# CSE 260
# Digital Logic Design

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Teaching Assistant ( to be appointed) will be there for your support

**Consultation Hours:**
\\tsr\Summer2019\CSE\DZK\CSE260\Lecture Slides

**Appointment by email !**

# Objective

- Distinguish between analog and digital system
- Understand the advantage and limitation of digital system
- Understand the meaning of digital logic

# Analog vs. Digital

- Analog data can vary over a continuous range of values. Example: speedometer

- Digital quantities can take on only discrete values (0 and 1, high and low). Example: Digital Computer, Decimal Digits, Alphabets

# Digital System

- A digital system is a combination of devices designed to manipulate physical quantities or information that are represented in digital form.

- “A discreet information processing system”
- Signals: Discreet information

# Advantage of digital system

- Greater accuracy or precision
- Easier to design (generality)
- Easier information storage
- Programmability (instructions)
- Speed
- Economical

# Limitation of digital technology

- The real world is mainly analog

# Overcome the limitation

- Convert the real world analog input data into digital one
- Process this digital data
- Then again convert into analog form

# Digital logic

- Design logic is a term used to denote the design and analysis of digital system

- Digital logic is concerned with the interconnection among digital components and modules

- Digital logic design is engineering and engineering means problem solving

# Number systems and codes

Digital Systems are built from circuits that process binary digits. BUT very few real-life problems are based on binary numbers.

SO a digital system designer must establish some correspondence between the binary digits processed by digital circuits and real-life numbers, events and conditions.

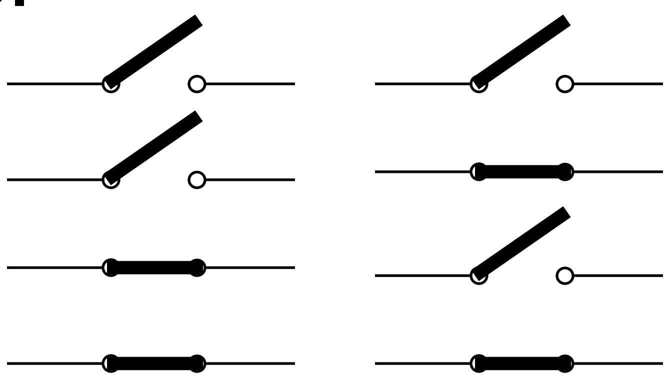
# Information representation

- Human decisions tends to be binary i.e. Yes or No
- Elementary storage units inside computer are *electronic switches*. Each switch holds one of two states: *on* (1) or *off* (0).


- We use a *bit* (*bi*nary digi*t*), 0 or 1, to represent the state.

# Information representation

- Storage units can be grouped together to cater for larger range of numbers. Example: 2 switches to represent 4 values.



0 (00)

1 (01)

2 (10)

3 (11)

# Information representation

- In general, $N$ bits can represent $2^N$ different values.
- For $M$ values, $\lceil \log_2 M \rceil$ bits are needed.

**1 bit → represents up to 2 values (0 or 1)**
**2 bits → rep. up to 4 values (00, 01, 10 or 11)**
**3 bits → rep. up to 8 values (000, 001, 010. …, 110, 111)**
**4 bits → rep. up to 16 values (0000, 0001, 0010, …, 1111)**

**32 values → requires 5 bits**
**64 values → requires 6 bits**
**1024 values → requires 10 bits**
**40 values → requires 6 bits**
**100 values → requires 7 bits**

# Positional Notations

- Decimal number system, symbols = { 0, 1, 2, 3, …, 9 }
- Position is important
- Example:$(7594)_{10} = (7x10^3) + (5x10^2) + (9x10^1) + (4x10^0)$
- In general, $(a_n a_{n-1} \ldots a_0)_{10} = (a_n \times 10^n) + (a_{n-1} \times 10^{n-1}) + \ldots + (a_0 \times 10^0)$
- $(2.75)_{10} = (2 \times 10^0) + (7 \times 10^{-1}) + (5 \times 10^{-2})$
- In general, $(a_n a_{n-1} \ldots a_0 . f_1 f_2 \ldots f_m)_{10} = (a_n \times 10^n) + (a_{n-1} x10^{n-1}) + \ldots + (a_0 \times 10^0) + (f_1 \times 10^{-1}) + (f_2 \times 10^{-2}) + \ldots + (f_m \times 10^{-m})$

# Other Number Systems

- Binary (base 2): weights in powers-of-2. Binary digits (bits): ***0,1.***
- Octal (base 8): weights in powers-of-8. Octal digits: ***0,1,2,3,4,5,6,7***
- Hexadecimal (base 16): weights in powers-of-16. Hexadecimal digits: ***0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F***

Note: when base is r, coefficient values range from 0 to r-1.

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| Binary | Octal | Decimal | Hexadecimal |
|---|---|---|---|
| 0000 | 0 | 0 | 0 |
| 0001 | 1 | 1 | 1 |
| 0010 | 2 | 2 | 2 |
| 0011 | 3 | 3 | 3 |
| 0100 | 4 | 4 | 4 |
| 0101 | 5 | 5 | 5 |
| 0110 | 6 | 6 | 6 |
| 0111 | 7 | 7 | 7 |
| 1000 | 10 | 8 | 8 |
| 1001 | 11 | 9 | 9 |
| 1010 | 12 | 10 | A |
| 1011 | 13 | 11 | B |
| 1100 | 14 | 12 | C |
| 1101 | 15 | 13 | D |
| 1110 | 16 | 14 | E |
| 1111 | 17 | 15 | F |

# Base-R to Decimal Conversion

*****Formula= $\sum$ digit * source_base $^{\text{position}}$**

- $(1101.101)_2 = 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-3}$
  $= 8 + 4 + 1 + 0.5 + 0.125$
  $= (13.625)_{10}$
- $(572.6)_8 = 5\times8^2 + 7\times8^1 + 2\times8^0 + 6\times8^{-1} = 320 + 56 + 2 + 0.75 = (378.75)_{10}$
- $(2A.8)_{16} = 2\times16^1 + 10\times16^0 + 8\times16^{-1} = 32 + 10 + 0.5 = (42.5)_{10}$
- $(341.24)_5 = 3\times5^2 + 4\times5^1 + 1\times5^0 + 2\times5^{-1} + 4\times5^{-2}$
  $= 75 + 20 + 1 + 0.4 + 0.16 = (96.56)_{10}$

# Decimal-to-Binary Conversion

Works for binary

- Method 1: *Sum-of-Weights Method*

*Note: Remembering the below value helps*

$2^8$, $2^7$, $2^6$, $2^5$, $2^4$, $2^3$, $2^2$, $2^1$, $2^0$

256, 128, 64, 32, 16, 8, 4, 2, 1

Works for all base

Method 2:

- *Repeated Division-by-2 Method* (for whole numbers)
- *Repeated Multiplication-by-2 Method* (for fractions)

# Sum-of-Weights Method

- Determine the set of binary weights whose sum is equal to the decimal number.

$(9)_{10} = 8 + 1 = 2^3 + 2^0 = (1001)_2$

$(18)_{10} = 16 + 2 = 2^4 + 2^1 = (10010)_2$

$(58)_{10} = 32 + 16 + 8 + 2 = 2^5 + 2^4 + 2^3 + 2^1$

$= (111010)_2$

$(0.625)_{10} = 0.5 + 0.125 = 2^{-1} + 2^{-3}$

$= (0.101)_2$

# Conversion between Decimal and other Bases

- Decimal to base-R
  - whole numbers: repeated division-by-R
  - fractions: repeated multiplication-by-R

# Repeated Division-by-2 Method

- To convert a whole number to binary, use successive division by 2 until the quotient is 0. The remainders form the answer, with the first remainder as the *least significant bit (LSB)* and the last as the *most significant bit (MSB)*.

| | | | |
|---|---|---|---|
| 2 | 43 | | |
| 2 | 21 | rem 1 | ← LSB |
| 2 | 10 | rem 1 | |
| 2 | 5 | rem 0 | |
| 2 | 2 | rem 1 | |
| 2 | 1 | rem 0 | |
| | 0 | rem 1 | ← MSB |

$(43)_{10} = (101011)_2$

# Repeated Multiplication-by-2 Method

- To convert decimal fractions to binary, repeated multiplication by 2 is used, until the fractional product is 0 (or until the desired number of decimal places). The carried digits, or *carries*, produce the answer, with the first carry as the MSB, and the last as the LSB.

$(0.3125)_{10} = (.0101)_2$

| | Carry | |
|---|---|---|
| $0.3125\times2=0.625$ | 0 | ←MSB |
| $0.625\times2=1.25$ | 1 | |
| $0.25\times2=0.50$ | 0 | |
| $0.5\times2=1.00$ | 1 | ←LSB |

# Note: difference between conversion of integer and fraction

**Integer**

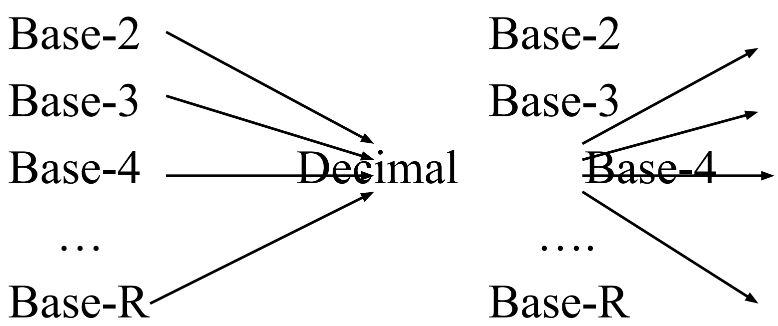
- Division by base of target no. system
- Remainders are accumulated
- By division we obtain LSB to MSB

**Fraction**

- Multiplication by base of target no. system
- Integers are accumulated
- By multiplication we obtain MSB to LSB

# Conversion between Bases

- In general, conversion between bases can be done via decimal:



# Binary-Octal/Hexadecimal Conversion

- Binary → Octal: Partition in groups of 3

  $(10\ 111\ 011\ 001\ .\ 101\ 110)_2 = (2731.56)_8$

- Octal → Binary: reverse

  $(2731.56)_8 = (10\ 111\ 011\ 001\ .\ 101\ 110)_2$

- Binary → Hexadecimal: Partition in groups of 4

  $(101\ 1101\ 1001\ .\ 1011\ 1000)_2 = (5D9.B8)_{16}$

- Hexadecimal → Binary: reverse

  $(5D9.B8)_{16} = (101\ 1101\ 1001\ .\ 1011\ 1000)_2$

# Exercise:

(1)Try converting this to

$(10110001101011.111100000110)_2$

a)octal b) hexadecimal

(2) Try converting these to binary

a) $(673.124)_8$

b) $(306.D)_{16}$

# Answers:

(1) a) $(26153.7406)_8$

(1) b) $(2C6B.F06)_{16}$

(2) a) $(110\ 111\ 011\ .\ 001\ 010\ 100)_2$

(2) b) $(0011\ 0000\ 0110\ .\ 1101)_2$

# Binary operations: Addition: Addition Rules w/Carries

**For 2 bit**

- 0 + 0 = 0 0 ( 0 with a 0 carry )
- 0 + 1 = 0 1 ( 1 with a 0 carry )
- 1 + 0 = 0 1 ( 1 with a 0 carry )
- 1 + 1 = 1 0 ( 0 with a 1 carry )

**For 3 bit**

- 0+0+0 = 0 0 (0 WITH 0 CARRY)
- 0+0+1 = 0 1 (1 WITH 0 CARRY)
- 0+1+1 = 1 0 (0 WITH 1 CARRY)
- 1+1+1 = 1 1 (1 WITH 1 CARRY)

# Adding Binary Numbers

$$\begin{array}{r} 28 \\ +\ 43 \\ \hline 71 \end{array} \qquad \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \qquad \begin{array}{r} {\color{red}0\ 1\ 1\ 1\ 0\ 0\ 0}\phantom{0} \\ 00011100 \\ +\ 00101011 \\ \hline 01000111 \end{array}$$

# Binary operations: Addition

- From right to left, we add each pair of digits
- We write the sum, and add the carry to the next column on the left

| | | | |
|---|---|---|---|
| | 1 | 9 | 8 |
| + | 2 | 6 | 4 |
| Sum | 4 | 6 | 2 |
| Carry | 0 | 1 | 1 |

| | | | |
|---|---|---|---|
| | 0 | 1 | 1 |
| + | 0 | 0 | 1 |
| Sum | 1 | 0 | 0 |
| Carry | 0 | 1 | 1 |

# Exercise

3(a) Add $(101101)_2$ with $(100111)_2$

# Solution

- 3(a) $(1010100)_2$

Working:

Augend: 101101

Addend: +100111

Sum: 1010100

# Addition of base-r

- Example:
  $(34)_5 + (41)_5 + (24)_5$

1+3+4+2= 10
10%5=0
10/5=2 (carry)

21

$$
\begin{array}{r}
(34)_5 \\
(41)_5 \\
+\ (24)_5 \\
\hline
(2\ 0\ 4)_5
\end{array}
$$

4+1+4= 9
9%5=4
9/5=1 (carry)

# Exercise

- Try:
  2 (a) $(FF)_{16}+(F1)_{16}$
  2 (b) $(66)_7 + (55)_7$

# Solution

- 2(a) $(1F0)_{16}$
- 2(b) $(154)_{7}$

# Binary Multiplication

- The multiplication of two binary numbers can be carried out in the same manner as the decimal multiplication.
- Unlike decimal multiplication, only two values are generated as the outcome of multiplying the multiplication bit by 0 or 1 in the binary multiplication. These values are either 0 or 1.
- The binary multiplication can also be considered as repeated binary addition. Therefore, the binary multiplication is performed in conjunction with the binary addition operation.

# Binary Multiplication

| A | B | A×B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Example : Perform the binary multiplication of the decimal numbers 12 and 10.

The equivalent binary representation of the decimal number 12 is 1100.
The equivalent binary representation of the decimal number 10 is 1010.

$$
\begin{array}{rrrrrrrr}
 & & & & 1 & 1 & 0 & 0 \\
 & & & \times & 1 & 0 & 1 & 0 \\
\hline
 & & & & 0 & 0 & 0 & 0 \\
 & & & 1 & 1 & 0 & 0 & \\
 & & 0 & 0 & 0 & 0 & & \\
 & 1 & 1 & 0 & 0 & & & \\
\hline
 & 1 & 1 & 1 & 1 & 0 & 0 & 0
\end{array}
$$

# Exercise

6 (a) Multiply 1011 with 101

# Solution

| | |
|---|---|
| **Multiplicand** | **1011** |
| **Multiplier** | **× 101** |
| **Partial Products** | **1011** |
| | **0000 -** |
| | **1011 - -** |
| **Product** | **110111** |

# Multiplication with base-r

- 2A3C
  xB7
  ------------
  127A4
  1D094
  ------------
  1E30E4

**Working**

7*C = 7*12 = 84 =(write 4, carry 5)
7*3+5 = 26 = 1A (write A, carry 1)
7*A+1 = 71 = 0x47 (write 7, carry 4)
7*2+4 = 18 = 0x12
This completes the 7*2A3C = 127A4 partial product.

B*C = 11*12 = 132 = (write 4, carry 8)
B*3+8 = 11*3+8 = 41 = (write 9, carry 2)
B*A+2 = 11*10+2 = 112 = (write 0, carry 7)
B*2+7 = 11*2+7 = 29 = 1D
This completes the B[0]*2A3C = 1D094[0] partial product, where I'm noting the [0] digits to remind us this is in the 16s column.

Adding the partial products: 127A4 + 1D0940
4+0 = 4
A+4 = E
7+9 = 16 = 0x10 (write 0, carry 1)
2+0+1 = 3
1+D = E
1 = 1

# Exercise

7 (a) Multiply $(34)_5$ with $(42)_5$
7 (b) Multiply $(25)_9$ with $(36)_9$

# Solution

7 (a) $(3133)_5$
7 (b) $(1033)_9$

# Binary operations: Subtraction Rules w/Carries

- For 2 bit
  - 0 - 0 = 0 0 ( 0 with a 0 carry )
  - 1 - 1 = 0 0 ( 0 with a 0 carry )
  - 1 - 0 = 0 1 ( 1 with a 0 carry )
  - 0 - 1 = ? ( ???)

# Subtracting Binary Numbers: approach 1

$$\begin{array}{r} 2 \\ -\ 1 \\ \hline 1 \end{array} \qquad \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \\ \end{array} \qquad \begin{array}{r} \overset{0}{\cancel{1}}\,\overset{2}{0} \\ -\ 0\cancel{1} \\ \hline 01 \end{array}$$

# Subtracting Binary Numbers: approach 2

$$\begin{array}{rr} & 4 \\ - & 1 \\ \hline & 3 \end{array} \qquad\qquad \begin{array}{rrrrr} & & & \cancel{1} & 1 \\ & & 0 & 1 & \cancel{1} \\ \longrightarrow & & \cancel{1} & 0 & 0 \\ \longrightarrow & - & 0 & 0 & \cancel{1} \\ \hline & & 0 & 1 & 1 \end{array}$$

# Exercise

5(b) Subtract $(100111)_2$ from $(101101)_2$

# Solution

5 (b) $(000110)_2$

Working:

**5(b)Subtraction**

| | |
|---|---|
| Minuend: | 101101 |
| Subtrahend: | −100111 |
| Difference: | 000110 |

# Subtraction of Base-r

$$\begin{array}{r} {\scriptstyle 3\quad 16\quad\;\;} \\ (\cancel{4}\ A\ 6)_{16} \\ -(1\ B\ 3)_{16} \\ \hline (2\ F\ 3)_{16} \end{array}$$

$$\begin{array}{r} 4\,6\;\; \\ (\cancel{5}4)_{6} \\ -\ (35)_{6} \\ \hline (15)_{6} \end{array}$$

# Exercise

4 (a) $(71)_8-(56)_8$

4(b) $(21)_3-(12)_3$

# Solution

- 4 a) $(13)_8$
- 4 b) $(2)_3$