

School of Mathematical Sciences  
National Institute of Science Education and Research

Problem Set - 6                      M 208

1. Prove that the Petersen graph is nonplanar ( without using Kuratowski and Wagner theorem).
2. Let  $G$  be a self dual graph on  $n$  vertices and  $m$  edges. Prove that  $2n = m + 2$ .
3. Let  $G$  be a graph of order  $n$ . Prove that  $\frac{n}{\alpha(G)} \leq \chi(G) \leq n - \alpha(G) + 1$ .
4. At a gathering of 8 employees of a company, which we denoted by  $A = \{a_1, a_2, \dots, a_8\}$ , it is decided that it would be useful to have these individuals meet in committees of three to discuss seven issues of importance to the company. The seven committees selected for this purpose are  $A_1 = \{a_1, a_2, a_3\}$ ,  $A_2 = \{a_2, a_3, a_4\}$ ,  $A_3 = \{a_4, a_5, a_6\}$ ,  $A_4 = \{a_5, a_6, a_7\}$ ,  $A_5 = \{a_1, a_7, a_8\}$ ,  $A_6 = \{a_1, a_4, a_7\}$ ,  $A_7 = \{a_2, a_6, a_8\}$ . If each committee is to meet during one of the time periods  $T_p = \{1 - 2pm, 2 - 3pm, 3 - 4pm, 4 - 5pm, 5 - 6pm, 6 - 7pm\}$ , then what is the minimum number of time periods needed for all seven committees to meet?
5. Find the chromatic index of Petersen graph.
6. Let  $G$  be a graph with  $m \geq 1$  edges. Prove that  $\chi'(G) \geq \frac{m}{\alpha'(G)}$ .