Reminder: Requirements for physical implementation of QC

DiVincenzo's criteria (2000):

1) Scalable, physical realization of a qubit
2) Ability to initialize qubits in a fiducial state, e.g. the ground state
3) Coherence time needs to be much greater than the gate time
4) Need universal set of gates
5) Need high-fidelity measurement of qubits

5.1 Universal set of gates

"Any unitary operator acting on a register of qubits can be decomposed into single-qubit gates and controlled NOT gates. "

Standard set of universal gates

Hadamard

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

Phase gate

$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

T gate

$$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$$

Single-qubit gates

Controlled NOT

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

2-qubit gate

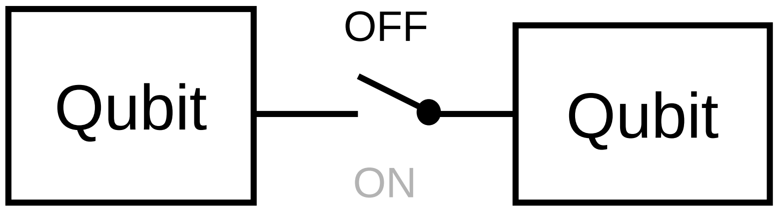
Nielsen, M. A. & Chuang, I. L. Quantum Computation and Quantum Information, *Cambridge University Press* (2000)

5.1 Universal set of gates

Discussion on the black board.

5.2 Realization of 2-qubit gates

Requirement: Controllable interaction between qubits.



Desirable Properties:

- Good ON/OFF ratio
- No additional decay/dephasing due to coupling
- Avoid leakage into non-computational states
- Robustness and ease of gate tune-up
- Scalability and hardware requirements

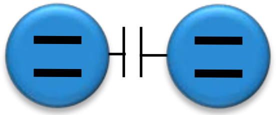
5.2 Controllable coupling in superconducting circuits

Conventional static coupling (Jaynes-Cummings):

$$H_{\mathrm{coupl}} = J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)$$

Realization:

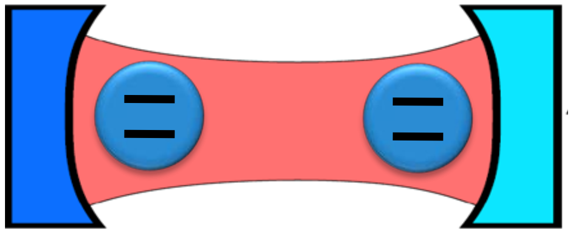
- Direct capacitive coupling



- Indirect coupling mediated by a BUS resonator

Blais et al., PRA **69**, 062320 (2004)

Majer et al., Nature **449** (2007)



Gates induced by

- ... tunable qubit frequency
- ... microwave drive
- ... parametric modulation of qubit frequency

Alternative tunable coupling schemes, e.g.:

- Gmon: Tunable coupling between transmons
  Chen et al., PRL 113, 220502 (2014)
- Tunable BUS cavity
  McKay et al., Phys. Rev. Applied 6, 064007
- Longitudinal coupling
  Didier et al., PRL 115, 203601 (2015)
  Billangeon et al., Phys. Rev. B 92, 020509 (2015)
  Richer et al., Phys. Rev. B 93, 134501 (2016)

Gates induced by

- ... parametric modulation
- ... tunable coupling
- ...

5.2 Examples of static qubit-qubit couplers

Direct capacitive coupling:

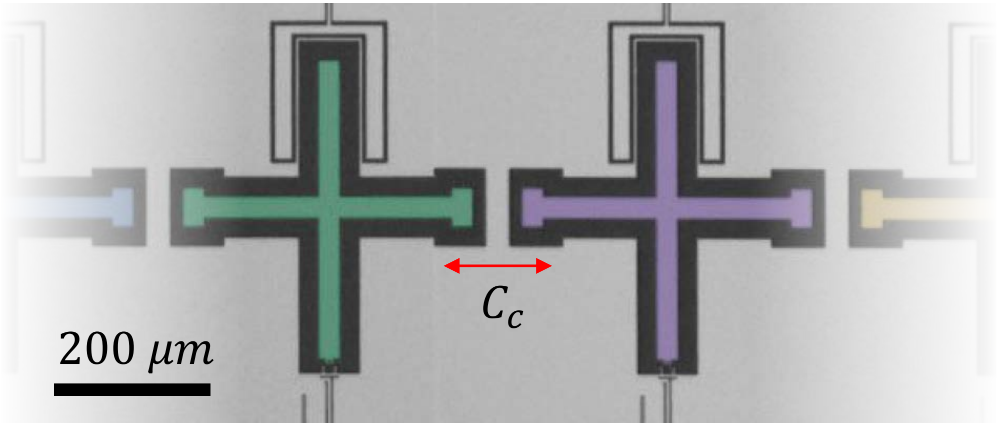


Image: Barends et al., Nature (2016)

Coupling mediated by a resonator

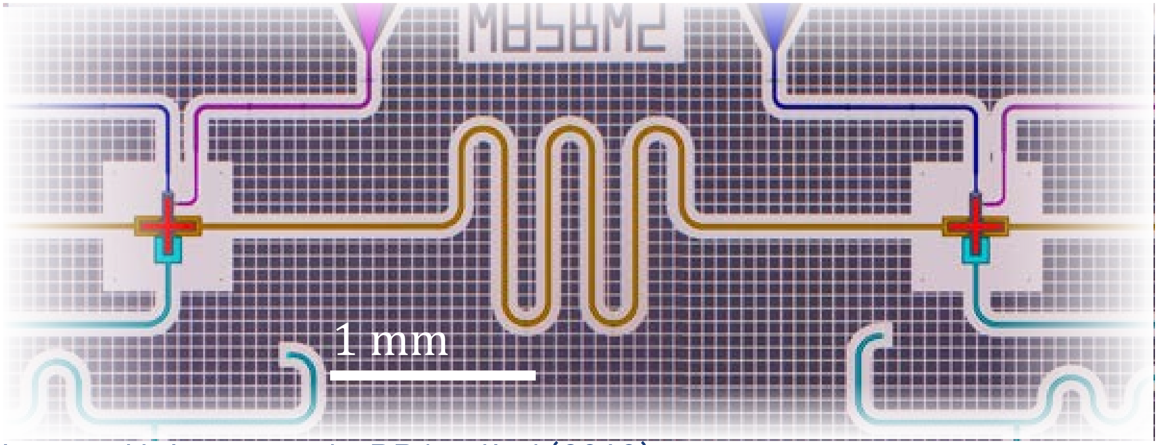


Image: Heinsoo et al., PRApplied (2018)

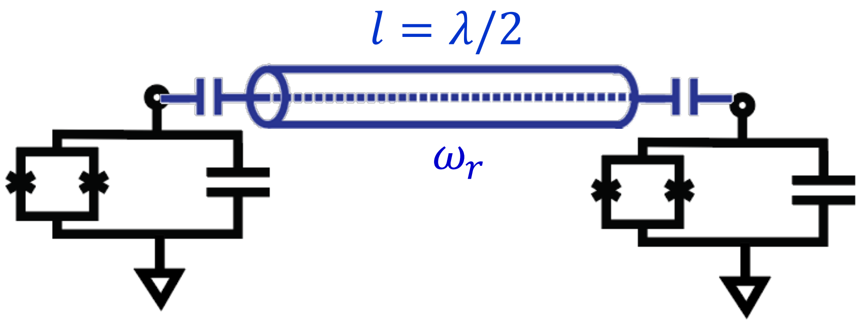
Properties:

- Coupling strength $J = \frac{C_c}{2\sqrt{C_1 C_2}}\sqrt{\omega_1 \omega_2}$ can be made large.
- Qubits are in near vicinity of each other.
- No need for an additional coupling element introducing an extra mode.

Properties:

- Coupling strength depends on the detuning between qubits and resonator.
- Multiple qubits can be coupled to a common resonator acting as a BUS
- Coupling mediated over "long" distance on the chip. Qubits can be spatially separated on the chip.

5.2 Resonator-mediated qubit-qubit coupling



Tavis-Cummings Hamiltonian

$$H = \omega_r a^\dagger a + \sum_{j=1,2} \frac{\omega_{aj}}{2} \sigma_{z_j} + \sum_{j=1,2} g_j (a^\dagger \sigma_{-_j} + a \sigma_{+_j})$$

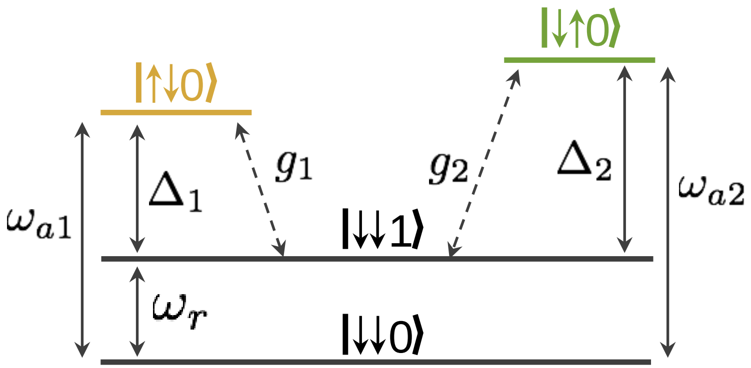
In the limit of $\omega_r - \omega_a \gg g$, perform Schrieffer-Wolff transformation to obtain:

$$H_{\mathrm{eff}} \approx \omega_r a^\dagger a + \sum_j \frac{1}{2} \left[\omega_{aj} + 2 \frac{g_j^2}{\Delta_j} \left(a^\dagger a + \frac{1}{2} \right) \right] \sigma_{z_j}$$

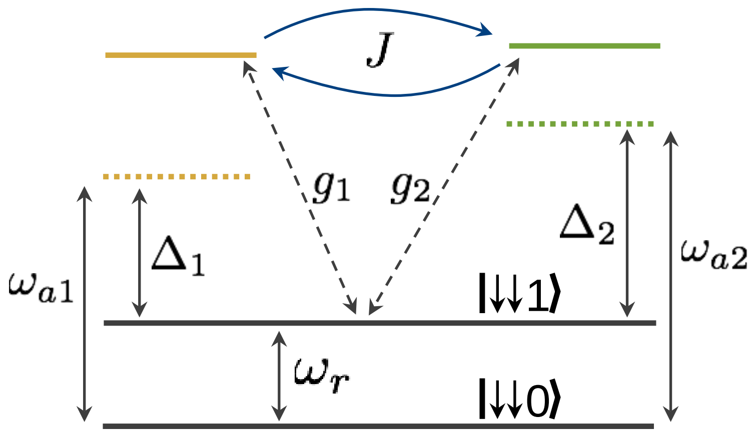
$$+ J \left(\sigma_{-_1} \sigma_{+_2} + \sigma_{+_1} \sigma_{-_2} \right)$$

with $J = \frac{g_1 g_2}{2} \left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2} \right)$ $\sim 1 - 50\ \mathrm{MHz}$

Detuned qubits: interaction suppressed

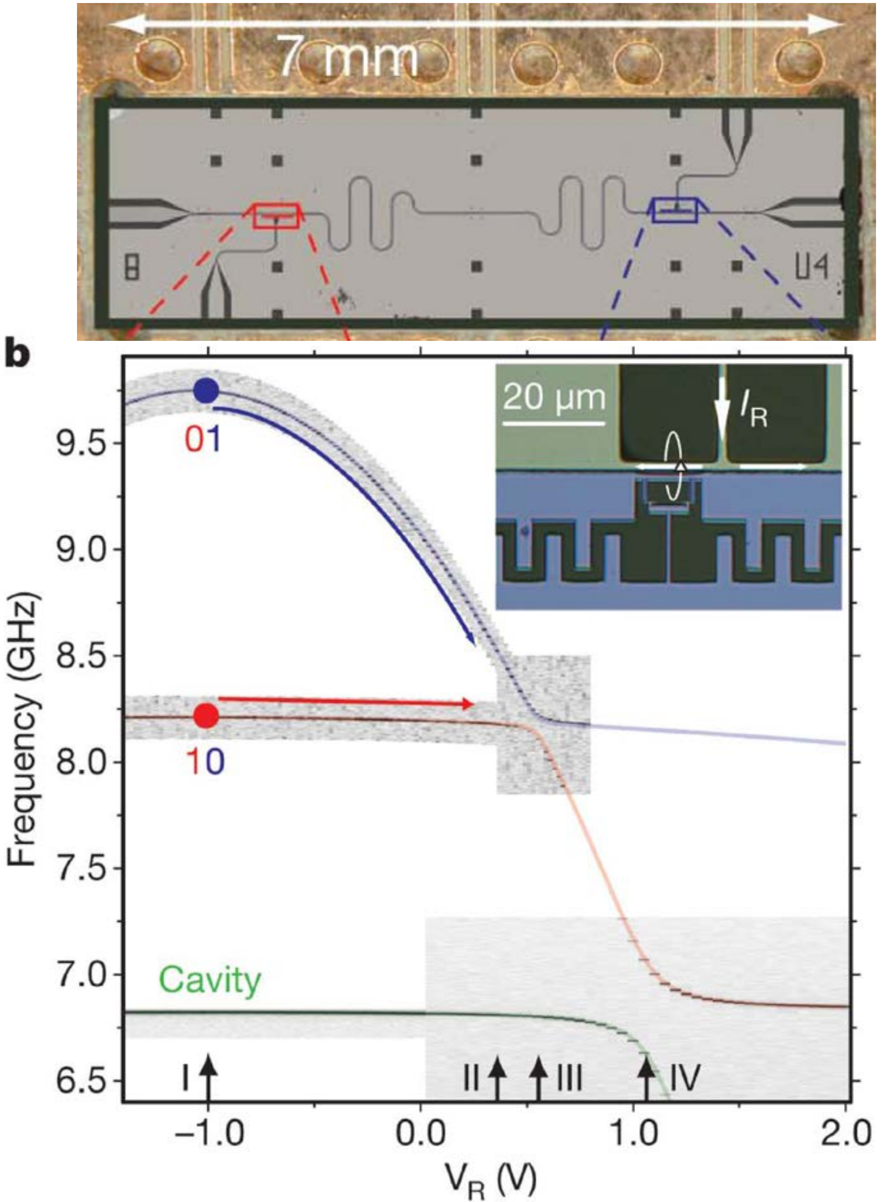


Qubits resonant: exchange interaction mediated by resonator.



A. Blais *et al.*, *Phys. Rev. A* **75**, 032329 (2007)

Cavity mediated qubit-qubit coupling

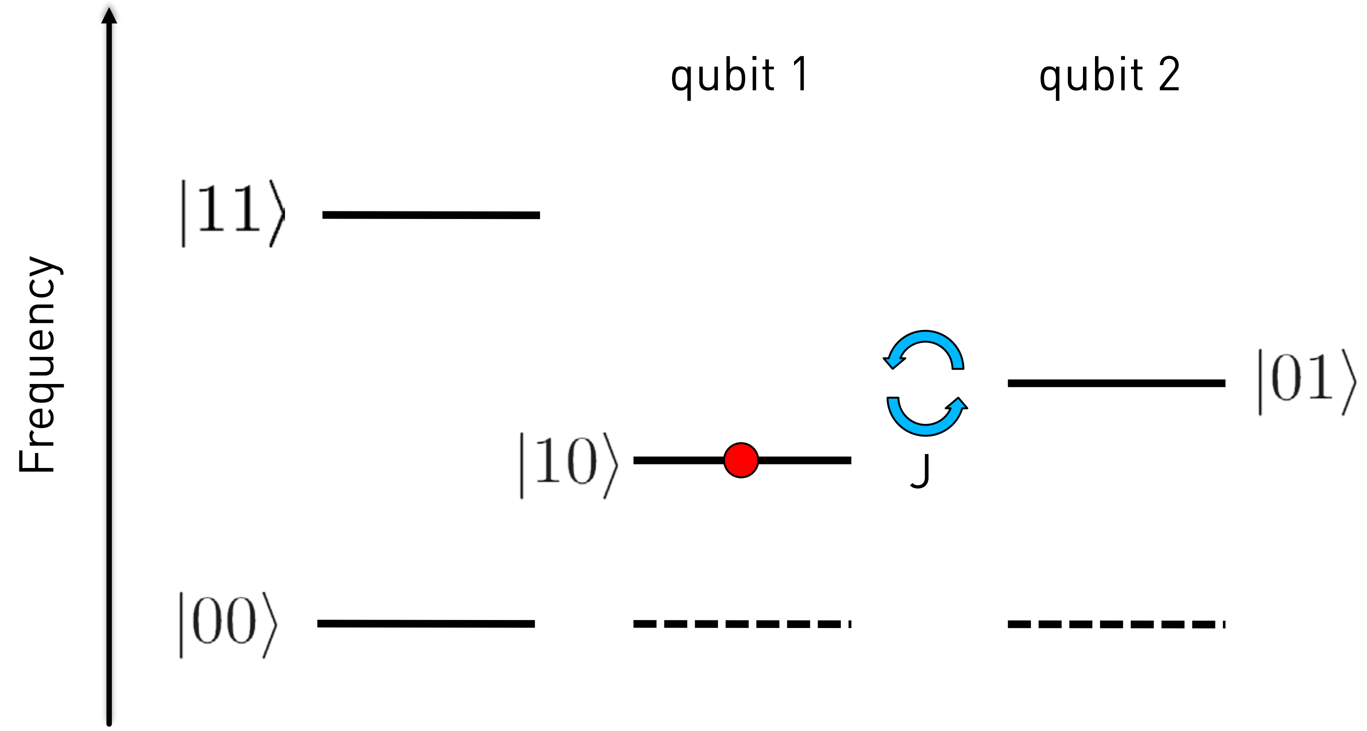


Spectroscopic observations:

- V_R proportional to magnetic flux in SQUID loop.
- Avoided crossing of qubit energy levels |01> and |10> at ~8.2 GHz.
- Splitting between the two states on resonance is *2J*.
- Resonant qubit-cavity coupling at ~6.8 GHz with splitting 2*g*.
- Finite cross-coupling between fluxline of qubit 1 (2) and SQUID loop of qubit 2 (1).

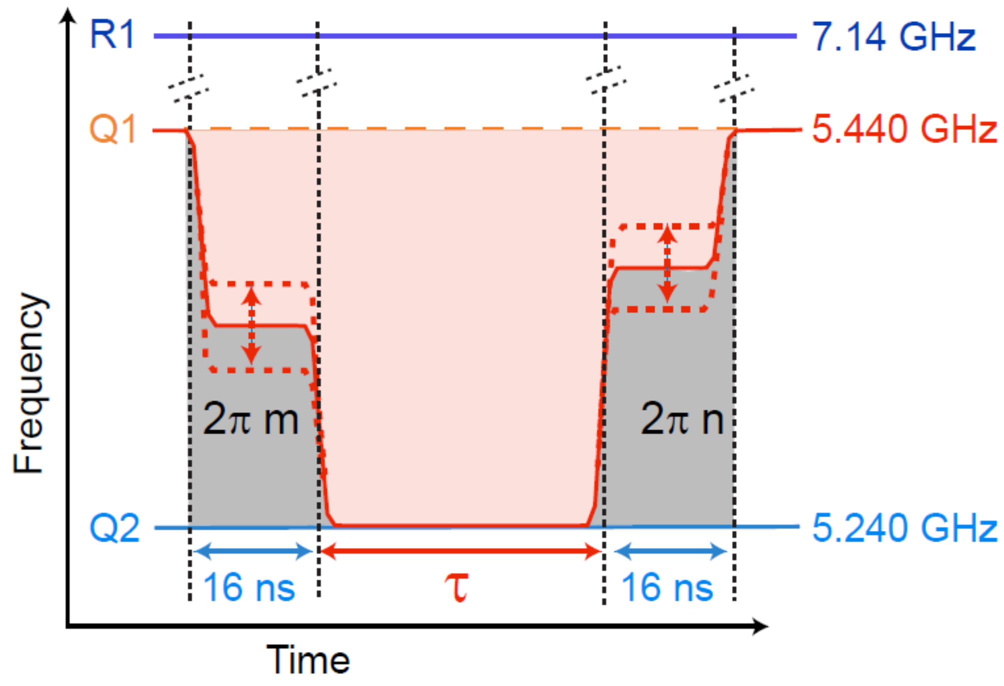
First demonstration: J. Majer *et al.*, *Nature* 449, 443 (2007)
This data: L. DiCarlo *et al.*, *Nature* 460, 240 (2009)

5.2 Exchange (SWAP) interaction between



Frequency tuning by magnetic flux:

- tunable interaction time τ
- compensation of dynamic phase



evolution of states during interaction:

Initial state intermediate state final state

$$|10\rangle \xrightarrow[\text{iSWAP}]{} i|01\rangle \longrightarrow -|10\rangle$$

$$\tau = \pi/(2J)$$

Salathé *et al.*, *PRX* **5**, 021027 (2015)

5.2 Conditional Z gate

How to realize a CNOT-type gate?

Condition Phase gate = C-Z

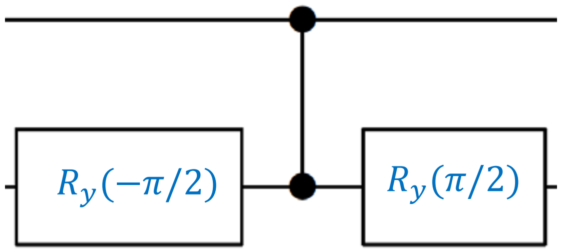
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Equivalent up to basis rotation of target qubit

$\longleftrightarrow$

CNOT = C-X

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



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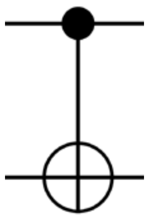
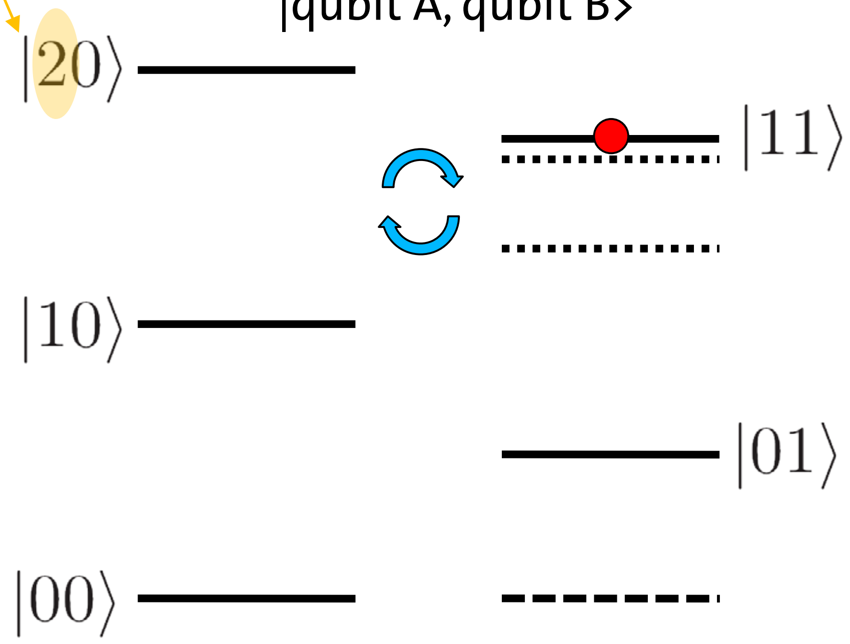


5.2 Controlled Phase Gate (11-20)

Make use of transmon states beyond 0, 1

2nd excited state ("f-level") of transmon

|qubit A, qubit B>



Tune levels into resonance using magnetic field

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$

Full 2π rotation induces phase factor -1

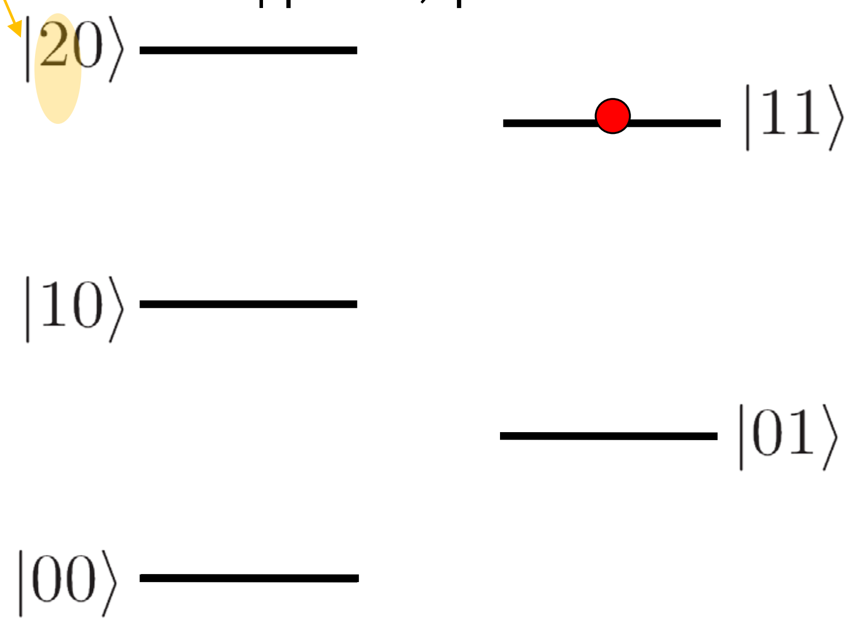
proposal: F. W. Strauch *et al., Phys. Rev. Lett.* **91**, 167005 (2003).
first implementation: L. DiCarlo *et al., Nature* **460**, 240 (2010).

5.2 Controlled Phase Gate

Make use of qubit states beyond 0, 1

2nd excited state ("f-level") of transmon

|qubit A, qubit B>



Qubits in states 01, 10 and 00 do not interact and thus acquire no conditional phase shift

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$
$$|01\rangle \longrightarrow |01\rangle$$
$$|10\rangle \longrightarrow |10\rangle$$
$$|00\rangle \longrightarrow |00\rangle$$

C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch *et al., Phys. Rev. Lett.* **91**, 167005 (2003).
first implementation: L. DiCarlo *et al., Nature* **460**, 240 (2010).

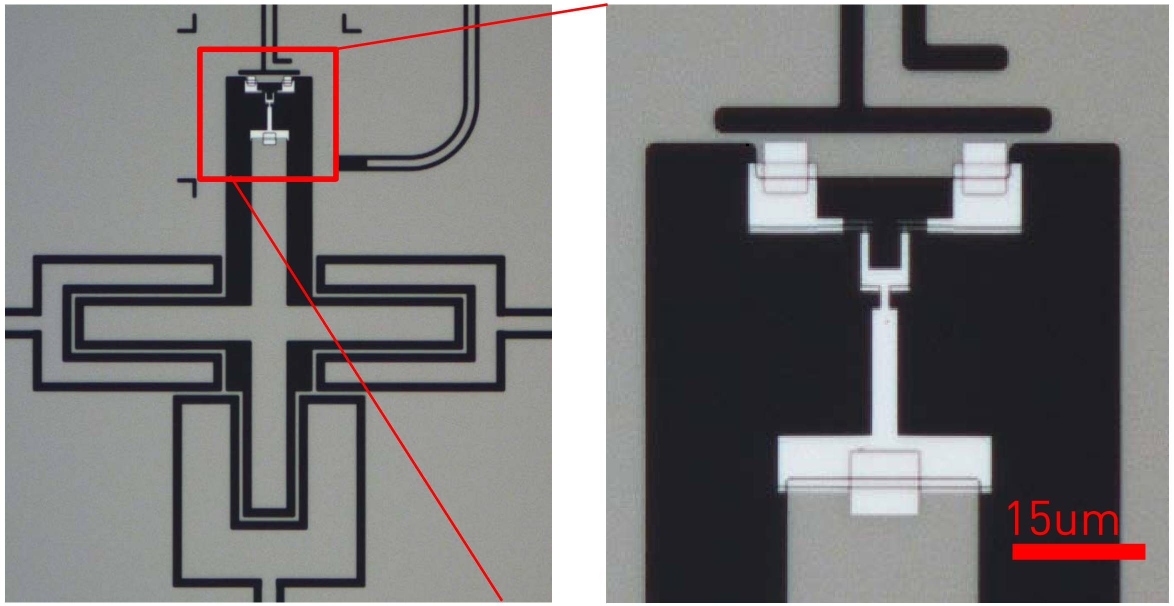
5.2 Controlled Phase gate: Effective Hamiltonian

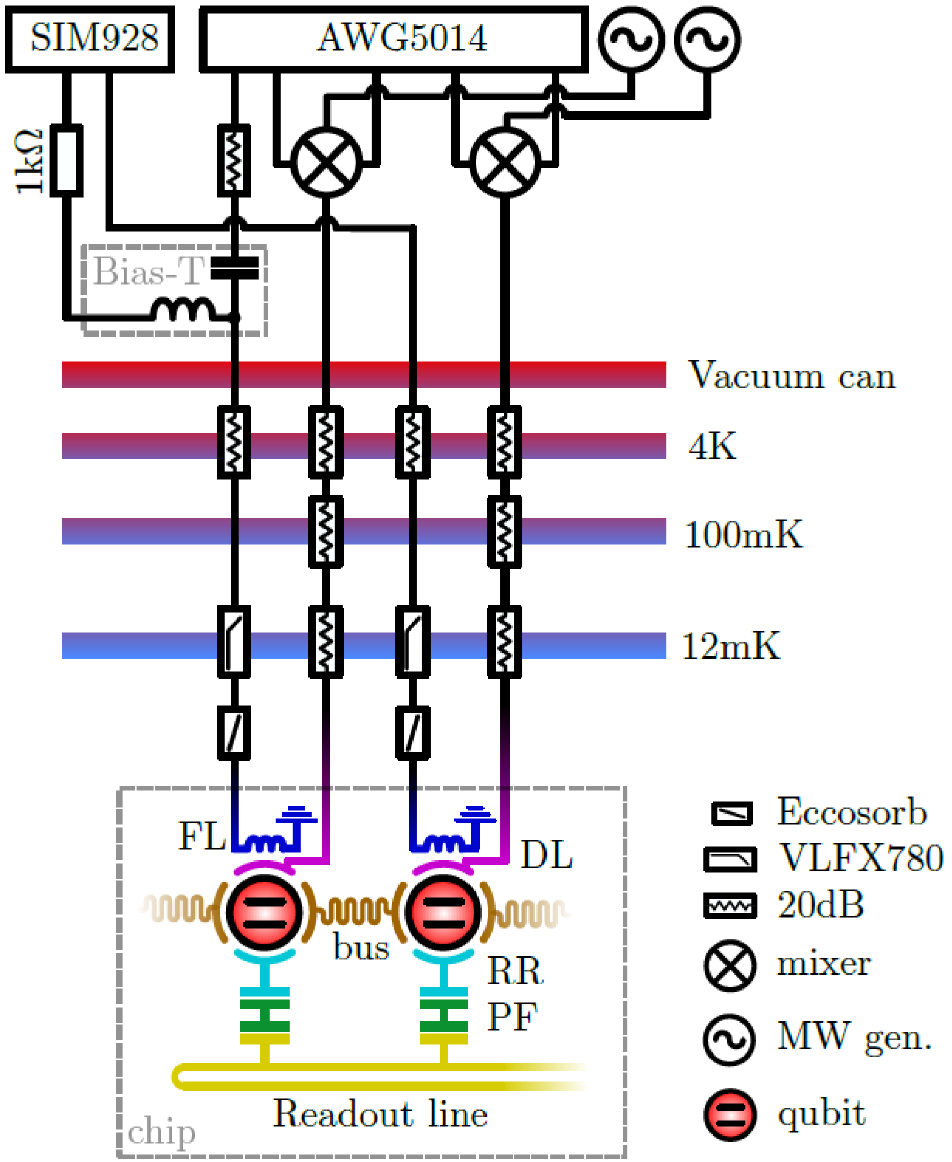
$$
H/\hbar = \begin{array}{c} \begin{array}{cccccc} & & & |20\rangle & |11\rangle & |02\rangle \\ & & & \downarrow & \downarrow & \downarrow \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_1 & J & 0 & 0 & 0 \\ 0 & J & \omega_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha + 2\omega_1 & \sqrt{2}J & 0 \\ 0 & 0 & 0 & \sqrt{2}J & \omega_1 + \omega_2 & \sqrt{2}J \\ 0 & 0 & 0 & 0 & \sqrt{2}J & -\alpha + 2\omega_2 \end{pmatrix} \end{array}
$$

- Typical parameters
  $(\omega_1,\ \omega_2,\ \alpha,\ J)/2\pi = (5, 6, 0.3, 0.005)\mathrm{GHz}$
- Assume higher frequency qubit (2) is suddenly tuned down for a time τ such that $\omega_2 - \alpha = \omega_1$ before returning back to initial frequency.
- Note: In the limit of large detuning compared to J, the basis states are approximate eigenstates of the Hamiltonian .

Exercise: Simulate time-evolution governed by $U = \exp\left(-\mathrm{i}\frac{\mathrm{H}}{\hbar}t\right)$of initial state $|00\rangle + |01\rangle + |10\rangle + |11\rangle$ and convince yourself that the gate time to reach the state $|00\rangle + |01\rangle + |10\rangle - |11\rangle$ (up to dynamic phase factors) is given by $\tau = \frac{\pi}{\sqrt{2}J}$.

5.2 Static (DC) and fast flux control





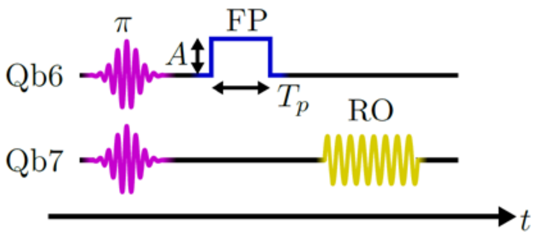
- Mutual inductance between flux line and SQUID typically 0.5-1 Φ_0/mA.
- Fast flux pulses generated by AWG. Static bias by DC source.
- Accurate pulse control at the qubit level requires to characterize the distortion of the pulse when propagating through the flux line and to compensate for it.
- Use qubit as a scope to measure the flux line response function*.

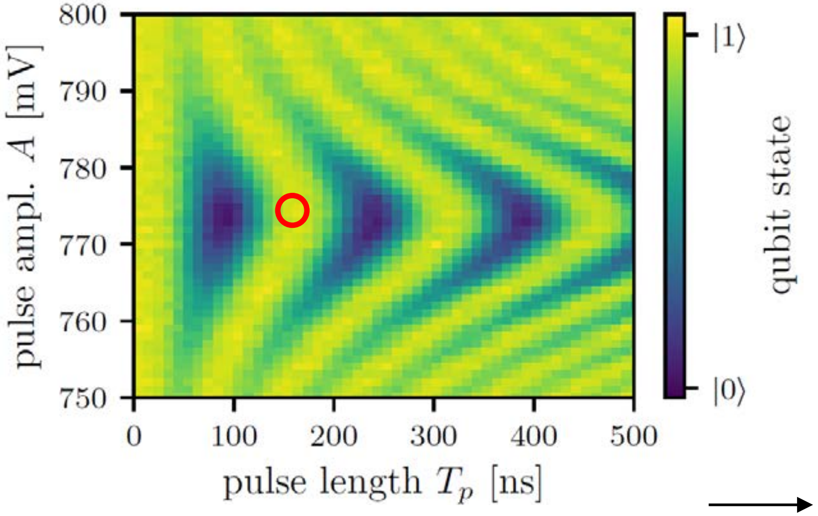
*See for example Jerger et al., PRL, 123, 120501 (2019)

5.2 Tune-up of the conditional phase gate

Conditions of c-phase gate:

1) Recovery of qubit population $pr = 1$





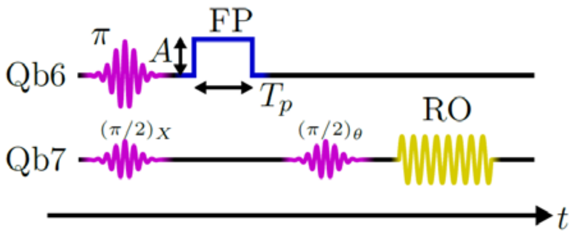
- Control Parameters T_p and A
- Tune-up:
  1) Optimize pr *w.r.t.* T_p
  2) Optimize ϕ_c *w.r.t.* A

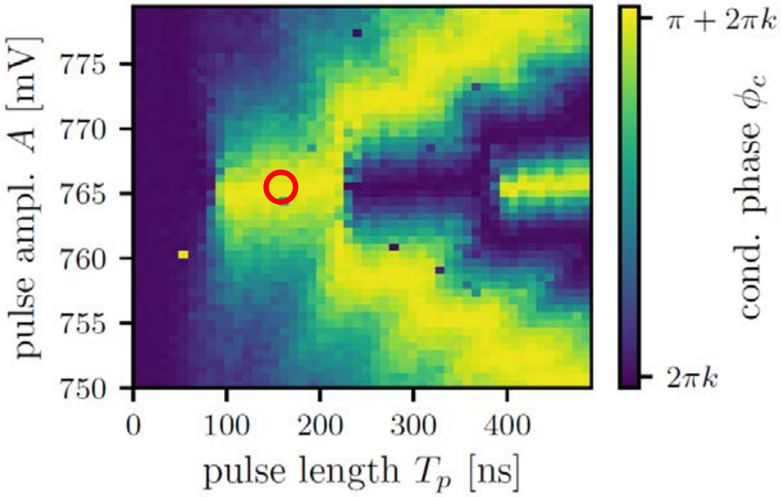
Gate characterization

- Bell state tomography
- Process tomography
- Randomized Benchmarking

→ How to measure the performance of a gate or entire process -> See next week.

2) Conditional phase $\phi_c = \pi$





5.2 Discussion conditional phase gate

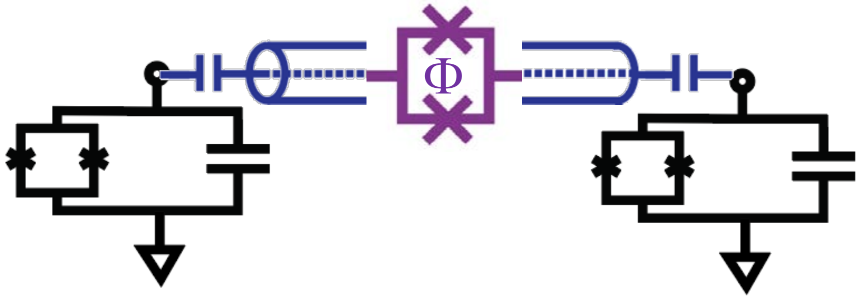
- Want: Fast gates ↔ Large coupling strength J.
- But: Residual coupling during idle ("OFF") state scales with $\chi \sim J^2/\Delta^2$, where Δ is the detuning between the two qubits.
- Challenge: Gate uses non-computational state $|02\rangle$: Finite probability to remain in this state after completion of the gate ("leakage error"). Sensitive to the precise shape of the control pulse (compare e.g. Rol et al., PRL 123, 120502 (2019)).
- Gate implementation requires flux-tunable qubits, rendering the qubits sensitive to flux noise during the gate.

⟶

- Option 1: Gate operation which does not require fast DC flux control:
  - Microwave induced gates (e.g. Chow et al., PRL 107, 080502 (2011))
  - Parametrically induced gates (e.g. Reagor et al., Science Advances, 4, 2 (2018))
- Option 2: Alternative coupling mechanisms using tunable couplers (e.g. McKay et al., PRApplied (2016)).

5.2 Two-qubit gates based on tunable couplers

Basic idea: Make the coupling element flux-tunable



$\omega_r(\Phi) \rightarrow J(\Phi)$

Tunable coupler frequency $\rightarrow$ Tunable coupling strength

Possible gate scheme:
Parametric modulation of flux $\Phi(t) = \Phi_{\mathrm{ac}} \sin(\omega_{\mathrm{ac}} t)$ to couple energy levels, which are detuned by $\Delta = \omega_{\mathrm{ac}}$.

Selected references: McKay et al., PRL (2015) , Kounalakis et al., NPJ QI (2018), Collodo et al., PRL 122, 183601 (2019)

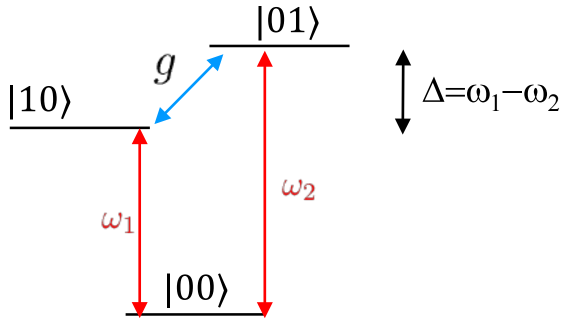
Sideband Coupling

2 qubits coupled to each other:

$$H/\hbar = \omega_1 \sigma_1^+ \sigma_1^- + \omega_2 \sigma_2^+ \sigma_2^- + J(\sigma_1^+ + \sigma_1^-)(\sigma_2^+ + \sigma_2^-)$$

rotating frame

$$H_{\text{int}}/\hbar = g\left(\underline{e^{-i(\omega_1+\omega_2)t}}\sigma_1^+\sigma_2^+ + \underline{e^{-i(\omega_1-\omega_2)t}}\sigma_1^+\sigma_2^-\right) + h.c.$$



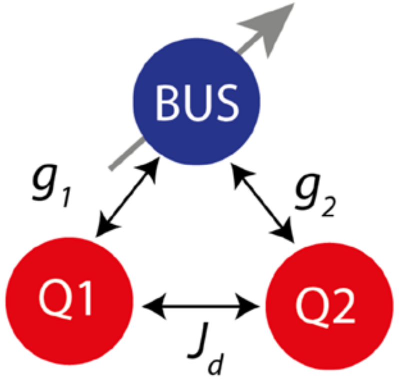
But: Fast oscillating terms -> drop out in rotating wave approximation for g << Δ

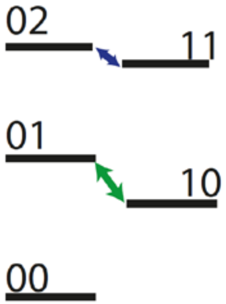
Solution: Modulate coupling at sum or difference frequency to cancel the fast-oscillating phase factors:

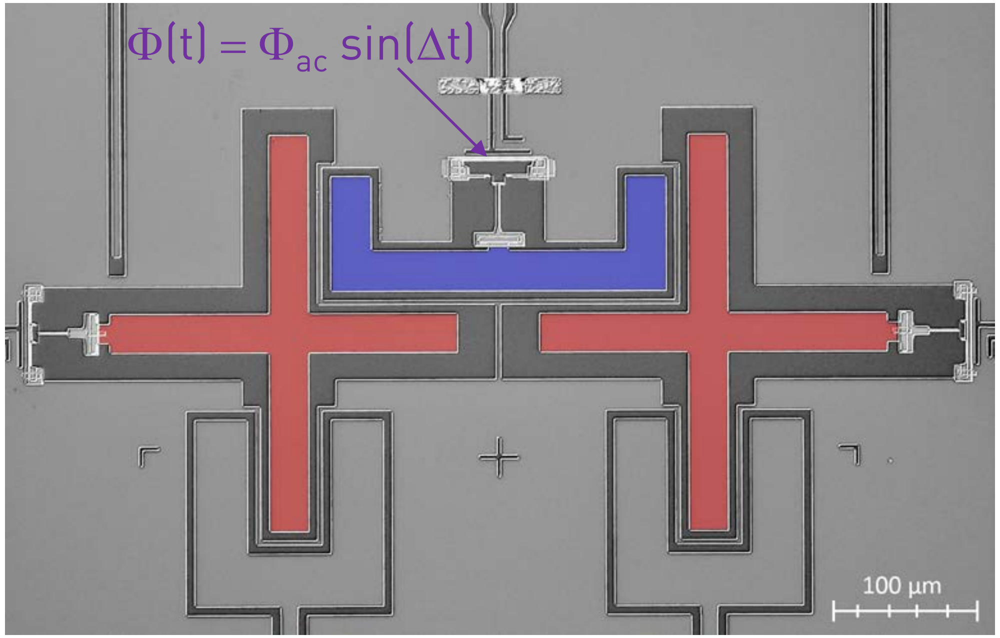
Red sideband: $g(t) = g_0 \sin\left((\omega_1 - \omega_2)t\right) \longrightarrow H_{\text{int}}/\hbar \approx g_0\left(\sigma_1^+\sigma_2^- + \sigma_1^+\sigma_2^-\right)$

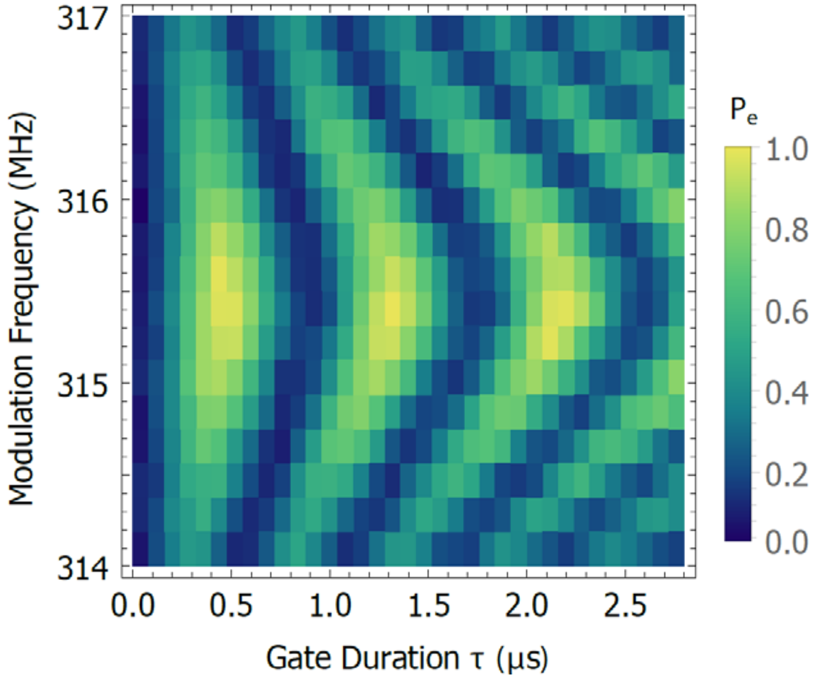
Blue sideband: $g(t) = g_0 \sin\left((\omega_1 + \omega_2)t\right) \longrightarrow H_{\text{int}}/\hbar \approx g_0\left(\sigma_1^+\sigma_2^+ + \sigma_1^-\sigma_2^-\right)$

Towards 2-Qubit gates with tunable couplers









M. Collodo, J. Herrmann et al., Quantum Device Lab (2019)

- Modulating the flux at the difference frequency of the two coupled levels (315 MHz in this example) leads to SWAP-type exchange of the population.
- Effective coupling strength proportional to modulation amplitude.

Conclusion and Outlook

Today

- Implementation of two-qubit gates in superconducting qubits.
- DiVincenzo criteria #1 - #5 ticked off!
- Covered basics of superconducting circuits.

Next week

- Tomography and Benchmarking of states, gates, and processes.

To come

- State-of-the-art SC experiments in Quantum Optics and Quantum Information Processing