XY model

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Contents

Introduction

1.1 Lattice models for phase transitions

A very successfull and general rage of models in statistical mechanics are the ones where the components of a system are allocated on an array of lattice sites, where every component interacts only wiht the closest ones to it (*nearest-neighbour interaction*). This kind of models turns out to describe well a very broad range of phase transitions including ferromagnetism transitions, gas-liquid and gas-solid transitions, superconductive transitions and so on. [1]

The case in consideration here will be the one for *ferromagnetic* transitions.

Being defined on a lattice means basically a discretization of space, a caveat that fit very well if the system in consideration is a metal whose atoms are situated in a crystal structure. The crystalline nature of metals enables to introduce another semplification in this kind of models: considering the atoms as harmonic oscillators swingind around the equilibrium point we can neglect the kinetic energy of these atoms and considering only the interactions between them. It has to be said that this kind of facilitation turns out to be good enough also for the other kinds of transitions listed above.

1.2 Hamiltonian of ferromagnetic lattice systems

Being interested in the magnetic interactions of atoms we consider them as a magnetic dipoles having a magnetic moment μ. Every magnetic dipole in the N lattice sites will have a magnitude $\mu = g\mu_B\sqrt{J(J+1)}$, with g being the Landè factor, μ_B the Bohr magneton and J being the total angolar momentum quantum number. The orientations of the magnetic momenta is given by the multiplicity of J: $(2J+1)$. This magnetic dipoles will interact witch each other and will interact with an eventual external field $\mathbf{B}$.

The net magnetization of the system $\overline{M}$ will depend on both the tempera-

[1] The foregoing discussion is heavily based on Pathria (1972) section 12.3

ture T and the field B ($\overline{M} \equiv \overline{M}(T, B)$). Studying the *spontaneous magnetization* $\overline{M}(0, B)$ of a *critical temperature* T_c can be defined as the threshold temperature for $\overline{M}(0, B) \neq 0$: above T_c the termal agitation will be to big to allow a spontaneous magnetization to form, then at T_c the system will perform a *ferromagnetic transition*.

Detailed studies beyond the scope of this tractation have shown that ferromagnetic properties arise when $J = \frac{1}{2}$ and so it can be assumed that this kind of phenomena are due to electons spin. Then we can rewrite the magnetic moment as $\mu = 2\mu_B\sqrt{s(s+1)}$, where $g = 2$ is the Landè factor for electrons and s is the spin of the electron.

From quantum mechanics considerations (see Bransden, Joachain, and Plivier 2003) it can be shown that the interaction energy between two electrons can be expressed as $K_{ij} \pm J_{ij}$ with i and j being the two indices indicating two neighboring electrons. The plus and minus sign are determined based on S the total spin of the the two electron: the plus corresponding to the case $S = 0$ and the minus to one with $S = 1$, where only J_{ij} is due to the spin configuration.

Then the energy difference between the "parallel" spin configuration and the "antiparallel" spin configuration is

$$\Delta E = E_{\uparrow\uparrow} - E_{\uparrow\downarrow} = -2J_{ij}$$

Defining the scalar product operator of the spin of the two electron as

$$\mathbf{s_j} \cdot \mathbf{s_i} = \frac{1}{2}S(S+1) - s(s+1)$$

is easy to show that the interaction energy between the spins can be written as

$$E_{ij} = \text{const.} - 2J_{ij}(\mathbf{s_i} \cdot \mathbf{s_j})$$

The exact value of the constant is irrilevant since any constant can be added to the potential energy. Still from quantum meachanical considerations can be seen J_{ij} fall very rapidly with the distance, supporting our initial caveat of regarding only the neirest-neighbors interactions.

1.3 *n*-vector model

1.3.1 Ising Model

1.3.2 Heisenberg Model

XY model

2.1 Definition

2.2 2D case

2.3 3D case

Montecarlo simulation

3.1 Monte Carlo Theory

3.2 Implementation

3.3 Results

Conclusion

Bibliography

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