

# Statistics I

Week 8: Graded Assignment Practice Session

# Plan for this session

- How to join?
  - Join on webex - click on link sent to you
  - Join on pear deck - [joinpd.com](https://joinpd.com) (enter code seen on top right)
  - Keep a notebook and pen ready for solving problems
- For every question - 15 minutes allotted
  - Question will be shown in a slide for solving - 5 minutes
  - If you are done solving, enter your answer at [joinpd.com](https://joinpd.com)
  - Presenter will provide a solution - 5 minutes
  - Questions and discussion - 5 minutes
- Prelude questions - 5 or 10 minutes allotted
  - Help to prepare for the main question

# Sample Question - your screen on joinpd.com

How to participate?  
joinpd.com  
code: see above

Description of the  
problem.

Question to be  
answered.

Desktop

Answer box

enter a number or  
a choice or some  
text

Mobile


Answer question

# Example Screenshots

How to participate?  
joinpd.com  
code: see above

## Laptop/Desktop

Q1 (a)



How to participate?  
joinpd.com  
code

Is this function  
even or odd  
or  
neither even nor odd?

Even

Odd

Neither even nor odd

Students choose an option

Pear Deck Interactive Slide  
Do not remove this bar

## Mobile

2:50 PM

app.peardeck.com/stud

Q3

A chemical substance A is the reactant in a chemical reaction which gets converted into a product B. The concentrations (in mol/L) of A and B depend on the reaction time  $t$  (min) as  $C_A(t) = 20 - 2t^2 - 42t + 90$ ,  $C_B(t) = 20 + 2t^2 + 44t$ .



How to participate?  
joinpd.com  
code

How much time (in min) elapses after the reaction starts before the concentrations of A and B become equal?

Students, enter a number

Portion for Answering

Answer Question

## Q.1

Rajat is having 15 marbles in his pocket. Out of which 6 are green, 3 are red and 6 are yellow.

Let  $X$  be a random variable that denotes the number of red marbles Rajat will draw randomly from his pocket.

He draws two marbles randomly from his pocket.

What is the value of  $P(X=2)$ ?

# Solution

Concept targeted in the question: Random Variable

$X$  = Random variable which represents number of random variables that Rajat will draw randomly.

So  $X$  can take values either 0 or 1 or 2 or 3.

Number of ways Rajat draw 2 marbles out of total 15 marbles =  $15C_2 = 105$

$P(X=2)$  that means probability that Rajat draws exactly 2 red marbles.

Number of ways Rajat can draw 2 red marbles out of total 3 red marbles =  $3C_2 = 3$

The probability that  $X$  can take value 2 =  $P(X=2) = 3/105$

# Q2

How to participate?  
joinpd.com  
code: see above

Throw a 6-sided biased die  
with following probability  
distribution :

X	1	2	3	4	5	6
P(X=x)	0.1	0.23	0.23	0.20	0.12	0.12

What is the probability  
that X takes value  
more than 3?

(enter in decimal)

# Solution:

Fundamental concept related to the problem: Probability Mass Function (pmf)

To check pmf:

- 1) Sum of probabilities of all outcomes should be 1.
- 2)  $P(X=x) \geq 0$  for all  $x$ .

X	1	2	3	4	5	6
P(X=x)	0.1	0.23	0.23	0.20	0.12	0.12

Experiment: Throw a biased die

Outcomes:  $\{1, 2, 3, 4, 5, 6\}$ , with different probabilities as given in a table.

Think: Can the given table be a pmf?

1.  $0.1 + 0.23 + 0.23 + 0.20 + 0.12 + 0.12 = 1$
2.  $P(X=1), P(X=2), \dots, P(X=6)$  are all greater than 0

The given table is a pmf



Solution:

$$P(X>3) = P(X=4) + P(X=5) + P(X=6) = 0.2 + 0.12 + 0.12 = 0.44$$

**OR**

$$P(X>3) = 1 - [P(X=2) + P(X=1) + P(X=0)] = 1 - [0.23 + 0.23 + 0.1] = 1 - 0.56 = 0.44$$

Q3

A discrete random variable  $X$  has the following pmf.

$$P(X = x) = \begin{cases} kx & \text{for } x = 1 \\ k(x - 1) & \text{for } x = 2 \\ k(x - 2) & \text{for } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of  $k$ ?  
(enter in decimal)



Students, enter a number!

## Solution:

Finding the individual properties by putting values of  $x$  in probability mass function given in question.

$$P(X = x) = \begin{cases} kx & \text{for } x = 1 \\ k(x - 1) & \text{for } x = 2 \\ k(x - 2) & \text{for } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

	$x=1$	$x=2$	$x=3$
$P(X=x)$	$k.1$ $=k$	$k.(2-1)$ $= k$	$k.(3-2)$ $= k$

## Solution:

Using the properties of probability mass function (pmf) as explained in previous question,

$$\sum P(X=x) = 1 \text{ and } P(X=x) \geq 0$$

$$\Rightarrow k+k+k = 1, k \geq 0$$

$$3k = 1, k \geq 0$$

$$k = 1/3$$

## Prelude to Q4

Let  $X$  be the random variable with the following pmf:

$$P(X = x) = \begin{cases} 0.1 & \text{for } x = 0 \\ 0.15 & \text{for } x = 1 \\ 0.3 & \text{for } x = 2 \\ 0.45 & \text{for } x = 3 \end{cases}$$

A new random variable  $Y$  is defined as  $2X + 3$ .

Find the values  $Y$  can take.



Students, enter a number!

## Solution: account for first step in a two-step experiment

From pmf for a random variable  $X$ , we know  $X$  can take values 0, 1, 2, 3.

To find the values  $Y$  can take, we need to insert values of  $X$  in a function  $2X + 3$ .

Therefore,  $Y = 2X + 3 = 2*0 + 3 = 3$ , where  $X = 0$

Similarly,  $Y = 2X + 3 = 2*1 + 3 = 5$ , where  $X = 1$

$Y = 2X + 3 = 2*2 + 3 = 7$ , where  $X = 2$

$Y = 2X + 3 = 2*3 + 3 = 9$ , where  $X = 3$

Therefore,  $Y = \{3, 5, 7, 9\}$

Q4

The cumulative distribution function for a discrete random variable is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.1 & \text{for } 0 \leq x < 1 \\ 0.25 & \text{for } 1 \leq x < 2 \\ 0.45 & \text{for } 2 \leq x < 3 \\ 0.65 & \text{for } 3 \leq x < 4 \\ 1.00 & \text{for } 4 \leq x \end{cases}$$

What is the probability mass function for  $Y = (2X + 1)$ ?



Students, enter a number!

## Solution: First step of a solution to find pmf of random variable $X$

Cumulative distribution function (cdf) is a addition of probabilities of a discrete random variable  $X$ .

From cdf given in question, we can see that  $X$  takes a constant value between an interval that is there are no values of probabilities in between an interval.



# Solution:

Given:  $F(x) = 0$ , for  $x < 0$  and  $F(x) = 0.1$ , for  $0 \leq x < 1$

Therefore,  $P(X < 0) = 0$

Similarly,  $P(X=0) = 0.1$

Since,  $P(X=0) + P(X=1) = 0.25$ ,

hence,  $P(X=1) = 0.25 - 0.1 = 0.15$

Similarly,

$P(X=2) = 0.2$

$P(X=3) = 0.2$

$P(X=4) = 0.35$

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.1 & \text{for } 0 \leq x < 1 \\ 0.25 & \text{for } 1 \leq x < 2 \\ 0.45 & \text{for } 2 \leq x < 3 \\ 0.65 & \text{for } 3 \leq x < 4 \\ 1.00 & \text{for } 4 \leq x \end{cases}$$

**Solution:** Second step of a solution to find pmf of a new random variable  $Y = 2X+1$

First, we will calculate the values that  $Y$  can take. From previous problem, we know how to find the values that a new random variable  $Y$  can take.

$$Y = \{1, 3, 5, 7, 9\}$$

## Solution:

Now, we calculate probabilities of each value which a new random variable can take.

$$P(Y=1) = P([2X+1] = 1) = P(X=0) = 0.1$$

$$P(Y=3) = P([2X+1] = 3) = P(X=1) = 0.15$$

$$P(Y=5) = P([2X+1] = 5) = P(X=2) = 0.2$$

$$P(Y=7) = P([2X+1] = 7) = P(X=3) = 0.2$$

$$P(Y=9) = P([2X+1] = 9) = P(X=4) = 0.35$$

$$P(Y = y) = \begin{cases} 0.1 & \text{for } y = 1 \\ 0.15 & \text{for } y = 3 \\ 0.2 & \text{for } y = 5 \\ 0.2 & \text{for } y = 7 \\ 0.35 & \text{for } y = 9 \end{cases}$$

Thank You