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Statistics I

Week 10: Graded Assignment Practice Session

Statistics I: Week 9 Graded Assignment Practice

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- Keep a notebook and pen ready for solving problems
- How to join?
  - Audio/screenshare on webex - click on link sent to you
    - Doubts? Use webex chat. Do not answer questions on webex chat.
  - Join on pear deck - joinpd.com (enter code seen on top right)
    - Answer questions only here
- For every question - 5 to 15 minutes allotted
  - Question will be shown in a slide for solving
  - If you are done solving, enter your answer at joinpd.com
  - Presenter will provide a solution
  - Questions and discussion

Example Screenshots

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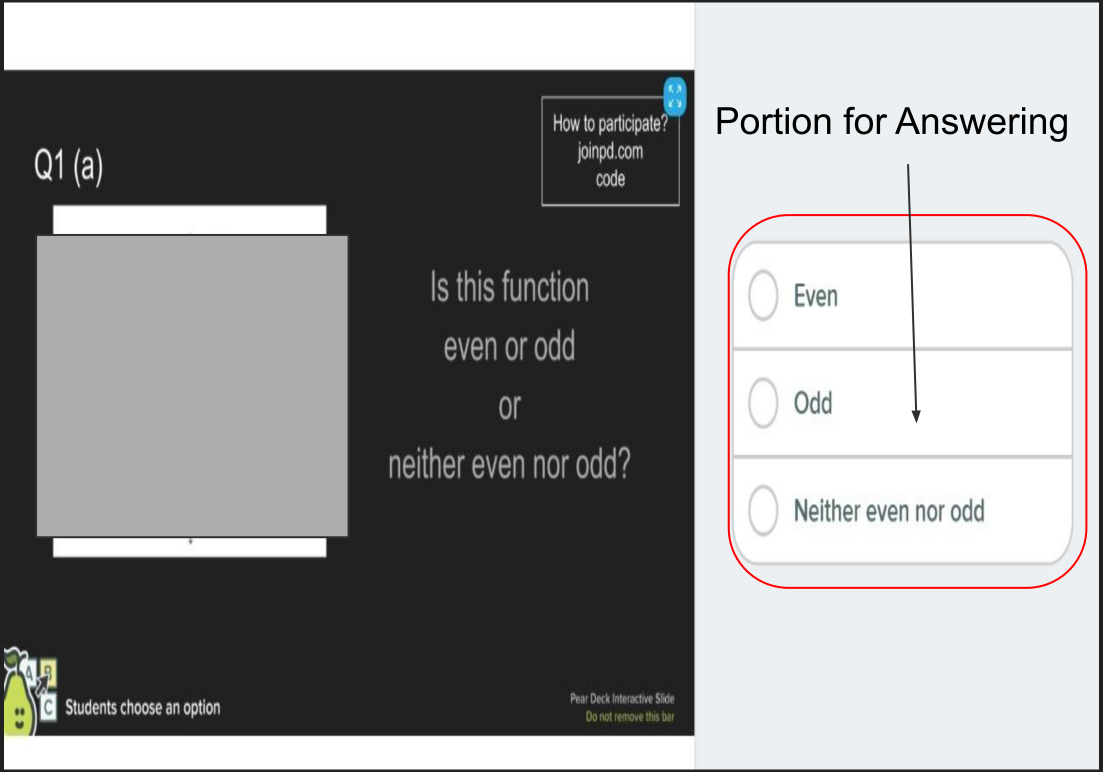
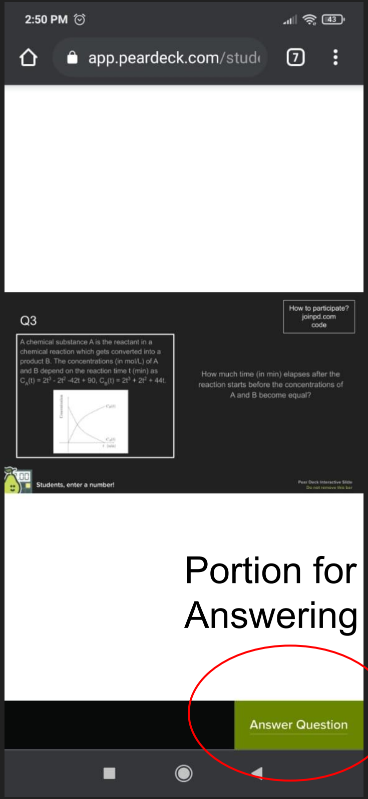
Mobile

Laptop/Desktop





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Q1

A discrete random variable X has a following pmf:

X	-2	-1	1	2
P(X=x)	0.2	0.3	0.15	0.35

What is the value of E(X)?

Students, enter a number!

Solution:

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Before going to find the expectation of a random variable, we need to check for valid pmf:

To check if pmf is valid:

1) Sum of probabilities of all outcomes should be 1.
2) $P(X=x) >= 0$ for all x.

X	-2	-1	1	2
P(X=x)	0.2	0.3	0.15	0.35

Is the given pmf valid?

1. $0.2 + 0.3 + 0.15 + 0.35 = 1$
2. $P(X=-2)$, $P(X=-1)$, $P(X=1)$, $P(X=2)$ are all greater than 0

Yes. The given pmf is valid.

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Solution: Expectation of a random variable

What is a expectation of a random variable?

→ It gives the weighted average of the possible values of the random variable.

Expectation of a random variable is given by,

$E(X) = \Sigma x P(X=x)$

$= (-2) * 0.2 + (-1) * 0.3 + (1) * 0.15 + (2) * 0.35$

$= 0.15$

Q2

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Let X be the random variable with the following pmf:

x	0	1	2	3
$P(X = x)$	0.1	0.15	0.3	0.45

A function of a random variable g(X) is defined as 2X + 3.

What is E[g(X)]?

Students, enter a number!

Solution

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From pmf for a random variable X, we know X can take values 0, 1, 2, 3.

To find the values g(X) can take, we need to insert values of X in the function (2X +3).

X = 0: g(0) = 2X + 3 = 2*0 + 3 = 3

X = 1: g(1) = 2X + 3 = 2*1 + 3 = 5

X = 2: g(2) = 2X + 3 = 2*2 + 3 = 7

X = 3: g(3) = 2X + 3 = 2*3 + 3 = 9

Therefore, g(X) takes values in { 3, 5, 7, 9 }

Solution: Expectation of a function of a random variable

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X takes values in { 0, 1, 2, 3}

g(X) takes values in { 3, 5, 7, 9}

P(g(0) = 3) = P(2X+3 = 3) = P(X= 0) = 0.1

P(g(1) = 5) = P(2X+3 = 5) = P(X=1) = 0.15

P(g(2) = 7) = P(2X+3 = 7) = P(X=2) = 0.3

P(g(3) = 9) = P(2X+3 = 9) = P(X=3) = 0.45

E[g(X)] = Σg(X) * P(g(X) = 3 * 0.1 + 5 * 0.15 + 7 * 0.3 + 9 * 0.45 = 7.2

Solution: Alternate method to find expectation

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From properties of mean,

E(X) = 0*0.1 + 1*0.15 + 2*0.3 + 3*0.45 = 2.1

E[g(X)] = E[(2X + 3)] = 2* E(X) + 3 = 2* 2.1 +3 =7.2

Q3

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A dairy firm predicted a next year's profit will follow following probability distribution:

X (in lakh rupees)	-1	0	1	2	3
P(X=x)	0.1	0.3	0.2	0.3	0.1

But dairy firm distributes 5% of profit to the milk vendors. The amount that a dairy firm retains is given by Y = 0.95X. Loss is indicated by negative sign.

What is the standard deviation of Y?

Students, enter a number!

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Solution:

Profit (in lakh rupees) of a dairy firm is a random variable which takes values -1, 0, 1, 2, 3.

The new random variable which is dependent on profit is the amount of money retained by a dairy firm is defined as Y = 0.95X.

Y can take values -0.95, 0, 0.95, 1.9, 2.85 as discussed in previous problem.

The probability distribution of Y can be shown below.

Y (in lakh rupees)	-0.95	0	0.95	1.9	2.85
P(Y=y)	0.1	0.3	0.2	0.3	0.1

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Solution: Expectation of a new random variable

Method 1:

$E[Y] = \Sigma y*P(Y=y) = (-0.95) * 0.1 + 0 * 0.3 + 0.95 * 0.2 + 1.9 * 0.3 + 2.85 * 0.1 = 0.95$

Method 2:

$E[X] = \Sigma x*P(X=x) = (-1) * 0.1 + 0 * 0.3 + 1 * 0.2 + 2 * 0.3 + 3 * 0.1 = 1$

$E[Y] = E[0.95X] = 0.95 * E[X] = 0.95 * 1 = 0.95$…...using properties of expectation

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Solution: Variance and standard deviation of a random variable

What is variance?

1. Expectation does not give the information about the variation or spread of the values that a random variable can take.
2. Variance gives the information about the spread of the values about the mean or average of a random variable.

Variance of a random variable can be given as,

Var(X) = E(X^2) - [E(X)]^2

Standard deviation of a random variable is given by,

SD(X) = √(Var(X)

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Solution: Variance and standard deviation

Method 1:

Step 1: Calculate E[Y^2]

E[Y^2] = Σy^2*P(Y=y) = (-0.95)^2*0.1 + 0*0.3 +0.95^2*0.2 + 1.9^2*0.3 + 2.85^2*0.1 = 2.166

Step 2: Calculate E[Y]

We know, E[Y] = 0.95

Y	-0.95	0	0.95	1.9	2.85
P(Y=y)	0.1	0.3	0.2	0.3	0.1

Step 3: Calculate variance

Var(Y) = E[Y^2] - (E[Y])^ 2 = 2.166 - 0.95^2 = 1.2635

Step 4: Calculate standard deviation

SD (Y) = √(Var(Y)) = √(1.2635) = 1.124

Solution: Variance and standard deviation

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Method 2:

Step 1: Calculate E[X^2]

E[X^2] = Σx^2*P(X=x) = (-1)^2*0.1 + 0*0.3 +1^2*0.2 + 2^2*0.3 + 3^2*0.1 = 2.4

Step 2: Calculate E[X]

We know, E[X] = 1

Step 3: Calculate variance

Var(X) = E[X^2] - (E[X])^ 2 = 2.4 - 1^2 = 1.4

X	-1	0	1	2	3
P(X=x)	0.1	0.3	0.2	0.3	0.1

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Solution:

Step 4: Calculate variance of a new random variable

Var(Y) = Var(0.95X) = 0.95^2 * Var(X) = 0.95^2 * 1.4 = 1.2635

Step 5: Calculate standard deviation

SD (X) = √(Var(X)) = √(1.2635) = 1.124

Thank You