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Statistics for Data Science-I

Week 11: Graded Assignment Practice Session

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Statistics I: Week 11 Graded Assignment Practice

- Keep a notebook and pen ready for solving problems

- How to join?
  - Audio/screenshare on webex - click on link sent to you
    - Doubts? Use webex chat. Do not answer questions on zoom chat.
  - Join on pear deck - joinpd.com (enter code seen on top right)
    - Answer questions only here

- For every question - 5 to 15 minutes allotted
  - Question will be shown in a slide for solving
  - If you are done solving, enter your answer at joinpd.com
  - Presenter will provide a solution
  - Questions and discussion

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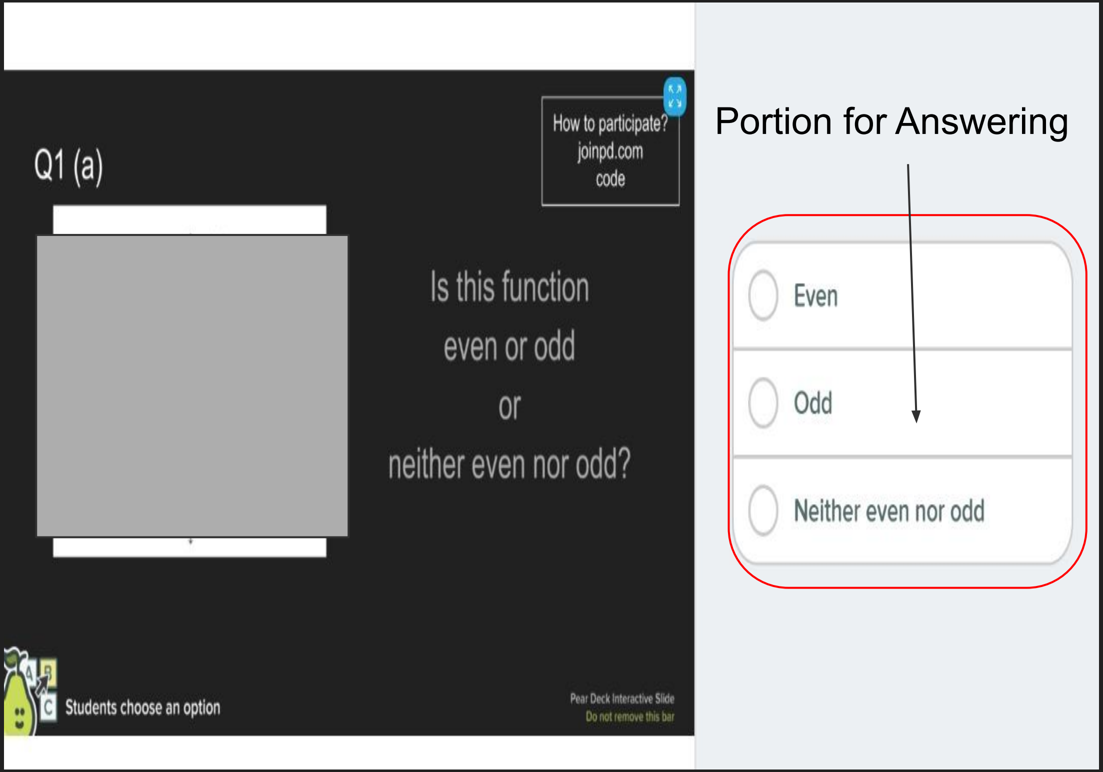
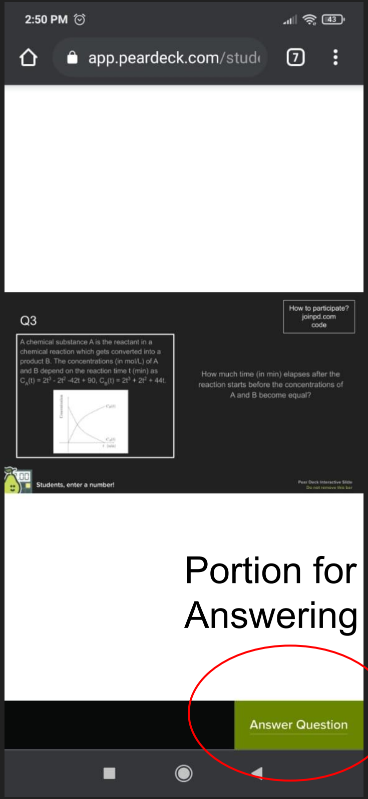
Example Screenshots

Laptop/Desktop



Mobile



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Tips to solve the question

Identify each trial and see whether you can classify it as success or fail.

See if the total number of trials (n) is fixed.

See if each of the trial is independent and identically distributed.

Independent trial means that each of the trial is independent from the other trials.

Identical trials means that probability of the success is same for each of the trials.

If an experiment satisfies all of the above it is a binomial experiment.

Given an experiment look whether we can classify each trial as success or failure. Also, check if all the trials are independent and identical. If both these conditions are satisfied, then it is a binomial experiment.

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Q1

Following are the experiments:

1) Tossing an unbiased coin for 100 times and counting the number of heads.
2) Tossing a fair coin for 100 times and counting the number of heads.
3) Choosing 5 balls randomly from an urn containing 20 yellow balls and 20 red balls without replacement and counting number of red balls chosen

Which of these is not a binomial experiment?

Students, write your response!

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Solution: Binomial random variable

Conditions:

Each trial should have either success or fail as outcomes.

n is fixed and all trials are independent and identical.

Experiment 1:

$n=100$ is fixed.

For each trial if we get head it is success and if we get tails it is failure.

Since the coin is unbiased $P(head)=P(tails)=\frac{1}{2}$ for each trial. So, each trial is independent and identical.

Solution: Binomial random variable

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Experiment 2:

$n=100$ is fixed.

For each trial if we get head it is success and if we get tails it is failure.

Since the coin is biased let P(head)=c, P(tails)=1-c. This value of P(head)=c, P(tails)=1-c holds true for all the trials. So, each trial is independent and identical.

Experiment 3:

$n=5$ is fixed

For each trial if we get red ball it is success else it is failure. So, outcomes for each trial is success or failure.

For 1st trial P(red ball)=20/(20+20)=½.

For 2nd trial P(red ball)= 19/39 if the first ball picked is red or P(red ball)=20/39 if the first picked is yellow. Either way the P(red ball) in second trial is not same as first trial. So, each trial is not independent and identical. Hence it is not a binomial experiment.

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Steps to solve the problem

Identify the event given in the question.

Calculate the probability (p) of the given event happening (success)

Identify the binomial random variable and number of trials.

Check whether it satisfies all the properties of binomial random variable.

Note down the expression for getting i successes in n trials of the experiment.

$$P(X = i) = \binom{n}{i} p^i (1-p)^{(n-i)}$$

Calculate the probability for the given condition of the problem

Q2

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An archery trainer wants to test the archery skills of newly joined students which includes some prodigies (skilled or talented). The trainer puts up a circular board of radius 20 metres. If the student shoots arrow in the region of radius less than 5 metres, then the trial is considered as a success. The student undergoes 3 trials and need to get at least 2 successes out of the 3 trials to pass the test. Prodigy is a student who has very good archery skills when compared to normal students.

What is the probability that a normal student will pass the test? Assume for a normal student there is equal chance that arrow will land anywhere on the board.

Students, enter a number!

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Solution: Binomial Random variable, Probability

Random experiment is shooting an arrow on to the circular board.

Binomial experiment is getting a success in throwing an arrow on to the region of radius less than 5 meters. If it lands elsewhere it is a failure.

Probability that an arrow will land in an particular region is directly proportional to the area of that region.

Area enclosed in a circle of radius r is pi*r*r, where pi is approximately 3.142

Area on the circular board which leads to success is 0<=r<5. Area= pi*5*5=25*pi.

Total area of board= pi*20*20 = 400*pi

P(Success)= Success area/Total area of circular board

= 25*pi/(400*pi)

=25/400

=0.0625

Solution: Binomial random variable, probability

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P(Success)=0.0625

P(Failure)= 1-0.0625=0.9375.

Given number of trials (n)=3

Let X be defined as the number of successes among the 3 trials.

The probability of getting i successes among the n trials is given by,

$$P(X=i) = \binom{n}{i} p^i (1-p)^{(n-i)}$$

The student should get at least 2 successes out of 3 trials to pass the test.

P(passing test) = P(X=2)+P(X=3)

Solution: Binomial random variable, probability

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P(passing test) = $\binom{3}{2}0.0625^2 \times 0.9375 + \binom{3}{3}0.0625^3 \times 0.9375^0$

P(passing test) = 0.01098 + 0.000244

P(passing test)= 0.011224= 1.12% approximately

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Steps to solve the question

Identify the binomial random variable.

Use the formula of expectation of a binomial random variable $E(X)=np$

Use the formula of variance of a binomial random variable $Var(X)= npq$

Where n is number of trials

p - Probability of success in a trial

q or (1-p) - Probability of failure in a trial

Use the equations of expectation and variance to find the unknown values of n and p.

Use these values of n and p to answer the given question of getting number of successes or failures.

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Q3

If the variance and expectation of getting number of tails in tossing an unbiased coin is 1.05 and 3.5 respectively.

What is the probability of getting 3 tails?

Students, enter a number!

Solution: Expectation and variance of binomial random variable

Experiment: Tossing an unbiased coin

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Let X be a binomial random variable that represents number of tails.

Let number of trials be n and probability of success(getting a tail) be p.

Given, E(X) = 3.5 and Var(X) = 1.05

Expectation and variance of a random variable is given by,

Expectation of a binomial random variable, $E(X) = n * p = 3.5$……………………....(1)

Variance of a binomial random variable, $Var(X) = n * p * (1-p) = 1.05$ …………..(2)

Solution:

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Divide equation (2) by (1),

$(1-p) = 0.3$

$p = 0.7$

Substituting value of p in equation (1) we will get,

$n=5$

Therefore, the probability of getting 3 tails in 5 tosses is given by,

$$P(X = 3) = \binom{5}{3} 0.7^3 0.3^2$$

$P(X=3) = 0.3087$

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Steps to solve the question

Identify the event of success and failure.

Identify the binomial random variable.

Identify total number of trials (n).

Note down the formula of getting i successes among n trials.

$$P(X = i) = \binom{n}{i} p^i (1 - p)^{(n-i)}$$

Use the formula and corresponding value of P(X=i) to find the value of p.

Use the value of p to answer the question.

Q4

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Rithika wants to test whether the coin she has is a fair coin or not. To test this, she conducted an experiment of tossing the coin 4 times. Binomial random variable X is defined as the total number of heads (i) after 4 tosses.

The pmf of binomial random variable is given in table below:

X	0	1	2	3	4
P(X=i)	0.1296	0.3456	0.3456	0.1536	0.0256

What is the probability of getting a head?

Students, enter a number!

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Solution: Variance, Binomial random variable

Given Rithika tossed a coin 4 times. So, n=4.

Also, P(X=4) = 0.0256.

From the formula, P(X=i) = $\binom{n}{i} p^i (1-p)^{(n-i)}$

n=4, i=4 So,

P(X=4) = $\binom{4}{4} p^4 (1-p)^0$

So, $p^4 = 0.0256$

$$\left(p^2\right)^2 = 0.0256$$

$$p^2 = 0.16$$

$$p = 0.4$$

$$P(X=0) = 0.1296$$

$$\binom{4}{0} p^0 (1-p)^4 = 0.1296$$

$$(1-p)^4 = 0.1296$$

$$(1-p)^2 = 0.36$$

$$(1-p) = 0.6$$

$$p = 0.4$$

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Solution: Binomial random variable

Alternate method:

Using expectation of a binomial random variable formula

$$E(X) = np$$

$$E(X) = \sum_{i=0}^{n} i * P(X = i)$$

So, E(X)=4*p

Also, E(X)= 0*0.1296+1*0.3456 +2*0.3456 + 3*0.1536 + 4*0.0256=1.6

Therefore, 4*p=1.6

p=(1.6)/4=0.4

Steps to solve the question

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Identify total number of trials (n).

Note down the formula of variance of a binomial random variable.

$Var(X)=np(1-p)$

Use the value of variance and n to solve for the unknown value(s) of p.

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Q5

If the variance of binomial random variable for n=8 is 1.92.

What are the possible values for p?

If more than two values are possible for p, enter the values of p separated by comma

Solution: Variance of a binomial random variable

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Given number of trials (n) =8

Var(X)= 1.92

We know variance of a binomial random variable is np(1-p)

np(1-p)=1.92

8*p(1-p)=1.92

p(1-p)=0.24

Solution: Variance of binomial random variable

$$p^2 - p - 0.24 = 0$$
$$p^2 - p - 0.6 * 0.4 = 0$$
$$p^2 - 0.6p - 0.4p - 0.6 * 0.4 = 0$$
$$p(p - 0.6p) - 0.4(p - 0.6) = 0$$
$$(p - 0.4)(p - 0.6) = 0$$
$$p = 0.4,\ 0.6$$

Note that for values of p = c or 1-c, we will have same variance. $0 \le c \le 1$

Thank You