TRANSACTIONS

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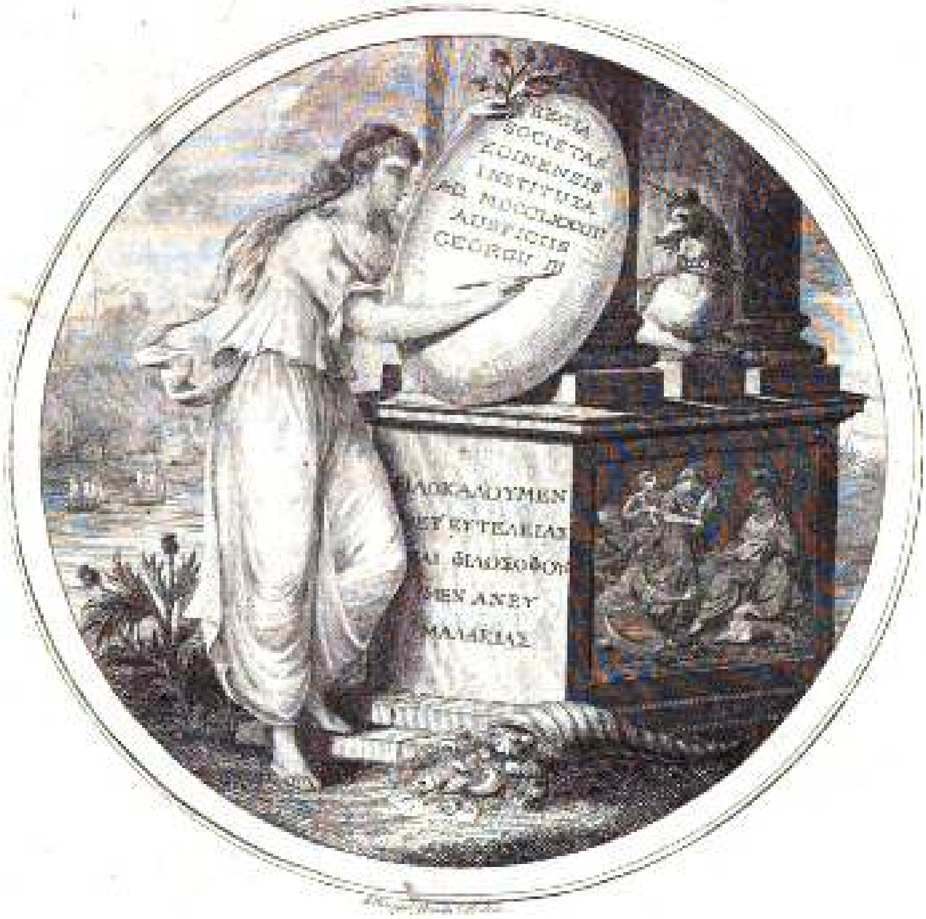

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$2 \sin^2\psi = 1 - \cos 2\psi$, and $\frac{\dot{\psi}}{\sqrt{1 - c^2 \sin^2\psi}} = \frac{(1+c')\dot{\psi}}{\sqrt{1 + c'^2 + 2c' \cos 2\psi}}$, c' being $= \frac{1-\sqrt{1-c^2}}{1+\sqrt{1-c^2}}$: these substitutions being made, we get

$$\frac{2\dot{\phi}\cos\phi}{\sqrt{1+c^2-2c\cos\phi}} = c \times \frac{(1+c')\dot{\psi}}{\sqrt{1+c'^2+2c'\cos 2\psi}} - c \times \frac{(1+c')\dot{\psi}\cos 2\psi}{\sqrt{1+c'^2+2c'\cos 2\psi}}$$

$+ 2\dot{\psi}\cos\psi$.

Suppose now, $\frac{1}{\sqrt{1+c'^2+2c'\cos 2\psi}} = A' - B'\cos 2\psi + c'\cos 4\psi -$ &c. it is evident, from what goes before, that, taking the fluents of the above fluxions, when ϕ and $\psi = \varpi$, we shall have $B \times \varpi = c \times (1 + c') \times (A' + \frac{B'}{2}) \times \varpi$, and so $B = c \times (1 + c') \times (A' + \frac{B'}{2})$.

The values of A' and B', in series according to the method of M. de la Grange, are

$$A' = 1 + \frac{1^2}{2^2}c'^2 + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}c'^4 + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2}c'^6 + \text{&c.}$$

$$\frac{1}{2}B' = (\frac{1}{2}c' + \frac{1}{2} \cdot \frac{1 \cdot 3}{2 \cdot 4}c'^3 + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}c'^5 + \text{&c.})$$

which series converge very fast, on account of the smallness of c' in respect of c.

If, however, it be required to find the value of B by series still more converging, we may easily do so: For it is manifest that B and B′ are similar functions of c and c': and that if we make $c'' = \frac{1-\sqrt{1-c'^2}}{1+\sqrt{1-c'^2}}$, $c''' = \frac{1-\sqrt{1-c''^2}}{1+\sqrt{1-c''^2}}$, and so on, and put

A″,

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