ANALYSIS 2 - HAUSAUFGABE 6

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Aufgabe 1

Aufgabe 2

Sei $\vec{v}(x,y,z) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ und $\vec{w}(x,y,z) = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$.

$$\text{div}\left(\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right) = \left(\text{rot}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}\right) \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \text{rot}\left(\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}\right)$$

$$\Leftrightarrow \text{div}\left(\begin{pmatrix} v_2w_3 - v_3w_2 \\ -v_1w_3 + v_3w_1 \\ v_1w_2 - v_2w_1 \end{pmatrix}\right) = \begin{pmatrix} \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \\ -\frac{\partial v_3}{\partial x} + \frac{\partial v_1}{\partial z} \\ \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial w_3}{\partial y} - \frac{\partial w_2}{\partial z} \\ -\frac{\partial w_3}{\partial x} + \frac{\partial w_1}{\partial z} \\ \frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial y} \end{pmatrix}$$

Ich glaube ab da ist es schon falsch.

$$\begin{aligned}
&\Leftrightarrow \frac{\partial}{\partial x}(v_2w_3 - v_3w_2) + \frac{\partial}{\partial y}(-v_1w_3 + v_3w_1) + \frac{\partial}{\partial z}(v_1w_2 - v_2w_1) \\
&= \frac{\partial}{\partial y}v_3w_1 - \frac{\partial}{\partial z}v_2w_1 - \frac{\partial}{\partial x}v_3w_2 + \frac{\partial}{\partial z}v_1w_2 + \frac{\partial}{\partial x}v_2w_3 - \frac{\partial}{\partial y}v_1w_3 \\
&- v_1\frac{\partial}{\partial y}w_3 + v_1\frac{\partial}{\partial z}w_2 + v_2\frac{\partial}{\partial x}w_3 - v_2\frac{\partial}{\partial z}w_1 - v_3\frac{\partial}{\partial x}w_2 + v_3\frac{\partial}{\partial y}w_1 \\
&\Leftrightarrow \frac{\partial}{\partial x}v_2w_3 - \frac{\partial}{\partial x}v_3w_2 - \frac{\partial}{\partial y}v_1w_3 + \frac{\partial}{\partial y}v_3w_1 + \frac{\partial}{\partial z}v_1w_2 - \frac{\partial}{\partial z}v_2w_1 \\
&= \frac{\partial}{\partial y}v_3w_1 - \frac{\partial}{\partial z}v_2w_1 - \frac{\partial}{\partial x}v_3w_2 + \frac{\partial}{\partial z}v_1w_2 + \frac{\partial}{\partial x}v_2w_3 - \frac{\partial}{\partial y}v_1w_3 \\
&- v_1\frac{\partial}{\partial y}w_3 + v_1\frac{\partial}{\partial z}w_2 + v_2\frac{\partial}{\partial x}w_3 - v_2\frac{\partial}{\partial z}w_1 - v_3\frac{\partial}{\partial x}w_2 + v_3\frac{\partial}{\partial y}w_1 \\
&\Leftrightarrow 0 \\
&= -v_1\frac{\partial}{\partial y}w_3 + v_1\frac{\partial}{\partial z}w_2 + v_2\frac{\partial}{\partial x}w_3 - v_2\frac{\partial}{\partial z}w_1 - v_3\frac{\partial}{\partial x}w_2 + v_3\frac{\partial}{\partial y}w_1
\end{aligned}$$

Irgendwie geht das nicht.

Aufgabe 3