ANALYSIS 2 - HAUSAUFGABE 6

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Aufgabe 1

Aufgabe 2

Sei $\vec{v}(x,y,z) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ und $\vec{w}(x,y,z) = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$.

$$\begin{aligned}
&\operatorname{div}\left(\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right) = \left(\operatorname{rot}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}\right) \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \operatorname{rot}\left(\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right) \\
\Leftrightarrow &\operatorname{div}\left(\begin{pmatrix} v_2w_3 - v_3w_2 \\ -v_1w_3 + v_3w_1 \\ v_1w_2 - v_2w_1 \end{pmatrix}\right) = \begin{pmatrix} \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \\ -\frac{\partial v_3}{\partial x} + \frac{\partial v_1}{\partial z} \\ \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial w_3}{\partial y} - \frac{\partial w_2}{\partial z} \\ -\frac{\partial w_3}{\partial x} + \frac{\partial w_1}{\partial z} \\ \frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial y} \end{pmatrix} \\
\Leftrightarrow &\frac{\partial(v_2w_3 - v_3w_2)}{\partial x} + \frac{\partial(-v_1w_3 + v_3w_1)}{\partial y} + \frac{\partial(v_1w_2 - v_2w_1)}{\partial z} \\
&= w_1\frac{\partial v_3}{\partial y} - w_1\frac{\partial v_2}{\partial z} - w_2\frac{\partial v_3}{\partial x} + w_2\frac{\partial v_1}{\partial z} + w_3\frac{\partial v_2}{\partial x} - w_3\frac{\partial v_1}{\partial y} \\
&- v_1\frac{\partial w_3}{\partial y} + v_1\frac{\partial w_2}{\partial z} + v_2\frac{\partial w_3}{\partial x} - v_2\frac{\partial w_1}{\partial z} - v_3\frac{\partial w_2}{\partial x} + v_3\frac{\partial w_1}{\partial y}
\end{aligned}$$

Aufgabe 3