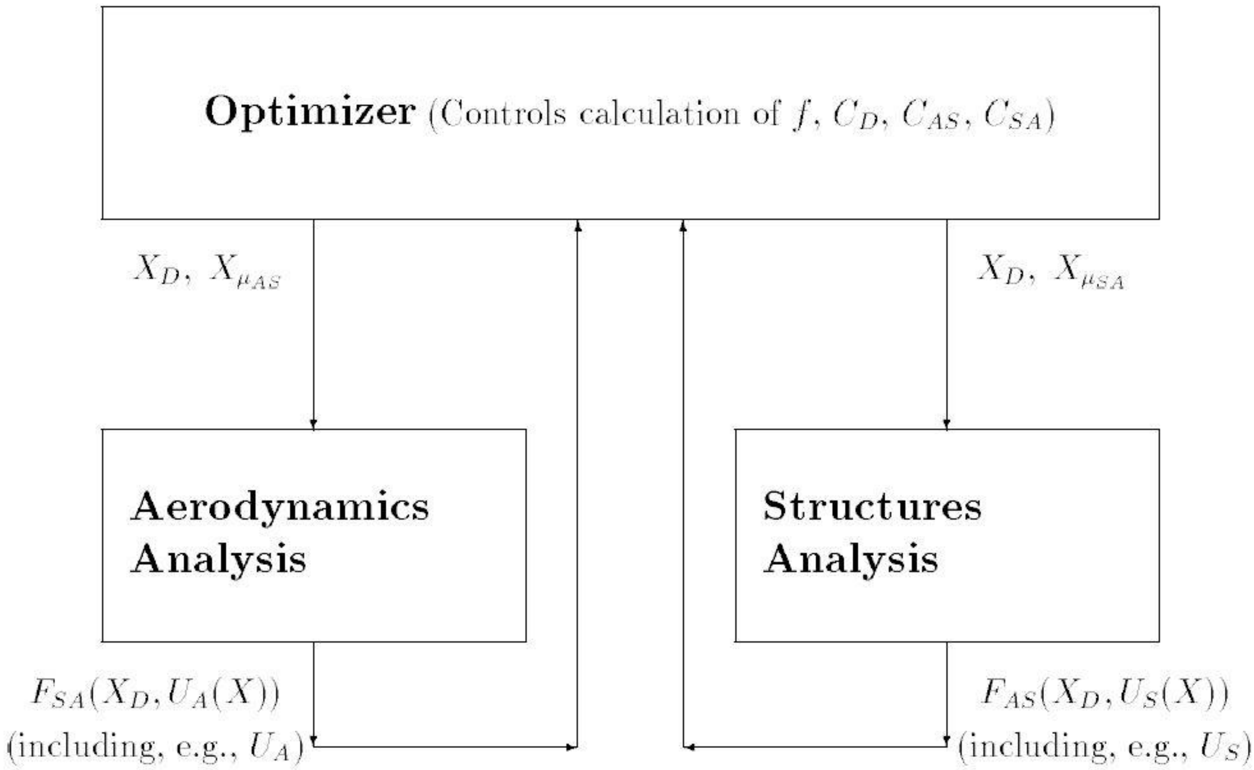

Figure 5: Diagram of IDF Architecture [1]

In this sample aeroelasticity diagram given in Figure 5, $U_A(X)$ is the output of the aerodynamic analysis that is used as input to structure analysis such as pressure and $U_A(X)$ is the output of the structure analysis that is used as input to aerodynamic analysis such as displacement. Conditions C_{AS} and C_{SA} of the Equation 2 are to check whether these outputs are equal to inputs of the related disciplines.

One advantage of IDF architecture is it allows parallel computation of the discipline analyses since each discipline works seperately and coupling is done by coupling variables ($U_A(X), U_S(X)$) which is obtained from result of discipline analysis. However, with large number of design variables, optimization problem becomes too large so it would be a problem to solve it efficiently.

4.3.Multidisciplinary Feasible

In multidisciplinary feasible(MDF) architecture, a complete multidisciplinary feasibility needs to be obtained for each iteration as it is seen in Equation 3. Therefore, a complete MDA is performed at every optimization iteration which means slower convergence rates for the optimization problem. The formulation of this architecture is given in Equation 3, the main difference of MDF formulation is clearly seen as the smaller number of optimization variables(only X_D) since no coupling variable is needed for optimization.

$$\begin{aligned} \text{minimize} \quad & \mathrm{f}(X_D, U_A(X_D), U_S(X_D)) \\ \text{with respect to} \quad & X_D \\ \text{subject to} \quad & C_D(X_D, U_A(X_D), U_S(X_D)) \geq 0 \end{aligned}$$

Equation 3 : MDF Formulation

Even complete optimization process takes longer time with MDF architecture, it has a significant advantage for computing objective function and constraints due to less number of optimization variables. On the other hand, MDF is also good for improving an engineering design instead of optimizing it because the output of MDF is always satisfies the consistency constraints. In other words, when the optimization loop of MDF is terminated earlier it gives a better design than the initial design which is significant from engineering point of view.

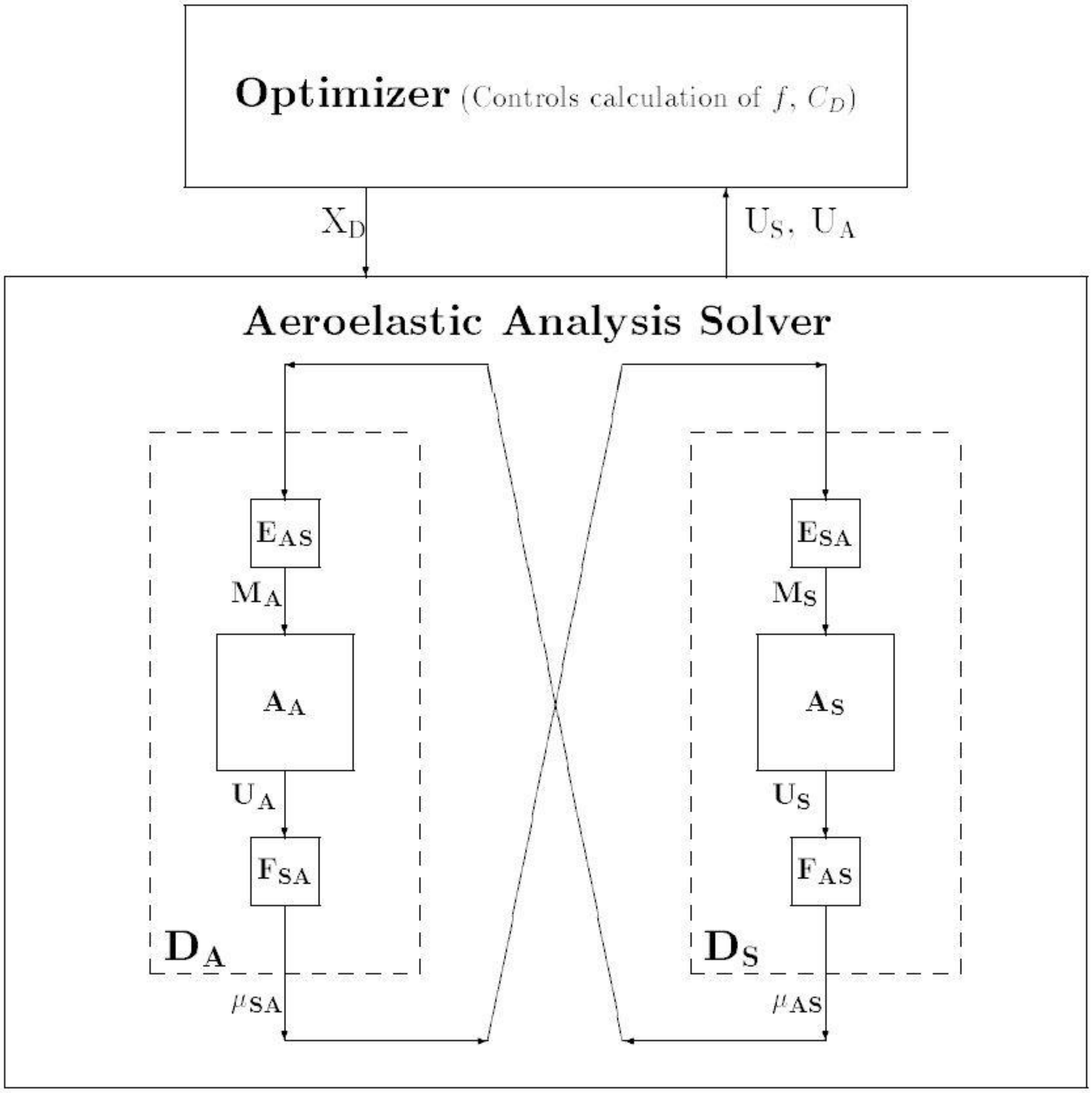

Figure 6: Diagram of MDF Architecture

References

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