Analysis 2 - Hausaufgabe 6

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Aufgabe 1

Aufgabe 2

Sei $\vec{v}(x,y,z) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ und $\vec{w}(x,y,z) = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$.

$$\begin{aligned}
&\operatorname{div}\left(\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right) = \left(\operatorname{rot}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}\right) \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \operatorname{rot}\left(\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right) \\
\Leftrightarrow\ &\operatorname{div}\left(\begin{pmatrix} v_2w_3 - v_3w_2 \\ -v_1w_3 + v_3w_1 \\ v_1w_2 - v_2w_1 \end{pmatrix}\right) = \begin{pmatrix} \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \\ -\frac{\partial v_3}{\partial x} + \frac{\partial v_1}{\partial z} \\ \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial w_3}{\partial y} - \frac{\partial w_2}{\partial z} \\ -\frac{\partial w_3}{\partial x} + \frac{\partial w_1}{\partial z} \\ \frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial y} \end{pmatrix} \\
\Leftrightarrow\ &\frac{\partial}{\partial x}(v_2w_3 - v_3w_2) + \frac{\partial}{\partial y}(-v_1w_3 + v_3w_1) + \frac{\partial}{\partial z}(v_1w_2 - v_2w_1) \\
&= \left(\frac{\partial}{\partial y}v_3\right)w_1 - \left(\frac{\partial}{\partial z}v_2\right)w_1 - \left(\frac{\partial}{\partial x}v_3\right)w_2 + \left(\frac{\partial}{\partial z}v_1\right)w_2 + \left(\frac{\partial}{\partial x}v_2\right)w_3 - \left(\frac{\partial}{\partial y}v_1\right)w_3 \\
&- v_1\left(\frac{\partial}{\partial y}w_3\right) + v_1\left(\frac{\partial}{\partial z}w_2\right) + v_2\left(\frac{\partial}{\partial x}w_3\right) - v_2\left(\frac{\partial}{\partial z}w_1\right) - v_3\left(\frac{\partial}{\partial x}w_2\right) + v_3\left(\frac{\partial}{\partial y}w_1\right) \\
\Leftrightarrow\ &\frac{\partial}{\partial x}v_2w_3 - \frac{\partial}{\partial x}v_3w_2 - \frac{\partial}{\partial y}v_1w_3 + \frac{\partial}{\partial y}v_3w_1 + \frac{\partial}{\partial z}v_1w_2 - \frac{\partial}{\partial z}v_2w_1 \\
&= \left(\frac{\partial}{\partial y}v_3\right)w_1 - \left(\frac{\partial}{\partial z}v_2\right)w_1 - \left(\frac{\partial}{\partial x}v_3\right)w_2 + \left(\frac{\partial}{\partial z}v_1\right)w_2 + \left(\frac{\partial}{\partial x}v_2\right)w_3 - \left(\frac{\partial}{\partial y}v_1\right)w_3 \\
&- v_1\left(\frac{\partial}{\partial y}w_3\right) + v_1\left(\frac{\partial}{\partial z}w_2\right) + v_2\left(\frac{\partial}{\partial x}w_3\right) - v_2\left(\frac{\partial}{\partial z}w_1\right) - v_3\left(\frac{\partial}{\partial x}w_2\right) + v_3\left(\frac{\partial}{\partial y}w_1\right) \\
\Leftrightarrow\ &\ldots
\end{aligned}$$

Komm grad nicht auf die Lösung

Aufgabe 3

(a)

$$\begin{aligned}
\text{div rot } \vec{v} &= \text{div rot } \begin{pmatrix} x^2 - y^2 \\ y^2 - z^2 \\ z^2 - x^2 \end{pmatrix} \\
&= \text{div } \begin{pmatrix} 2z \\ 2x \\ 2y \end{pmatrix} \\
&= 0 + 0 + 0 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{rot grad } f &= \text{rot grad } e^{xy} \\
&= \text{rot } \begin{pmatrix} ye^{xy} \\ xe^{xy} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} 0 - 0 \\ 0 - 0 \\ e^{xy} + xye^{xy} - e^{xy} - xye^{xy} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\end{aligned}$$

(b)

$$\begin{aligned}
\text{div grad } f &= \text{div grad } e^{xy} \\
&= \text{div } \begin{pmatrix} ye^{xy} \\ xe^{xy} \\ 0 \end{pmatrix} \\
&= y^2 e^{xy} + x^2 e^{xy}
\end{aligned}$$

$$\begin{aligned}
\Delta f &= \text{div grad } f \\
&\overset{\text{siehe oben}}{=} y^2 e^{xy} + x^2 e^{xy}
\end{aligned}$$

(c)

$$
\begin{aligned}
fg &= xe^{xy} + y^2 e^{xy} + z^3 e^{xy} \\
f\vec{v} &= \begin{pmatrix} x^2 e^{xy} - y^2 e^{xy} \\ y^2 e^{xy} - z^2 e^{xy} \\ z^2 e^{xy} - x^2 e^{xy} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{grad} fg &= \operatorname{grad} xe^{xy} + y^2 e^{xy} + z^3 e^{xy} \\
&= \begin{pmatrix} e^{xy} + xye^{xy} + y^3 e^{xy} + yz^3 e^{xy} \\ x^2 e^{xy} + 2ye^{xy} + y^2 xe^{xy} + xz^3 e^{xy} \\ 3z^2 e^{xy} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{div} f\vec{v} &= \operatorname{div} \begin{pmatrix} x^2 e^{xy} - y^2 e^{xy} \\ y^2 e^{xy} - z^2 e^{xy} \\ z^2 e^{xy} - x^2 e^{xy} \end{pmatrix} \\
&= 2xe^{xy} + yx^2 e^{xy} - y^3 e^{xy} + 2ye^{xy} + y^2 xe^{xy} - z^2 xe^{xy} + 2ze^{xy}
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{rot} f\vec{v} &= \operatorname{rot} \begin{pmatrix} x^2 e^{xy} - y^2 e^{xy} \\ y^2 e^{xy} - z^2 e^{xy} \\ z^2 e^{xy} - x^2 e^{xy} \end{pmatrix} \\
&= \begin{pmatrix} z^2 xe^{xy} - x^3 e^{xy} + 2ze^{xy} \\ -z^2 ye^{xy} + 2xe^{xy} + x^2 ye^{xy} \\ y^3 e^{xy} - z^2 ye^{xy} - (x^3 e^{xy} - 2ye^{xy} - y^2 xe^{xy}) \end{pmatrix}
\end{aligned}
$$

Bitte nochmal nachrechnen.