Problem Set 8

Problem 1: Measurement induced dephasing

Objective: Understand the concepts of measurement induced dephasing and pointer states.

We consider the dispersive Hamiltonian of a coupled qubit-cavity system

$$\mathcal{H}/\hbar = \omega_{\text{res}} a^\dagger a + \omega_{ge}\sigma^+\sigma^- + \chi\sigma_z a^\dagger a, \tag{1}$$

where ω_{res} is the resonator frequency, a the ladder operator, ω_{ge} the qubit frequency, $\chi = 2\pi \cdot 1$ MHz the dispersive shift, and $\sigma^+ = |e\rangle\langle g|$, $\sigma^- = |g\rangle\langle e| = (\sigma^+)^\dagger$ and $\sigma_z = |g\rangle\langle g| - |e\rangle\langle e|$ the Pauli matrices.
We perform readout by applying a driving pulse to the resonator, of amplitude Ω_0, between time $t_i = 0$ and $t_f = 1\ \mu$s, corresponding to a drive Hamiltonian $\mathcal{H}_d/\hbar = \Omega(t))(a^\dagger + a)$, see Fig. 1. We pick $t_f \gg 1/\kappa$, with $\kappa = 2\pi \cdot 2$ MHz $\approx (80\text{ ns})^{-1}$ the external linewidth of the cavity, such that we are in a quasi steady-state.

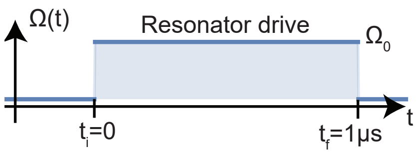

Figure 1: Pulse scheme used for readout of the qubit.

We account for resonator decay by solving the Lindblad master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}(c), \tag{2}$$

where the first part is the standard Schrödinger equation, and $\mathcal{L}(c) = c\rho c^\dagger - \frac{1}{2}(c^\dagger c\rho + \rho c^\dagger c)$ accounts for decay, with a collapse operator $c = \sqrt{\kappa}a$. Collapse operators can be passed as the fourth argument of the function mesolve in qutip.

a) Show that the dispersive Hamiltonian $\mathcal{H}$ can be seen as a qubit-state dependent cavity frequency $\omega_{\text{res}} \pm \chi$. Argue from your derivations in Problem Set 5 that a two-sided cavity, with a large asymmetry $\kappa \gg \kappa_{\text{in}}$, has a response of the form $S_{21g(e)}(\omega) = \frac{\sqrt{\kappa\kappa_{\text{in}}}}{\kappa/2 + i(\omega - \omega_{\text{res}} \mp \chi)}$. Plot the cavity transmission versus frequency for each of the qubit ground and excited state. At a drive frequency of ω_{res}, plot the steady-state $S_{21g(e)}(\omega_{\text{res}})$ in the complex plane.

b) Convince yourself that the Hamiltonian $\mathcal{H}$ in a rotating frame of both the cavity and the qubit reads $\mathcal{H}/\hbar = \chi\sigma_z a^\dagger a$. With the provided python script, simulate the mean state-dependent intra-cavity field $\alpha_{g(e)} = \langle a\rangle_{g(e)}$, as well as the evolution an initial qubit state $|\psi_0\rangle = |g + e\rangle/\sqrt{2}$. Argue that the amplitude of the integrated output field is $\tilde{\alpha}_{g(e)} \approx \sqrt{\kappa t_f}\alpha_{g(e)}$ resulting in the measurement contrast $s = |\tilde{\alpha}_e - \tilde{\alpha}_g| = \sqrt{\kappa t_f}|\alpha_g - \alpha_e|$. Plot the coherence (entry ρ_{ge} of the qubit density matrix) versus s and show that it is independent of the exact parameters κ and χ. Explain the observation.

c) Argue that the qubit state ψ_0 evolves after the measurement to a correlated state $(|\tilde{\alpha}_g g\rangle + |\tilde{\alpha}_e e\rangle)/\sqrt{2}$. Discuss the implications of this state for large measurement contrast.