4.8 Amplification and Detection of microwave radiation

Reminder: Typical qubit readout is based on the dispersive interaction with a readout resonator (see 4.7 and slides). Dispersive approximation valid for moderate photon numbers $n \lesssim n_{crit} \approx \frac{\alpha}{4\chi}$.

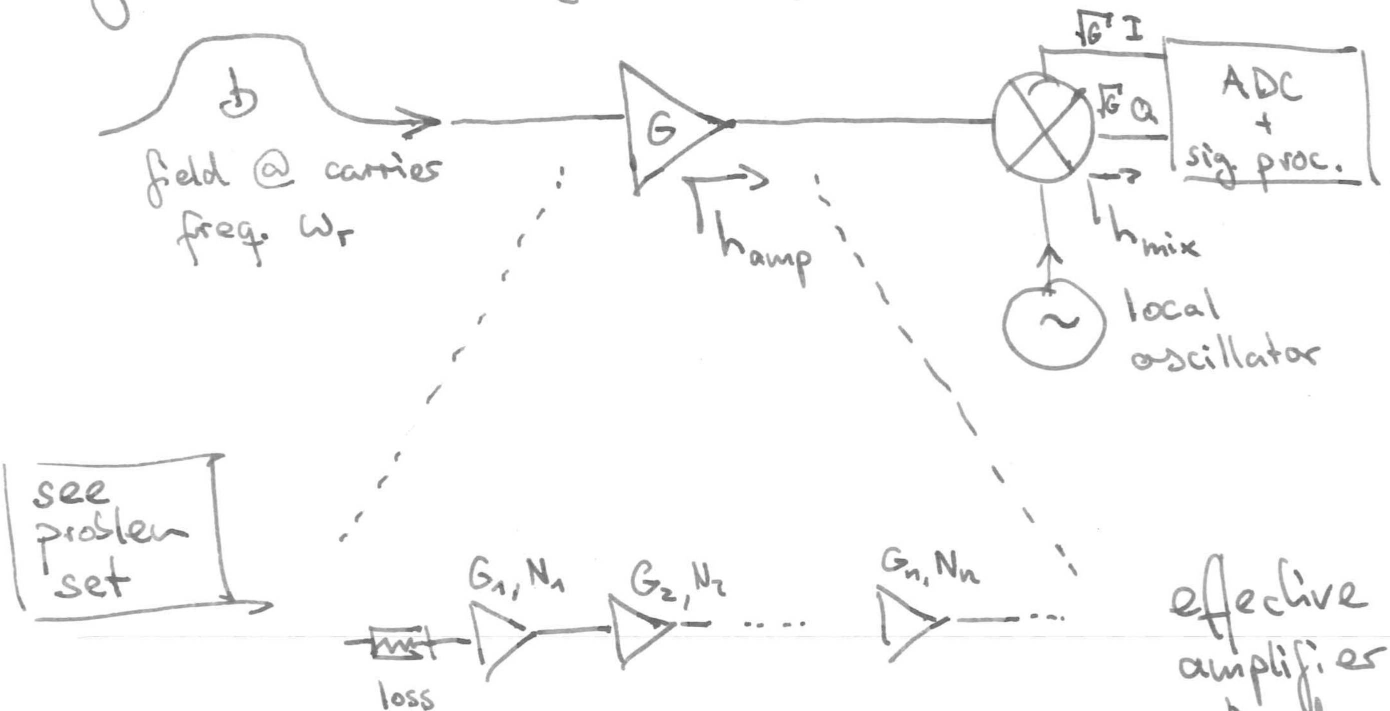
Readout signal at output of resonator typically small:

$$P_{out} = \underbrace{\hbar \omega_r \kappa}_{\sim 10^{-16}\,W} \underbrace{\langle a^\dagger a \rangle}_{\sim \mathcal{O}(1)} \approx -130\,\mathrm{dBm}$$

How to detect in the absence of commercially available photon detectors in GHz frequency range?

$\Rightarrow$ Linear amplification + analog/digital signal processing

Generic model of linear field detection



effective amplifier realized as a chain of multiple amplifiers.

Generic amplifier transforms

$$b \to \sqrt{G}\, b + \sqrt{G-1}\, h^{+}_{amp}$$

compare Caves 1982.

exercise: check that output field satisfies bosonic commutation relation.

Mixer splits the signal internally and multiplies each output with $\sin(\omega_r t)$ and $\cos(\omega_r t)$, respectively, by mixing with local oscillator field, to obtain quadrature amplitudes I, Q.

Splitting the signal requires to introduce an additional mode h_{mix} such that

$$\boxed{I + iQ = b + h^{+}}$$ complex amplitude

with noise mode

$$h = \sqrt{\frac{G-1}{G}}\, h_{amp} + \sqrt{\frac{1}{G}}\, h_{mix}$$

For large gain $G \gg 1$, the total noise is dominated by the amplifier noise

$$h \approx h_{amp}$$

even for $\langle h^{+}_{mix} h_{mix} \rangle \gg 1$ at room temperature.

The detection efficiency is defined as $\eta = \frac{1}{1+\langle h^{+}h \rangle}$, with $\langle h^{+}h \rangle \equiv N_{noise}$ being the average number of (typically thermal) photons in the effective noise mode h.

Discussion and comments:

1) Even for ideal case of h being in the vacuum state

$$\overline{\Delta I^2} = \overline{\Delta Q^2} = \frac{1}{2} > 0$$

=> Consequence of Heisenberg's uncertainty principle in combination with simultaneous amplification & measurement of two conjugate field quadratures.

2) Alternatively, amplification of single quadrature possible using a „squeezer"

$$b \rightarrow e^{i\varphi}\sqrt{G}\, b + e^{-i\varphi}\sqrt{G-1}\, b^{+}$$

... φ controls which quadrature component is (de-)amplified.

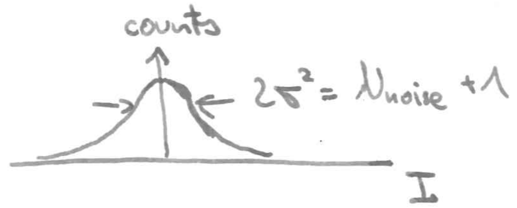
3) Amplifier noise is typically thermal

$$\rho_h = \frac{1}{Z} e^{-\frac{\hbar\omega \hat{n}}{k_B T_{noise}}}, \quad \text{with } N_{noise} = n_{BE}(T_{noise}) \approx \frac{k_B T_{noise}}{\hbar\omega}$$

and uncorrelated from signal

$$\rho = \rho_b \otimes \rho_h .$$

Gaussian distribution



4) Corresponding measurement operator in POVM formalism

$$\hat{\Pi}(\alpha) = |\alpha\rangle\langle\alpha| , \quad \alpha = I + iQ$$

for $N_{noise} = 0$

coherent state

4.9 Measurement & Decoherence

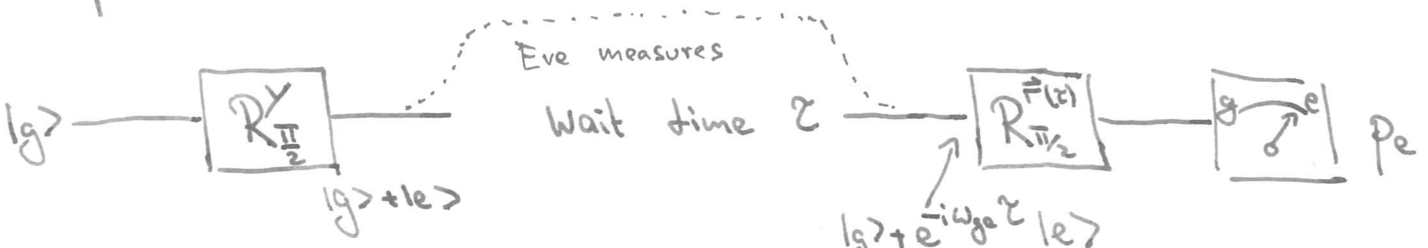
Let's go back to the qubit measurement.

Q: What happens to the qubit while being dispersively measured?

Consider the following Gedanken experiment:

"Alice" performs Ramsey experiment

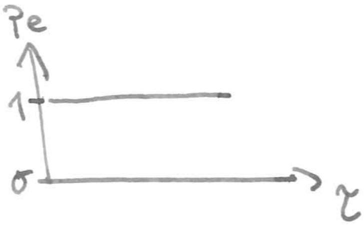
"Eve" performs dispersive measurement during wait time



Dispersive interaction $\chi\sigma_z a^\dagger a$

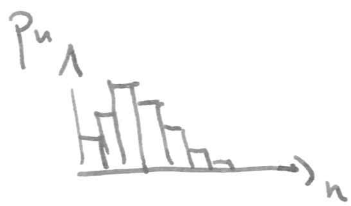
$\omega_{ge} \rightarrow \omega_{ge} - 2n\chi$

for perfect qubit



How many photons n are in the resonator while reading out with a coherent field?

$$P_n = |\langle n|\alpha\rangle|^2 = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$



Poisson distribution

What is the qubit state after time τ for Alice, who does not know the measurement record of Eve?

$\Rightarrow$ She cannot know exactly, because n fluctuates probabilistically!

Interpretation:

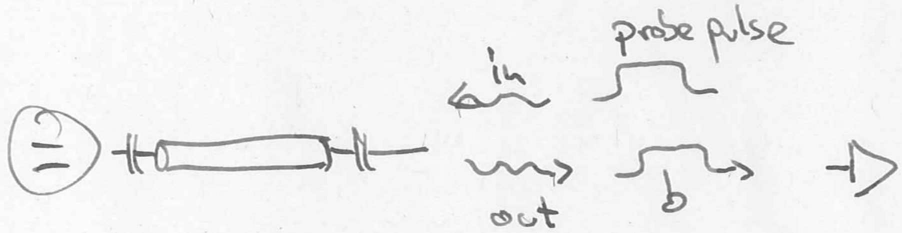
- Eve gains information about observable Z, i.e. projects into a corresponding eigenstate.
- Alice aims at probing the phase of the Bloch vector in xy-plane.
- $[Z, X] \neq 0 \quad \Rightarrow$ incompatible!
- Measurement in Z basis „dephases" the qubit.

Measurement induced dephasing can be used to quantify the efficiency of the detection chain η.

Consider the case of an actual experiment in which we take both roles that of Alice and Eve.

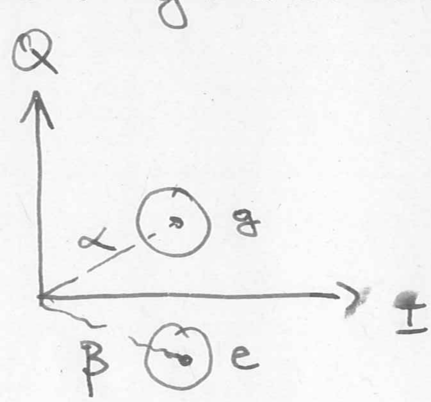
prepare

$|g\rangle + |e\rangle$



Output mode b and qubit become entangled/correlated:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\,\alpha\rangle + |e\,\beta\rangle\right)$$

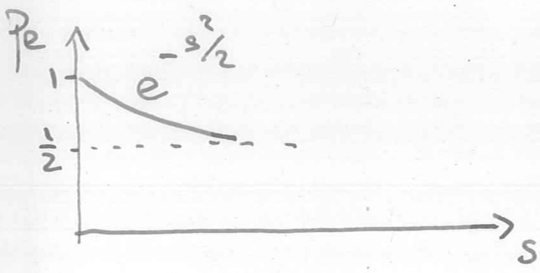


Take partial trace over field degree of freedom to obtain

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & e^{-s^2/2} \\ e^{-s^2/2} & 1 \end{pmatrix}$$

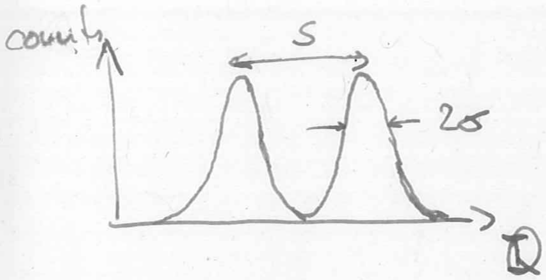
reduced density matrix of qubit.

Ramsey experiment with measurement interleaved ... [next page]



$\Rightarrow$ use to calibrate s.

Comparison with the histogram (projected on Q quadrature)



$$\eta = \frac{1}{1+N_{noise}} = \frac{1}{2\sigma^2} = \frac{SNR^2}{2s}$$

allows to determine η, which includes all types of inefficiencies such as cable losses, amplifier noise, finite internal Q of resonator, filter inficiencies.