Directed Acyclic Graphs (DAGs)

Madhavan Mukund
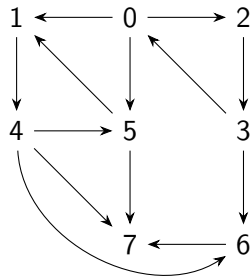
https://www.cmi.ac.in/~madhavan

Mathematics for Data Science 1
Week 11

Directed cycles

- In a directed graph, a cycle must follow same direction
  - $0 \to 2 \to 3 \to 0$ is a cycle
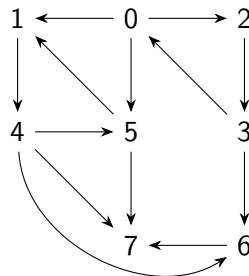 - $0 \to 5 \to 1 \leftarrow 0$ is not

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  - Forward edges
  - Back edges
  - Cross edges

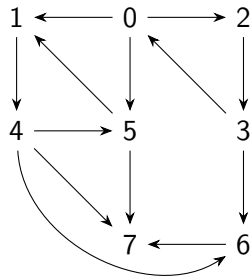# Directed cycles

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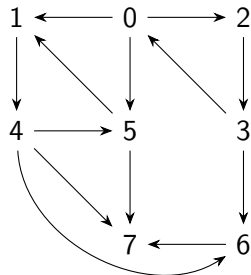- Only back edges generate cycles
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- Why bother about directed cycles?

Tasks and dependencies

- Startup moving into new office space

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- Major tasks for completing the interiors
  - Lay floor tiles
  - Plaster the walls
  - Paint the walls
  - Lay conduits (pipes) for electrical wires
  - Do electrical wiring
  - Install electrical fittings
  - Lay telecom conduits
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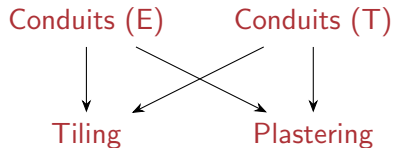
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Conduits (E)   Conduits (T)

Tiling   Plastering

Painting
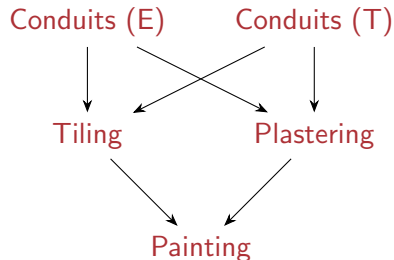
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Fittings (E)

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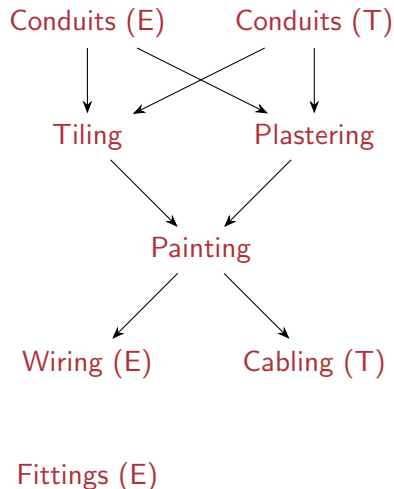Tiling   Plastering

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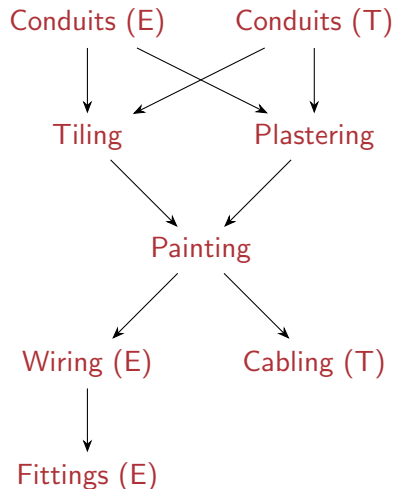Wiring (E)   Cabling (T)

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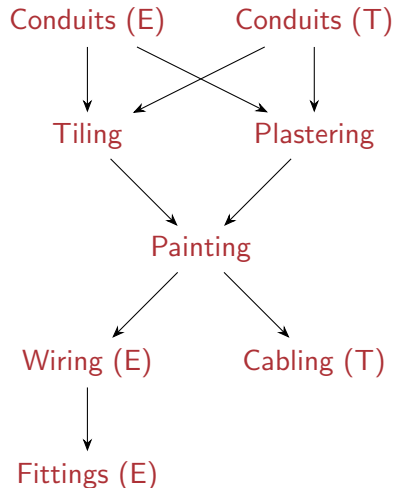Painting

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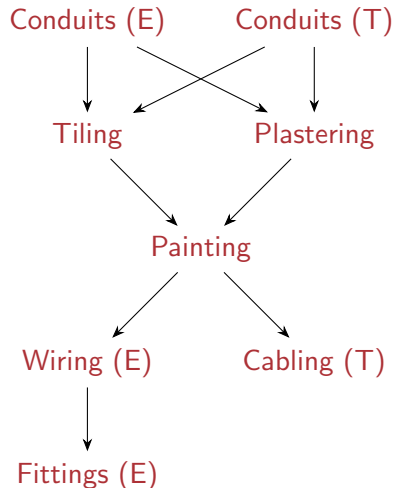 - Edge (t, u) if task t has to be completed before task u

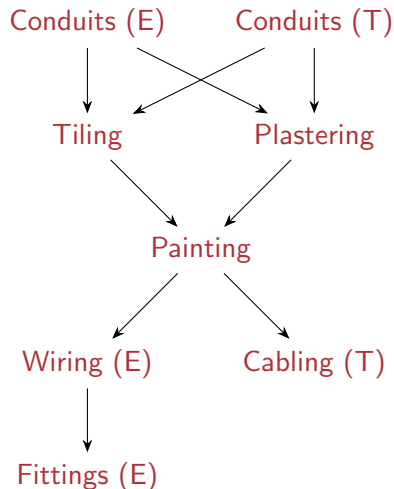- Schedule the tasks respecting the dependencies

Typical questions

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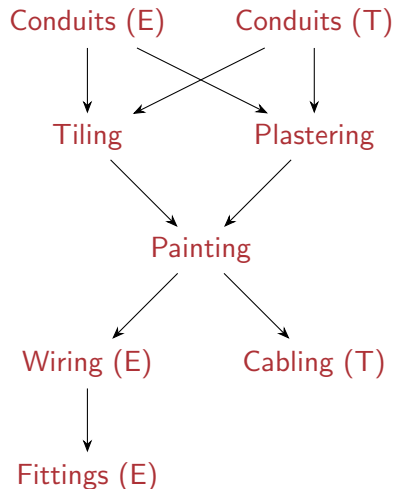
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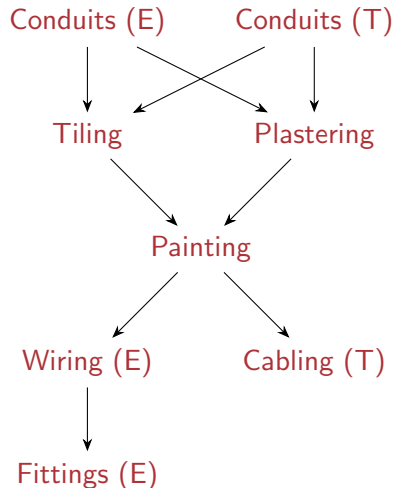   - Conduits (T) – Conduits (E) – Plastering – Tiling – Painting – Wiring (E) – Fittings (E) – Cabling (T)
    - . . .
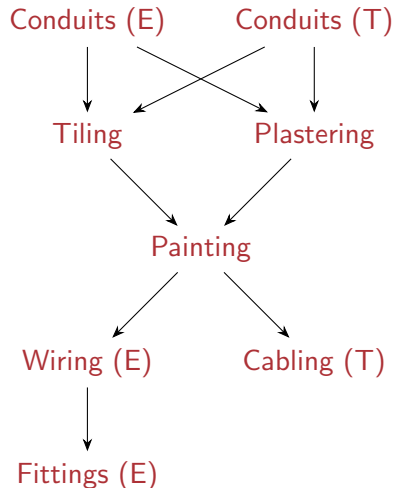
- How long will the work take?

Directed Acyclic Graphs

- Formally, we have a directed acyclic graph (DAG)

- $G = (V, E)$, a directed graph without directed cycles

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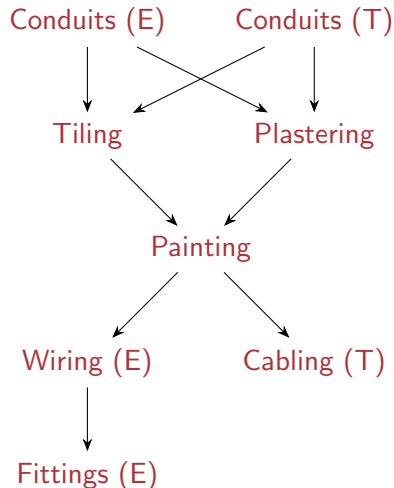
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- How long with the work take?
  - Find the longest path in the DAG
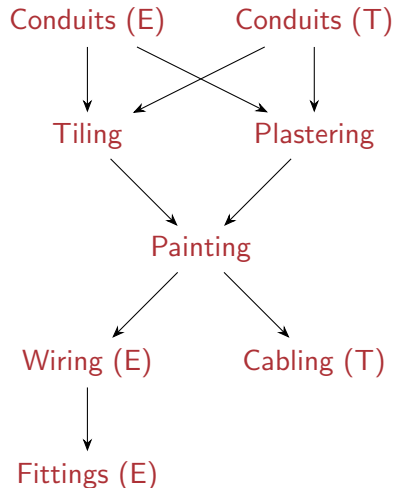
Summary

- Directed acyclic graphs are a natural way to represent dependencies

- Arise in many contexts
    - Pre-requisites between courses for completing a degree
    - Recipe for cooking
    - Construction projects
    - . . .

- Problems to be solved on DAGS
    - Topological sorting
    - Longest paths