## Statistics I

Week 8: Graded Assignment Practice Session

#### Plan for this session

- How to join?
  - Join on webex click on link sent to you
  - Join on pear deck joinpd.com (enter code seen on top right)
  - Keep a notebook and pen ready for solving problems
- For every question 15 minutes allotted
  - Question will be shown in a slide for solving 5 minutes
  - If you are done solving, enter your answer at joinpd.com
  - Presenter will provide a solution 5 minutes
  - Questions and discussion 5 minutes
- Prelude questions 5 or 10 minutes allotted
  - Help to prepare for the main question

### Sample Question - your screen on joinpd.com

How to participate? joinpd.com code: see above

Description of the problem.

Question to be answered.

Desktop

Answer box

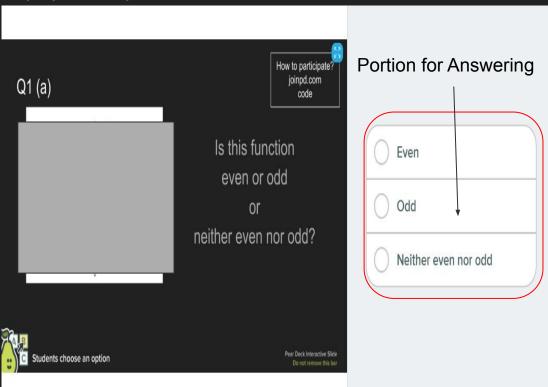
enter a number or a choice or some text

Mobile

Answer question

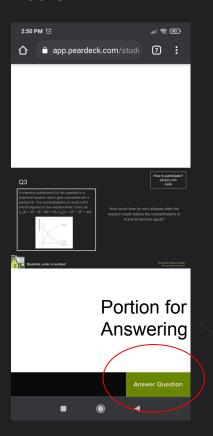
### **Example Screenshots**

Laptop/Desktop



How to participate? joinpd.com code: see above

#### Mobile



#### Q.1

Rajat is having 15 marbles in his pocket. Out of which 6 are green, 3 are red and 6 are yellow.

Let X be a random variable that denotes the number of red marbles Rajat will draw randomly from his pocket.

He draws two marbles randomly from his pocket.

What is the value of P(X=2)?

Concept targeted in the question: Random Variable

X = Random variable which represents number of random variables that Rajat will draw randomly.

So X can take values either 0 or 1 or 2 or 3.

Number of ways Rajat draw 2 marbles out of total 15 marbles = 15C2 = 105

P(X = 2) that means probability that Rajat draws exactly 2 red marbles.

Number of ways Rajat can draw 2 red marbles out of total 3 red marbles = 3C2=3

The probability that X can take value 2 = P(X=2) = 3/105

Throw a 6-sided biased die with following probability distribution:

Х	1	2	3	4	5	6
P(X= x)	0.1	0.23	0.23	0.20	0.12	0.12

What is the probability that X takes value more than 3?

(enter in decimal)

How to participate? joinpd.com code: see above

Fundamental concept related to the problem: Probability Mass Function (pmf)

#### To check pmf:

- 1) Sum of probabilities of all outcomes should be 1.
- 2) P(X=x) >= 0 for all x.

X	1	2	3	4	5	6
P(X=x)	0.1	0.23	0.23	0.20	0.12	0.12

Experiment: Throw a biased die

Outcomes: { 1, 2, 3, 4, 5, 6 }, with different probabilities as given in a table.

Think: Can the given table be a pmf?

- 1. 0.1 + 0.23 + 0.23 + 0.20 + 0.12 + 0.12 = 1
- 2. P(X=1), P(X=2),.. P(X=6) are all greater than 0

The given table is a pmf

$$P(X>3) = P(X=4) + P(X=5) + P(X=6) = 0.2 + 0.12 + 0.12 = 0.44$$

#### OR

$$P(X>3) = 1 - [P(X=2) + P(X=1) + P(X=0)] = 1 - [0.23 + 0.23 + 0.1] = 1 - 0.56 = 0.44$$

Q3

A discrete random variable X has the following pmf.

What is the value of k? (enter in decimal)

$$P(X = x) = \begin{cases} kx & \text{for } x = 1 \\ k(x - 1) & \text{for } x = 2 \\ k(x - 2) & \text{for } x = 3 \\ 0 & \text{otherwise} \end{cases}$$



#### Solution:

Finding the individual properties by putting values of x in probability mass function given in question.

$$P(X = x) = \begin{cases} kx & \text{for } x = 1\\ k(x - 1) & \text{for } x = 2\\ k(x - 2) & \text{for } x = 3\\ 0 & \text{otherwise} \end{cases}$$

	x=1	x=2	x=3
P(X=x)	k.1	k.(2-1)	k.(3-2)
	=k	= k	= k

k = 1/3

Using the properties of probability mass function(pmf) as explained in previous question,

$$\Sigma P(X=x) = 1 \text{ and } P(X=x) >= 0$$
  
 $\Rightarrow k+k+k = 1, k>= 0$   
 $3k = 1, k>= 0$ 

Let X be the random variable with the following pmf:

$$P(X = x) = \begin{cases} 0.1 & \text{for } x = 0 \\ 0.15 & \text{for } x = 1 \\ 0.3 & \text{for } x = 2 \\ 0.45 & \text{for } x = 3 \end{cases}$$

A new random variable Y is defined as 2X + 3.

Find the values Y can take.

# Solution: account for first step in a two-step experiment

From pmf for a random variable X, we know X can take values 0, 1, 2, 3.

To find the values Y can take, we need to insert values of X in a function 2X +3.

Therefore, 
$$Y = 2X + 3 = 2*0 + 3 = 3$$
, where  $X = 0$ 

Similarly, 
$$Y = 2X + 3 = 2*1 + 3 = 5$$
, where  $X = 1$ 

$$Y = 2X + 3 = 2*2 + 3 = 7$$
, where  $X = 2$ 

$$Y = 2X + 3 = 2*3 + 3 = 9$$
, where  $X = 3$ 

Therefore,  $Y = \{3,5,7,9\}$ 

Q4

The cumulative distribution function for a discrete random variable is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.1 & \text{for } 0 \le x < 1 \\ 0.25 & \text{for } 1 \le x < 2 \\ 0.45 & \text{for } 2 \le x < 3 \\ 0.65 & \text{for } 3 \le x < 4 \\ 1.00 & \text{for } 4 \le x \end{cases}$$

What is the probability mass function for Y = (2X +1)?

# Solution: First step of a solution to find pmf of random variable X

Cumulative distribution function (cdf) is a addition of probabilities of a discrete random variable X.

From cdf given in question, we can see that X takes a constant value between an interval that is there are no values of probabilities in between an interval.

Given: F(x) = 0, for x<0 and F(x) = 0.1, for 0<=x<1

Therefore, P(X<0) = 0

Similarly, P(X=0) = 0.1

Since, P(X=0) + P(X=1) = 0.25,

hence, P(X=1) = 0.25 - 0.1 = 0.15

Similarly,

$$P(X=2) = 0.2$$

$$P(X=3) = 0.2$$

$$P(X=4) = 0.35$$

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.1 & \text{for } 0 \le x < 1 \\ 0.25 & \text{for } 1 \le x < 2 \\ 0.45 & \text{for } 2 \le x < 3 \\ 0.65 & \text{for } 3 \le x < 4 \\ 1.00 & \text{for } 4 \le x \end{cases}$$

## Solution: Second step of a solution to find pmf of a new random variable Y = 2X+1

First, we will calculate the values that Y can take. From previous problem, we know how to find the values that a new random variable Y can take.

$$Y = \{1,3,5,7,9\}$$

Now, we calculate probabilities of each value which a new random variable can take.

$$P(Y=1) = P([2X+1] = 1) = P(X=0) = 0.1$$
  
 $P(Y=3) = P([2X+1] = 3) = P(X=1) = 0.15$   
 $P(Y=5) = P([2X+1] = 5) = P(X=2) = 0.2$   
 $P(Y=7) = P([2X+1] = 7) = P(X=3) = 0.2$   
 $P(Y=9) = P([2X+1] = 9) = P(X=4) = 0.35$ 

$$P(Y = y) = \begin{cases} 0.1 & \text{for } y = 1 \\ 0.15 & \text{for } y = 3 \\ 0.2 & \text{for } y = 5 \\ 0.2 & \text{for } y = 7 \\ 0.35 & \text{for } y = 9 \end{cases}$$

### Thank You