Random Variable

→ Not interested in the details of experimental result, but in the value of some numerical quantity.

Ex:- Rolling 2 dice simultaneously

$$Sum = 7$$

Q. How many outcomes will result in sum = 7

$X \to$ random varible

$$P(X=7) = 6/36$$

* Discrete random variable

→ can take on at most a countable no. of possible values

→ Irrespective of the values, the no. should be countable.

* Continuous random varibles

→ non-contable

ex – height of person, speed of vehicle

Pmf:

$$P(x_i) = P(X = x_i)$$

X	x_1	x_2	x_3	x_4	$\dots$	x_n
$P(X=x_i)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	$\dots$		$P(x_n)$

Properties

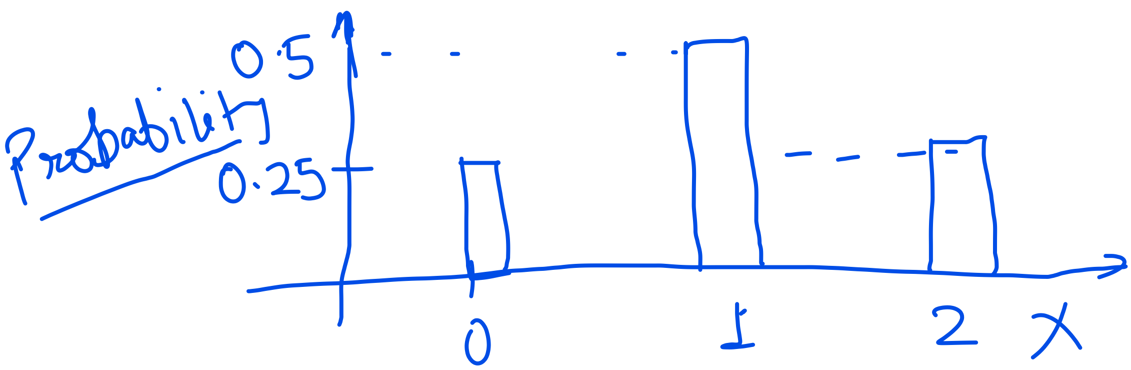
i.) $P(x_i) \geq 0$

ii.) $P(x) = 0$ for all other values of x

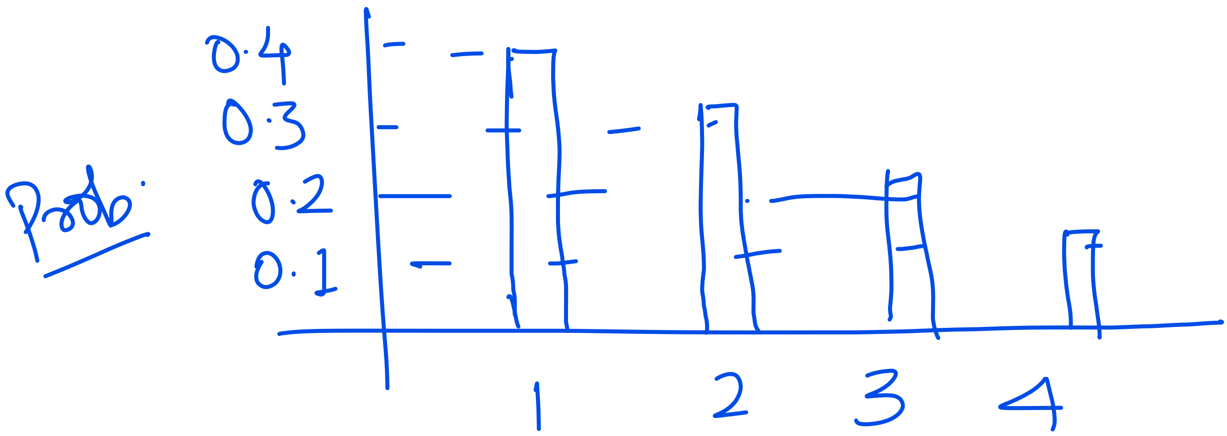
iii.) $\sum_{i=1}^{\infty} P(x_i) = 1$

Graph of discrete pmf

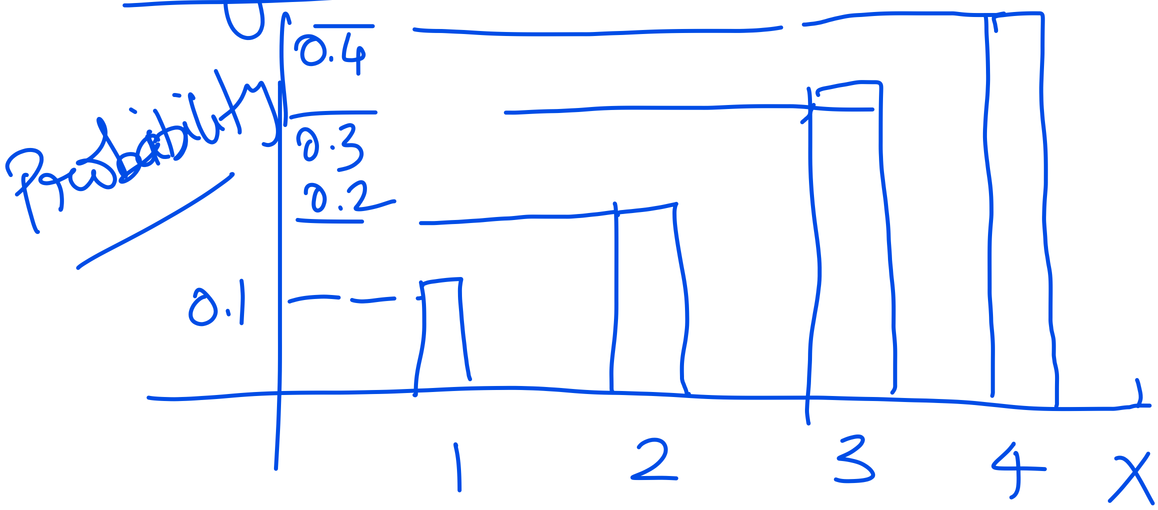
X	0	1	2
$P(X=x_i)$	0.25	0.5	0.25



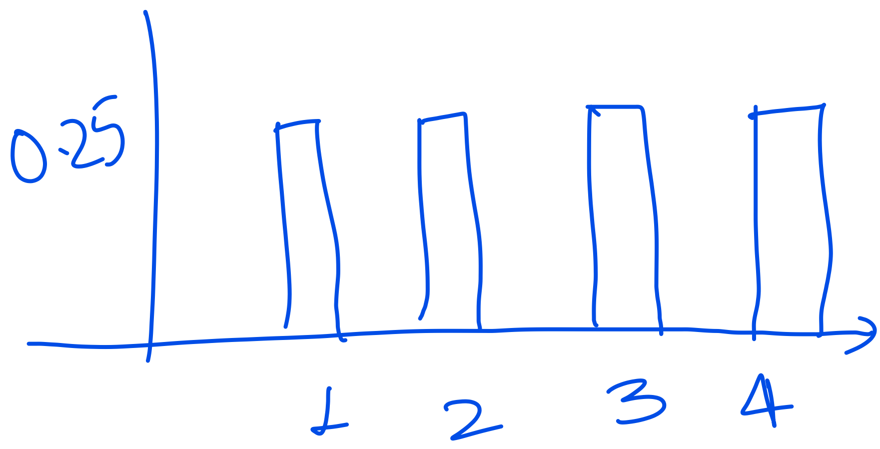
Positive skewed

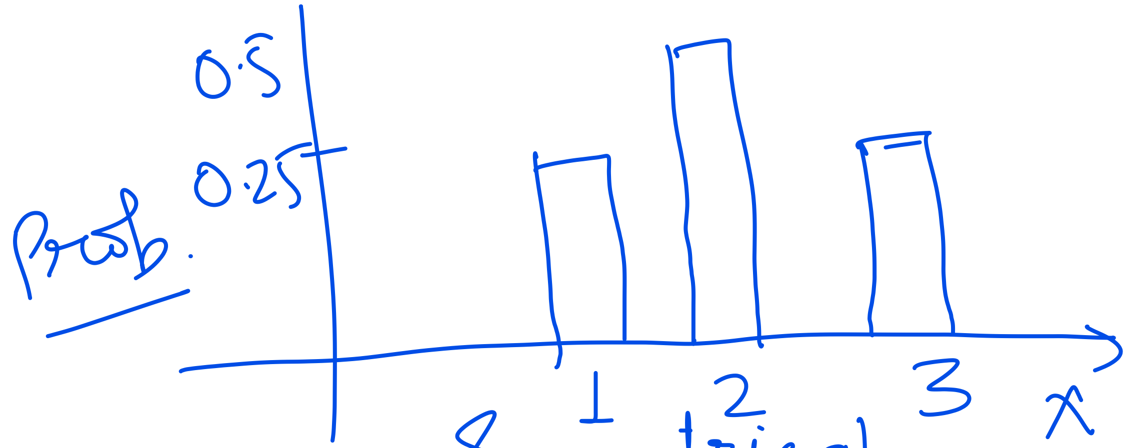


Negative skewed



Uniform discrete





Symmetrical

cdf

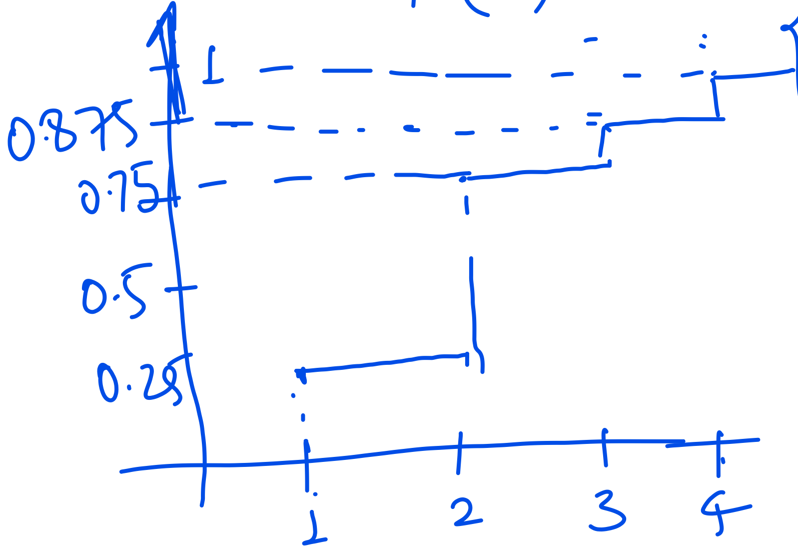
Accumulate the probability at different points.

$$F : \mathbb{R} \to [0, 1]$$

$$F(a) = P(X \leq a)$$

X	1	2	3	4
$P(X=x_i)$	0.25	0.5	0.125	0.125

$$F(a) = \begin{cases} 0 & a < 1 \\ 0.25 & 1 < a < 2 \\ 0.75 & 2 \leq a < 3 \\ 0.875 & 3 \leq a < 4 \\ 1 & a \geq 4 \end{cases}$$



Expectation :

$$E(X) = \sum_{i=1}^{\infty} x_i \, P(X=x_i)$$

" long-run-average "

→ Average of large no. of rolls

→ It's value can be different from the outcomes.

* Expected value of a bernoulli random variable

$$E(X) = p$$

* Expected value for discrete uniform random variable

$$E(X) = \frac{n+1}{2}$$

⇒ Expectation of a fn of a random variable

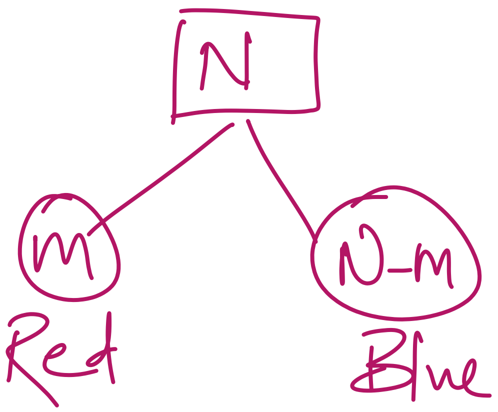
$$E(g(X)) = \sum g(x_i) \, P(X=x_i)$$

Eg:- $E(ax+b) = aE(x)+b$

Properties

$$E(X+Y) = E(X)+E(Y)$$

Hypergeometric random varible



$n \rightarrow$ sample to be chosen

$X \rightarrow$ no. of red balls selected

$$P(X=i) = \frac{\left({}^{m}C_i\right)\left({}^{N-m}C_{n-i}\right)}{{}^{N}C_n}$$

$$\boxed{E(X) = \frac{nm}{N}}$$

Variance

measures of spread.

$$E(X) = \mu$$

$$Var(X) = E(X-\mu)^2$$

$$= E(X^2 + \mu^2 - 2\mu X)$$

$$= E(X^2) + \mu^2 - 2\mu E(X)$$

$$= E(X^2) - \mu^2$$

$$= E(X^2) - [E(X)]^2$$

★ Bernoulli random varible

$$V(X) = p(1-p)$$

★ Discrete uniform random variable

$$E(X) = \frac{(n+1)}{2}$$

$$E(X^2) = \frac{(n+1)(2n+1)}{6}$$

$$V(X) = \frac{n^2-1}{12}$$

Properties

$$Var(cX) = c^2 Var(X)$$

$$Var(X+c) = Var(X)$$

$$Var(aX \pm b) = a^2 Var(X)$$

$$Var(-aX) = a^2 Var(X)$$

Ex:-

$$Var(X+X) = Var(2X)$$

$$= 4Var(X)$$

$$\neq Var(X) + Var(X)$$

* $Var(X+Y) \neq Var(X) + Var(Y)$

If X & Y are independent

then

$$Var(X+Y) = Var(X) + Var(Y)$$

Standard deviation

$$SD(X) = \sqrt{Var(X)}$$

positive ~~s~~square root of variance

Properties

$$SD(cX) = c\,SD(X)$$

$$SD(X+c) = SD(X)$$

* Bernoulli trial

$$S = \{\text{Success}, \text{failure}\}$$

$$\text{Success} \downarrow 1 \qquad \text{failure} \downarrow 0$$

Experiment whose outcomes can be classified as $\{S, F\}$

X	0	1
$P(X=x_i)$	$1-p$	p

$$E(X) = p$$

$$V(X) = p(1-p)$$

At $p = 0.5$, variance is max^m

$\downarrow$

Uncertainty is very high

IID

Independent & identically distributed Bernoulli trials

→ Should be independent of each other

→ In each trial, the outcomes should be identical.

Binomial

n independent trials $\langle$ S (p), F $(1-p)$

$X \rightarrow$ total no. of success in n trial

$X \sim Bin(n, p)$

$$\boxed{P(X=i) = {}^{n}C_{i}\,(p)^{i}\,(1-p)^{(n-i)}}$$

* right skewed if $p < 0.5$
* left skewed if $p > 0.5$
* Symmetric if $p = 0.5$

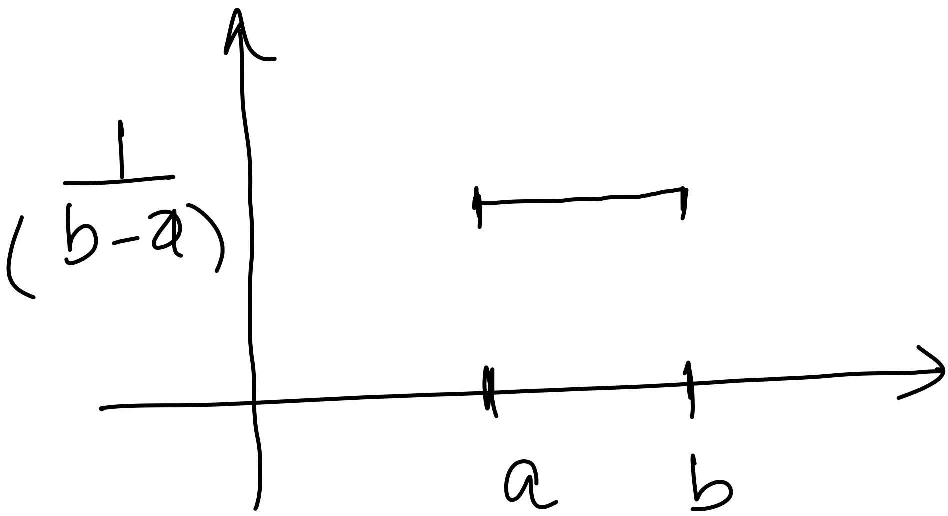
} n smaller

For large n, it approaches symmetry.

$$E(x) = np$$
$$V(x) = np(1-p)$$

Continuous Uniform distribution

$$X \sim U(a,b)$$



$$f(x) = K \quad a \le x \le b$$

$$\int_a^b f(x)\,dx = 1$$

Verify $f(x)$ is a pdf $X \sim U(0,1)$

① $f(x) \geq 0$ for $0 < x < 1$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

<u>cdf</u> $X \sim U(a,b)$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a,b) \\ 1 & x \geq b \end{cases}$$

* <u>Properties</u> :- $E(X) = \frac{a+b}{2}$

$$V(X) = \frac{(b-a)^2}{12}$$

$$E(X^2) = \frac{a^2+b^2+ab}{3}$$

* For - Non Uniform & Triangular distribution

$\Rightarrow$ Calculate the area under pdf.

Exponential distribution

pdf $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

cdf

$$\boxed{F(a) = P(X \leq a) = 1 - e^{-\lambda a}}$$

$$E(X) = 1/\lambda$$

$$V(X) = 1/\lambda^2$$

$$E(X^n) = \frac{n}{\lambda} E(X^{n-1})$$