

Advanced Course Computer Science

## Music Processing

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## Tempo and Beat Analysis



## Introduction

### Musical Properties:

- Harmony
- Melody
- Rhythm
- Timbre

## Introduction

### Musical Properties:

- Harmony
- Melody
- **Rhythm: Tempo and beat analysis**
- Timbre

## Introduction

Example 1: Britney Spears – Oops!...I Did It Again

Tempo: 100 BPM



## Introduction

Example 2: Queen – Another One Bites The Dust

Tempo: 110 BPM



## Introduction

Example 3: Burgmueller – Op100-2

Tempo: 130 BPM



## Introduction

Example 4: Chopin – Mazurka Op. 68-3

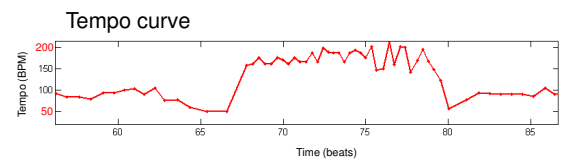
Tempo:



## Introduction

Example 4: Chopin – Mazurka Op. 68-3

Tempo: 50-200 BPM



## Introduction

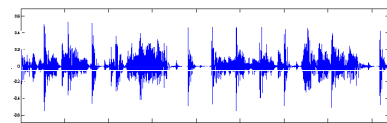
Example 5: Borodin – String Quartet No. 2

Tempo: 120-140 BPM (roughly)



## Introduction

Given a recording of a musical piece

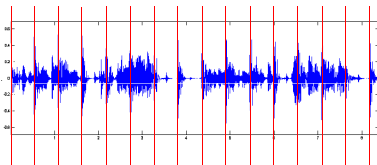


determine the periodic sequence of beat positions:

*Tapping the foot to a piece of music*

## Introduction

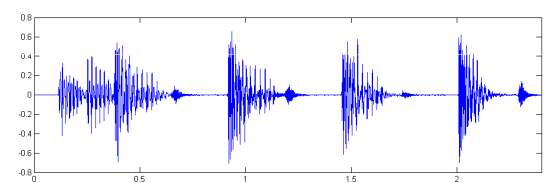
Given a recording of a musical piece



determine the periodic sequence of beat positions:

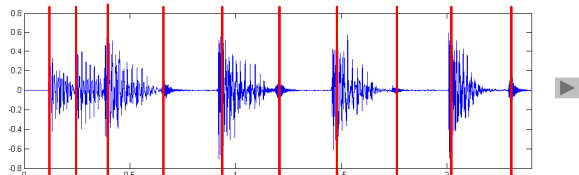
*Tapping the foot to a piece of music*

## Introduction



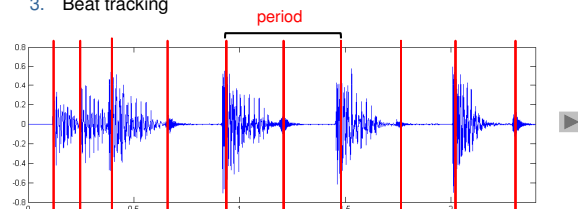
## Introduction

1. Note onset detection
2. Tempo estimation
3. Beat tracking



## Introduction

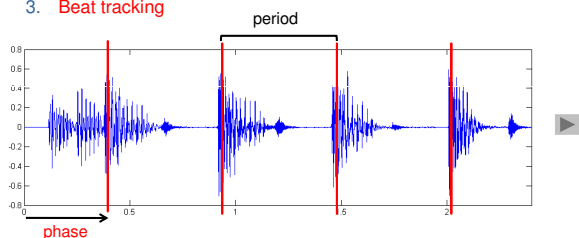
1. Note onset detection
2. Tempo estimation
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Tempo (BPM) := 60 / period (seconds)

## Introduction

1. Note onset detection
2. Tempo estimation
3. Beat tracking



## Introduction

### Beat

Sequence of equally spaced impulses, which periodically occur in music. The perceptually most salient pulse (foot tapping rate).

### Tempo

The tempo of a piece is the inverse of the beat period. Instead of frequency in Hz, we think beats per minute (BPM).

## Introduction

- Tempo and beat are fundamental properties of music
- The beat provides the temporal framework of music (musical meaningful time axis)
- Beat-synchronous audio features
- Rhythmic similarity for music recommendation, genre classification, music segmentation
- Music transcription
- Commercial applications
  - automatic DJ / mixing
  - light effects



## Introduction

### Tasks

1. Note onset detection
2. Tempo estimation
3. Beat tracking

## Overview

### Tasks

1. Note onset detection
2. Tempo estimation
3. Beat tracking

### Challenges

- Non-percussive music
- Soft note onsets
- Time-varying tempo

## Overview

### Tasks

1. **Note onset detection**
2. Tempo estimation
3. Beat tracking

### Challenges

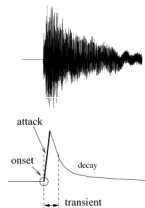
- Non-percussive music
- Soft note onsets
- Time-varying tempo

## Note Onset Detection

- Finding perceptually relevant impulses in a music signal
- Musical accents, note onsets

### Onset:

- The exact time, a note is hit
- One of the three parameters defining a note (pitch, onset, duration)
- Change of properties of sound:
  - Energy or Loudness
  - Pitch or Harmony
  - Timbre



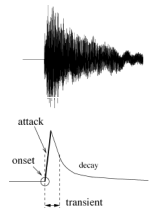
[Bello et al. 2005]

## Note Onset Detection

- Finding perceptually relevant impulses in a music signal
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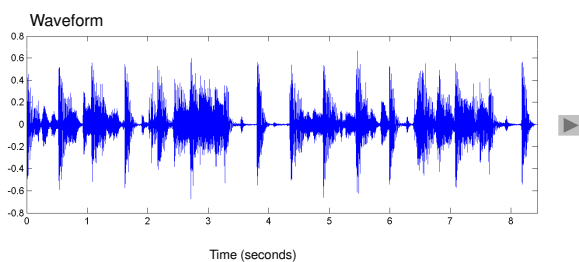
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[Bello et al. 2005]

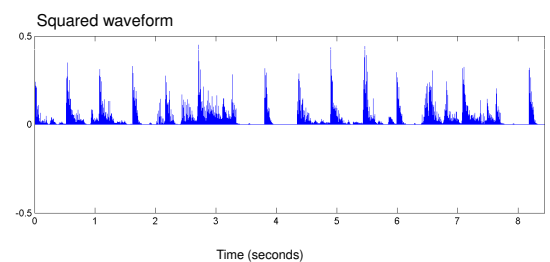
## Note Onset Detection

- Amplitude Squaring
- Windowing
- Differentiation
- Half wave rectification



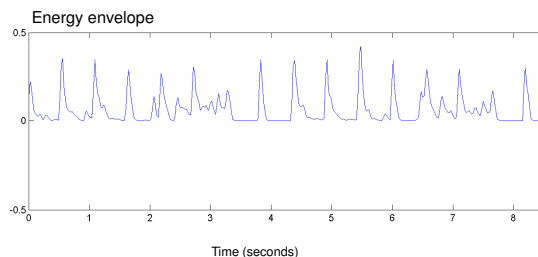
## Note Onset Detection

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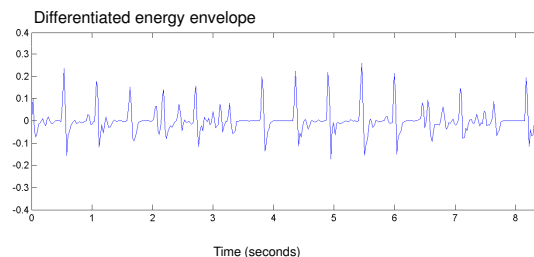
## Note Onset Detection

- Amplitude Squaring
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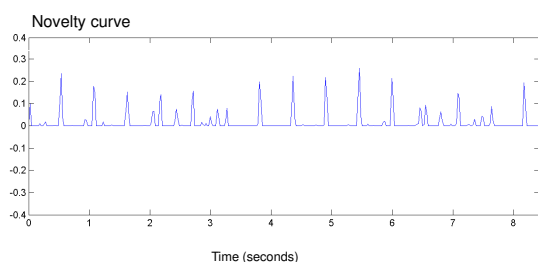
## Note Onset Detection

- Amplitude Squaring
  - Windowing
  - Differentiation
  - Half wave rectification
- capture energy changes

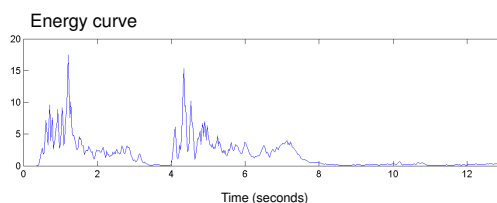


## Note Onset Detection

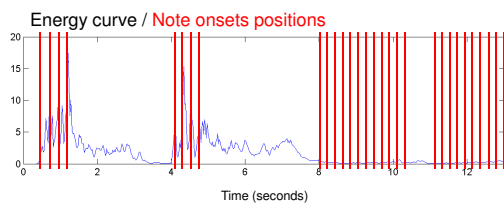
- Amplitude Squaring
  - Windowing
  - Differentiation
  - Half-wave rectification
- only energy increases are relevant for note onsets



## Note Onset Detection



## Note Onset Detection



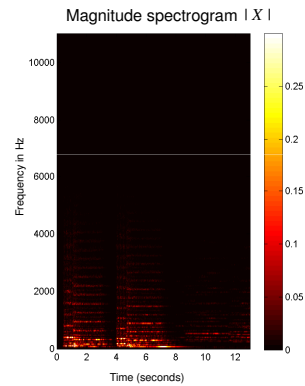
## Onset Detection

- Energy curves only work for percussive music
- Many instruments have weak note onsets (strings)
- No energy increase observable in complex mixtures
- More refined methods addressing different signal properties:
  - Change of spectral content
  - Change of pitch
  - Change of harmony

## Onset Detection

- Energy curves only work for percussive music
- Many instruments have weak note onsets (strings)
- No energy increase observable in complex mixtures
- More refined methods addressing different signal properties:
  - Change of spectral content
  - Change of pitch
  - Change of harmony

## Onset Detection



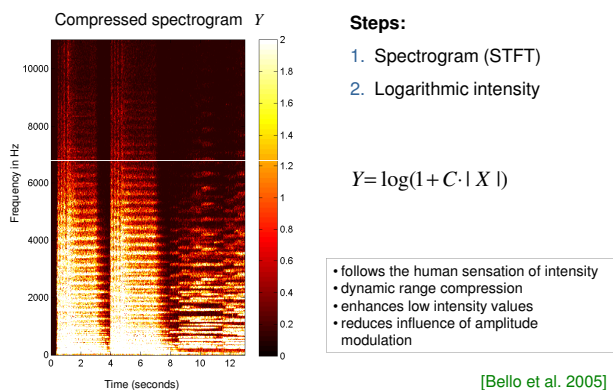
### Steps:

1. Spectrogram (STFT)

- allows for detecting local energy increases in certain frequency ranges
- pitch, harmony, or timbre changes are captured

[Bello et al. 2005]

## Onset Detection

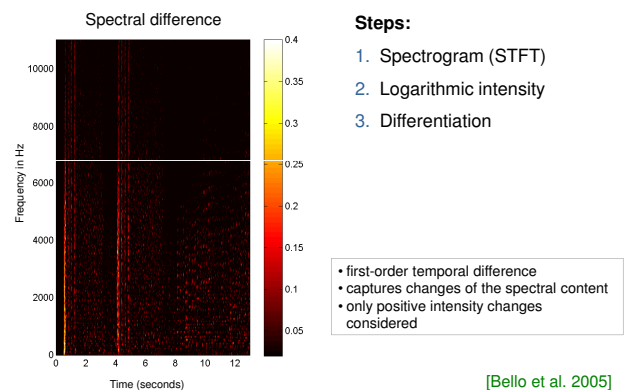


### Steps:

1. Spectrogram (STFT)
2. Logarithmic intensity

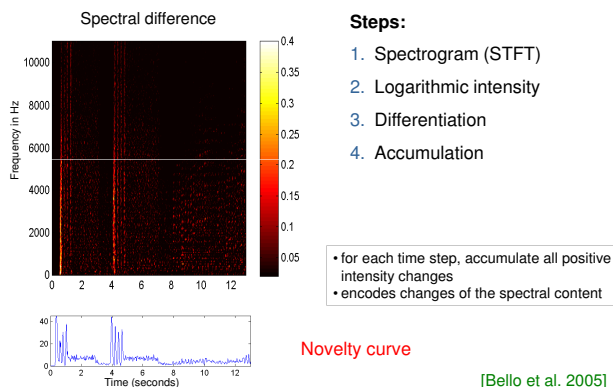
[Bello et al. 2005]

## Onset Detection



[Bello et al. 2005]

## Onset Detection



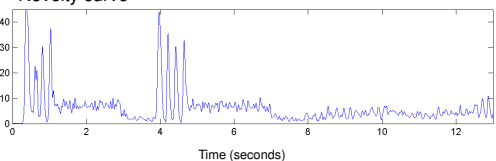
[Bello et al. 2005]

## Onset Detection

### Steps:

1. Spectrogram (STFT)
2. Logarithmic intensity
3. Differentiation
4. Accumulation

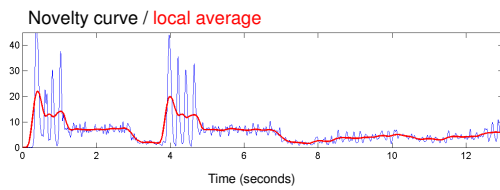
### Novelty curve



## Onset Detection

### Steps:

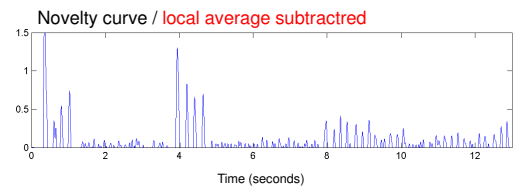
1. Spectrogram (STFT)
2. Logarithmic intensity
3. Differentiation
4. Accumulation



## Onset Detection

### Steps:

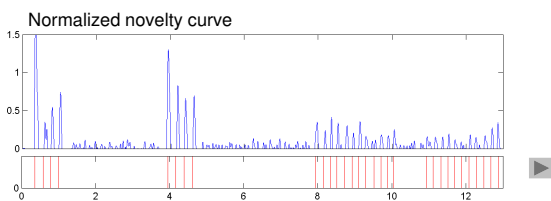
1. Spectrogram (STFT)
2. Logarithmic intensity
3. Differentiation
4. Accumulation
5. Mean Subtraction



## Onset Detection

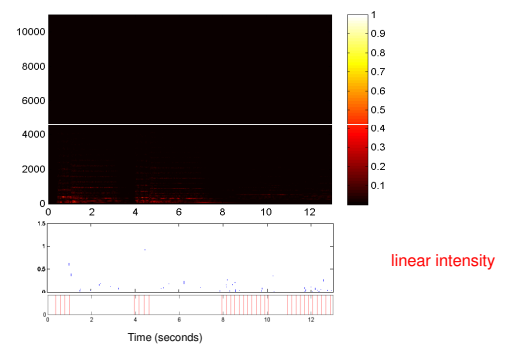
### Steps:

1. Spectrogram (STFT)
2. Logarithmic intensity
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4. Accumulation
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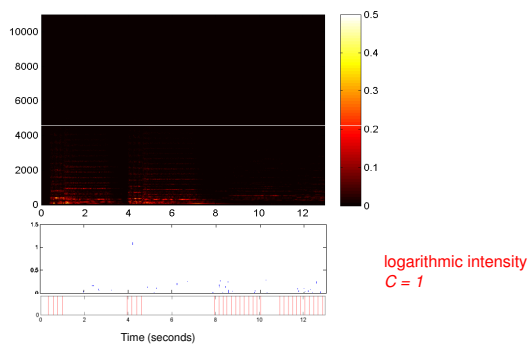
## Onset Detection

- Logarithmic compression is essential



## Onset Detection

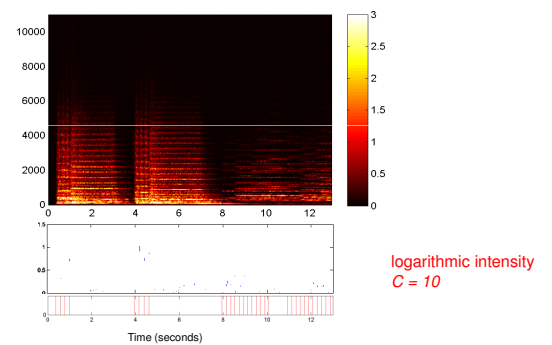
- Logarithmic compression is essential



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## Onset Detection

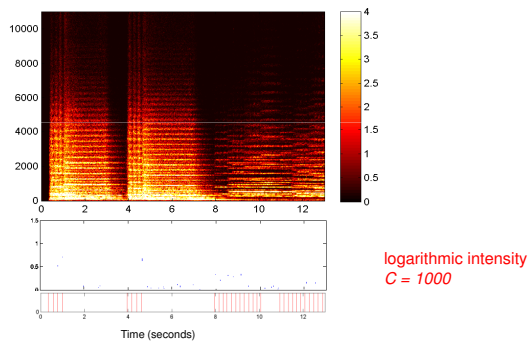
- Logarithmic compression is essential



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Onset Detection

- Logarithmic compression is essential



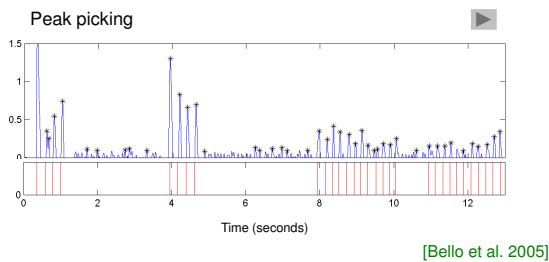
Onset Detection

- Spectrogram  $X = (X(t, k))_{t, k}$   $t \in [1 : T]$   
 $k \in [1 : K]$
- Compressed Spectrogram  $Y := \log(1 + C \cdot |X|)$   $C > 1$
- Novelty curve  $\Delta : [1 : T - 1] \rightarrow \mathbb{R}$

$$\Delta(t) := \sum_{k=1}^K |Y(t + 1, k) - Y(t, k)|_{\geq 0}$$

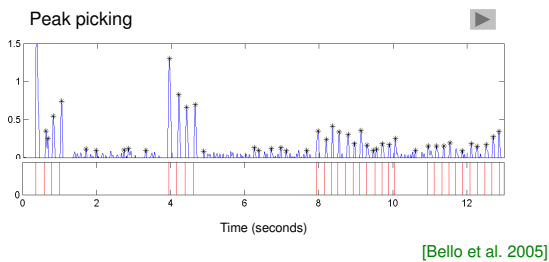
Onset Detection

- Peaks of the novelty curve are note onset candidates
- Extraction of note onsets by peak-picking methods (thresholding)

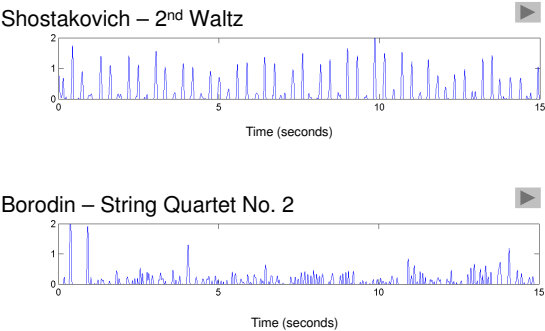


Onset Detection

- Peaks of the novelty curve are note onset candidates
- Extraction of note onsets by peak-picking methods (thresholding)
- Peak-picking is a very fragile step in particular for soft onsets (strings)
- How to distinguish between true onset peaks and spurious peaks?



Onset Detection



Onset Detection

- Drumbeat
- Going Home
- Lyphard melodie
- Por una cabeza
- Donau



## Onset Detection, Summary

- Compute a novelty curve that captures changes of certain signal properties
  - Energy
  - Spectrum
  - Pitch, harmony, timbre
- Energy based methods work for percussive music only
- Peaks of the novelty curve indicate note onset candidates
- Extraction of note onsets by peak-picking methods (thresholding)
- Peak-picking is a very fragile step in particular for soft onsets (strings)

[Bello et al. 2005]

## Overview

### Tasks

1. Note onset detection
2. Tempo estimation
3. Beat tracking

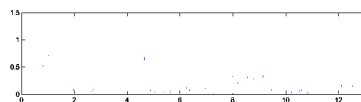
## Tempo Estimation

[Peeters 2007]

- The beat is a *periodic* sequence of impulses
- Reveal periodic structure of the note onsets
- Avoid the explicit determination of note onsets (no peak picking)
- Analyze the novelty curve with respect to periodicities

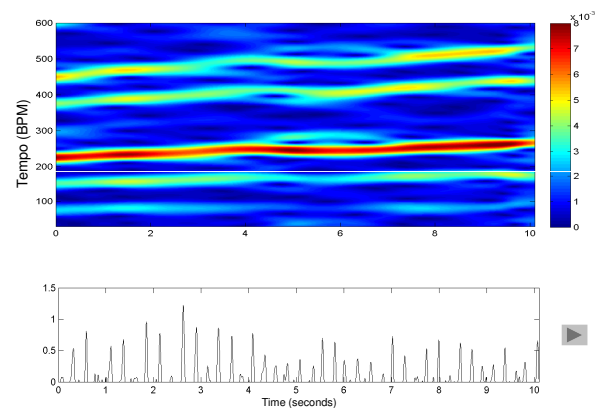
Methods for frequency / tempo estimation:

1. Fourier Transform
2. Autocorrelation



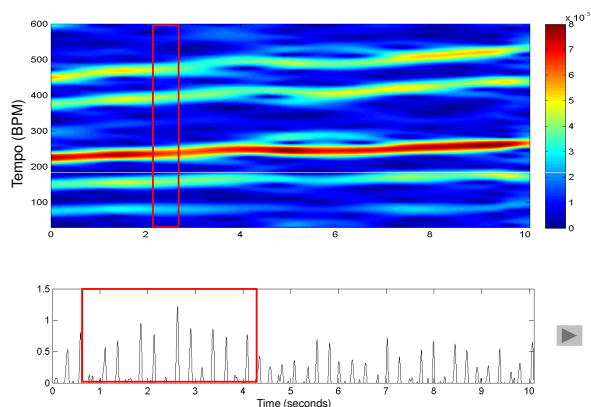
## Fourier-Tempogram

[GroscheMueller 2009]



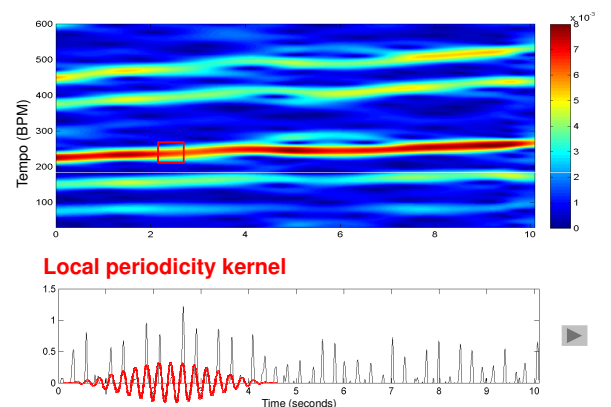
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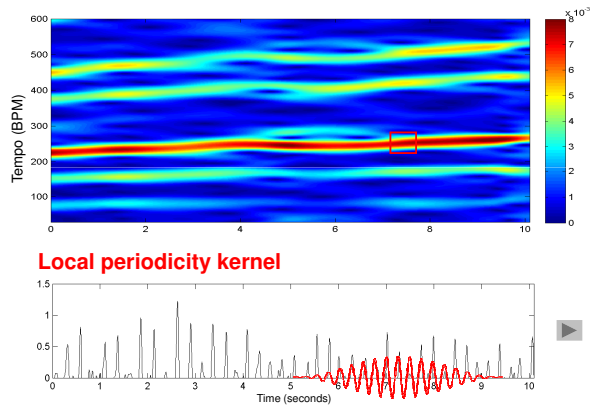
## Fourier-Tempogram

[GroscheMueller 2009]



## Fourier-Tempogram

[GroscheMueller 2009]



## Fourier-Tempogram

[GroscheMueller 2009]

- A time / tempo representation that encodes the local tempo of the piece
- A spectrogram (STFT) of the novelty curve
- Frequency axis is interpreted as tempo in BPM instead of frequency in Hz
- Reveals periodicities of the note onsets

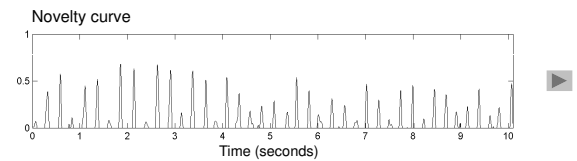
## Fourier-Tempogram

[GroscheMueller 2009]

- Fourier coefficient  $\mathcal{F}(t, \omega) = \sum_{n \in \mathbb{Z}} \Delta(n) \cdot W(n - t) \cdot e^{-2\pi i \omega n}$   
window function  $W : \mathbb{Z} \rightarrow \mathbb{R}$  centered at  $t = 0$   $\omega \in \mathbb{R}_{\geq 0}$
- Fourier tempogram  $\mathcal{T} : [1 : T] \times \Theta \rightarrow \mathbb{C}$   
 $\mathcal{T}^F(t, \tau) = \mathcal{F}(t, \tau/60)$   
for the tempo parameter  $\tau = 60 \cdot \omega$  in BPM  
and the set of tempo parameters  $\Theta \subset \mathbb{R}_{>0}$   $\Theta = [30:600]$

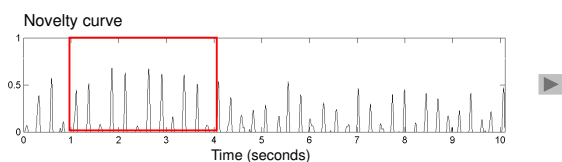
## Autocorrelation-Tempogram

[Peeters 2007]



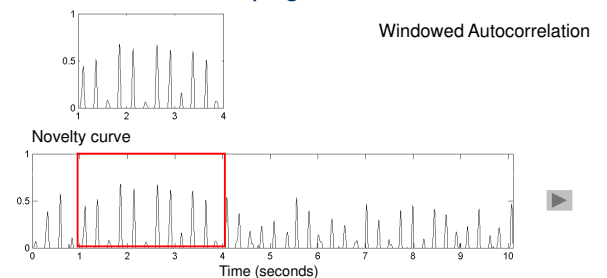
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[Peeters 2007]



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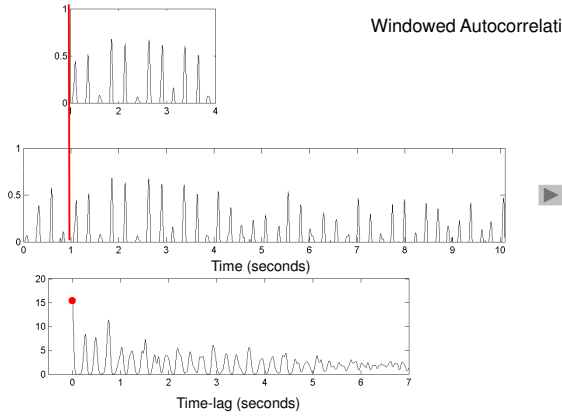


Compare the novelty curve with time-shifted copies of itself

## Autocorrelation-Tempogram

[Peeters 2007]

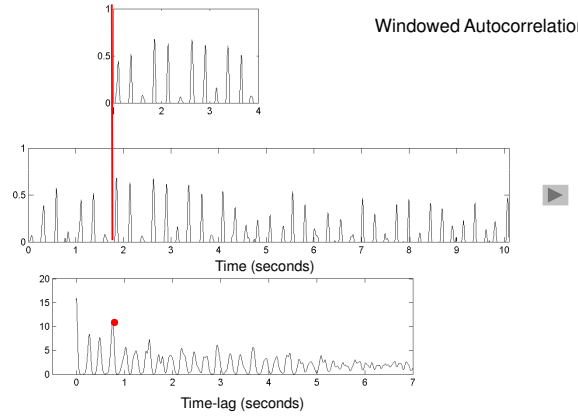
Windowed Autocorrelation



## Autocorrelation-Tempogram

[Peeters 2007]

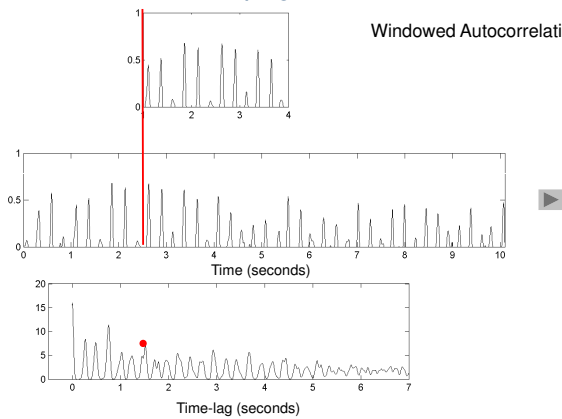
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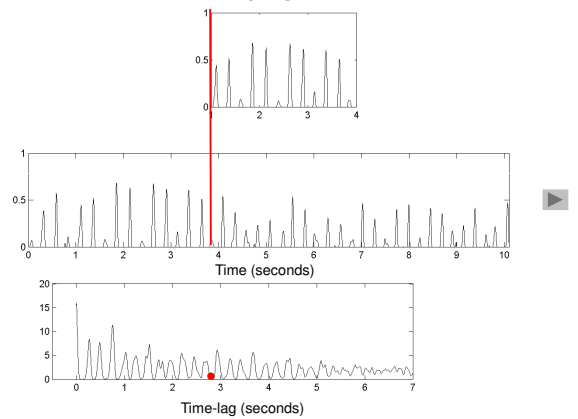
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Windowed Autocorrelation



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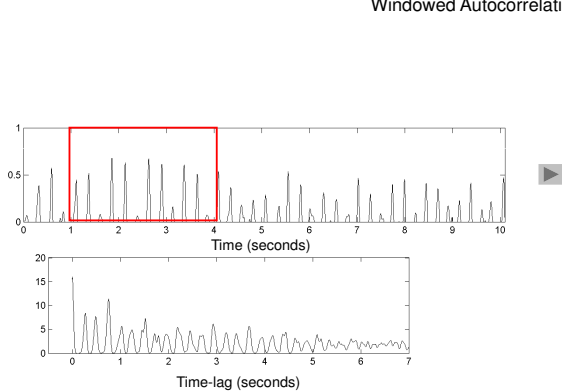
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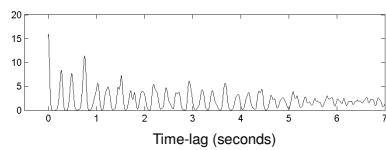


## Autocorrelation-Tempogram

[Peeters 2007]

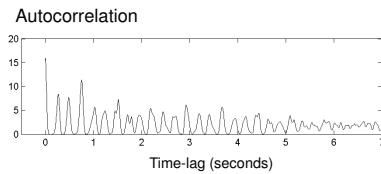
- High values for time lags with high correlation
- Reveals periodic self-similarities
- Maximum for a lag of zero (no shift)

Autocorrelation



## Autocorrelation-Tempogram [Peeters 2007]

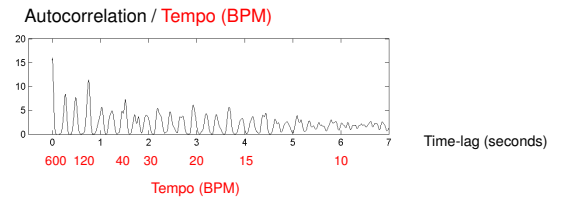
- High values for time lags with high correlation
- Reveals periodic self-similarities
- Maximum for a lag of zero (no shift)
- Time-lag is not intuitive for music signals



## Autocorrelation-Tempogram [Peeters 2007]

- Convert time-lag into tempo in BPM

$$\text{Tempo (in BPM)} = 60 / \text{Lag (in sec)}$$

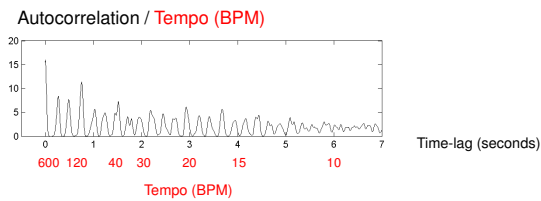


## Autocorrelation-Tempogram [Peeters 2007]

- Convert time-lag into tempo in BPM

$$\text{Tempo (in BPM)} = 60 / \text{Lag (in sec)}$$

- Still not a meaningful tempo axis

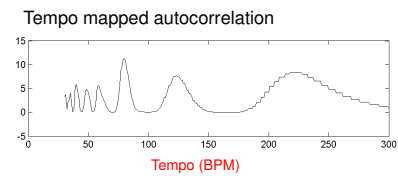


## Autocorrelation-Tempogram [Peeters 2007]

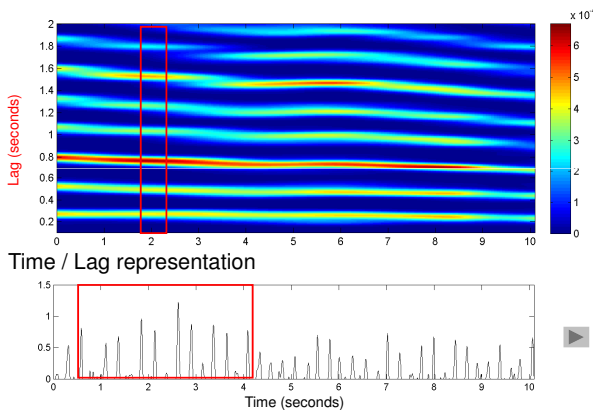
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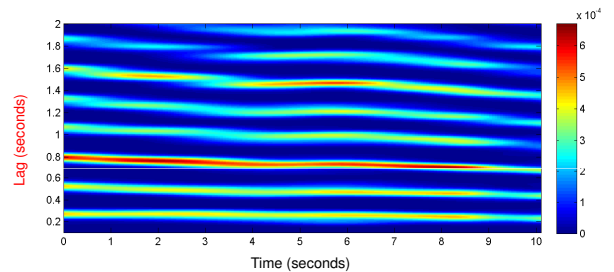
- Interpolate to a linear tempo axis in a musically meaningful tempo range



## Autocorrelation-Tempogram [Peeters 2007]

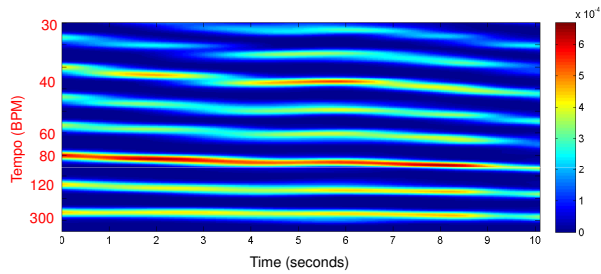


## Autocorrelation-Tempogram [Peeters 2007]



## Autocorrelation-Tempogram

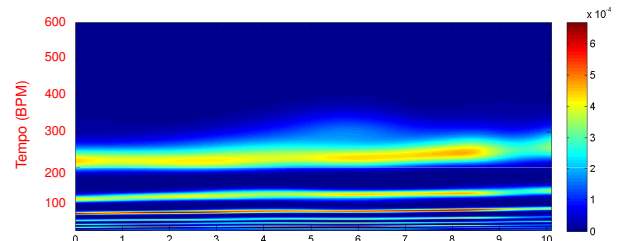
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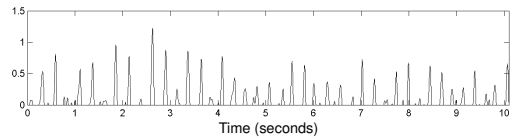
Time – Lag is not musically meaningful

## Autocorrelation-Tempogram

[Peeters 2007]



Rescaled to linear tempo axis: Tempogram



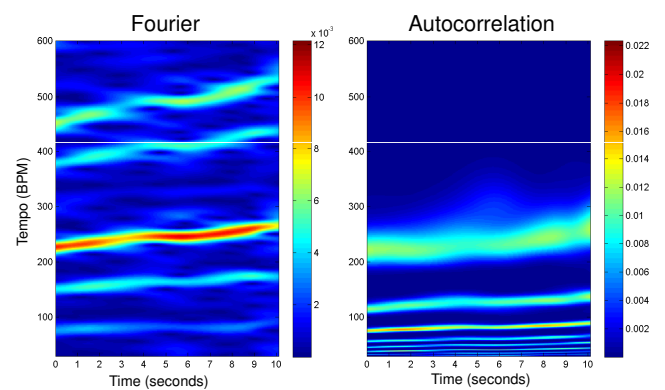
## Autocorrelation-Tempogram

[Peeters 2007]

- Autocorrelation  $\mathcal{A}(t, \ell) = \sum_{n \in \mathbb{Z}} \Delta(n) \Delta(n + \ell) \cdot W(n - t)$   
window function  $W : \mathbb{Z} \rightarrow \mathbb{R}$  centered at  $t = 0$   $\ell \in [0 : N]$
- Autocorrelation tempogram  
 $\mathcal{T}^A(t, \tau) = \mathcal{A}(t, 60/\tau)$

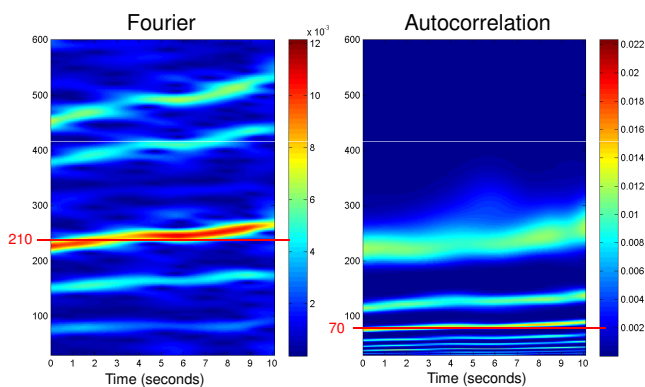
## Tempograms

[Peeters 2007]



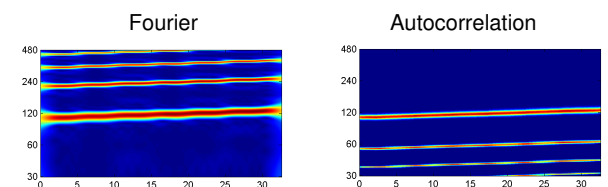
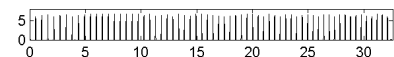
## Tempograms

[Peeters 2007]



## Tempograms

[Peeters 2007]



## Tempogram

[Peeters 2007]

Time-tempo representations that encode the local tempo of the piece over time

### Fourier

- Compare the novelty curve with templates consisting of sinusoidal kernels each representing a specific tempo
- Reveals periodic sequences of peaks
- Emphasizes harmonics, i.e. multiples of the tempo:  
Tatum - Level

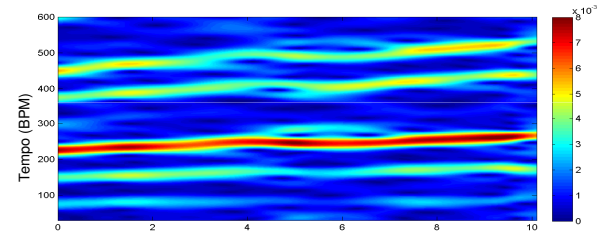
### Autocorrelation

- Compare the novelty curve with time-shifted copies of itself
- Reveals periodic self-similarities
- Emphasizes subharmonics, i.e. fractions of the tempo:  
Measure - Level

## Tempo Estimation

[Peeters 2007]

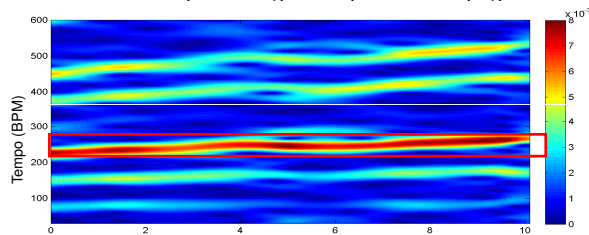
- Extract musically meaningful tempo from tempograms



## Tempo Estimation

[Peeters 2007]

- Extract musically meaningful tempo from tempograms

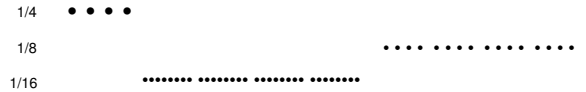


- Local maximum of tempogram is correct in many cases

## Tempo Estimation

[GroscheMueller 2009]

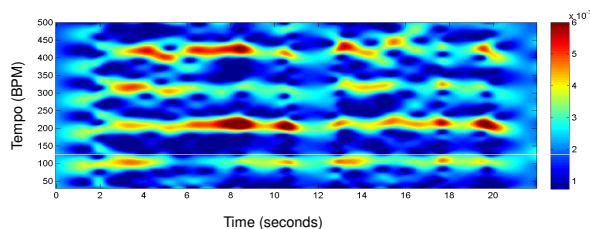
Piano Etude Op. 100 No. 2 by Burgmüller



What if the pulse level is changing?

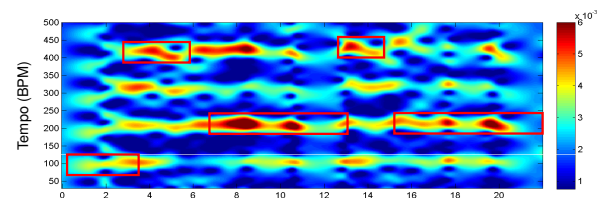
## Tempo Estimation

[GroscheMueller 2009]



## Tempo Estimation

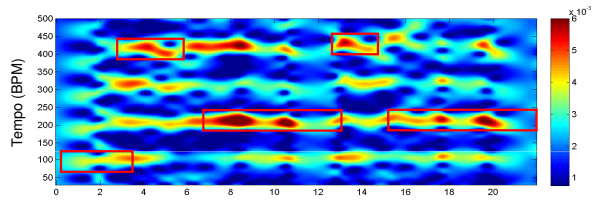
[GroscheMueller 2009]



Switching of predominant pulse level

## Tempo Estimation

[GroscheMueller 2009]



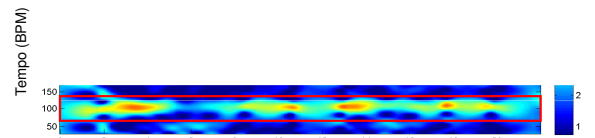
Switching of predominant pulse level

We can restrict the analysis to certain pulse levels



## Tempo Estimation

[GroscheMueller 2009]

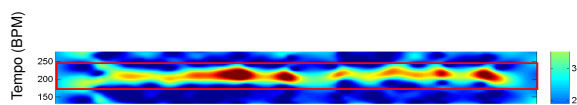


Prior knowledge: 1/4 note pulse level



## Tempo Estimation

[GroscheMueller 2009]

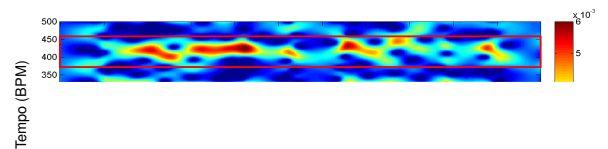


Prior knowledge: 1/8 note pulse level



## Tempo Estimation

[GroscheMueller 2009]

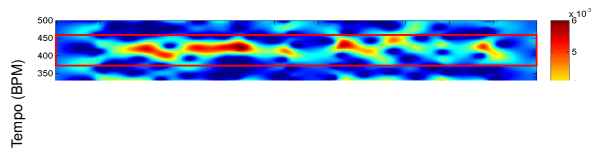


Prior knowledge: 1/16 note pulse level



## Tempo Estimation

[GroscheMueller 2009]



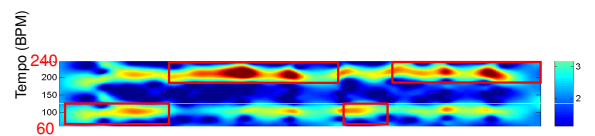
Prior knowledge: 1/16 note pulse level

Without prior knowledge?



## Tempo Estimation

[Peeters 2007]

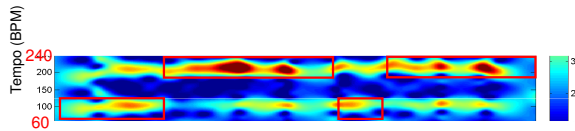


Restrict the tempo to a certain range:

For most pieces the tempo will be in the range of 60 to 240 BPM (close to the human heartbeat ~120 BPM)

## Tempo Estimation

[Peeters 2007]

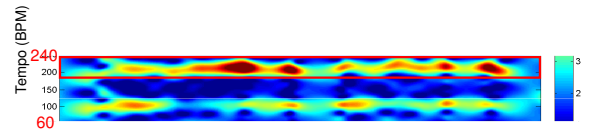


### Prevent pulse level changes:

Assuming smooth tempo changes: the tempo of a piece will not change abruptly  
Compute a tempo curve that constrains the local tempo estimates to a single pulse level

## Tempo Estimation

[Peeters 2007]

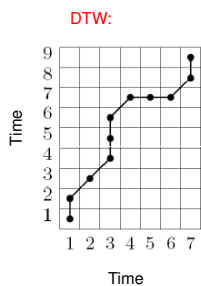


### Prevent pulse level changes:

Assuming smooth tempo changes: the tempo of a piece will not change abruptly  
Compute a **tempo curve** that constrains the local tempo estimates to a single pulse level and finds the best sequence of local tempi

## Tempo Estimation

[Peeters 2007]



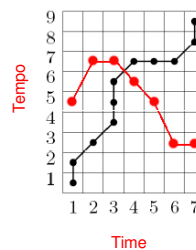
- Boundary conditions: find path from (1,1) to (M,N)
- Monotonicity: monotone in both axes
- Step size condition: from (n,m) only to (n+1,m), (n,m+1) or (n+1, m+1)

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## Tempo Estimation

[Peeters 2007]

Tempocurve determination:



- Boundary conditions: find path from (1,.) to (M,.)
- Monotonicity: monotone in **time axis**
- Step size condition: depending on **allowed tempo change**

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## Overview

### Tasks

- Note onset detection
- Tempo estimation
- Beat tracking**

## Beat Tracking

[GroscheMueller 2009]

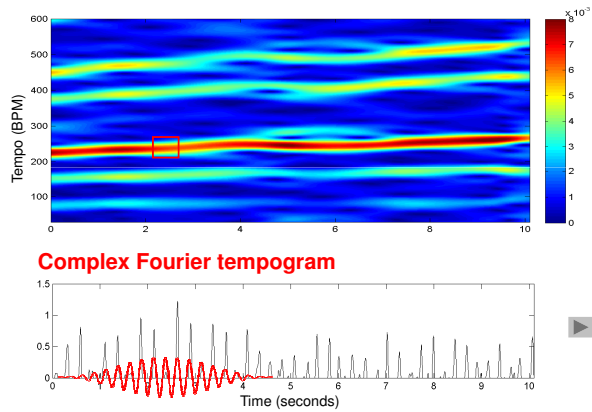
- Given the tempo, find the best sequence of beats
- Complex Fourier tempogram contains magnitude and **phase** information
- The magnitude encodes how well the novelty curve resonates with a periodicity kernel of a tempo
- The **phase** aligns the periodicity kernels with the peaks of the novelty curve

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## Beat Tracking

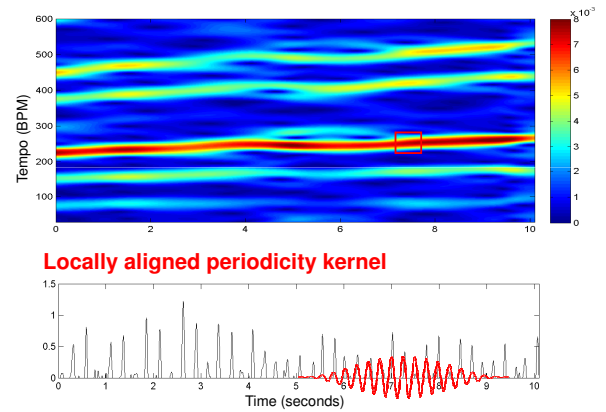
[GroscheMueller 2009]



Complex Fourier tempogram

## Beat Tracking

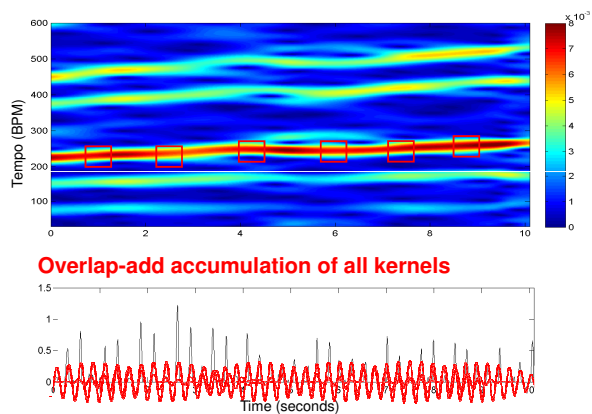
[GroscheMueller 2009]



Locally aligned periodicity kernel

## Beat Tracking

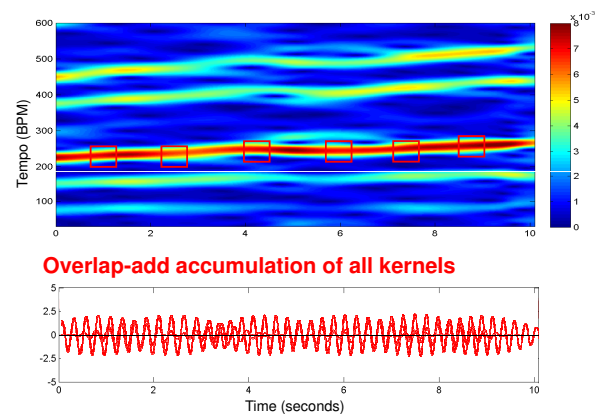
[GroscheMueller 2009]



Overlap-add accumulation of all kernels

## Beat Tracking

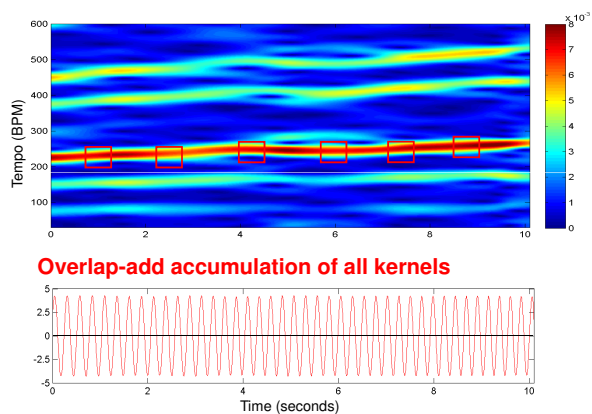
[GroscheMueller 2009]



Overlap-add accumulation of all kernels

## Beat Tracking

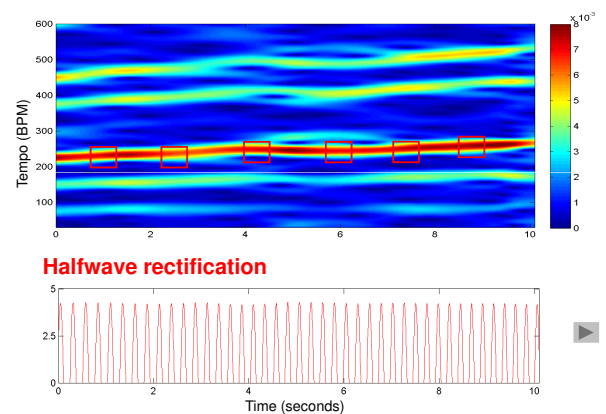
[GroscheMueller 2009]



Overlap-add accumulation of all kernels

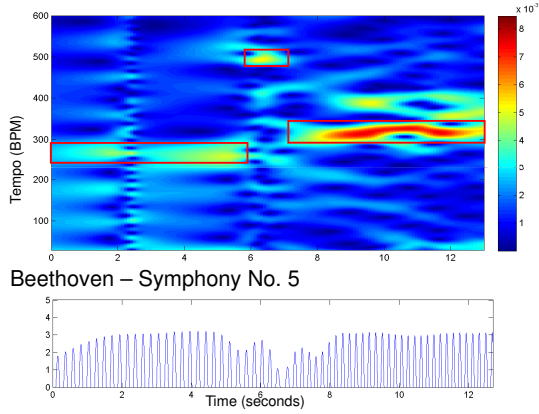
## Beat Tracking

[GroscheMueller 2009]

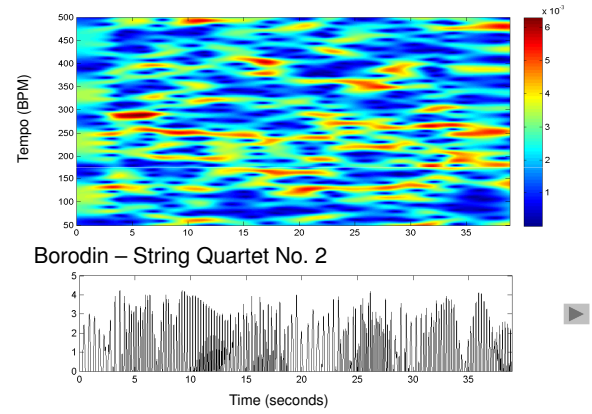


Halfwave rectification

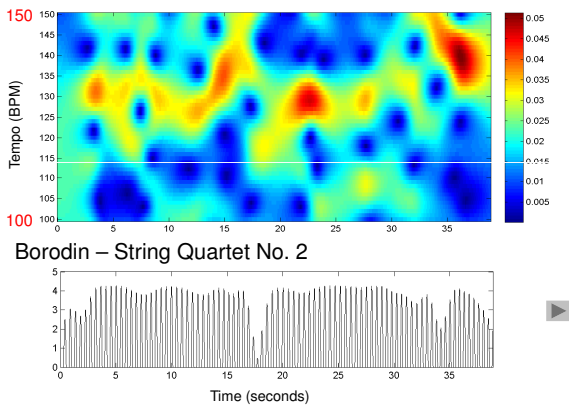
## Beat Tracking



## Beat Tracking

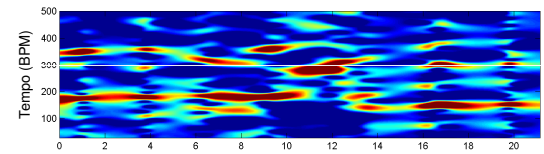


## Beat Tracking



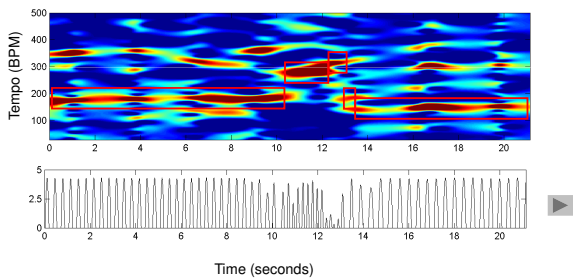
## Beat Tracking

Brahms Hungarian Dance No. 5



## Beat Tracking

Brahms Hungarian Dance No. 5



## Beat Tracking

[GroscheMueller 2009]

- Local tempo at time  $t$  :  $\tau_t \in \Theta$   $\Theta = [60:240]$  BPM
- Phase  $\varphi_t := -\frac{1}{2\pi} \arccos\left(\frac{\text{Re}(\mathcal{T}(t, \tau_t))}{|\mathcal{T}(t, \tau_t)|}\right)$
- Sinusoidal kernel  $\kappa_t : \mathbb{Z} \rightarrow \mathbb{R}$   

$$\kappa_t(n) := W(n - t) \cos(2\pi(\tau_t/60 \cdot n - \varphi_t)) \quad n \in \mathbb{Z}$$
- Periodicity curve  $\Gamma : [1 : T] \rightarrow \mathbb{R}_{\geq 0}$   

$$\Gamma(n) = \left| \sum_{t \in [1 : T]} \kappa_t(n) \right|_{\geq 0} \quad n \in [1 : T]$$

## Summary

1. Onset Detection
  - Novelty curve (*something is changing*)
  - Indicates note onset candidates
  - Hard task for non-percussive instruments (strings)
2. Tempo Estimation
  - Fourier tempogram
  - Autocorrelation tempogram
  - Musical knowledge (tempo range, continuity)
3. Beat tracking
  - Find most likely beat positions
  - Exploiting phase information from Fourier tempogram

## References

[GroscheMueller 2009]

Peter Grosche and Meinard Müller  
Computing predominant local periodicity information in music recordings.  
Proceedings of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA), New Paltz, New York, USA, 2009.

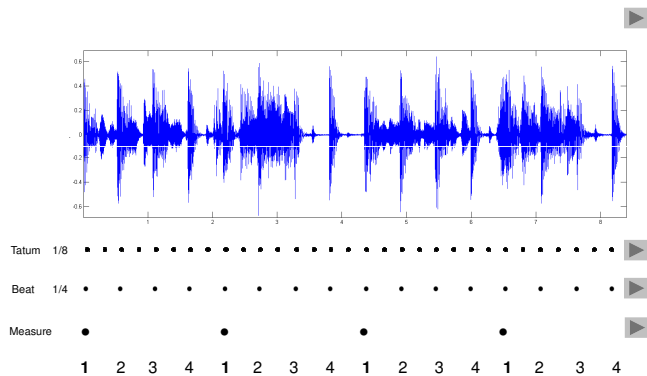
[Peeters 2007]

Geoffroy Peeters  
Template-based estimation of time-varying tempo  
Eurasip Journal on Applied Signal Processing, (Special Issue on Music Information Retrieval Based on Signal Processing) 2007.

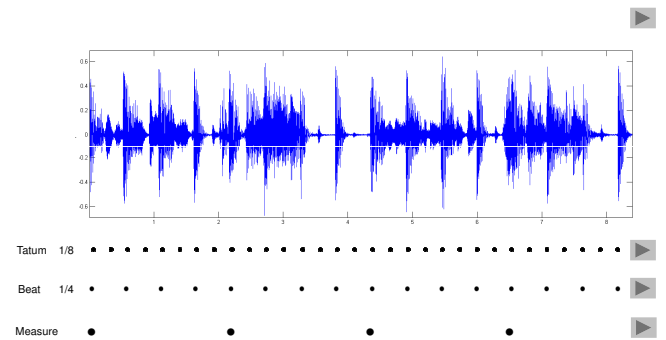
[Bello et al. 2005]

J. P. Bello, L. Daudet, S. Abdallah, C. Duxbury, M. Davies, M. B. and Sandler  
A tutorial on onset detection in music signals.  
IEEE Transactions on Speech and Audio Processing, 2005.

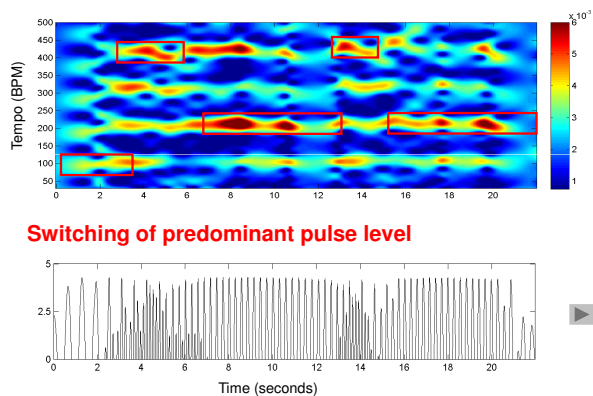
## Introduction



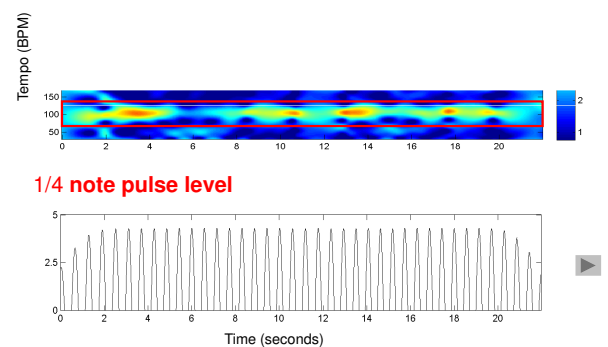
## Introduction



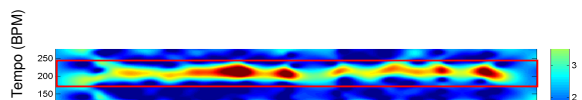
## Beat Tracking



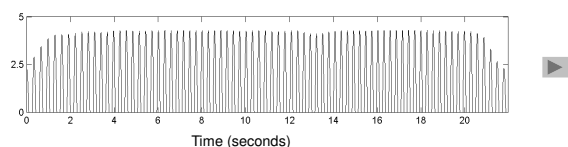
## Beat Tracking



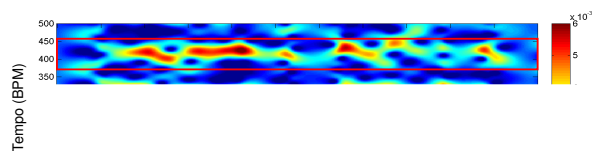
## Beat Tracking



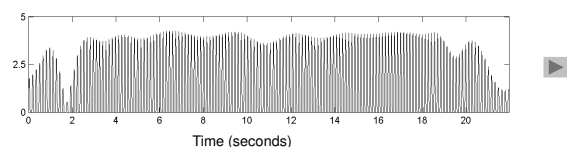
1/8 note pulse level



## Beat Tracking



1/16 note pulse level



## Beat Tracking

- Queen – Another One Bites The Dust ▶
- Shostakovich – 2<sup>nd</sup> Waltz ▶
- Beethoven – Symphony No. 5 ▶
- Borodin – String Quartet No. 2 ▶

## Examples: Strong or weak rhythm?

- Queen – Another One Bites The Dust ▶
- Shostakovich – 2<sup>nd</sup> Waltz ▶
- Beethoven – Pathetique ▶
- Beethoven – Symphony No. 5 ▶
- Borodin – String Quartet No. 2 ▶