

Tabulation Method

The Quine-McCluskey Method

K-map going big!

CDE									
		000	001	011	010	110	111	101	100
AB	00								
	01								
	11								
	10								

5- variable Karnaugh map (Gray code)

		$x = 0$				$x = 1$			
		00	01	11	10	00	01	11	10
$x = 0$	00	m_0	m_1	m_3	m_2	m_{16}	m_{17}	m_{19}	m_{18}
	01	m_4	m_5	m_7	m_6	m_{20}	m_{21}	m_{23}	m_{22}
	11	m_{12}	m_{13}	m_{15}	m_{14}	m_{28}	m_{29}	m_{31}	m_{30}
	10	m_8	m_9	m_{11}	m_{10}	m_{24}	m_{25}	m_{27}	m_{26}
$x = 1$	00	m_{32}	m_{33}	m_{35}	m_{34}	m_{48}	m_{49}	m_{51}	m_{50}
	01	m_{36}	m_{37}	m_{39}	m_{38}	m_{52}	m_{53}	m_{55}	m_{54}
	11	m_{44}	m_{45}	m_{47}	m_{46}	m_{60}	m_{61}	m_{63}	m_{62}
	10	m_{40}	m_{41}	m_{43}	m_{42}	m_{56}	m_{57}	m_{59}	m_{58}

(a) Map organization

FIGURE 4.11
Six-variable map

6 variable k-map

- K-map works well upto 4 variable, but when number of variable is more than that trouble starts as it gets difficult to recognize patterns leading to wrong selection
- Quine McCluskey method or the tabulation method is more systematic and works well for any number of variable

The starting point of the tabulation method is the list of minterms that specify the function. The first tabular operation is to find the prime implicants by using a matching process. This process compares each minterm with every other minterm. If two minterms differ in only one variable, that variable is removed and a term with one less literal is found. This process is repeated for every minterm until the exhaustive search is completed. The matching-process cycle is repeated for those new terms just found. Third and further cycles are continued until a single pass through a cycle yields no further elimination of literals. The remaining terms and all the terms that did not match during the process comprise the prime implicants. This tabulation method is illustrated by the following example.

Part 1:Determination of the prime implicant

Forming the tabulation method

Example

$$F(w,x,y,z)=\sum(1,4,6,7,8,9,10,11,15)$$

$$F=\{0001,0100,0110,0111,1000,1001,1010,1011,1111\}$$

Step 1: group the minterms based on the no. of '1's in them

Group A: single '1': {0001,0100,1000}

Group B: two '1's: {0110,1001,1010}

Group C: three '1's: {0111,1011}

Group D: four '1's: {1111}

	Step 1				
1	0001				
4	0100				
8	1000				
6	0110				
9	1001				
10	1010				
7	0111				
11	1011				
15	1111				

- **Step 2:** Any 2 minterm that differ by 1 variable is combined and the different variable is removed. The minterms of a group is compared with minterms of immediate next group (this is because 2 term differing by more than 1 bit cannot match). If 2 matching minterms are found, a '✓' (tick) is place besides them

	Step 1	Step 2			
1	0001	(1,9)	_001		
4	0100	(4,6)	01_0		
8	1000	(8,9)	100_		
		(8,10)	10_0		
6	0110	(6,7)	011_		
9	1001	(9,11)	10_1		
10	1010	(10,11)	101_		
7	0111	(7,15)	_111		
11	1011	(11,15)	1_11		
15	1111				

- Step 3:

Terms from step 2 has 3 options. '1' means variable is unprimed, '0' means variable is primed and '_' means variable is not included in the term.

Now compare terms only if they have '_' in the same position. Once two term match place a '✓' beside each

	Step 1	Step 2			Step 3		
1	0001	√	(1,9)	_001		(8,9,10,11)	10__
4	0100	√	(4,6)	01_0		(8,9,10,11)	10__
8	1000	√	(8,9)	100_	√		
			(8,10)	10_0	√		
6	0110	√	(6,7)	011_			
9	1001	√	(9,11)	10_1	√		
10	1010	√	(10,11)	101_	√		
7	0111	√	(7,15)	_111			
11	1011	√	(11,15)	1_11			
15	1111	√					

Prime Implicant

- The remaining term (those without '√') in Step 2 and the terms from Step 3 are called the prime implicants. These terms are candidates suitable for forming the simplified form of the expression

Here prime implicants are:

(1,9)	_001	X'Y'Z
(4,6)	01_0	W'XZ'
(6,7)	011_	W'XY
(7,15)	_111	XYZ
(11,15)	1_11	WYZ
(8,9,10,11)	10__	WX'

- K-MAP using prime implicant shows that it can be further simplified to

$$F(w,x,y,z) = X'Y'Z + W'XZ' + XYZ + WX'$$

To achieve further simplification we need to find the essential prime implicant and remove the domination rows and dominated columns.

		yz		y	
		00	01	11	10
w	x				
00		1	1		1
01					
11				1	1
10		1		1	1

Diagram illustrating a 4x4 Karnaugh Map (K-MAP) for the function $F(w,x,y,z)$. The map is labeled with variables w and x on the left, and y and z on the top. The map shows the following cells containing 1s:

- Row 00: Columns 00, 01, and 10.
- Row 11: Columns 11 and 10.
- Row 10: Columns 00, 11, and 10.

The map is grouped into four prime implicants (PIs) indicated by boxes:

- Group 1: $X'Y'Z$ (Cells 00, 01, 10, 11).
- Group 2: $W'XZ'$ (Cells 00, 01, 10, 11).
- Group 3: XYZ (Cells 11, 10, 01, 00).
- Group 4: WX' (Cells 10, 11, 00, 01).

Part 2: Selection of prime implicants- the setup

Optimizing the tabular method

given minterms
in the question

Create the large table

Prime
Implicants

	1	4	6	7	8	9	10	11	15
$x'y'z$	\times					\times			
$w'xz'$		\times	\times						
$w'xy$			\times	\times					
xyz				\times					\times
wyz								\times	\times
wx'					\times	\times	\times	\times	

$x'y'z$ (1, 9) \rightarrow so put \times in both 1 and 9
Similarly, for all, put \times in corresponding spots.

So solution is

$$F(W,X,Y,Z) = X'Y'Z + W'XZ' + WX' + XYZ$$

This matched the result from K-MAP!

Example 2

$$F(a,b,c,d)=\sum m(2,5,6,11,12,14,15)+ \sum d(0,3,4)$$

Note: avoid the don't care terms in the set-up(optimization) phase.

	Step 1	Step 2				Step 3	
0	0000	√	(0,2)	00-0	√	(0,2,4,6)	0--0
			(0,4)	0-00	√	(0,4,2.6)	0--0
2	0010	√	(2,3)	001-		(4,6,12,14)	-1-0
4	0100	√	(2,6)	0-10	√	(4,12,6,14)	-1-0
			(4,5)	010-			
			(4,6)	01-0	√		
			(4,12)	-100	√		
3	0011	√	(3,11)	-011			
5	0101	√	(6,14)	-110	√		
6	0110	√	(12,14)	11-0	√		
12	1100	√					
11	1011	√	(11,15)	1-11			
14	1110	√	(14,15)	111-			

Don't care terms are avoided here.

	2	5	6	11	12	14	15
$A'D'$	\otimes		*				
BD'			*		\otimes	*	
$A'B'C$	x						
$A'BC'$	\otimes	\otimes					
$B'CD$				*	\otimes		
ACD				\otimes			*
ABC					\otimes	*	*
	✓	✓	✓	✓	✓	✓	✓

$BD' \otimes + A'BC' + ACD + A'D'$

So solution is

$$F(W,X,Y,Z) = BD' + A'BC' + A'D' + ACD$$

Try it yourself

- $F(w,x,y,z) = \sum(0,1,2,8,10,11,14,15)$

Solution

(a)							(b)							(c)						
$w x y z$							$w x y z$							$w x y z$						
0	0	0	0	0	0	✓	0, 1	0	0	0	–			0, 2, 8, 10	–	0	–	0		
							0, 2	0	0	–	0	✓		0, 8, 2, 10	–	0	–	0		
1	0	0	0	1		✓	0, 8	–	0	0	0	✓		10, 11, 14, 15	1	–	1	–		
2	0	0	1	0		✓								10, 14, 11, 15	1	–	1	–		
8	1	0	0	0		✓	2, 10	–	0	1	0	✓								
							8, 10	1	0	–	0	✓								
10	1	0	1	0		✓	10, 11	1	0	1	–	✓								
11	1	0	1	1		✓	10, 14	1	–	1	–	✓								
14	1	1	1	0		✓														
15	1	1	1	1		✓	11, 15	1	–	1	1	✓								
							14, 15	1	1	1	–	✓								

Prime implicant optimization

$$F = w'x'y' + x'z' + wy$$

	$w'x'y'$	$x'z'$	wy
0000	1	1	
0001	1		
0010		1	
1000		1	
1010		1	1
1011			1
1110			1
1111			1

All rows have single '1', so all are essential prime implicant hence no further optimization can be done. Answer is:

$$F = w'x'y' + x'z' + wy$$