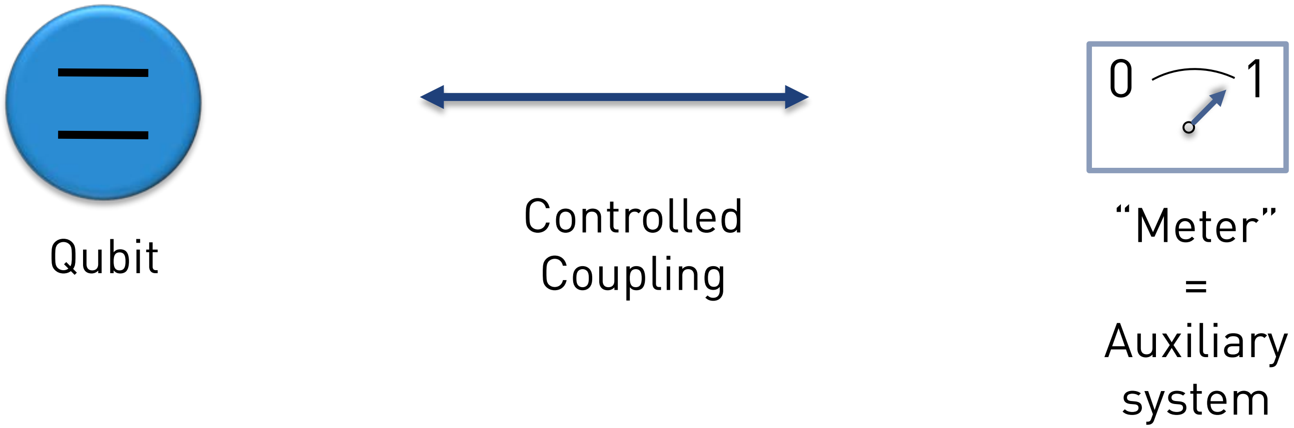
Today's lecture

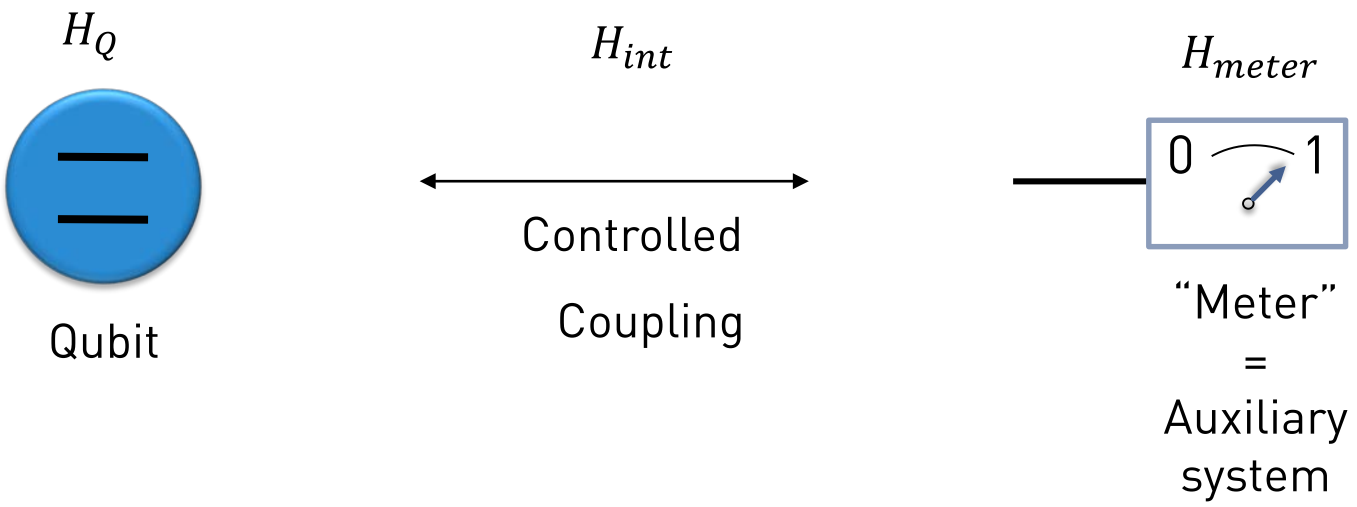
Simulation of dissipative quantum dynamics: Master equation

Measurement of Superconducting Qubits

- General properties of quantum measurements
- Circuit Quantum Electrodynamics and the Jaynes-Cummings Hamiltonian.
- Dispersive limit and readout of superconducting qubits.



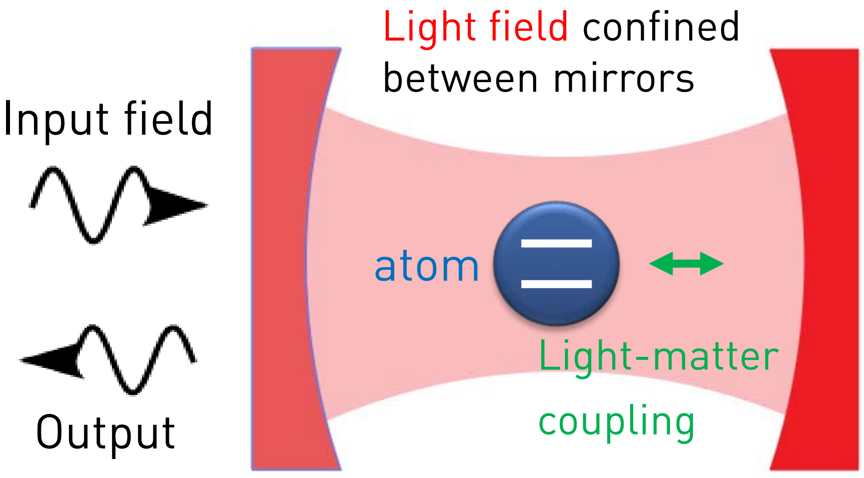
4.5 General properties of quantum measurements



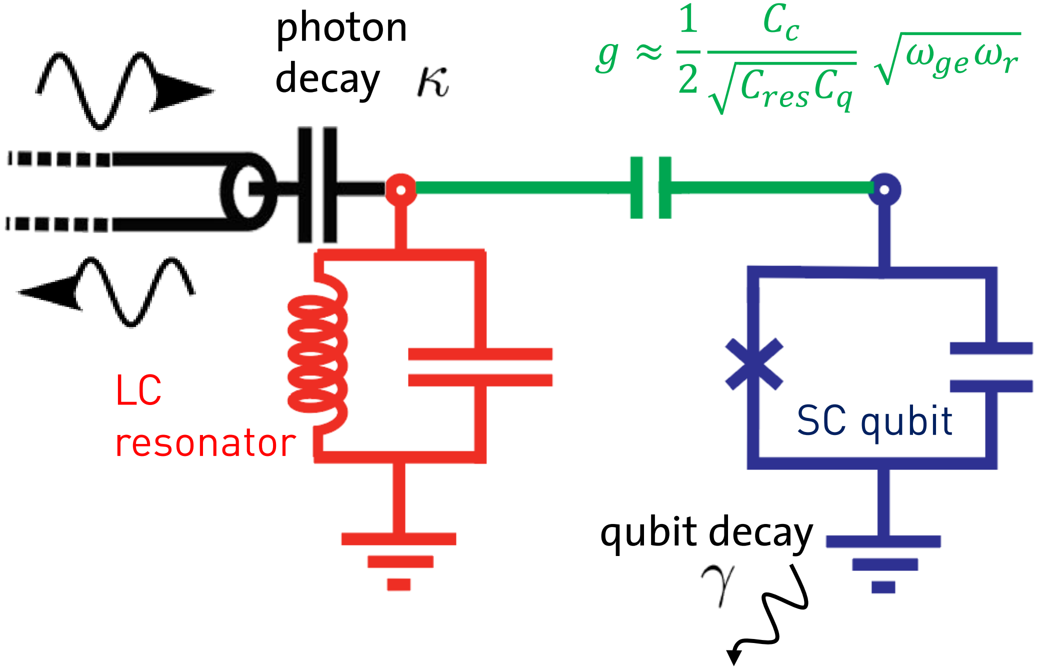
Desirable properties:

- Projective and Quantum non-demolition (QND)
  - Coupling to the meter does not change the state of the qubit $[H_Q, H_{int}] = 0.$
  - Repeated measurement yields the same outcome.
- Good ON/OFF ratio
  - $[H_{int}, H_{meter}] = 0$ during "OFF"
  - $[H_{int}, H_{meter}] \neq 0$ during "ON"
- No spontaneous decay/excitation due to measurement apparatus
- Fast and high fidelity

4.6 Circuit QED



Circuit equivalent



$$g \approx \frac{1}{2}\frac{C_c}{\sqrt{C_{res}C_q}}\sqrt{\omega_{ge}\omega_r}$$

System Hamiltonian (compare chapter 2):

$$H_{\text{sys}}/\hbar = \omega_r a^\dagger a \quad + \quad \omega_{ge} b^\dagger b - \frac{\alpha}{2}(b^\dagger)^2 b^2 \quad - \quad g(a - a^\dagger)(b - b^\dagger)$$

- Rotating wave approximation (RWA)
- Two-level approximation

$$= \underbrace{\boxed{\omega_r a^\dagger a}}_{\text{Resonator field}} + \underbrace{\boxed{\frac{\omega_{ge}}{2}\sigma^z}}_{\text{qubit}} \underbrace{\boxed{+ g(a^\dagger \sigma^- + a\sigma^+)}}_{\text{coupling}}$$

Jaynes-Cummings Hamiltonian

S. Haroche & J. Raimond, Exploring the Quantum, OUP Oxford (2006)

4.6 Circuit QED: Resonant case and dispersive limit

Jaynes-Cummings Hamiltonian:

$$H/\hbar = \boxed{\omega_r a^\dagger a} + \boxed{\frac{\omega_{ge}}{2}\sigma^z} + \boxed{g(a^\dagger\sigma^- + a\sigma^+)}$$

quantized field — qubit — coupling

Strong coupling regime: $g > \gamma,\ \kappa$

What happens in the limit of large detuning?

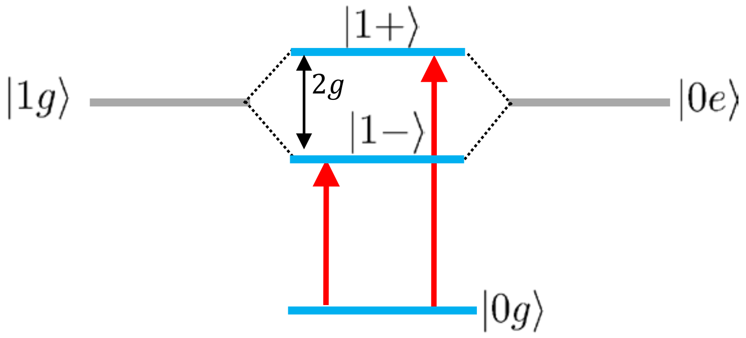
$|\Delta| = |\omega_{ge} - \omega_r| \gg g$

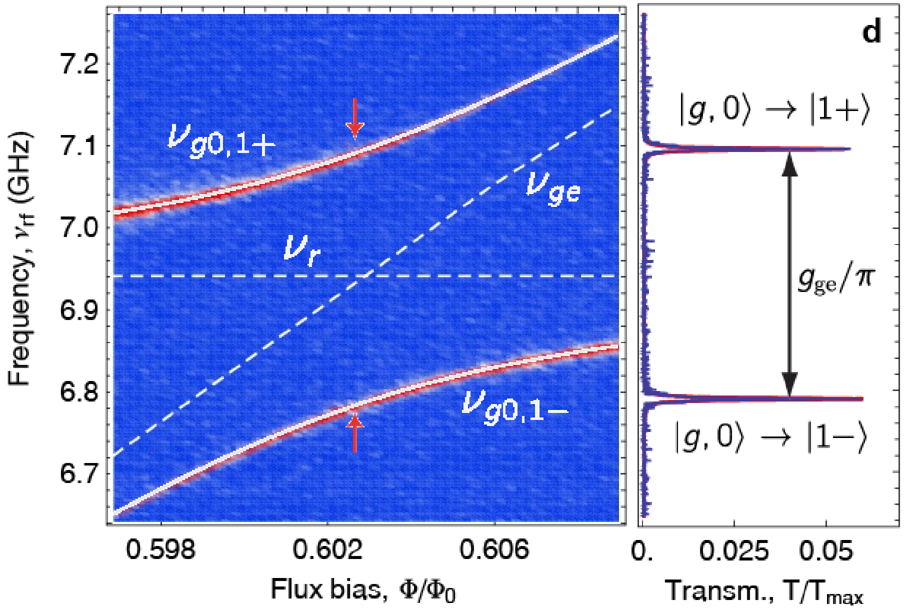
$$\boxed{\chi\sigma_z a^\dagger a}$$

Dispersive coupling

- Limit of large detuning is referred to as the dispersive limit. No resonant exchange of excitations.
- In the dispersive regime coupling Hamiltonian commutes with qubit Hamiltonian.

Energy level diagram for resonant case $\omega_r = \omega_{ge}$:



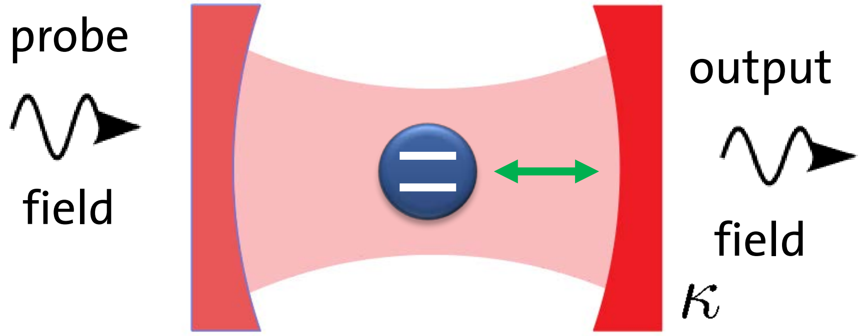


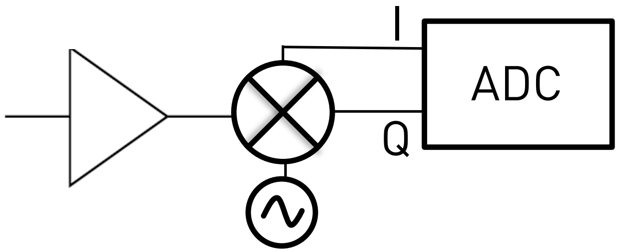
A. Wallraff *et al.*, *Nature* 431, 162 (2004), J. Fink et al., Nature (London) 454, 315 (2008)

4.7 Derivation of Dispersive Hamiltonian

Discussion on the black board.

4.7 Principle of Dispersive Qubit Measurement





$$A\, e^{i\phi} = I + i\, Q$$

signal amplitude

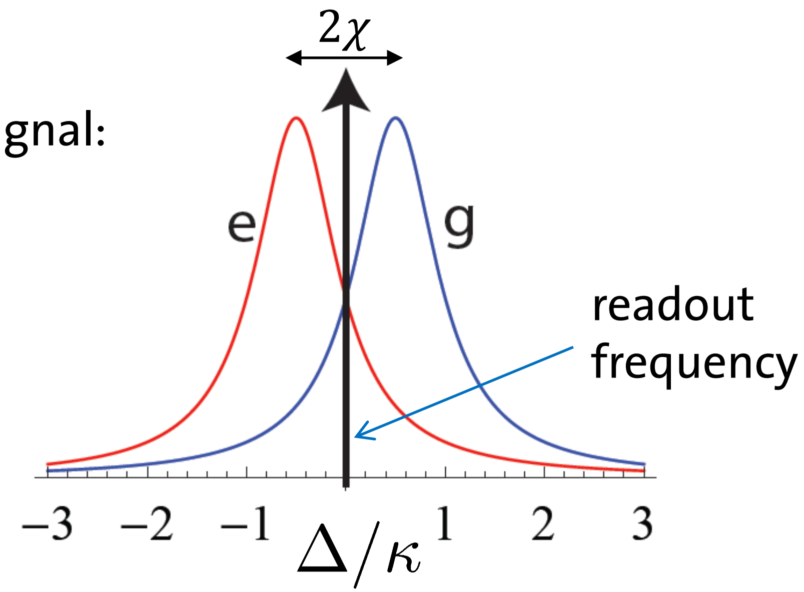
Phase

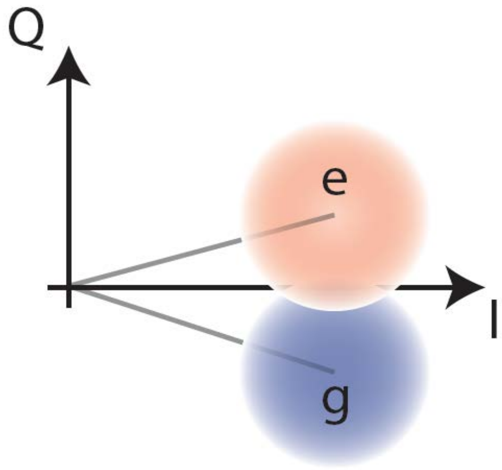
In-phase and quadrature components

In the limit of large detuning $\omega_{\mathrm{r}} - \omega_{ge} \gg g$:

$$H/\hbar \approx (\omega_{\mathrm{r}} + \chi\sigma_z)a^\dagger a$$,with $\chi \approx -\alpha \frac{g^2}{\Delta(\Delta-\alpha)}$

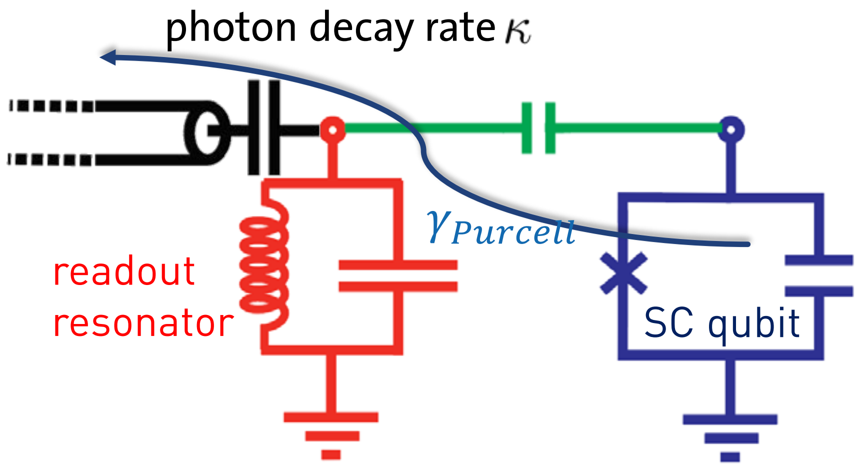
Amplitude of transmitted signal:





A. Wallraff *et al., Phys. Rev. Lett.* 95, 060501 (2005).
R. Vijay *et al., Phys. Rev. Lett.* 106, 110502 (2011).

Purcell decay and protection

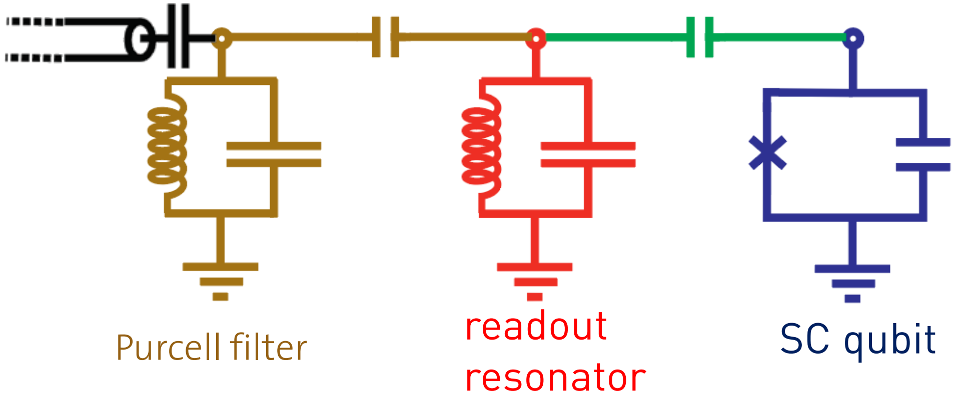


What about decay of the qubit into the measurement line via the resonator?

- In the limit of large detuning we find

$$\gamma_{Purcell} \approx \kappa \frac{g^2}{\Delta^2} \approx \kappa \frac{|\chi|}{\alpha}$$

- Calculate e.g. using the methods discussed in chapter 4.4.
- BUT: Fast readout requires large κ and $|\chi|$.



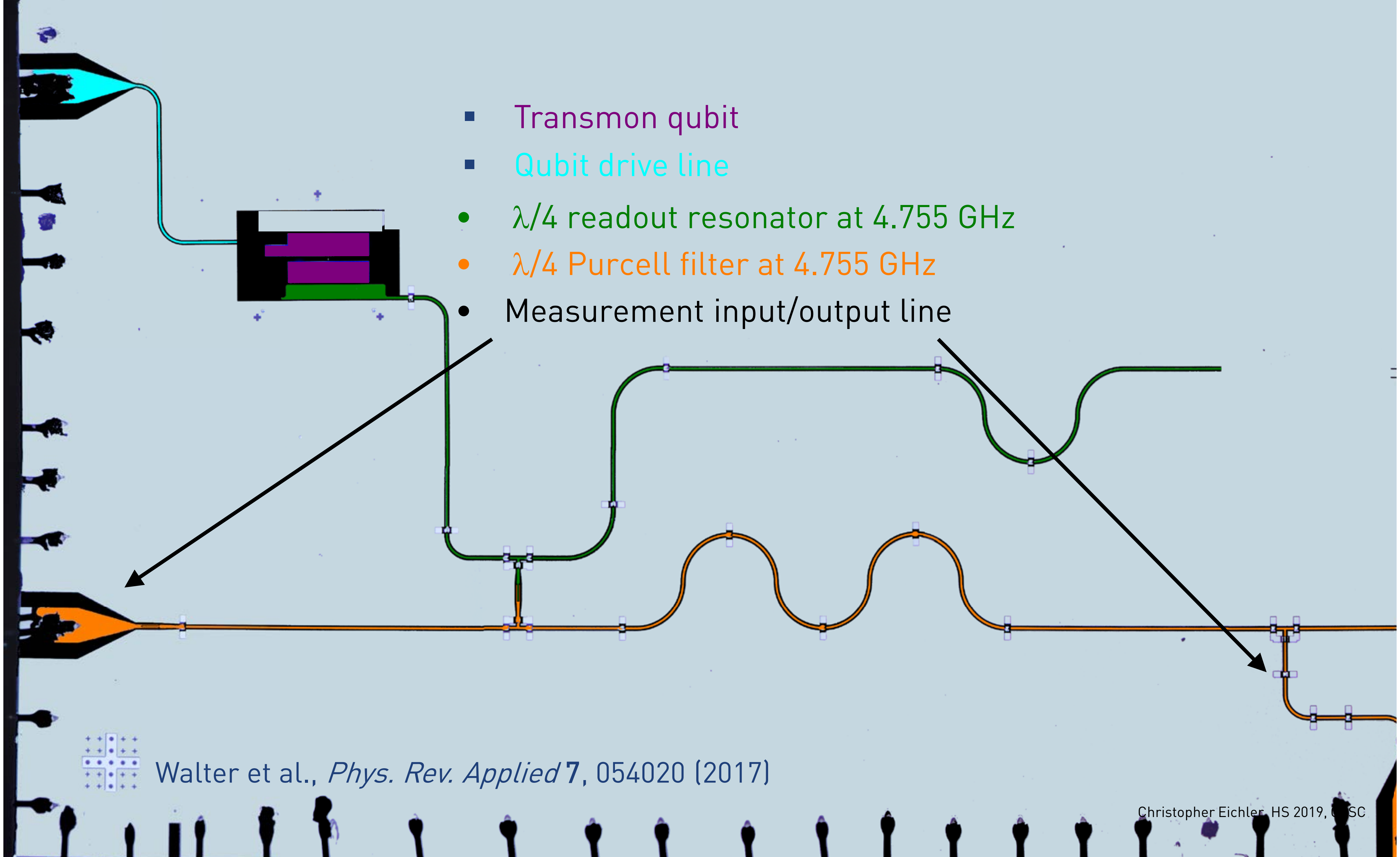
- Solution: Include an additional filter, called "Purcell filter" to suppress qubit decay while allowing for large κ and $|\chi|$.
- Purcell filter can be realized e.g. as an an additional LC- resonator (see schematic).
- In this case

$$\gamma_{Purcell} \propto 1/\Delta^4$$

is strongly suppressed.

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

Transmon and Readout Circuit optimized for fast Readout

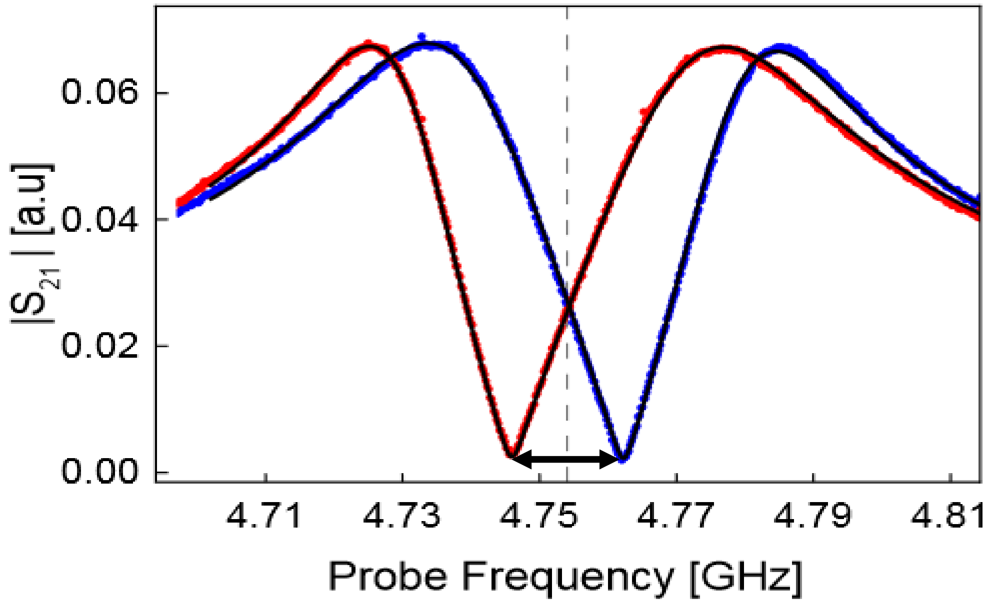


- Transmon qubit
- Qubit drive line
- λ/4 readout resonator at 4.755 GHz
- λ/4 Purcell filter at 4.755 GHz
- Measurement input/output line

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

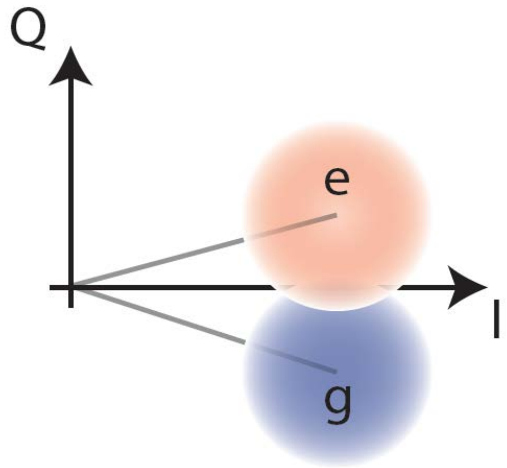
Readout Resonator Response

Transmission amplitude or readout resonator extracted through Purcell filter for qubit prepared in ground (g) or excited (e) state :



Parameter fit (input-output model):

- Purcell filter $\kappa_p/2\pi$ = 64 MHz
- Readout resonator $\kappa_r/2\pi$ = 37.5 MHz
- State dependent resonator shift $2\chi/2\pi \simeq$ -16 MHz

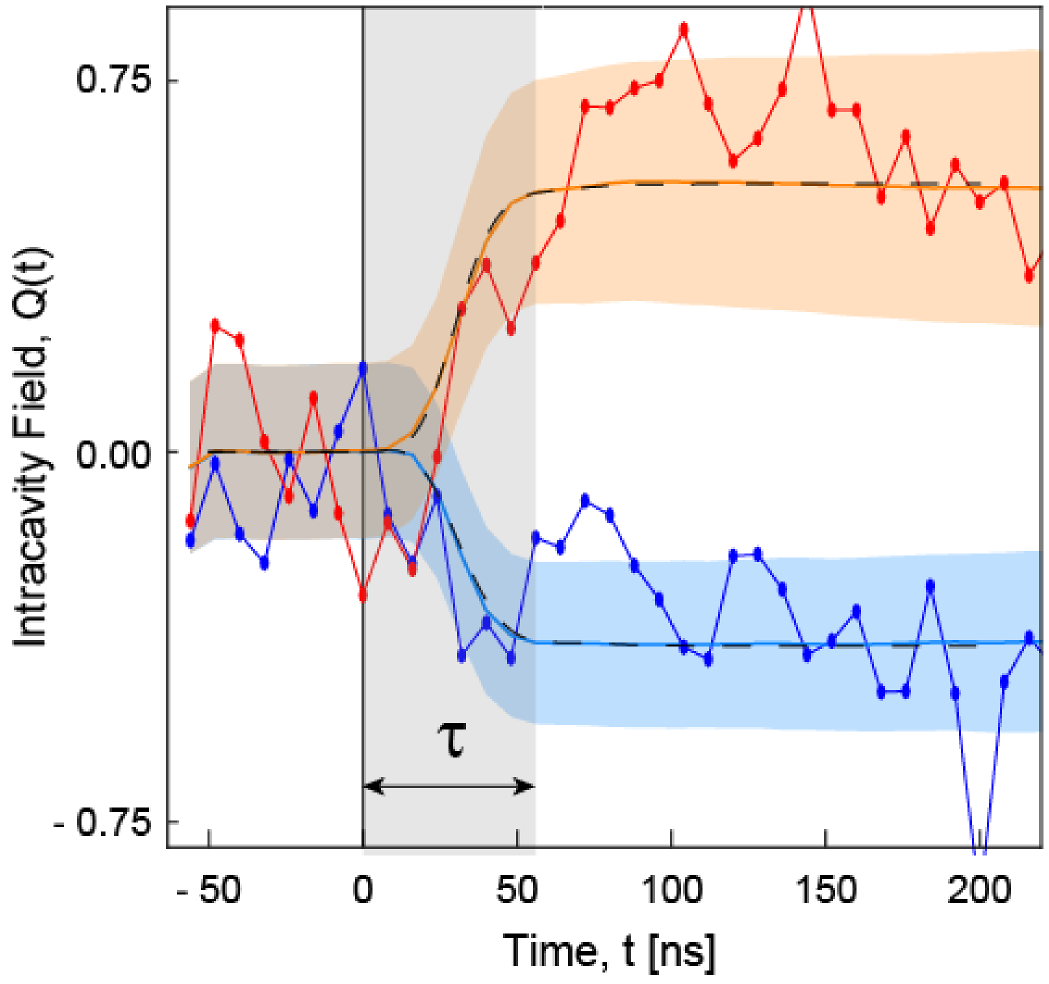


In ground/excited state:

- Data measured after state prep. (*,*)
- Fit to resonator response model (-)

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

Time Dependence of Measured Quadrature



Quantities:

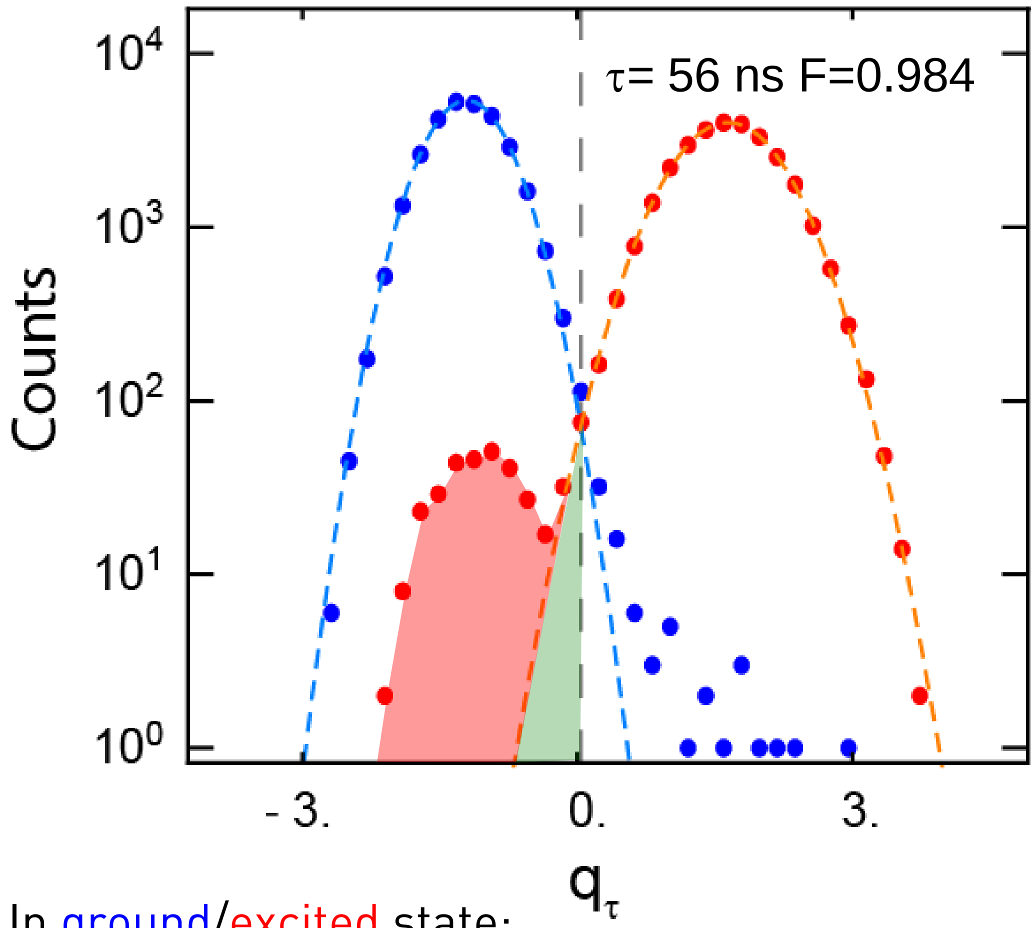
- Single ground state (g) trace
- Average and Stdv of g traces
- Simulated dynamics (-)
- Single excited state (e) trace
- Average and Stdv of e traces
- Simulated dynamics (-)
- Integration time τ

Observations:

- Fast rise of measurement signal (< 50 ns) due large χ (and κ)
- Small decay of average excited state trace due to Purcell protected T_1
- Little increase of average ground state trace due to measurement induced mixing

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

Histograms of Integrated Quadrature Signals



In ground/excited state:

- Data of 30k preparations each (*,*)
- Fitted Gaussian distribution (-,-)
- Constant threshold (---)

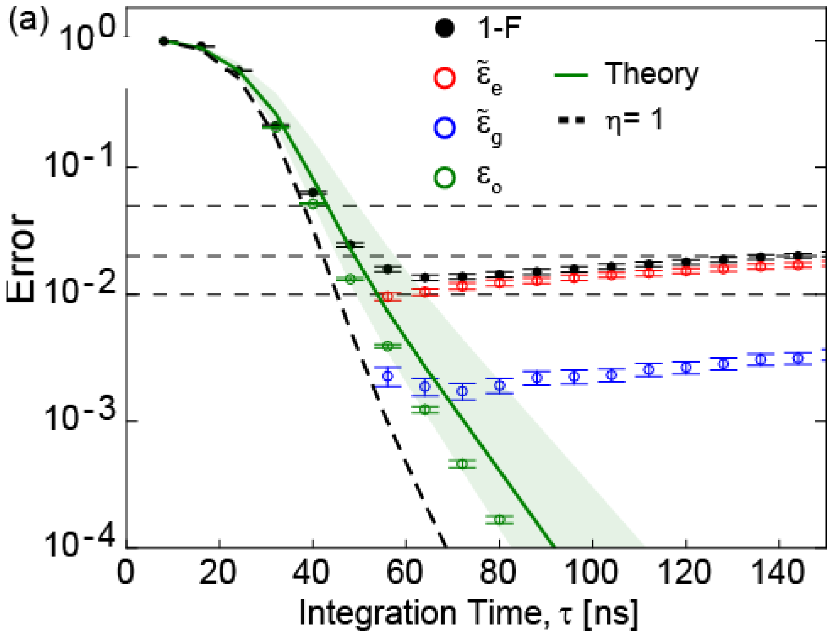
- Transmission quadrature integrated with opt. filter.
- Definition of errors and fidelities in ground/excited state:
  - Overlap error: $\varepsilon_{o,g/e}$
  - Transition, preparation (and other) errors: $\tilde{\varepsilon}_{g/e}$
  - Total error $\varepsilon_{g/e} = \varepsilon_{o,g/e} + \tilde{\varepsilon}_{g/e}$
- For measurement of unknown state:
  - Total error $\varepsilon = \varepsilon_g + \varepsilon_e$
  - Total fidelity $F = 1 - \varepsilon$

Note:

- Alternative fidelity metric calculates the *average probability of correct assignment*. For a single qubit this probability is $1 - \varepsilon/2$.

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

Measurement Error *vs.* Integration Time



Discussion:

- Fast state discrimination with overlap error drop to 1 % in only < 50 ns
- Excited state error < 0.96 %
- Ground state error < 0.23 %
- Max. total fidelity > 98 % limited by qubit T_1

Improved Understanding:

- Power Dependency
  - overlap vs measurement induced errors
- Pulse Shaping (Two-Step)
- Improved measurement efficiency (36 dB gain)
- 99.2% total Fidelity reached
  - Knowledge for improvement

Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)