# Remarks on Dispersive Readout

- Measurement rate scales with $\kappa$ and $\chi$.
- Measurement fidelity limited by SNR:
- **Signal** amplitude bound by the maximum number of photons $n \lesssim n_{crit} = \Delta^2/4g^2$ in the resonator before dispersive approximation breaks down. Alternative coupling mechanisms, which aim to overcome this limitation are under development.
- **Noise** ultimately limited by vacuum noise (for linear detectors). Low-noise (parametric) amplifiers required to get close to this fundamental limit.
- Use pulse shaping and advanced signal processing to optimize fidelity and speed of readout.

Questions to be continued today:

- How to detect microwave frequency radiation?
- Linear amplification at the quantum noise limit.
- How to quantify the efficiency of the measurement chain?

Challenges:

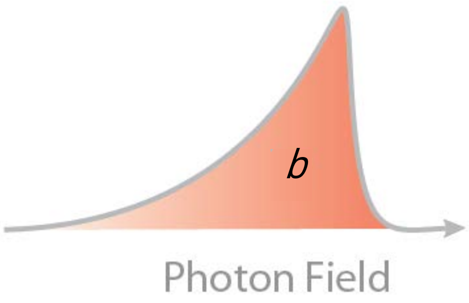
- Typical signals are weak
  $$P_{out} = \hbar\omega \, \langle b_{out}^{+} b_{out} \rangle = \hbar\omega \, \kappa \langle a^{+} a \rangle$$
  $$\sim 10^{-16}\, W = -130\ \mathrm{dBm}$$
- No (commercially available) photon counters in this frequency range.

Most common approach:

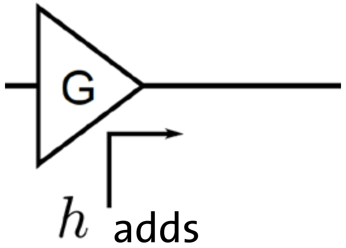
- Use linear amplification in combination with analog/digital signal processing for detection.

# Generic model for linear field detection

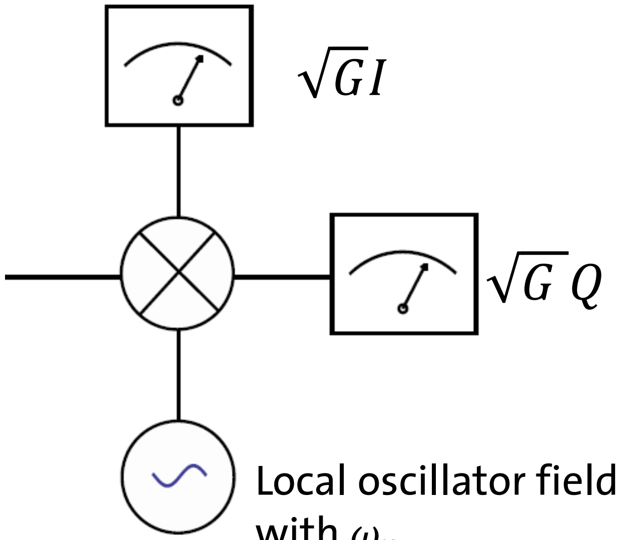
Amplitude detector = Analog and digital signal processing




With carrier frequency $\omega_r$




adds vacuum/thermal noise




Local oscillator field with $\omega_r$

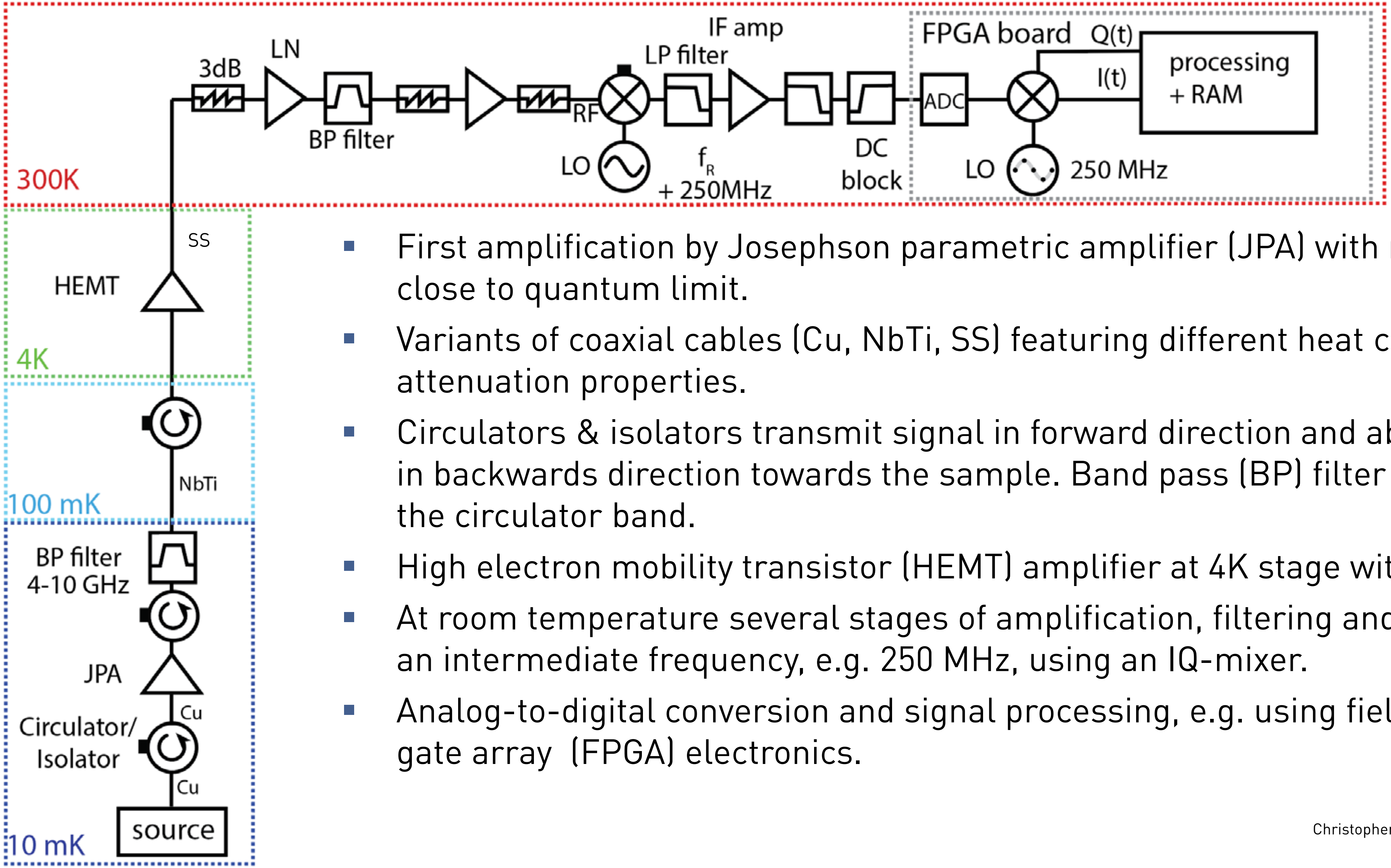
Complex amplitude: $\boxed{I + i\,Q = b + h^{+}}$

"signal" "noise"

Detection effiiciency: $\eta = 1/(1 + \langle h^{\dagger} h \rangle)$

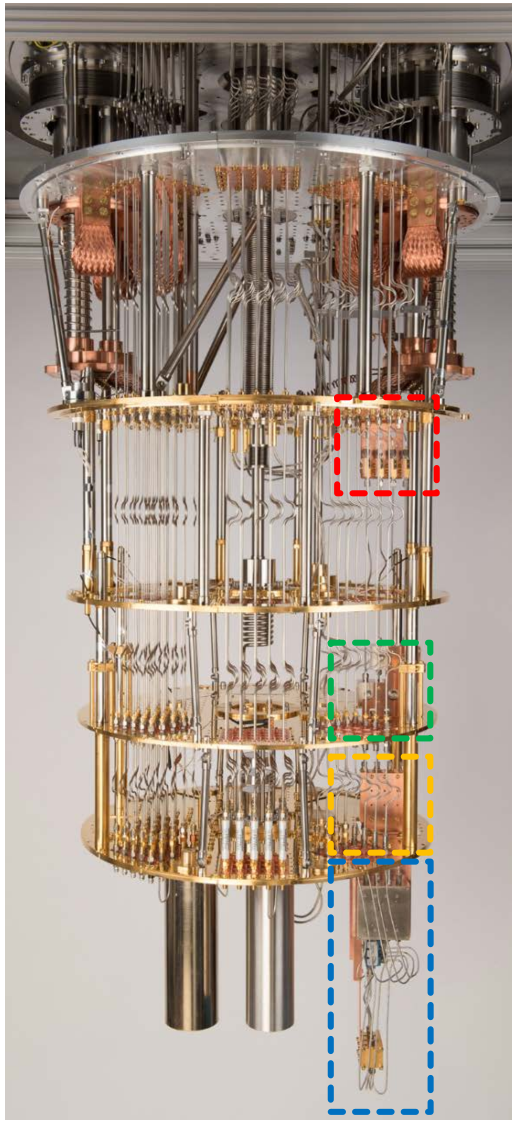
Eichler et al., *PRA* 86, 032106 (2012)
M. P. da Silva et al., *PRA* 82, 043804 (2010).
C. M. Caves, *PRD* 26, 1817 (1982).

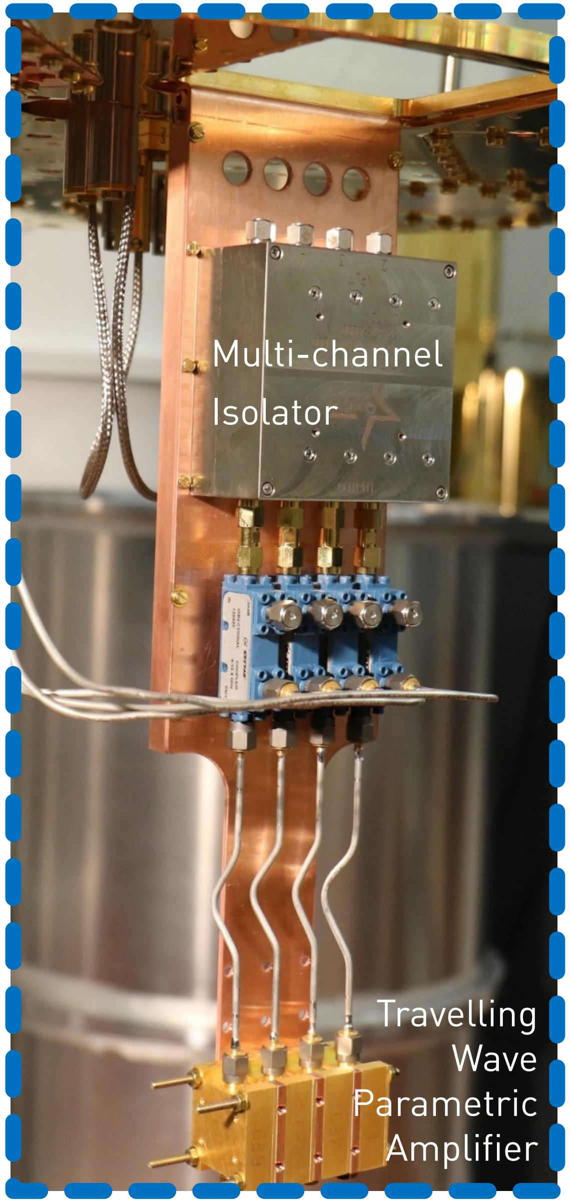
# Detection setup

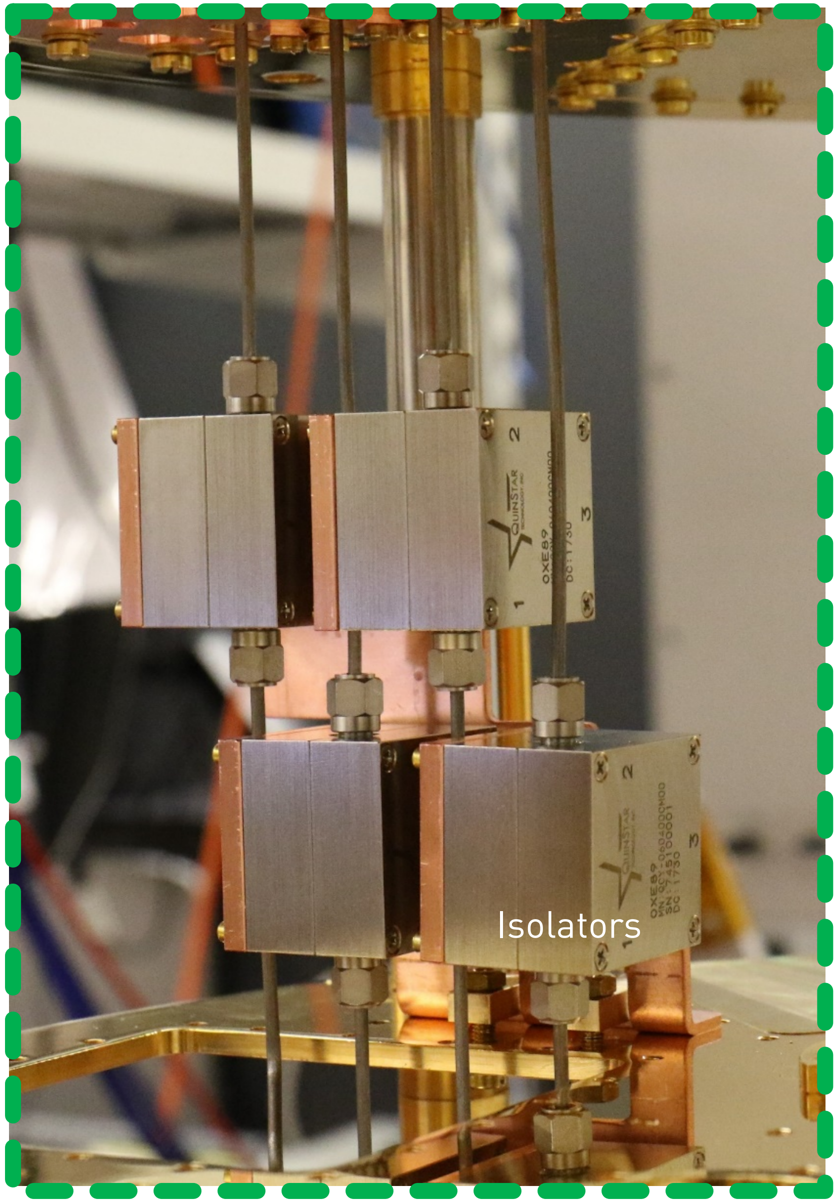



- First amplification by Josephson parametric amplifier (JPA) with noise performance close to quantum limit.
- Variants of coaxial cables (Cu, NbTi, SS) featuring different heat conductance and attenuation properties.
- Circulators & isolators transmit signal in forward direction and absorb noise travelling in backwards direction towards the sample. Band pass (BP) filter block noise outside the circulator band.
- High electron mobility transistor (HEMT) amplifier at 4K stage with typical $N_{noise} \sim 10$.
- At room temperature several stages of amplification, filtering and down-conversion to an intermediate frequency, e.g. 250 MHz, using an IQ-mixer.
- Analog-to-digital conversion and signal processing, e.g. using field programmable gate array (FPGA) electronics.

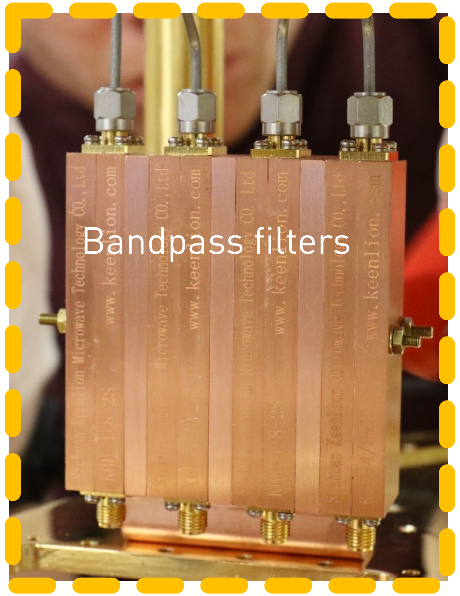
# Output Lines





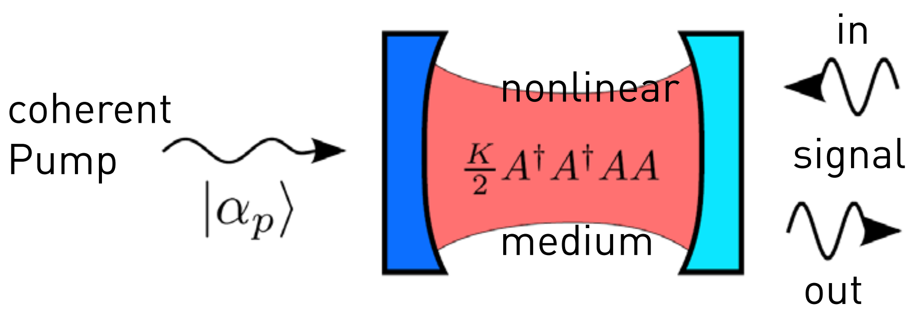





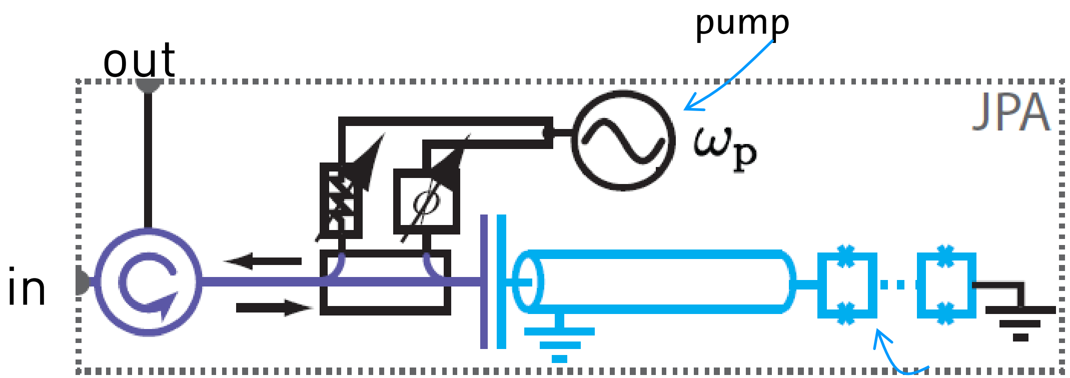



S. Krinner *et al.*, *EPJ Quantum Technology* ***6***:2 (2019)

# Josephson Parametric Amplifier (JPA)

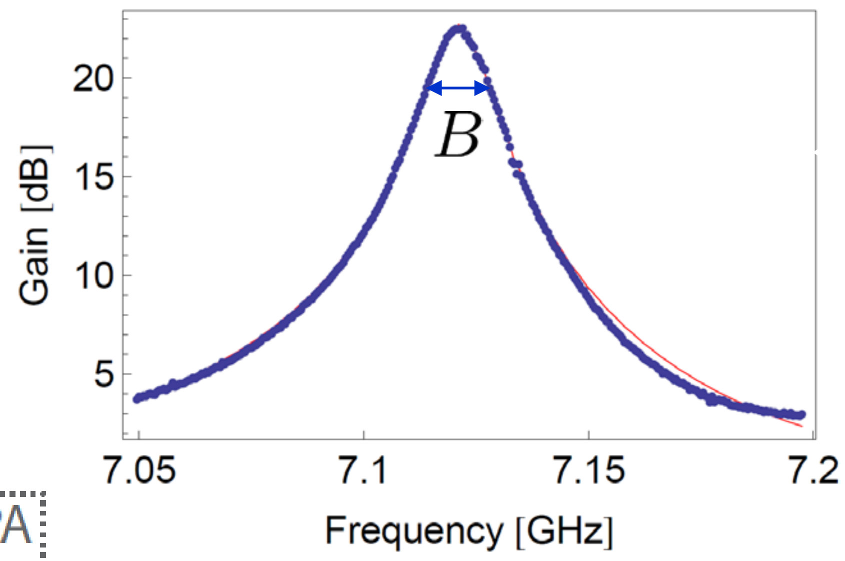



Circuit QED implementation:




SQUIDs provide nonlinearity!

measured gain + theory fit




Gain bandwidth limit:
$B = \kappa/\sqrt{G}$

Yurke *et al., PRL* 60, 764 (1988)
Castellanos-Beltran et al., *Nat. Phys.* 4, 929 (2008)
Eichler *et al., PRL* 107, 113601 (2011)
Eichler and Wallraff, EPJ 1, (2014)

# Parametric amplifier: Basic Theory

System Hamiltonian

$$H_{\mathrm{JPA}} = \hbar\tilde{\omega}_0 A^\dagger A + \hbar\frac{K}{2}\left(A^\dagger\right)^2 A^2$$

Typical parameter regime:

$K << \kappa$

Equation of Motion:

$$\dot{A} = -i\tilde{\omega}_0 A - iKA^\dagger AA - \frac{\kappa+\gamma}{2}A + \sqrt{\kappa}A_{\mathrm{in}}(t) + \sqrt{\gamma}b_{\mathrm{in}}(t)$$

Decompose field into "pump" and "signal":

$$A_{\mathrm{in}}(t) = \left(a_{\mathrm{in}}(t) + \alpha_{\mathrm{in}}\right)e^{-i\omega_p t}$$

(quantum) signal

Classical pump

Find steady-state solution for pump field.
Linearized equation for signal field (assuming $|\alpha_{\mathrm{in}}|^2 \gg |a_{\mathrm{in}}|^2$ )
Frequency domain $t \rightarrow \Delta$. Here, $\Delta$ is the detuning from the pump.

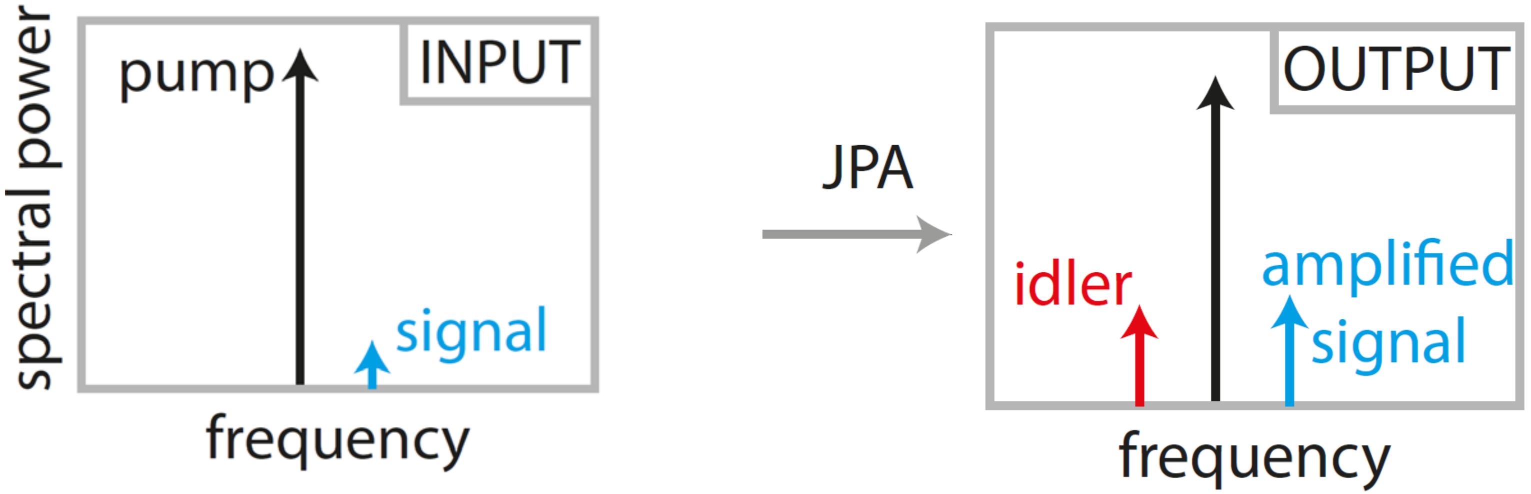
$$\boxed{a_{\mathrm{out},\Delta} = g_{S,\Delta}a_{\mathrm{in},\Delta} + g_{I,\Delta}a^\dagger_{\mathrm{in},-\Delta}}$$

Gain:

$$G_\Delta \equiv |g_{S,\Delta}|^2 = |g_{I,\Delta}|^2 + 1$$

Yurke and Buks, *J. Lightwave Tech.* 24, 5054 (2006)
Eichler and Wallraff, EPJ 1, (2014)

# Parametric amplifier: IN vs. OUT spectrum




- Four-wave mixing: $\boxed{2\omega_p = \omega_s + \omega_i}$

Scattering picture: "Two pump photons are converted into a signal-idler pair"

- Signal and idler are highly correlated
  -> Vacuum squeezing
- Ideally no added noise from coupling to other modes ("Quantum limited"). Noise temperature set by the radiation temperature at the input of the amplifier.
- Many different variants of basic working principle
  Single mode devices, multi-mode devices, travelling-wave amplifiers
  4-wave mixing, 3-wave mixing, multi-pump schemes