Statistics I

Week 8: Solve with instructor (Graded Assignment)

Plan for this session

- How to join?
  - Join on webex - click on link sent to you
  - Join on pear deck - joinpd.com (enter code seen on top right)
  - Keep a notebook and pen ready for solving problems

- For every question - 15 minutes allotted
  - Question will be shown in a slide for solving - 5 minutes
  - If you are done solving, enter your answer at joinpd.com
  - Presenter will provide a solution - 5 minutes
  - Questions and discussion - 5 minutes

- Prelude questions - 5 or 10 minutes allotted
  - Help to prepare for the main question

How to participate?
joinpd.com
code: see above

Sample Question - your screen on joinpd.com

Description of the problem.

Question to be answered.

Desktop

Answer box

enter a number or a choice or some text

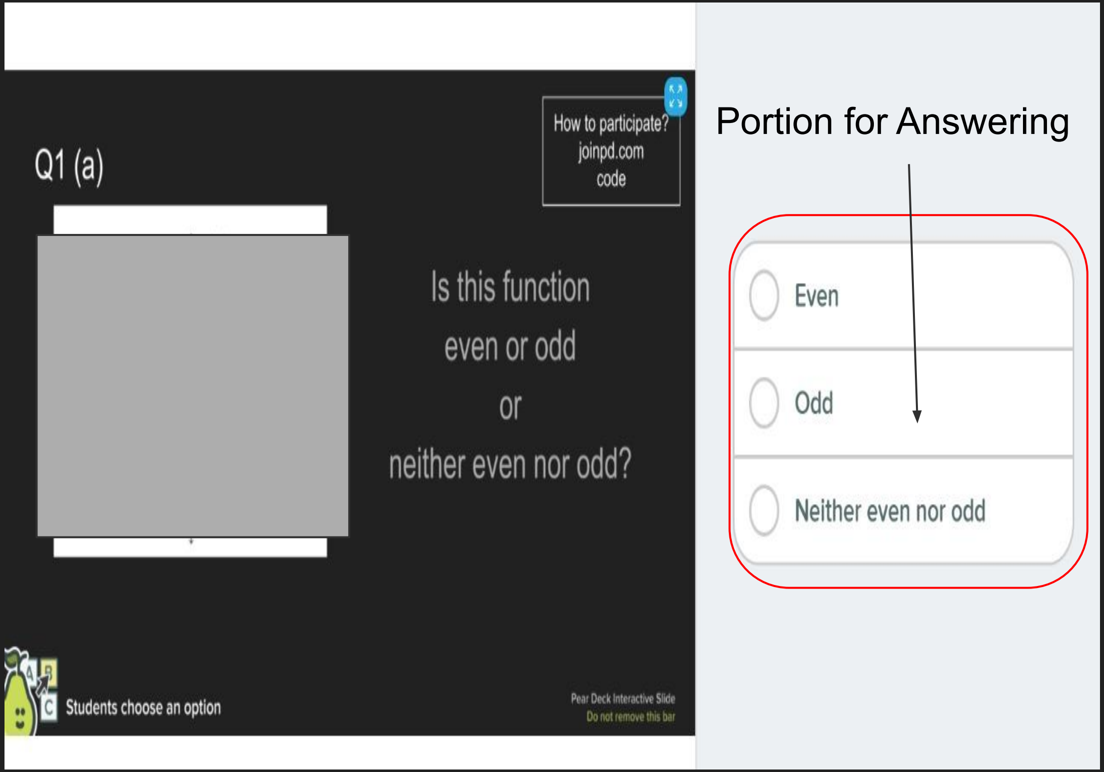
Mobile

Answer question

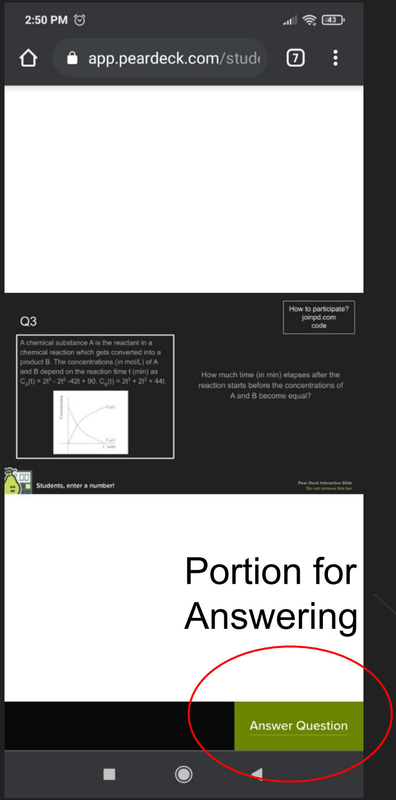
Example Screenshots

How to participate?
joinpd.com
code: see above

Laptop/Desktop



Mobile



Test Problem

How to participate?
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code: see above

Throw a 6-sided die.

Hint: (no of favourable outcomes) / (no of total outcomes)

What is the probability that you will get a multiple of 3?

(enter in decimal)

How to participate?
joinpd.com
code: see above

Solution: how to find probability under uniform distribution?

Experiment: Throw a die

Outcomes: { 1, 2, 3, 4, 5, 6 }, uniform distribution (each outcome is equally likely)

Event A: Multiple of 3 => Favourable outcomes = { 3, 6 }

P(A) = (no of favourable outcomes) / (no of total outcomes)

= 2 / 6 = 1 / 3

How to participate?
joinpd.com
code: see above

Prelude 1 to Q1

An urn contains 5 blue balls and 8 red balls. A ball is chosen at random.

Hint: (no of favourable outcomes) / (no of total outcomes)

What is the probability that the ball is blue?

(enter in decimal)

Students, enter a number!

How to participate?
joinpd.com
code: see above

Solution: how to find probability under uniform distribution?

Experiment: Pick a ball from an urn with 5 blue and 8 red balls

Outcomes: { B1, B2, B3, B4, B5, R1, R2, R3, R4, R5, R6, R7, R8 }, uniform distribution (each outcome is equally likely)

Event A: Blue ball => Favourable outcomes = { B1, B2, B3, B4, B5 }

P(A) = (no of favourable outcomes) / (no of total outcomes)

= 5 / 13

Once you get enough practice, you should do such calculations in one step!

Prelude 2 to Q1

How to participate?
joinpd.com
code: see above

An urn contains 5 red and 13 blue balls. Two balls are drawn one after another at random without replacement. The first ball is red.

Hint: Account for first ball and then… same as before

What is the probability that the second ball is blue?

(enter in decimal)

How to participate?
joinpd.com
code: see above

Solution: account for first step in a two-step experiment

Originally, there are 5 red and 13 blue balls in the urn.

After first ball is drawn, there are 4 red and 13 blue balls in the urn.

Second ball is drawn at random - uniform distribution

P(Second ball blue | First ball red)

= (no of favourable outcomes) / (no of total outcomes)

= 13 / 17

How to participate?
joinpd.com
code: see above

Q1

An urn contains 7 red and 13 blue balls. Two balls are drawn one after another at random without replacement.

Hint: Account for all possibilities for first ball and then combine the terms correctly.

What is the probability that the second ball is blue?

(enter in decimal)

How to participate?
joinpd.com
code: see above

Solution: account for first step in a two-step experiment and combine terms

Two balls are drawn one after another without replacement.

Event: Second ball is blue

Step I: Incorporate all possibilities for first ball and write out event

Second ball is blue = (First ball is blue and Second ball is blue)

or

(First ball is red and Second ball is blue)

Solution: rules

How to participate?
joinpd.com
code: see above

Step II: Get to know the rules

Prob(empty event) = 0, Prob(sure event) = 1

Prob(Event1 or Event2) = P(Event1) + P(Event2) - P(Event1 and Event2)

Prob(Event1 and Event2) = P(Event1) P(Event2 | Event1)

P(Event2 | Event1) = Conditional probability of Event2 given Event1

= Account for Event1, change the possible outcomes and compute prob for Event2 using modified set of outcomes

How to participate?
joinpd.com
code: see above

Solution: apply the rules

Step III: Apply the rules for the event (at start: 7 red and 13 blue)

Second ball is blue (B2) = (First ball is blue (B1) and B2)

or

(First ball is red (R1) and B2)

Prob(B2) = P(B1 and B2) + P(R1 and B2) - P(B1 and B2 and R1 and B2)

B1 and B2 and R1 and B2: empty event (why?)

P(R1 and B2) = P(R1) P(B2 | R1) = 7/20 x 13/19

P(B1 and B2) = P(B1) P(B2 | B1) = 13/20 x 12/19

Answer: 13/20

How to participate?
joinpd.com
code: see above

Q2

In a population, 45% are female and 55% are male. Among all the females, 9% are left-handed and among all the males 11% are left-handed. A person is randomly chosen from the population.

What is the probability that the person is not left-handed?

(enter in decimal)

Students, enter a number!

How to participate?
joinpd.com
code: see above

Solution

Step I: Incorporate all possibilities and write out event

Not Left-handed = (Male and Not Left-handed)

or

(Female and Not Left-handed)

In short, NL = (M and NL) or (F and NL)

Solution: apply the rules

How to participate?
joinpd.com
code: see above

Step II: Apply the rules for the event

Rule: P(not Event) = 1 - P(Event)

$$P(NL) = P(M \text{ and } NL) + P(F \text{ and } NL) - P(M \text{ and } NL \text{ and } F \text{ and } NL)$$

$$= P(M)\, P(NL \mid M) + P(F)\, P(NL \mid F)$$

$$= 0.55 \times (1 - 0.11) + 0.45 \times (1 - 0.09)$$

$$= 0.899$$

Prelude 1 to Q3

Student writes an exam with two sections. The probability of passing Section 1 is 0.8 and the probability of passing Section 2 is 0.9.

Hint: Trick question….

What is the probability that the student passes both sections?

(enter in decimal)

How to participate?
joinpd.com
code: see above

Solution

P(Section 1 pass) = 0.8

P(Section 2 pass) = 0.9

P(Section 1 pass and Section 2 pass)

= P(Section 1 pass) P(Section 2 pass | Section 1 pass)

= 0.8 x ???

We do not have the conditional probability!!! This problem is not fully specified.

Prelude 2 to Q3

How to participate?
joinpd.com
code: see above

Student writes an exam with two sections. The probability of passing Section 1 is 0.8 and the probability of passing Section 2 is 0.9. The events of passing Sections 1 and 2 are independent.

Hint: Use independence.

What is the probability that the student passes both sections?

(enter in decimal)

Students, enter a number!

Solution

How to participate?
joinpd.com
code: see above

Event1 and Event2 are independent if

$$P(\text{Event1 and Event2}) = P(\text{Event1})P(\text{Event2})$$

Also: $P(\text{Event1} \mid \text{Event2}) = P(\text{Event1})$ or $P(\text{Event2} \mid \text{Event1}) = P(\text{Event2})$

$P(\text{Section 1 pass}) = 0.8$, $P(\text{Section 2 pass}) = 0.9$

Since the two events are independent,

$$\begin{aligned} &P(\text{Section 1 pass and Section 2 pass}) \\ &\quad = P(\text{Section 1 pass})\ P(\text{Section 2 pass}) \\ &\quad = 0.8 \times 0.9 = 0.72 \end{aligned}$$

How to participate?
joinpd.com
code: see above

Q3

A student appeared for an aptitude test of a company that has a total of 3 sections. The student will pass the aptitude test if he passes in either Section 1 and 2 or Section 1 and 3. The probabilities of the student passing Sections 1, 2, and 3 are p, 1/4, and 1/3, respectively, and these are independent events.

Hint: Write out event and use independence.

Find the value of p if probability of passing the aptitude test is 1/3.

(enter in decimal)

Solution

How to participate?
joinpd.com
code: see above

Step I: Incorporate all possibilities and write out event

Pass = (Section 1 pass and Section 2 pass)

or

(Section 1 pass and Section 3 pass)

In short, Pass = (Pass1 and Pass2) or (Pass1 and Pass3)

How to participate?
joinpd.com
code: see above

Solution: apply the rules

Step II: Apply the rules for the event and use independence

P(Pass) = P(Pass1 and Pass2) + P(Pass1 and Pass3)
- P(Pass1 and Pass2 and Pass1 and Pass3)

P(Pass1 and Pass2 and Pass1 and Pass3) = P(Pass1 and Pass2 and Pass3)
= P(Pass1) P(Pass2) P(Pass3) = p x 1/4 x 1/3 = p/12

P(Pass1 and Pass2) = p/4, P(Pass1 and Pass3) = p/3

So, P(pass) = p/4 + p/3 - p/12 = p/2

Finally, p/2 = 1/3 => p = 2/3

Prelude 1 to Q4

How to participate?
joinpd.com
code: see above

In a manufacturing firm, machines M1, M2, and M3 makes 25%, 55%, and 20% of the screws, respectively. It is known from the past experience that 1%, 3%, and 2% of the screws will be defective if made by machines M1, M2, and M3, respectively.

Hint: Write out the event

If a screw is chosen at random, find the probability that it is defective.

(enter in decimal)

Students, enter a number!

Solution: Write out the event

How to participate?
joinpd.com
code: see above

Def: Screw is defective, M1: Screw is from Machine 1

M2: Screw is from Machine 2, M3: Screw is from Machine 3

Def = (M1 and Def) or (M2 and Def) or (M3 and Def)

How to handle Event1 or Event2 or Event3?

How to participate?
joinpd.com
code: see above

Solution

Rule for disjoint or mutually exclusive events:

If Event1 and Event2 = empty, Event1 and Event3 = empty, Event2 and Event3 = empty, (events are mutually exclusive or disjoint)

$$P(\text{Event1 or Event2 or Event3}) = P(\text{Event1}) + P(\text{Event2}) + P(\text{Event3})$$

Solution

How to participate?
joinpd.com
code: see above

Def = (M1 and Def) or (M2 and Def) or (M3 and Def)

(M1 and Def), (M2 and Def), (M3 and Def): disjoint

$$
\begin{aligned}
P(\text{Def}) &= P(\text{M1 and Def}) + P(\text{M2 and Def}) + P(\text{M3 and Def}) \\
&= P(\text{M1})\, P(\text{Def} \mid \text{M1}) + P(\text{M2})\, P(\text{Def} \mid \text{M2}) + P(\text{M3})\, P(\text{Def} \mid \text{M3}) \\
&= 0.25 \times 0.01 + 0.55 \times 0.03 + 0.20 \times 0.02 = 0.023
\end{aligned}
$$

Prelude 2 to Q4

How to participate?
joinpd.com
code: see above

In a population, 45% are female and 55% are male. Among all the females, 9% are left-handed and among all the males 11% are left-handed. A person is randomly chosen from the population.

Hint: Bayes’ rule

If the person is left-handed, what is the (conditional) probability that the person is male?

(enter in decimal)

How to participate?
joinpd.com
code: see above

Solution: Bayes' rule

P(Event1 and Event2) = P(Event1) P(Event2 | Event1)
= P(Event2) P(Event1 | Event2)

Question asks for P(Male | Left-handed). Put Event1 = Male (M) and Event2 = Left-handed (L) in Bayes' rule.

$$P(M)\,P(L \mid M) = P(L)\,P(M \mid L)$$

$$P(M) = 0.55,\ P(L \mid M) = 0.11,\ P(L) = ?$$

How to participate?
joinpd.com
code: see above

Solution: apply the rules

L = (M and L) or (F and L)

P(L) = P(M and L) + P(F and L) - P(M and L and F and L)

= P(M) P(L | M) + P(F) P(L | F)

= 0.55 x 0.11 + 0.45 x 0.09

= 0.101

So, P(M | L) = P(M) P(L | M) / P(L) = 0.55 x 0.11 / 0.101 = 0.599

How to participate?
joinpd.com
code: see above

Q4

In a manufacturing firm, machines M1, M2, and M3 makes 25%, 55%, and 20% of the screws, respectively. It is known from the past experience that 1%, 3%, and 2% of the screws will be defective if made by machines M1, M2, and M3, respectively.

Hint: Bayes' rule

If a screw is chosen at random and found to be defective, find the probability that it is made by machine M1.

(enter in decimal)

Students, enter a number!

Solution

How to participate?
joinpd.com
code: see above

Def: Screw is defective, M1: Screw is from Machine 1

$$P(M1)\, P(Def \mid M1) = P(Def)\, P(M1 \mid Def)$$

$$P(M1) = 0.25,\ P(Def \mid M1) = 0.01,\ P(Def) = 0.023$$

So, $P(M1 \mid Def) = 0.01 \times 0.25 / 0.023 = 5/46 = 0.109...$

Thank You

Q.3

Suppose we roll a pair of fair dice, so that each of the 36 possible outcomes are equally likely. Let A be the event that the sum of the outcomes of rolling the pair of dice is 6, B be the event that the sum of the outcomes of rolling the pair of dice is 7, and C be the event that the sum of the outcomes of rolling the pair of dice is even. Choose the correct statements from the following.

a) Events A and B are mutually exclusive.
b) Events A and C are mutually exclusive.
c) Events A and B are independent.
d) Events A and C are independent.

solution:

A = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}, P(A) = 5/36

B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}, P(B) = 6/36

C = {(1, 1), (1, 3), (2, 2), (3, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1),(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (4,6), (6, 4), (5, 5), (6, 6)}, P(C) = 18/36

Intersection of A and B is empty.

A is contained in C.

A and B are disjoint, so they cannot be independent.

A intersection C is A itself, so P(A intersection C) is not equal to P(A).P(C).

Q.6

An urn contains a total of 10 balls, out of which 3 are black and the remaining are red. Suppose a sample of 4 balls is drawn from the urn one by one (with replacement). What is the conditional probability that the first, third, and fourth ball will be red given that exactly 3 balls out of the 4 in the sample are red?

Solution:

Let A be the event that the 1st, 3rd, and 4th balls are red.

Let B be the event that in the sample of 4, three balls are red.

Now $P(A \mid B) = P(A \cap B) / P(B)$

For $(A \cap B)$, we have only 1 choice: Red Black Red Red

Probability of getting a red ball = 0.7

Probability of getting a black ball = 0.3

$P(A \cap B) = (0.7)^3 * 0.3$

$P(B) = 4C3\ (0.7)^3\ (0.3)$

Now $P(A \mid B) = 1/(4C3) = ¼ = 0.25$.