Constrained Diffusion Limited Aggregation in 3 Dimensions

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Abstract

Diffusion Limited Aggregation (DLA) has usually been studied in 2 dimensions as a model of fractal growth processes such as branching, lightning, snowflakes, mineral deposits, and coral. Here, the basic principles are extended into 3 dimensions and used to create believable models of root systems. A straightforward approach is introduced for constraining the growth of the 3 dimensional DLA by a surface or containing it within a vessel.

DLA in 3 dimensions

The rules for forming DLA structures are very simple and were first introduced in 2 dimensions by T.A. Witten and L.M. Sander around 1981 [1]. A particle is introduced into an environment at a random position, it moves around randomly until it encounters the existing structure (initially just a single stationary particle) at which stage it adheres at the point of contact. There are many refinements discussed in the literature [2] that improve the performance [2] and allow for variations in the geometric and statistical structure [3]. These are outside the scope of this discussion except to note that most of the efficiencies and variations that apply in 2 dimensions can be readily extended into 3 dimensions.

Extending diffusion limited aggregation to 3 dimensions [4] is relatively straightforward but there are some important differences and options not available in 2 dimensions. In 2 dimensions it is common practice to grow structures on a discrete grid, namely the pixel image space that will eventually form the DLA image. A significant aspect of the algorithm used here is not to form the DLA on a grid of finite resolution in 3 dimensional but rather on a continuum. A particle adheres to the existing structure if it comes within some minimum distance of any part of the existing structure. The process of adhering to the existing DLA structure involves extending a branch from the closest position on the existing structure to the new particle position. The minimum distance before the particle adheres becomes a convenient variable that determines how sparse or thick the DLA becomes. In 2 dimensional DLA this is often controlled by introducing a probability of adhesion (stickiness), this can also be implemented in 3 dimensions with much the same effect.
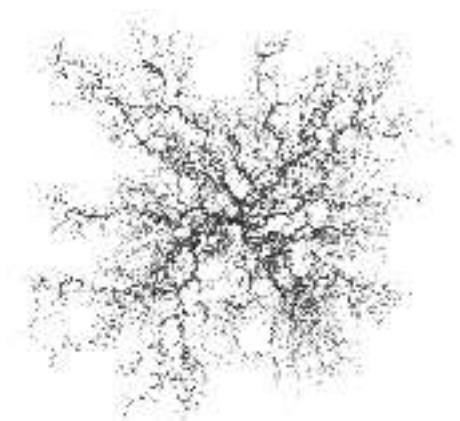

Figure 1: Diffusion Limited Aggregation extended into 3 dimensions, without constraints and particles introduced randomly on the surface of a sphere surrounding the initial seed particle.

It is possible to control the shape of the DLA by varying the spatial distribution at which particles randomly enter the environment. The classical 2 dimensional DLA introduces the particles at random positions on a circle with a radius just larger than the bounds of the existing structure. This leads to structures that tend to grow evenly; the equivalent in 3 dimensions is used in Figure 1 where new particles are introduced to the simulation randomly on the surface of a sphere. Directed growth as illustrated in Figure 2, is achieved by introducing particles along one axis. Other ways of controlling the overall form of the DLA is to apply additional forces on the structure such as gravity or wind, or to slowly rotate the DLA while it is being formed.

Figure 2: Directed DLA growth formed by introducing particles randomly within a small disk positioned along one of the coordinate axes.

Constraint surfaces

In order to support quite general constraint surfaces, a simple but industry standard file format was chosen to describe the geometry, namely the STL format. The STL format was developed for STereo-Lithography and is in common usage by the rapid prototyping industry and as such it is supported as an export format by many 3D modeling packages. The STL format describes a surface as a collection of triangles in 3 dimensions along with a normal that determines which side of the triangle is "inside" and which is "outside", a common requirement for rapid prototyping of closed solids.

Constraining the DLA growth by a surface or solid described by an STL dataset occurs at the stage when the new particle is to be attached to the existing structure. If the branch that is to be extended from the existing structure to the new particle intersects a triangle then it either isn't added, or the intersection with the triangle is calculated and a branch is added that just touches the triangle. In the former case the simulation continues, that is, the particle moves randomly to a new position. This approach tends to result in the structure following the constraint surface, a desirable behavior when simulating root structures.

Results

Figure 3 shows a very simple constraint surface made up of a cylinder with an open top. The particles are added randomly on a sphere about the origin (the center of the missing top), no consideration is given to the cylinder until the particle adheres to the existing structure.

Figure 3: DLA constrained within an open can (a cylinder sealed at one end but open at the other end). Particles are added randomly on the surface of a sphere with the origin at the centre of the top face of the cylinder.

Figure 4 shows a more complicated constraint surface, the DLA structure is constrained to lie within a scanned bust of a human subject, with added antlers. In this case the surface consists of approximately 23000 triangular faces. Since most of the time in a typical DLA simulation is spent in the random walk phase, a linear search through the list of triangles doesn't adversely affect the performance since the search is only performed when a particle is close enough to adhere to the DLA structure.

An additional option when using a constraint surface is to relate the spatial distribution of new particles to reflect the geometry of the surface in some way. In Figure 4 the distribution of the new particles matches the radial density of the head model, this results in more par-ticles being added around the antlers which otherwise would probably not have filled given the narrow gap between them and the head.

The examples shown here are all rendering using PovRay, a ray tracing package with a powerful scene description and scripting language that is both free and supported across multiple computer platforms. PovRay supports a geometric primitive called a "sphere-sweep" defined as the solid space a sphere of variable radius occupies as it moves along a spline curve. This primitive is used to define each root strand, the positions at which the particles adhere during the DLA aggregation define the control points of the spline curve and the radius of the sphere is varied to give a root thickness that decreases with distance from the start of the branch.

Figure 4: DLA constrained to lie within a 3 dimensional, digitally scanned human bust (with antlers). Particles are added randomly with a distribution proportional to the radial mesh density of the constraint surface. There are two initial seed particles, one in the throat and the other just behind the eyes.

Conclusion

The techniques described here provide the basis for an efficient DLA growth simulation which can be constrained by a surface described with a simple but well documented 3 dimensional file format that is supported by many 3D modeling packages. The DLA structures formed using the algorithm described, look like root structures and could thus be used to form models for a number of computer graphics applications. In addition the structures are grown in time and therefore can be used to create animation sequences.

References

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