Alternated Julia Sets

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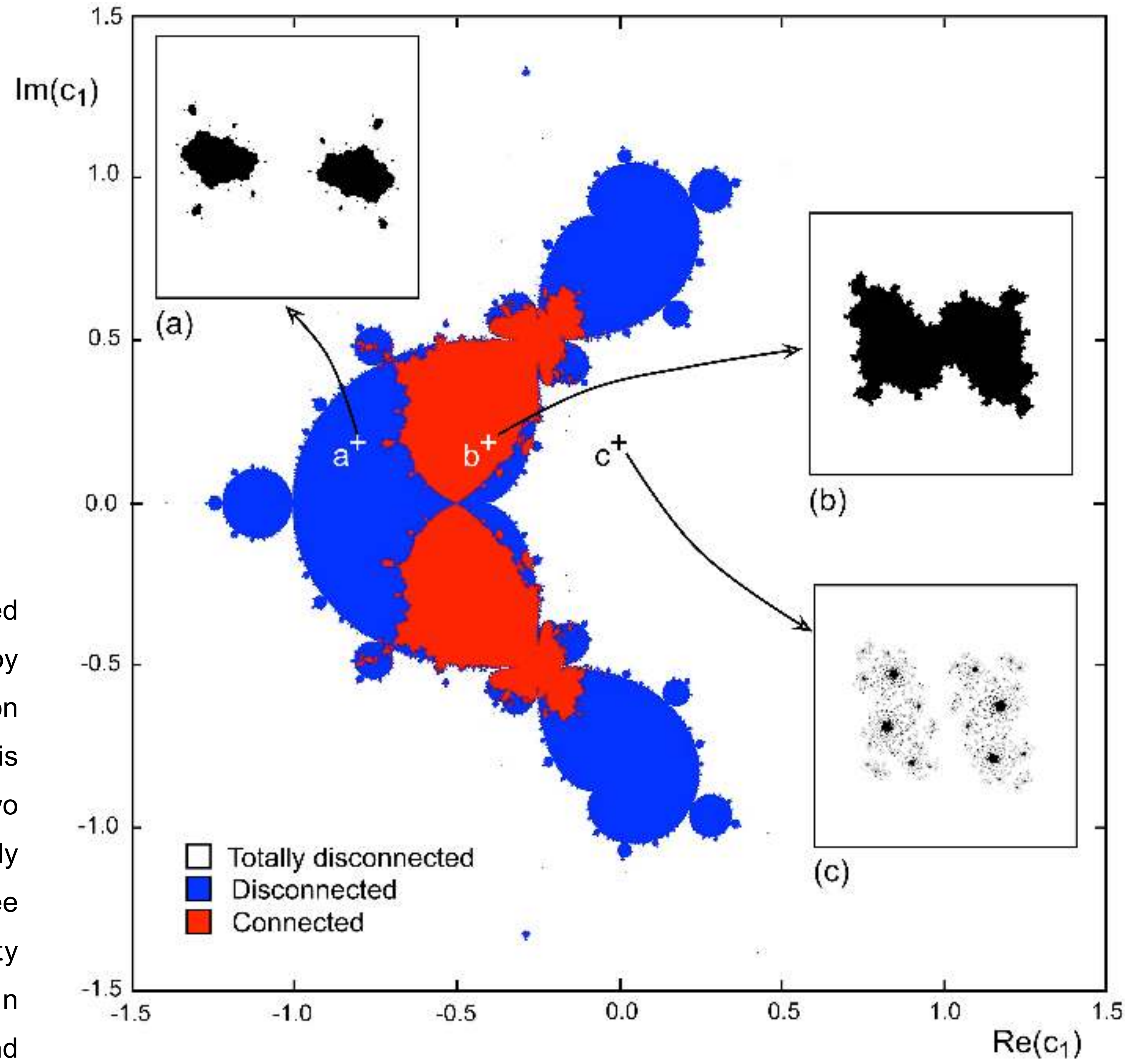
The connectivity domains of the alternated Julia sets are computationally studied, by using computer graphics and visualisation algorithms. The alternated Julia sets is defined by switching the dynamics of two quadratic Julia sets. As proved analytically they exhibit, as for polynomials of degree greater than two, the disconnectivity property in addition to the known dichotomy property (connectedness and totally disconnectedness) which characterizes the standard Julia sets. The underlying connectivity domains are four-dimensional fractal bodies. To visualize these sets, hyperplane slices through these bodies are considered, using volume rendering algorithms or by taking further planar slice.

We study analytically and numerically the connectedness properties of alternated Julia sets. These are defined by alternating two known complex quadratic maps

$$z_{n+1} = z_n^2 + c_i, \quad i = 1,2, \quad z_0, c_i \in \mathbf{C}, \quad n \in \mathbf{N}.$$

We draw and explore, with the aid of leading edge computer graphics algorithms, the geometric locus of the points representing all the three states: connectedness, disconnectedness and totally disconnectedness. In order to study these 4 dimensional sets we employ a range of techniques for visualizing high dimensional objects more commonly reserved for studying 3D volumes arising from MRI or CT scans, techniques known as volume visualization.



Planar section obtained for Re(c2) = -0.4, Im(c2) = 0. The points A(-0.8,0.2), B(-0.4,0.2), and C(0,0.2) respectively belong to the disconnected, connected and totally disconnected respectively alternated Julia sets.