Outline

Last week (lecture 1):

- Organization of the course
- Brief introduction to Quantum Information Science and Quantum Computing

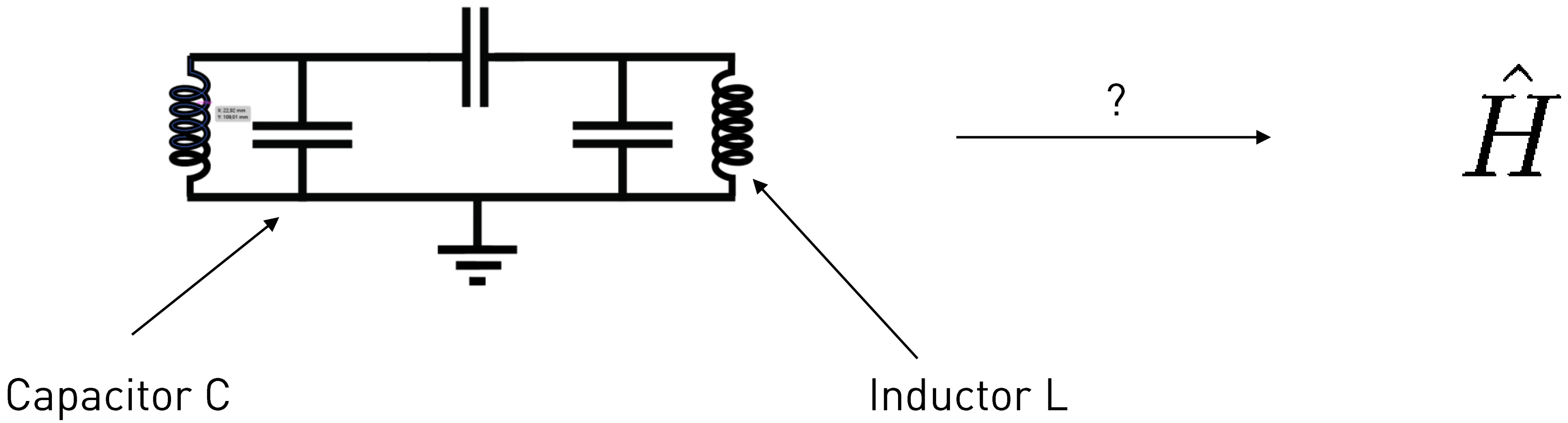
This week (lecture 2):

- Quantization of electrical circuits
  - Step 1: "Given an electrical circuit composed of inductors and capacitors, find the corresponding system Hamiltonian."
  - Later add …
    - … **resistors** to model controlled and uncontrolled coupling to the environment.
    - … **Josephson junctions** as a nonlinear circuit element.

Big picture: Build a quantum computer from electrical circuit elements.

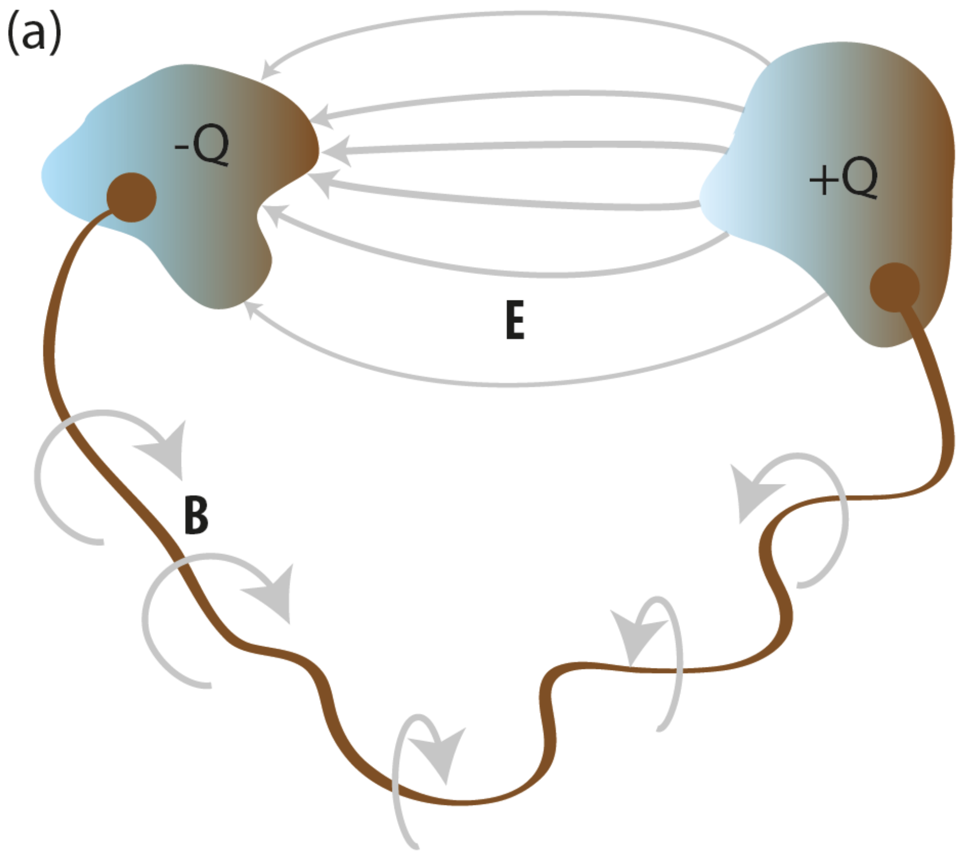
2) Quantization of electrical circuits

Goal: Given an electric circuit composed of inductors and capacitors, find the system Hamiltonian *H.*



Understand: In which regime is a lumped element representation of an electrical circuit justified?

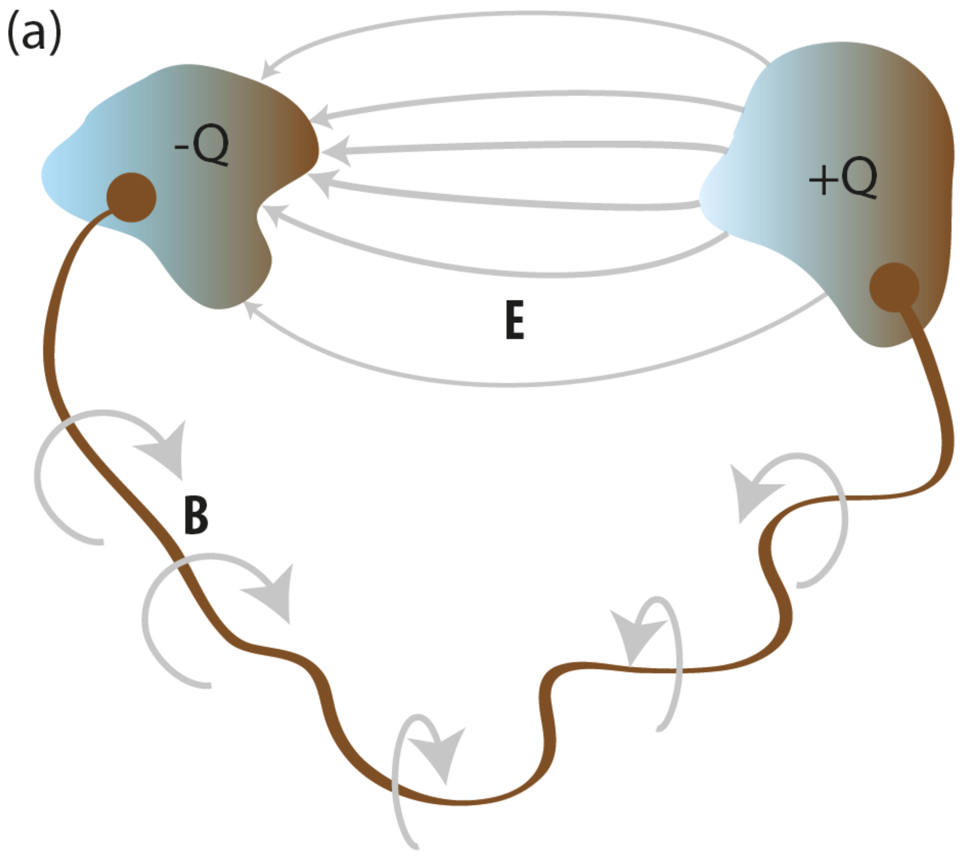
2.1 Electrical circuits in the lumped element regime



We consider the system shown in Figure 2.1(a), which consists of two metallic islands connected by a wire. For simplicity, we assume that there is no resistive loss in the metal[1] and that the surrounding dielectric medium is the vacuum. From Maxwell's first equation $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ it follows that a net charge difference $2Q$ between the two objects leads to an electrical field $\mathbf{E}$. If we prohibit, for the moment, a flow from one to the other island the charges on each side will arrange on the surface such that the metal stays free of electrical field inside. In this static situation, the energy required for moving one charge through the electrical field from one to the other side is path-independent and therefore it is convenient to define the position independent voltage V as this energy per unit charge. The voltage V is proportional to the charge difference $2Q$ between the two islands $V = Q/C$, where C is the constant capacitance between the islands, which only depends on the shape and geometrical arrangement of the two objects as well as the dielectric medium in between. In this static situation the total energy stored in the electrical field is thus given by $E_{\mathrm{el}} = Q^2/2C$.

C. Eichler, PhD thesis (2013)

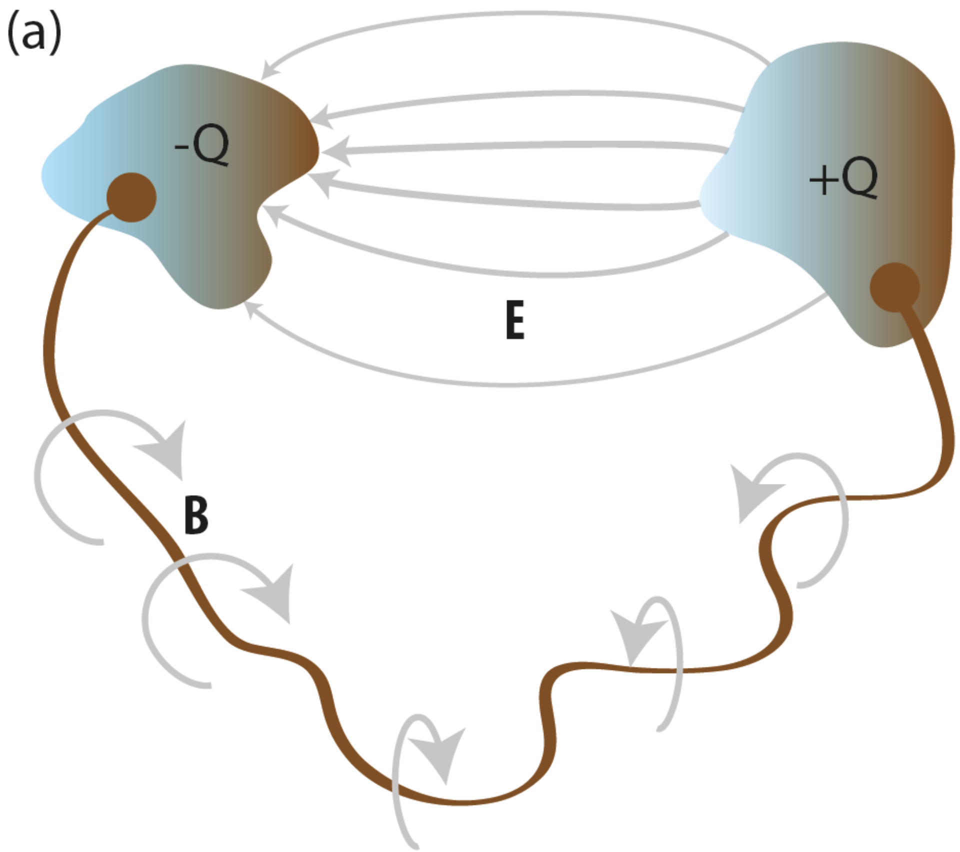
2.1 Electrical circuits in the lumped element regime



The time required for relaxing into a quasi-static field configuration for an object of characteristic size d is typically on the order of $\tau \sim d/c$ where c is the speed of light in the relevant dielectric medium. If τ is much smaller than the timescale on which the total charge $Q(t)$ on the object changes, the spatial charge configuration as well as the electrical field lines can follow quasi-instantaneously. If this is the case, the finite extent of the object is fully captured by the single parameter C. The object can thus be represented by a pointlike – or lumped – effective element. As long as $Q(t)$ changes on timescales much larger than τ we have $E_{\mathrm{el}}(t) = Q(t)^2/2C$ at all times. Note that for typical metals the inverse plasma frequency of the electron gas density is also very small such that waves of the electron gas density do not get excited at the frequencies we are interested in [Girvin11].

C. Eichler, PhD thesis (2013)

2.1 Electrical circuits in the lumped element regime



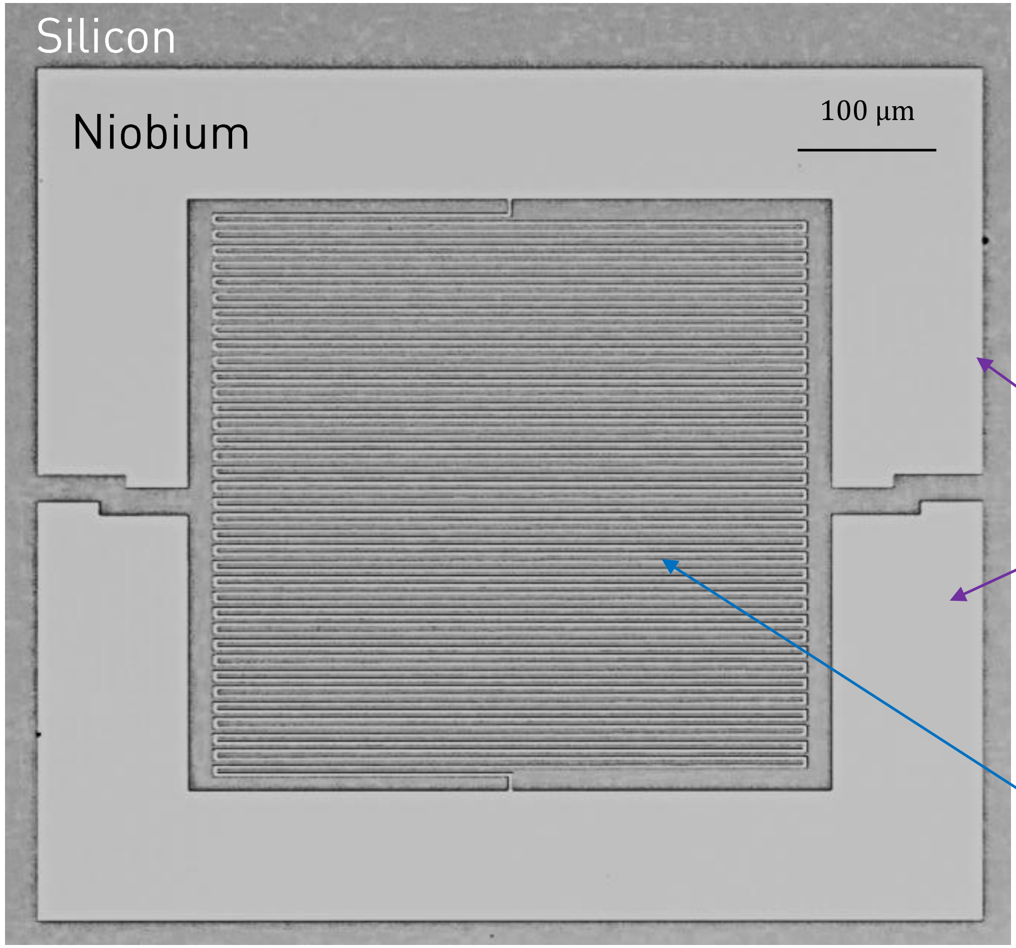
If charge carriers can flow from one to the other object through the wire, the associated current density $\mathbf{j}$ produces a magnetic field $\mathbf{B}$ according to $\nabla\times\mathbf{B} = \mu_0\mathbf{j}$ (see Figure 2.1). Assuming that the wire is short compared to the characteristic wavelength of the system, the current I is constant along the wire and given by the change of charges on the islands $I = \dot{Q}$. Similarly to the electrical field case, there is a constant proportionality between the square of the current and the energy stored in the magnetic field $E_{\mathrm{mag}} = L\dot{Q}^2/2$. The inductance L is an effective parameter which is constant and determined by the geometry of the wire and the surrounding magnetic susceptibility. The total energy $E_{\mathrm{tot}} = Q^2/2C + L\dot{Q}^2/2$ stored in the magnetic and electric fields only depends on the dynamics of the single charge variable Q, which can therefore be interpreted as a collective variable representing the effective dynamics of all the involved charge carriers. Note that the kinetic energy contribution of the moving charge carriers is typically very small compared to the energy stored in the magnetic field and is therefore negligible. However, for thin superconducting wires of nanoscale

C. Eichler, PhD thesis (2013)

2.4 Planar Lumped Element Resonators

- Resonators fabricated from a 150nm thin Niobium film on a Silicon substrate.
- Inductance and capacitance controlled by designed geometry.
- Characteristic size much smaller tan characteristic wavelength.
- Accurate prediction of resonance frequency needs to take stray capacitance of inductor into account.
- Measured in a metallic cavity with an sender (IN) and receiver (OUT) antenna.

Problem set 2: Estimate capacitance of a planar capacitor.



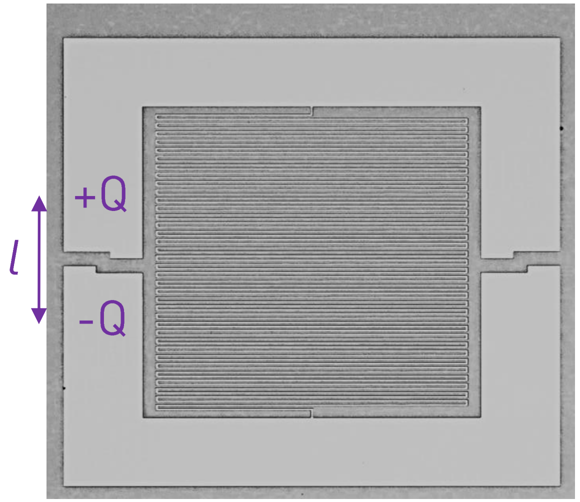
Resonance frequencies. typically between 4-8 GHz

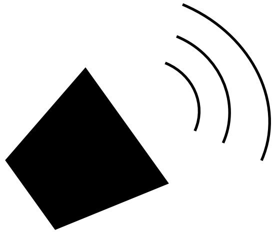
Two plates forming a capacitor. Typical values: C ~ 50 -500 fF

Meandering wire forming an inductor. Typical values L ~ 2 - 20 nH

2.5 Coupling Planar Lumped Element Resonators to an E-field

Apply Electromagnetic field at resonator frequency ω_0





Electric field $\underline{E}$

- Coupling mediated by dipole-field interaction

$$H_{\mathrm{d}} = \underline{d} \cdot \underline{E}$$

- Dipole moment

$$\hat{\underline{d}} = \int \mathrm{d}^3 r \, \underline{r} \, \rho(\underline{r}) \equiv \hat{Q} \, \underline{l}$$

- With average charge separation *l.*
- For an LC resonator we thus obtain

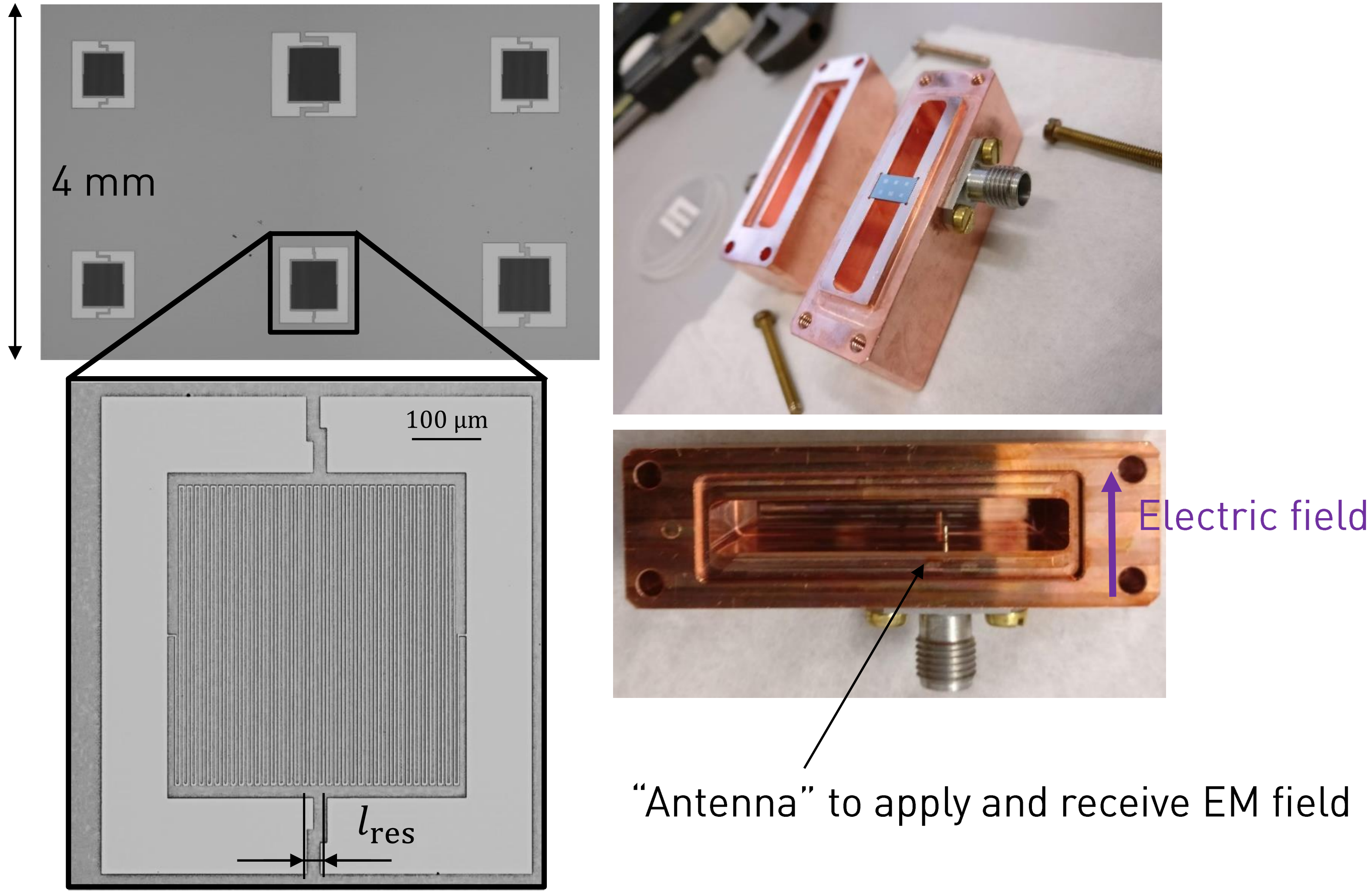
$$H_{\mathrm{d}} = \underline{l} \cdot \underline{E} \, Q_{\mathrm{zpf}} (a + a^\dagger)$$

Note: Applying H_{d} for short duration and with $\underline{E}(t) = \underline{E}_0 \cos(\omega_0 t)$ generates a coherent state:

$$|0\rangle \rightarrow |\alpha\rangle$$

Problem set 2: Estimate dipole moment of a planar *LC* resonator.

2.5 Lumped Element Resonators in 3D Cavities

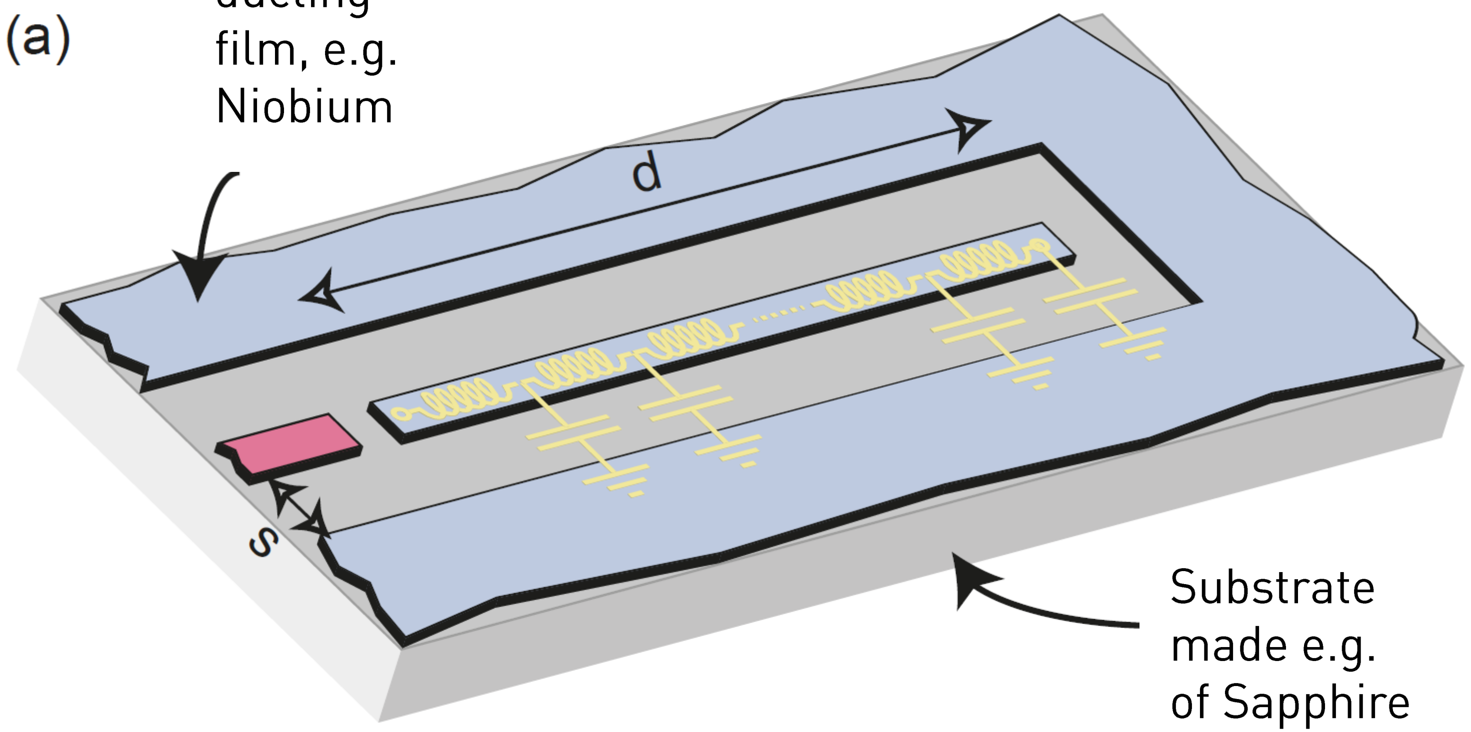


Properties of the system:

- multiple lumped element with different frequencies on a chip
- 3-Dimenesional cavity, typically made of copper or aluminium
- TE101 mode of the cavity couples to the dipole moment of the resonators
- Inner conductor of the connector acts as an antenna to send and receive electromagnetic fields to/from the resonators
- Typical use case: Establish well-controlled environment to test material-related loss mechanisms.

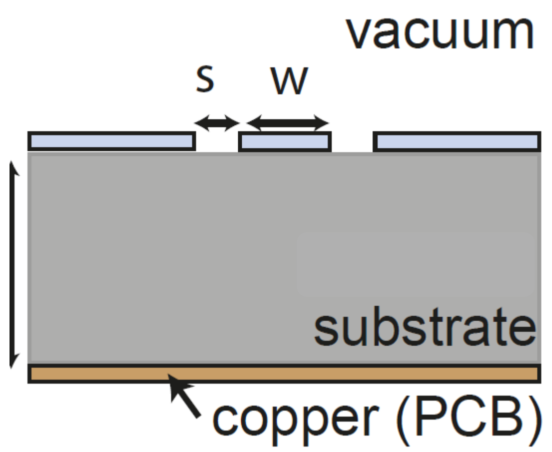
2.7 Distributed coplanar waveguide resonators

Supercon
ducting
film, e.g.
Niobium



Substrate
made e.g.
of Sapphire
or Silicon

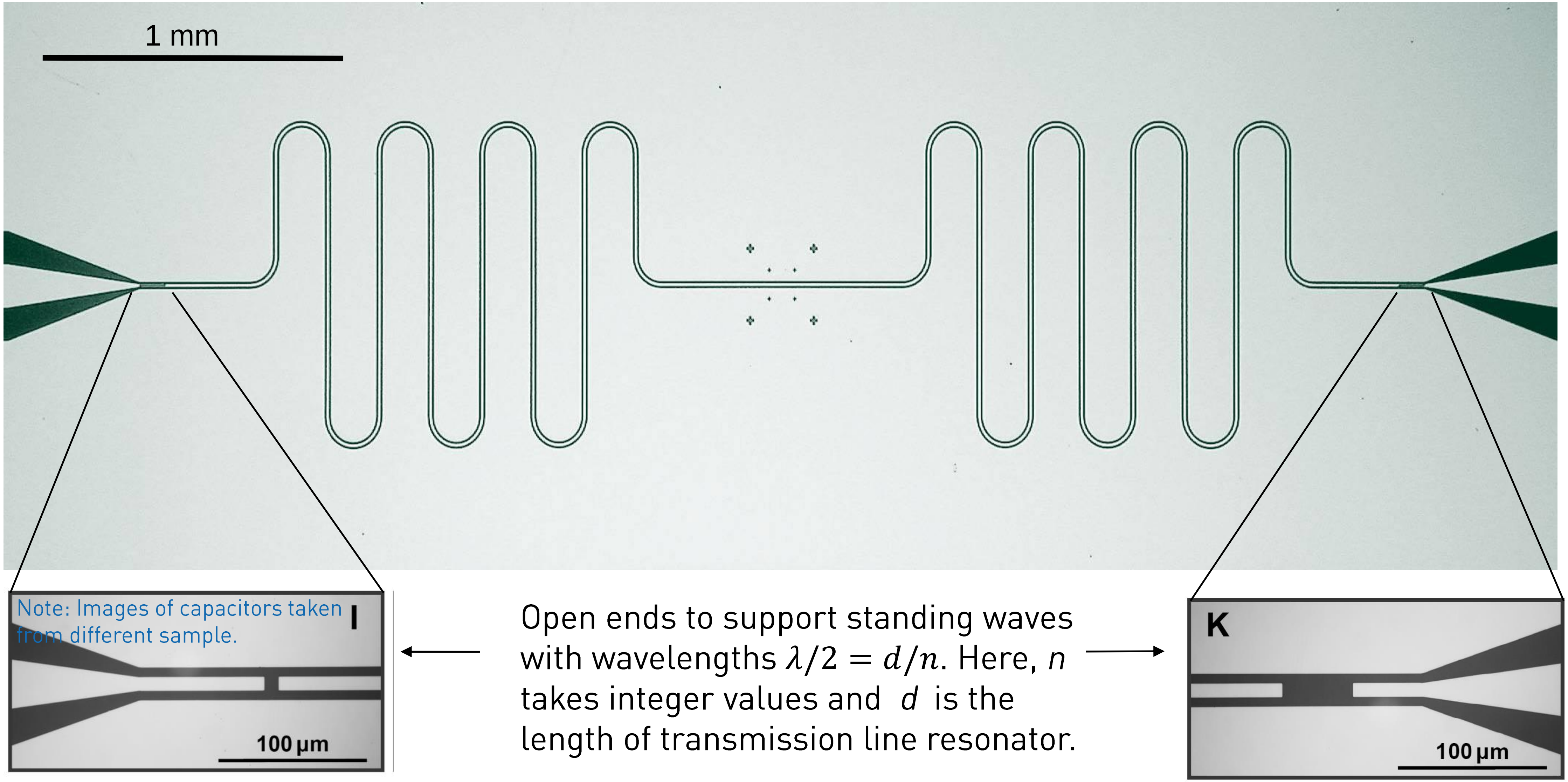
Discussion see
blackboard.



Typical dimensions:

$$s = 4.5\ \mu m$$
$$w = 10\ \mu m$$
$$d \sim 5\ mm$$

2.7 Distributed coplanar waveguide resonators



Open ends to support standing waves with wavelengths $\lambda/2 = d/n$. Here, n takes integer values and d is the length of transmission line resonator.

M. Goeppl et al., J. Appl. Phys. 104, 113904 (2008)