Lecture 11

Last week(s)

- Set of universal gates: Decomposition into single- and two-qubit gates
- Implementation of two-qubit gates in SC circuits.
- Q&E session

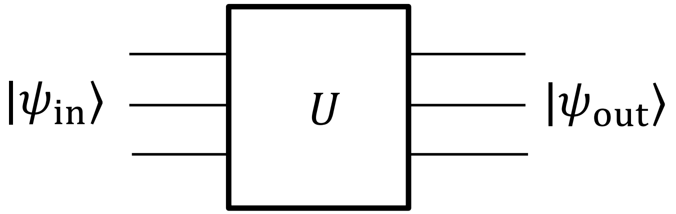
How to verify performance?

Today

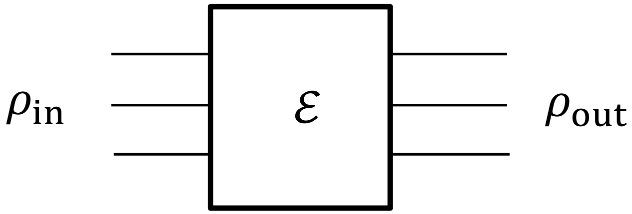
- Tomography and Benchmarking of states, gates, and processes.

6) Characterization & Benchmarking of Quantum Processes

Ideally



Realistically

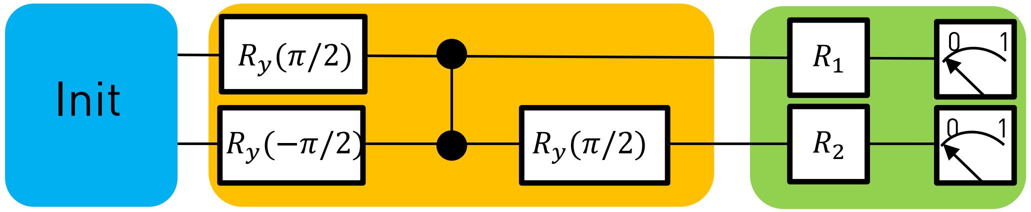


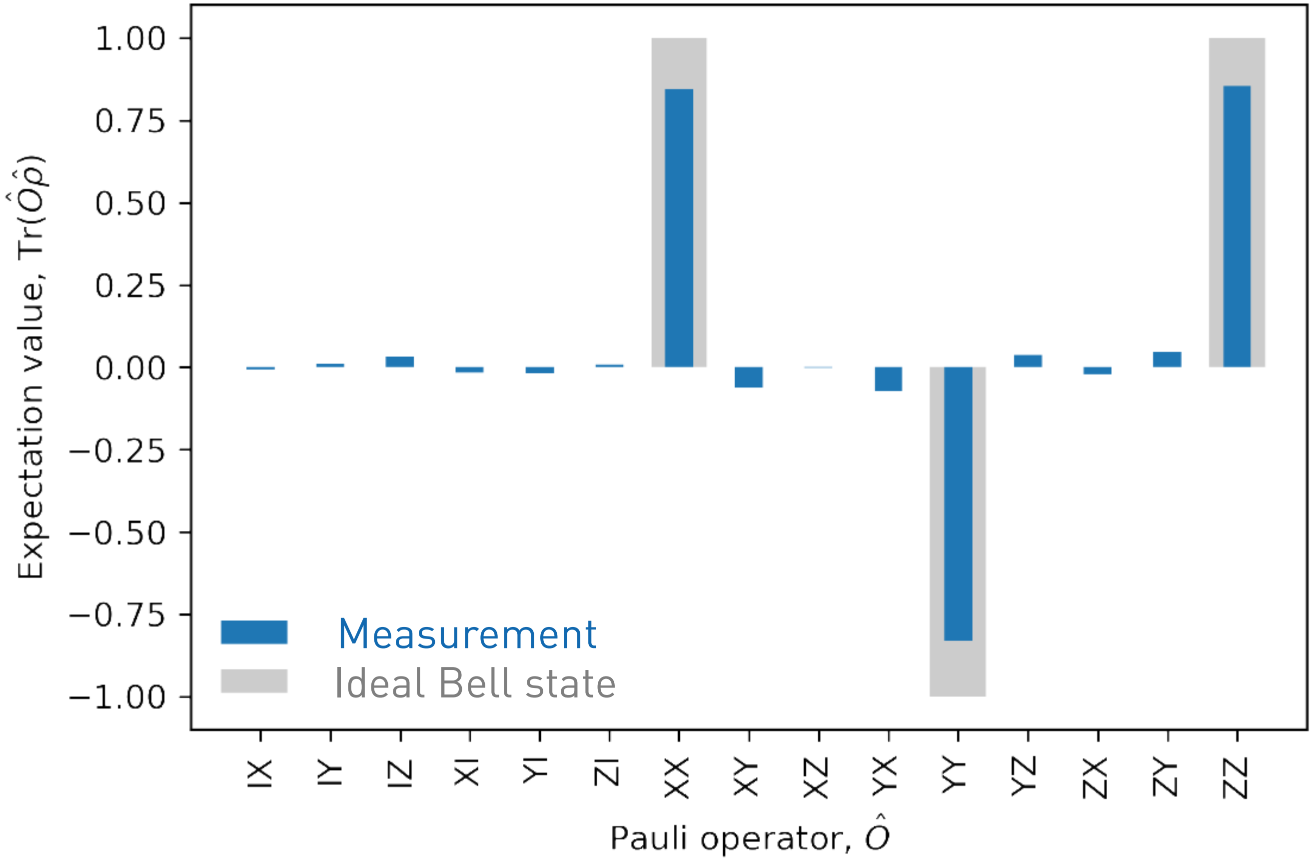
Questions:

- General properties of the map $\mathcal{E}$?
- How to measure the map $\mathcal{E}$?
  - State and process tomography
- Measure of distance between quantum states and processes: Fidelity
- How to benchmark quantum gates with fidelities close to one?
  - Randomized Benchmarking

Discussion on blackboard

6.1 State tomography: Example Bell state



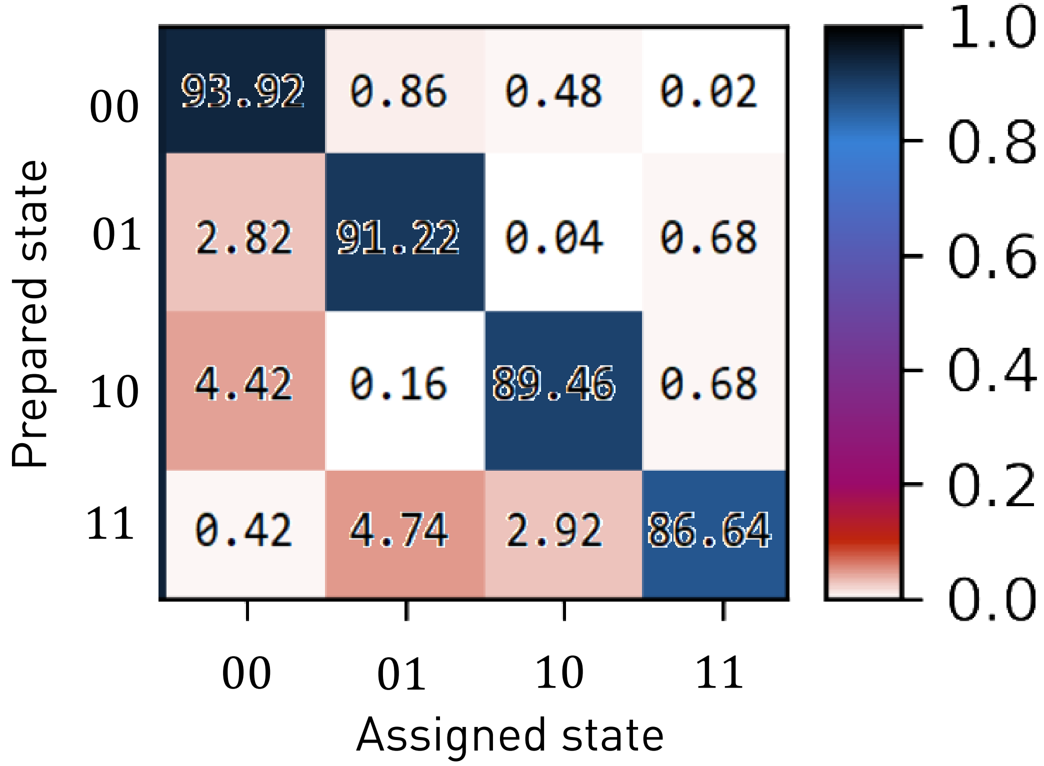


- Initialize qubits in ground state
- Prepare Bell state
- Apply basis rotation pulses R_i to to effectively measure along x, y, z axes.
- Observations
  - Single-qubit expectations vanish. Characteristic feature of entangled Bell state.
  - Measured XX, YY, ZZ correlations exhibit reduced contrast compared to ideal state.
  - Deviations from ideal state due to (i) finite state preparation error and (ii) finite measurement fidelity.

How to account for finite readout efficiency?

6.1 State tomography: Finite readout fidelity

Readout assignment probability matrix M



- Possible measurement outcomes $i \in \{00,01,10,11\}$.
- Assume an unknown state with measured probabilities p_i to obtain one of the four possible outcomes.
- Use assignment probability matrix M to estimate

$$\tilde{p}_i = \sum_j M_{ij}^{-1} p_j$$

Probabilities compensating for finite readout fidelity.

Multiplication with inverse assignment probability matrix

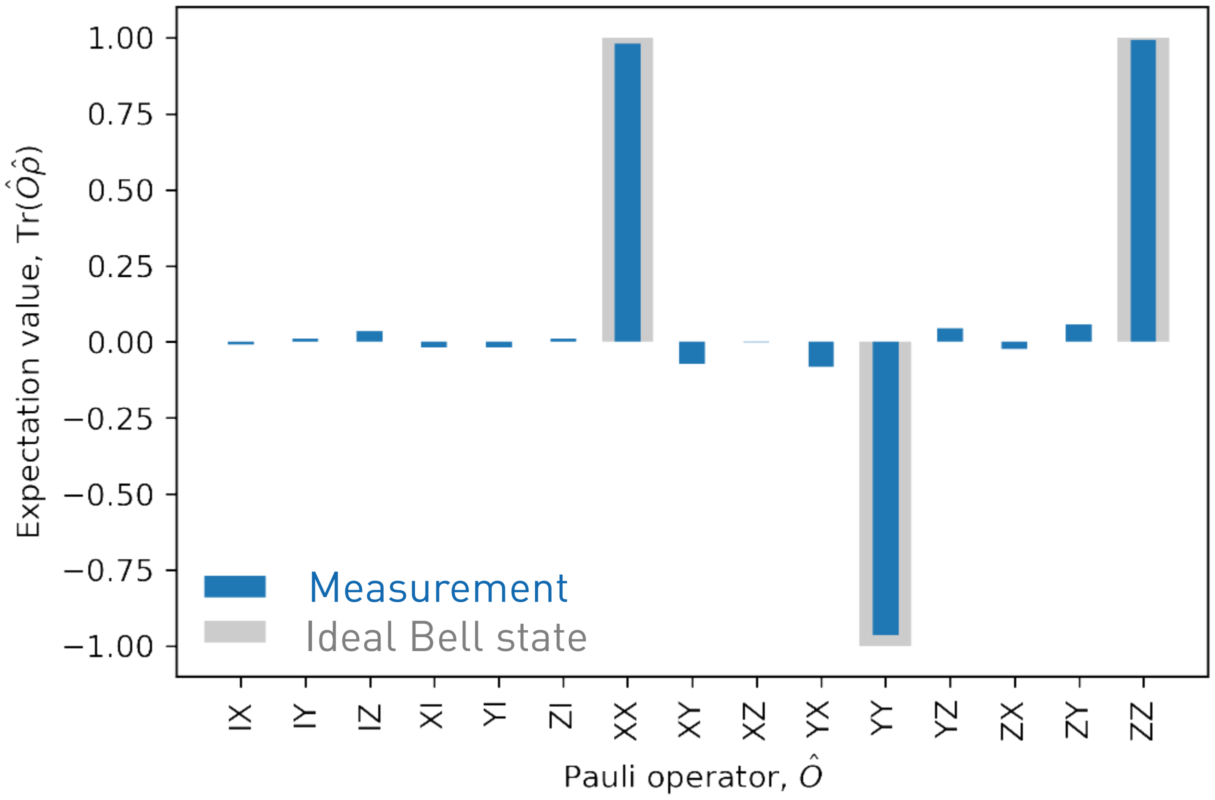
Measured probability to be in state i.

- Comments:
  - Relies on the assumption that ideal reference states can be prepared.
  - In general, need MLE to ensure positivity of $\tilde{p}$.

6.1 State tomography: Mostly likely density matrix

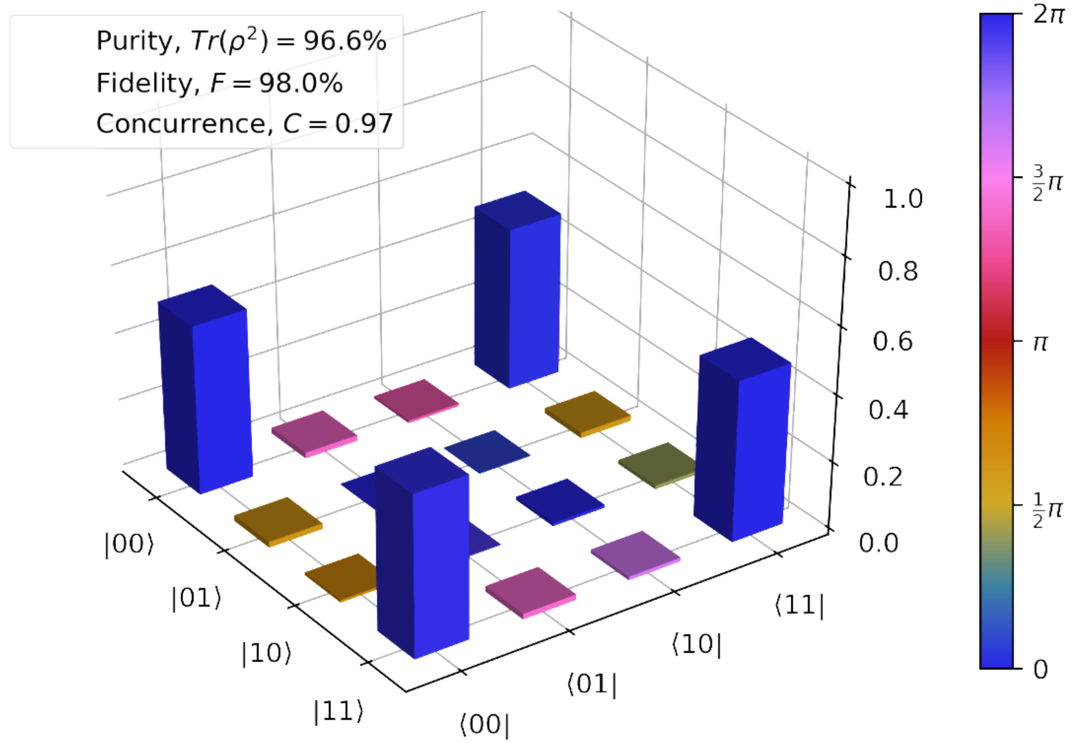
Pauli set after compensating for readout infidelity.

- Expectation values based on $\widetilde{p}_i$ display larger contrast.
- All expectation values are close to target, indicating small errors in the Bell state preparation.



Density matrix reconstruction

- Minimize negative Log-likelihood function, subject to the constraint that ρ has positive eigenvalues.
- Fidelity $F = \sqrt{\langle\psi|\rho|\psi\rangle} = 0.98$



6.2 Process tomography

Discussion on blackboard

6.2 Process tomography

One possible representation of a quantum process: Pauli transfer matrix $\mathcal{R}$.

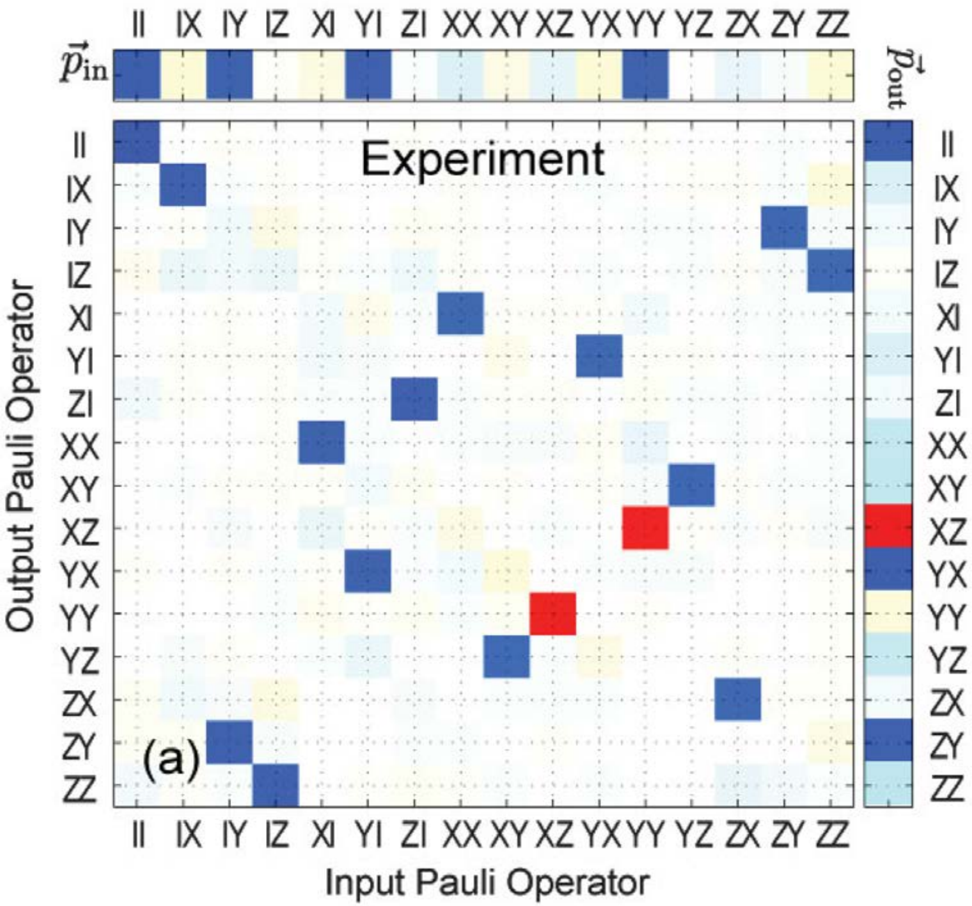
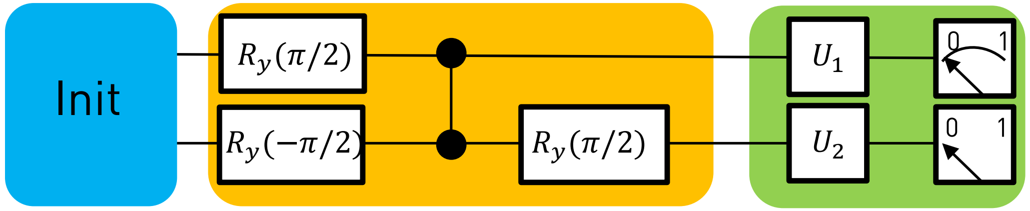


Figure from Chow et al., 109, 060501 PRL (2012)

- Concept of Bloch vector $\vec{r}$ can be generalized to multi-qubit states.
- Elements of $\vec{r}$ are given by muti-qubit Pauli expectation values $\langle \sigma_{i_1}\sigma_{i_2} \ldots \rangle$ with $\sigma_i \in \{I, X, Y, Z\}$, which fully characterize a quantum state.
- Pauli transfer matrix specifies quantum process by relating arbitrary input Bloch vector to the corresponding output Bloch vector according to
$$\vec{r}_{out} = \mathcal{R}\,\vec{r}_{in}$$
- Example shows most likely R measured in an experiment for a CNOT gate.
- Process fidelity, computed as $F = \frac{\mathrm{Tr}[\mathcal{R}_{ideal}^T \mathcal{R}]/d+1}{d+1}$,corresponds to average fidelity of output state.

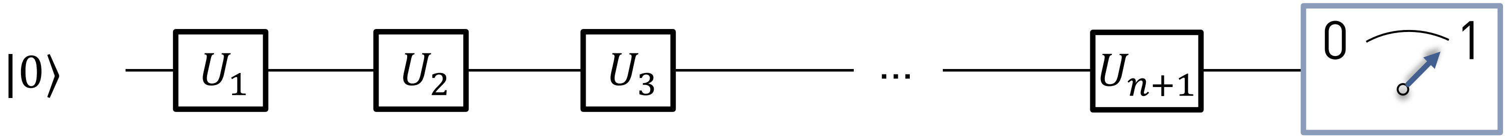
6.2 State and process tomography: Discussion



- Result of a tomographic measurement is sensitive to "state-preparation and measurement" (SPAM) errors.
- For processes, e.g. gates, which are very close to target, it can be challenging to distinguish errors in the process from SPAM errors.
- How to quantify the (small) errors in a process/gate in the presence of finite initialization and readout fidelity?
- Strategy: Amplify errors by applying sequences of multiple gates before measurement.
  - Additional advantage: Tests how repeatable operations are.
- → Widely used approach: Randomized benchmarking.

6.3 Randomized benchmarking

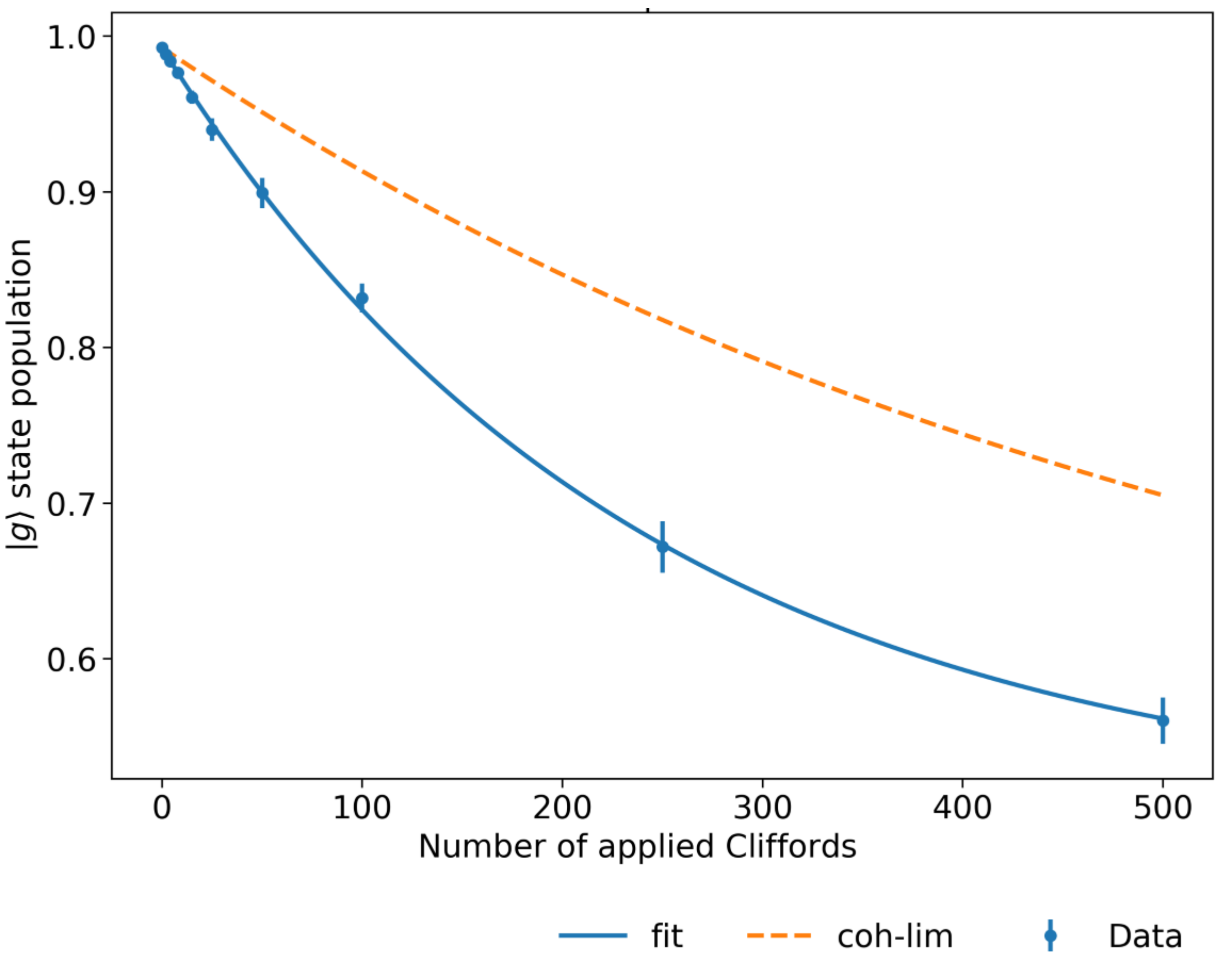
- For simplicity, consider single-qubit case.
- Apply sequence of n gate operations U_i before measuring.



- Gates U_i are chosen randomly from the Clifford group, mapping an element of the Pauli group to an element of the Pauli group. For a single qubit there are 24 Clifford gates.
- Last gate U_{n+1} is chosen such that in the absence of errors state is brought back to initial state, i.e. $U_{n+1} \ldots U_2 U_1 = I$.
- Average over m different such sequences.
- Success probability p_0 to recover the initial state decays exponentially with # of gates $p_0 \propto \alpha^n$, with depolarization parameter α.
- The error per gate is given by $\epsilon_{RB} = (1-\alpha)\frac{(d-1)}{d}$ where d is the dimension of Hilbert space (d=2 for a single qubit).

Knill et al., PRA **77**, 012307 2008

6.3 Randomized benchmarking: Example single qubit gates



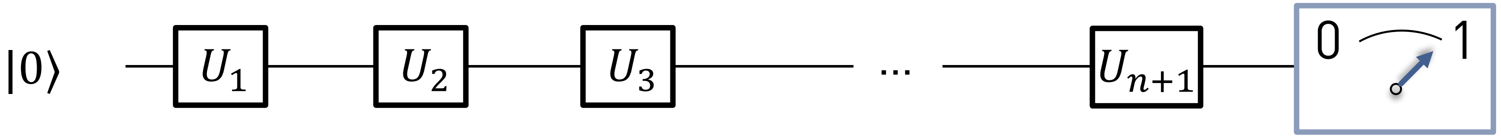
- Pulse duration typically between 15 to 50 ns. Leakage into second excited state avoided by using (DRAG*) pulse parametrization.
- Use Clifford decomposition in terms of X rotations and virtual Z gates (see McKay et al., PRA (2017)).
- Population of ground state decays exponentially.
- Fitted depolarization parameter $\alpha \approx$99.6% and gate error $\epsilon \approx$ 0.2% in this example.
- Orange dashed line indicates the limit expected when only considering qubit decoherence.
- Deviation from coherence limit hints at finite control errors, e.g. resulting in leakage to the f-level.

*Motzoi et al., PRL 103, 110501 (2009)

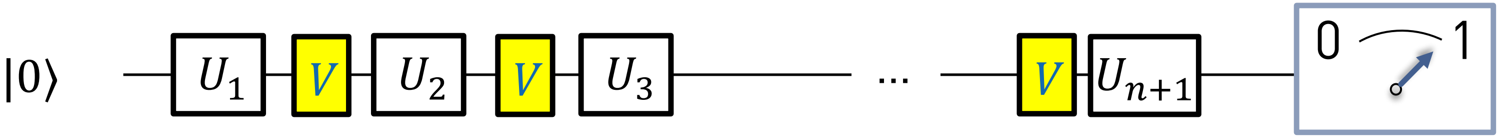
6.3 Interleaved Randomized benchmarking (IRB)

Question how to characterize the fidelity of one particular Clifford gate V?

- Compare decay of standard RB sequence …



- …with result of a 2nd experiment, in which gate V gets interleaved with random Clifford gates.



- Difference between the depolarization parameters α_{RB} and α_{IRB} results in an estimate for the error $\epsilon_V = \frac{d}{d+1}(1 - \frac{\alpha_{IRB}}{\alpha_{RB}})$ per gate V.

- Randomized Benchmarking can be generalized to multi-qubit gates (see problem set 10).

Magesan et al., PRL 109, 080505 (2012)