# Theoretische Grundlagen der Informatik 3: Hausaufgabenabgabe 9

Tutorium: Sebastian , Mi 14.00 - 16.00 Uhr


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## Aufgabe 1

(i)

$$
\begin{aligned}
\varphi_1 &:= \neg(\exists x \exists y E(x,y) \wedge \neg\exists x \forall y \exists z(\neg E(x,z) \vee f(x,y) = z)) \rightarrow \exists x E(x, f(y,x)) \\
&\equiv (\exists x \exists y E(x,y) \wedge \neg\exists x \forall y \exists z(\neg E(x,z) \vee f(x,y) = z)) \vee \exists x E(x, f(y,x)) \\
&\equiv (\exists x \exists y E(x,y) \wedge \forall x \exists y \forall z \neg(\neg E(x,z) \vee f(x,y) = z))) \vee \exists x E(x, f(y,x)) \\
&\equiv (\exists x \exists y E(x,y) \wedge \forall x \exists y \forall z(E(x,z) \wedge \neg(f(x,y) = z)))) \vee \exists x E(x, f(y,x)) \\
&\equiv (\exists x_1 \exists y_2 E(x_1,y_2) \wedge \forall x_2 \exists y_2 \forall z_1(E(x_2,z_1) \wedge \neg(f(x_2,y_2) = z_1)))) \vee \exists x_3 E(x_3, f(y,x_3)) \\
&\equiv \exists x_1 \exists y_2 \forall x_2 \exists y_2 \forall z_1 \exists x_3((E(x_1,y_2) \wedge (E(x_2,z_1) \wedge \neg(f(x_2,y_2) = z_1)))) \vee E(x_3, f(y,x_3)))
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\varphi_2 &:= \exists y \forall z(E(x,z) \wedge (E(y,z) \rightarrow \forall x(E(f(x,y),z) \wedge \neg\forall y R(x,y)))) \\
&\equiv \exists y \forall z(E(x,z) \wedge (\neg E(y,z) \vee \forall x(E(f(x,y),z) \wedge \neg\forall y R(x,y)))) \\
&\equiv \exists y \forall z(E(x,z) \wedge (\neg E(y,z) \vee \forall x(E(f(x,y),z) \wedge \exists y \neg R(x,y)))) \\
&\equiv \exists y_1 \forall z_1(E(x_1,z_1) \wedge (\neg E(y_1,z_1) \vee \forall x_2(E(f(x_2,y_1),z_1) \wedge \exists y_1 \neg R(x_2,y_1))) \\
&\equiv \exists y_1 \forall z_1 \forall x_2(E(x_1,z_1) \wedge (\neg E(y_1,z_1) \vee (E(f(x_2,y_1),z_1) \wedge \neg R(x_2,y_1))))
\end{aligned}
$$

## Aufgabe 2

$$
\begin{aligned}
\phi_1(\mathcal{N}) &:= \exists\ x\ (y = x + x) \\
\phi_2(\mathcal{N}) &:= \exists\ x\ (y = x \cdot x) \\
\phi_3(\mathcal{R}) &:= x = y \cdot y \\
\phi_4(\mathcal{R}) &:= \exists\ m\ \forall\ n\ (m \cdot n = m \wedge m = x + y) \\
\phi_5(\mathcal{R}) &:= \exists\ m\ \exists\ n\ (n \cdot n = m \wedge y = x + m) \\
\phi_6(\mathcal{R}) &:= (u'' = u \cdot u' - v \cdot v') \wedge (v'' = u' \cdot v + u \cdot v')
\end{aligned}
$$