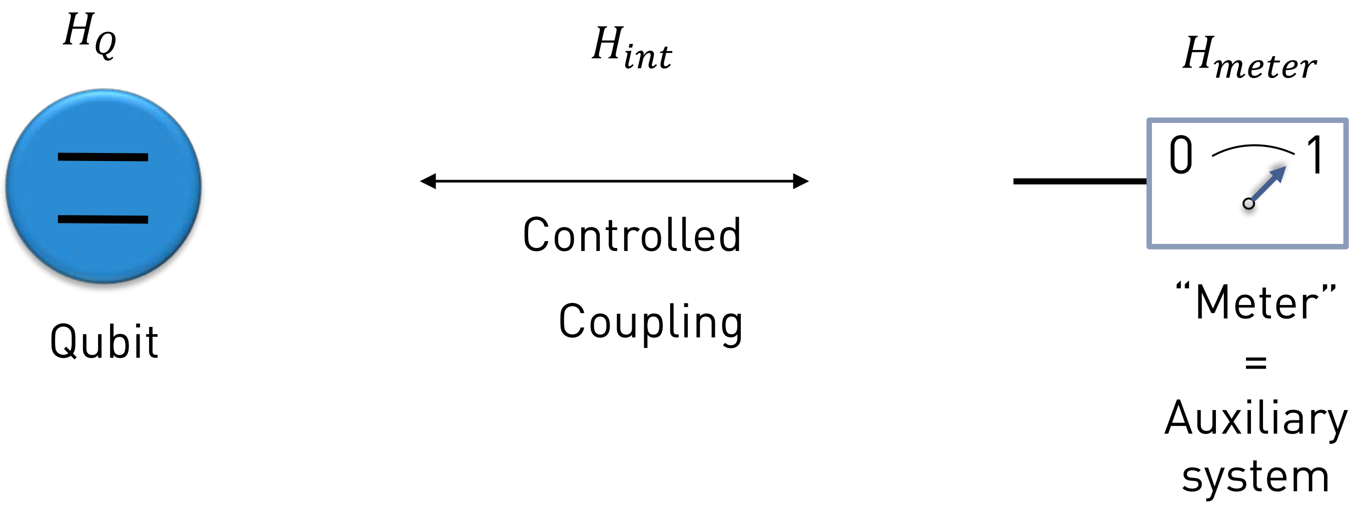
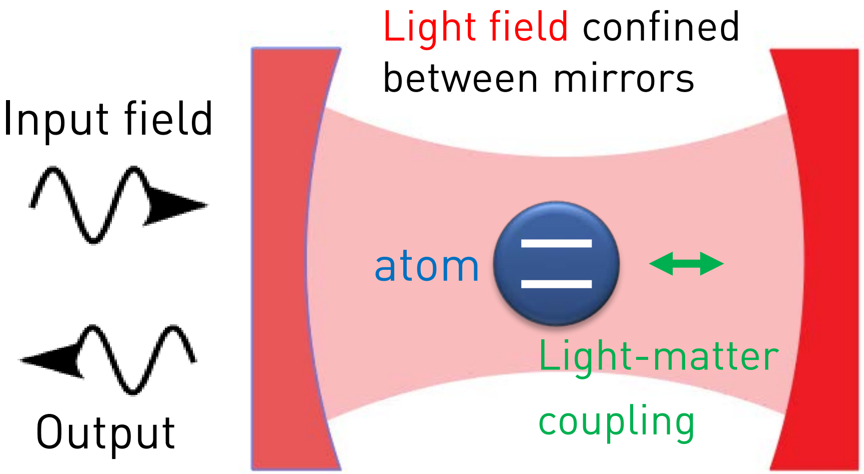
1.1 General properties of quantum measurements



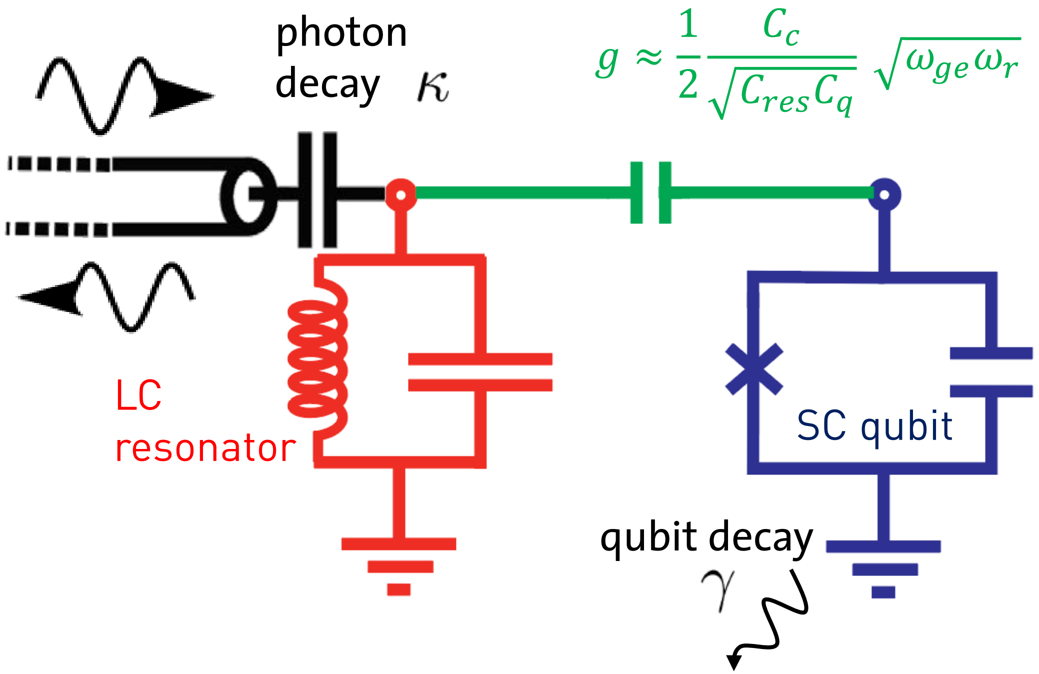
Desirable properties:

- Projective and Quantum non-demolition (QND)
  - Coupling to the meter does not change the state of the qubit $[H_Q, H_{int}] = 0$.
  - Repeated measurement yields the same outcome.
- Good ON/OFF ratio
  - $[H_{int}, H_{meter}] = 0$ during "OFF"
  - $[H_{int}, H_{meter}] \neq 0$ during "ON"
- No spontaneous decay/excitation due to measurement apparatus
- Fast and high fidelity

1.2 Circuit QED



Circuit
equivalent



System Hamiltonian (compare chapter 2):

$$H_{\text{sys}}/\hbar = \omega_r a^\dagger a \quad + \quad \omega_{ge} b^\dagger b - \frac{\alpha}{2}(b^\dagger)^2 b^2 \quad - \quad g(a - a^\dagger)(b - b^\dagger)$$

- Rotating wave approximation (RWA)
- Two-level approximation

$$= \underbrace{\boxed{\omega_r a^\dagger a}}_{\text{Resonator field}} + \underbrace{\boxed{\frac{\omega_{ge}}{2}\sigma^z}}_{\text{qubit}} \underbrace{\boxed{+ g(a^\dagger \sigma^- + a\sigma^+)}}_{\text{coupling}}$$

Jaynes-Cummings Hamiltonian

S. Haroche & J. Raimond, Exploring the Quantum, OUP Oxford (2006)

1.3 Circuit QED: Resonant case and dispersive limit

Jaynes-Cummings Hamiltonian:

$$H/\hbar = \underbrace{\boxed{\omega_r a^\dagger a}}_{\text{quantized field}} + \underbrace{\boxed{\frac{\omega_{ge}}{2}\sigma^z}}_{\text{qubit}} + \underbrace{\boxed{g(a^\dagger\sigma^- + a\sigma^+)}}_{\text{coupling}}$$

Strong coupling regime: $g > \gamma,\ \kappa$

What happens
in the limit of large detuning?

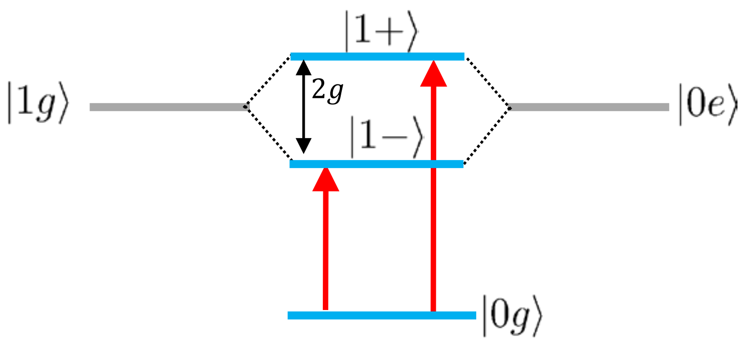
$$|\Delta| = |\omega_{ge} - \omega_r| \gg g$$

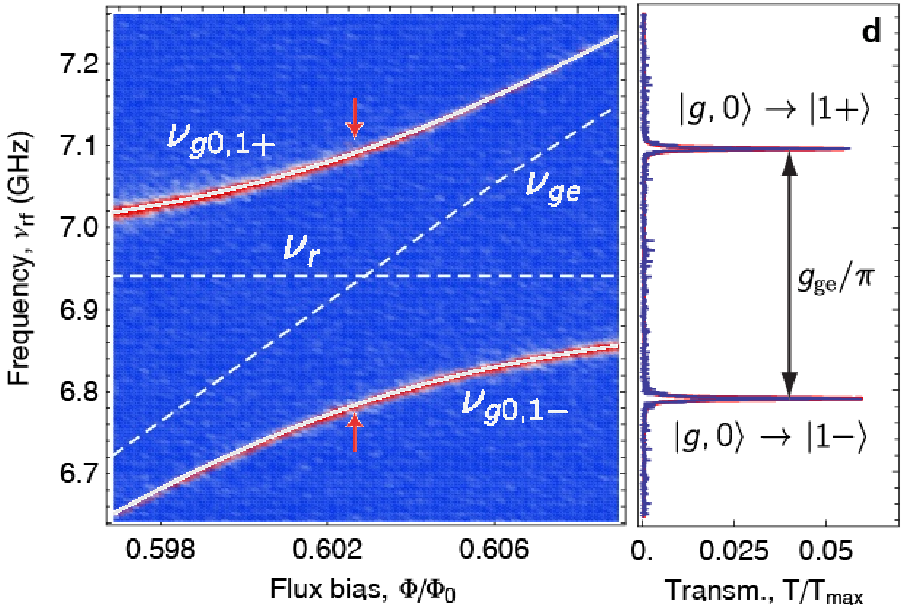
$$\boxed{\chi\sigma_z a^\dagger a}$$

Dispersive coupling

- Limit of large detuning is referred to as the dispersive limit. No resonant exchange of excitations.
- In the dispersive regime coupling Hamiltonian commutes with qubit Hamiltonian.

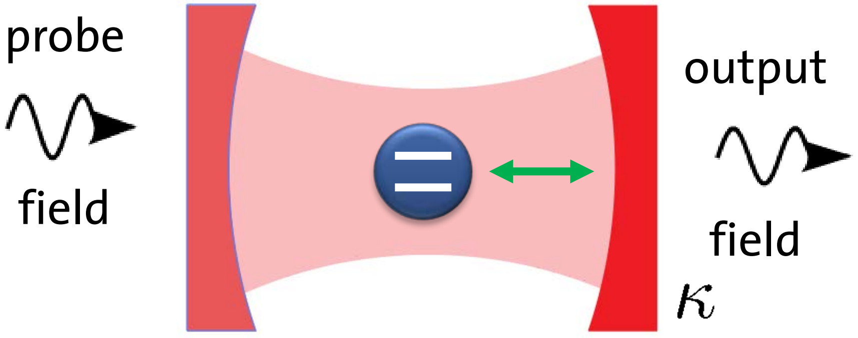
Energy level diagram for resonant case $\omega_r = \omega_{ge}$:

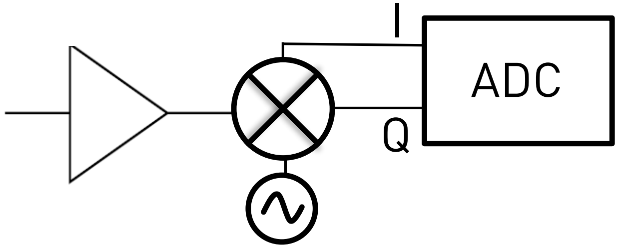




A. Wallraff *et al.*, *Nature* 431, 162 (2004), J. Fink et al., Nature (London) 454, 315 (2008)

1.4 Principle of Dispersive Qubit Measurement





$$A\, e^{i\phi} = I + i\, Q$$

signal amplitude

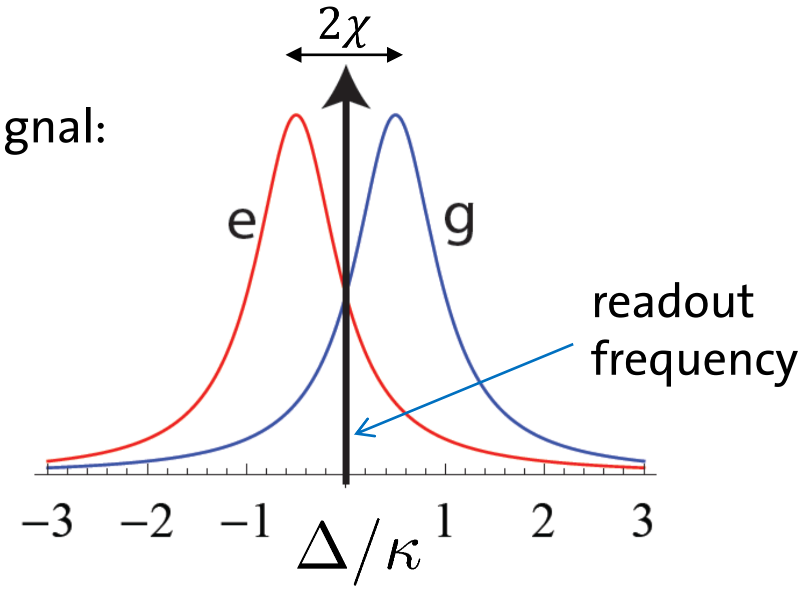
Phase

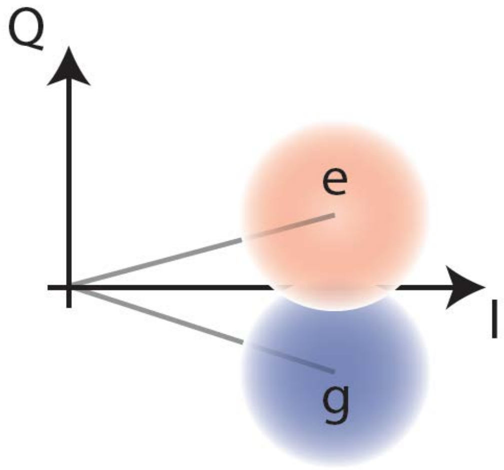
In-phase and quadrature components

In the limit of large detuning $\omega_{\mathrm{r}} - \omega_{ge} \gg g$:

$$H/\hbar \approx (\omega_{\mathrm{r}} + \chi\sigma_z)a^\dagger a$$,with $\chi \approx -\alpha \frac{g^2}{\Delta(\Delta-\alpha)}$

Amplitude of transmitted signal:

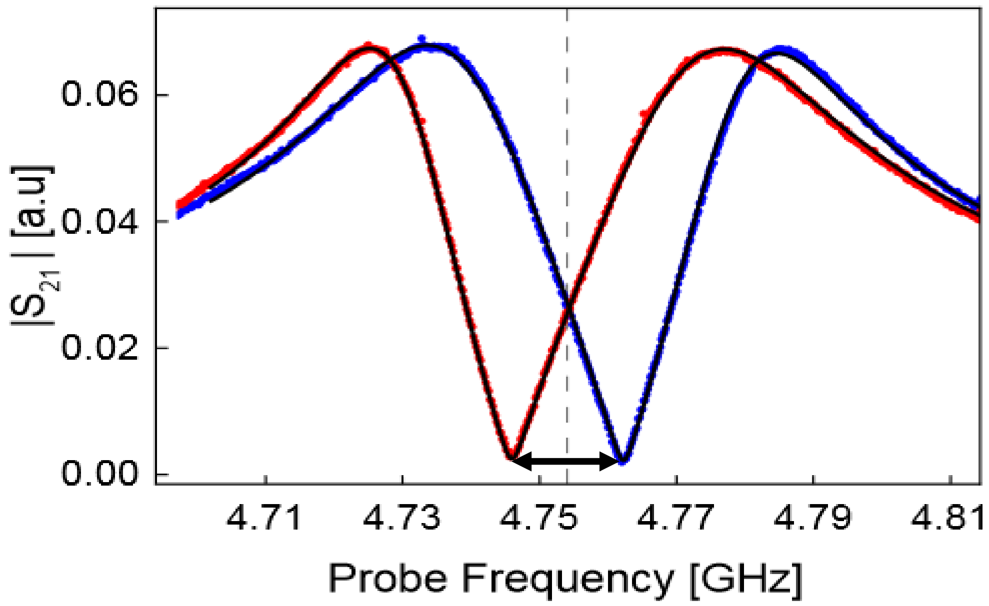




A. Wallraff *et al., Phys. Rev. Lett.* 95, 060501 (2005).
R. Vijay *et al., Phys. Rev. Lett.* 106, 110502 (2011).

1.5 Readout Resonator Response

Transmission amplitude or readout resonator extracted through Purcell filter for qubit prepared in ground (g) or excited (e) state :



Parameter fit (input-output model):

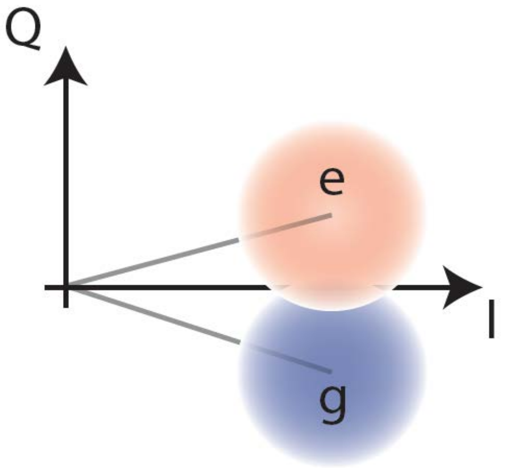
Readout resonator $\kappa_r/2\pi$ = 37.5 MHz

State dependent resonator shift $2\chi/2\pi \simeq$ -16 MHz

In ground/excited state:

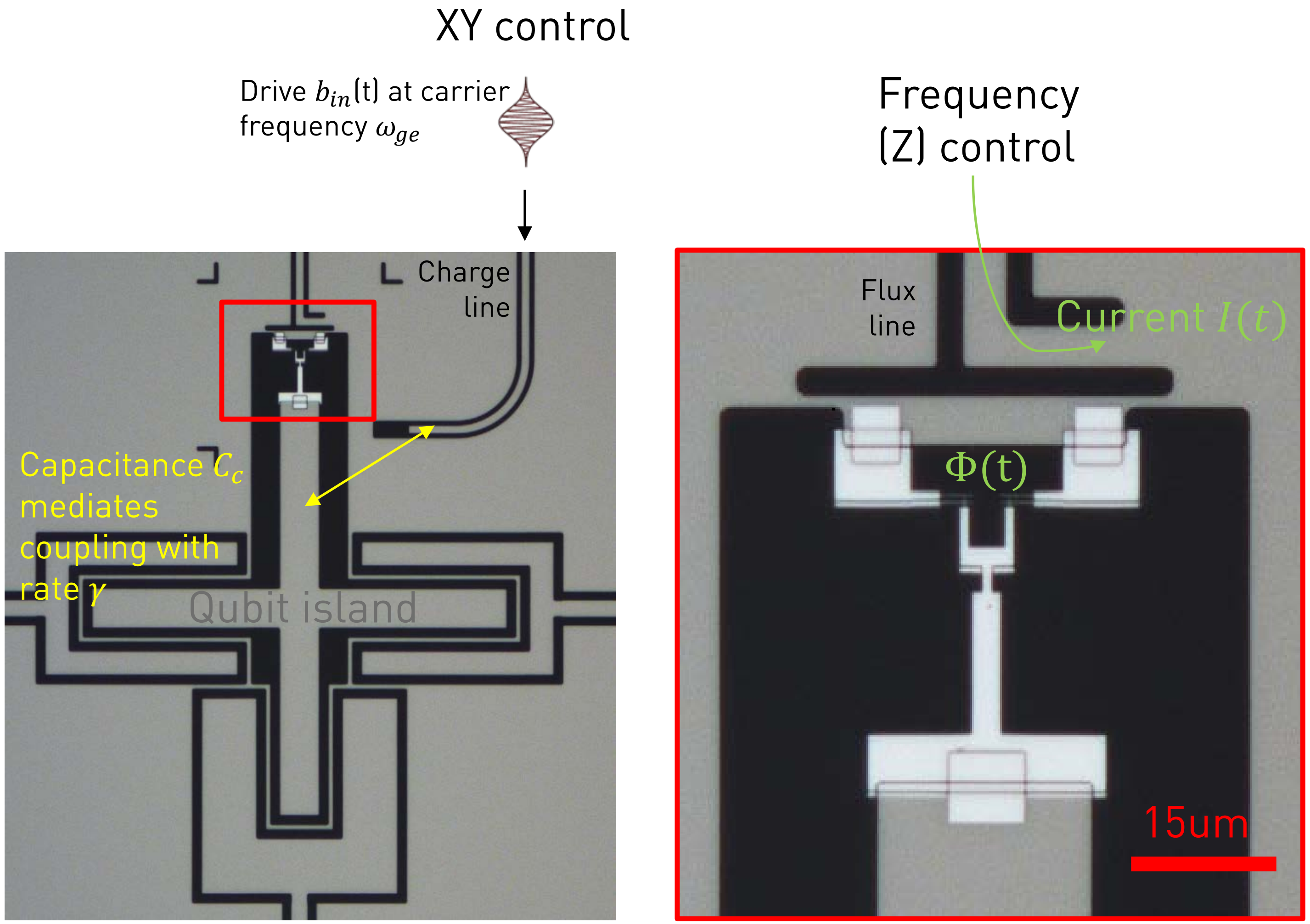
Data measured after state prep. (*,*)

Fit to resonator response model (-)



Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

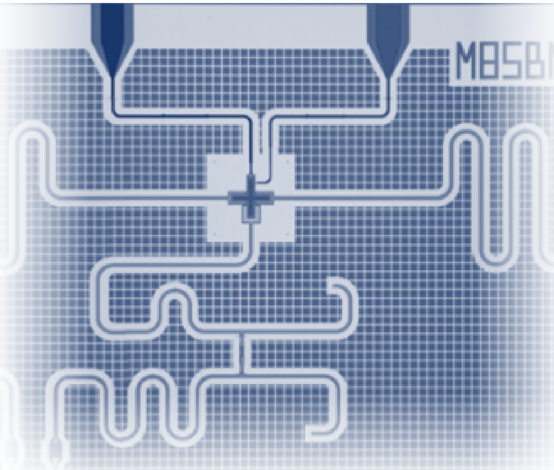
2.1 Control and Characterization of superconducting qubits

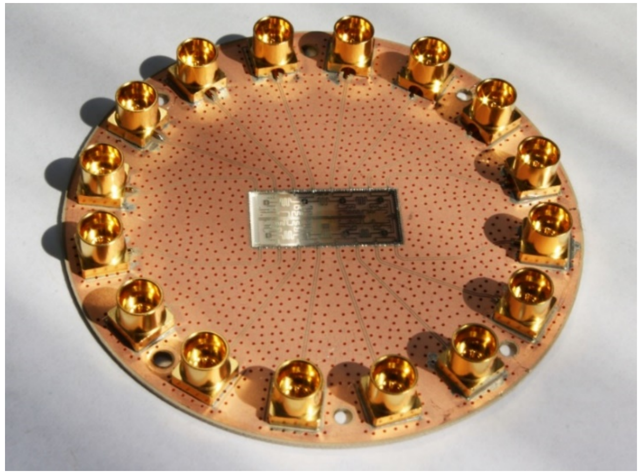


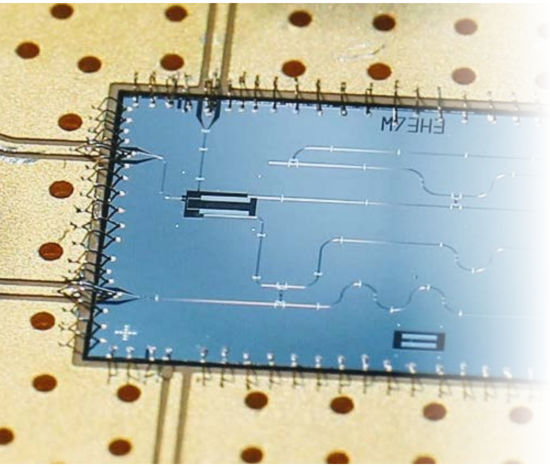
On-chip control

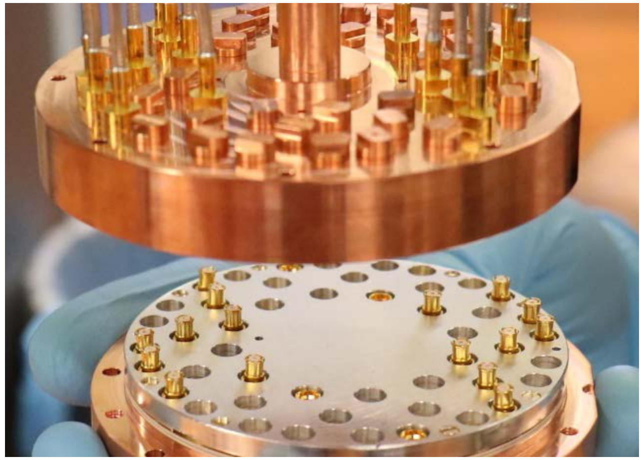
- Microwave drive b_{in}(t) resonant with qubit frequency rotates Bloch vector about X and Y axis. Drive power $P_{in} = \hbar\omega b_{in}^{+} b_{in}$.
- Arbitrary waveform generators (AWG) used to generate pulses, up-converted to the MW frequency band by mixing with a local oscillator field.
- Coupling rate to charge line $\gamma = \frac{C_c^2 Z_0 \omega^2}{C_\Sigma}$ imposes decay and therefore needs to be $\gamma \ll 1/T_1$.
- Tunability of the qubit achieved by sending a current $I(t)$ to a separate control line generating a magnetic flux $\Phi(t)$in the SQUID loop.
- Used for both static (DC) control of the qubit frequency and for applying pulses on nanosecond timescales.

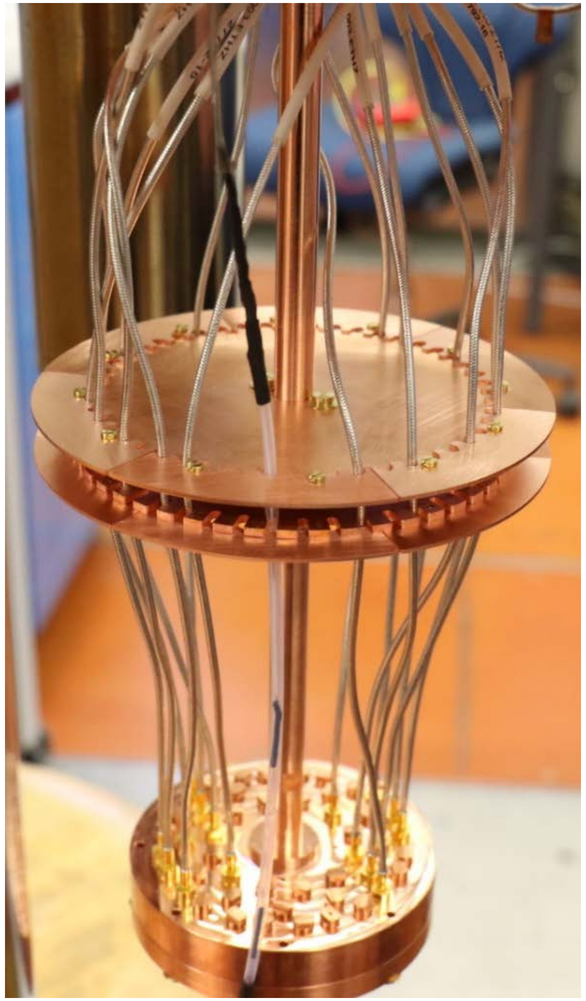
2.1 Control and Characterization of superconducting qubits

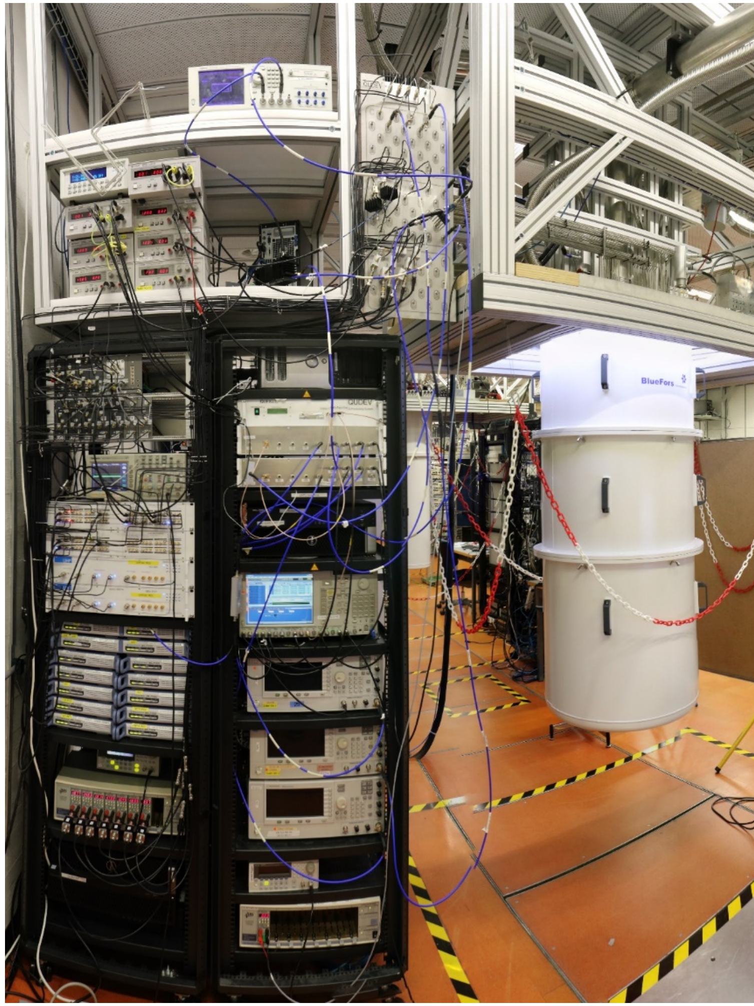




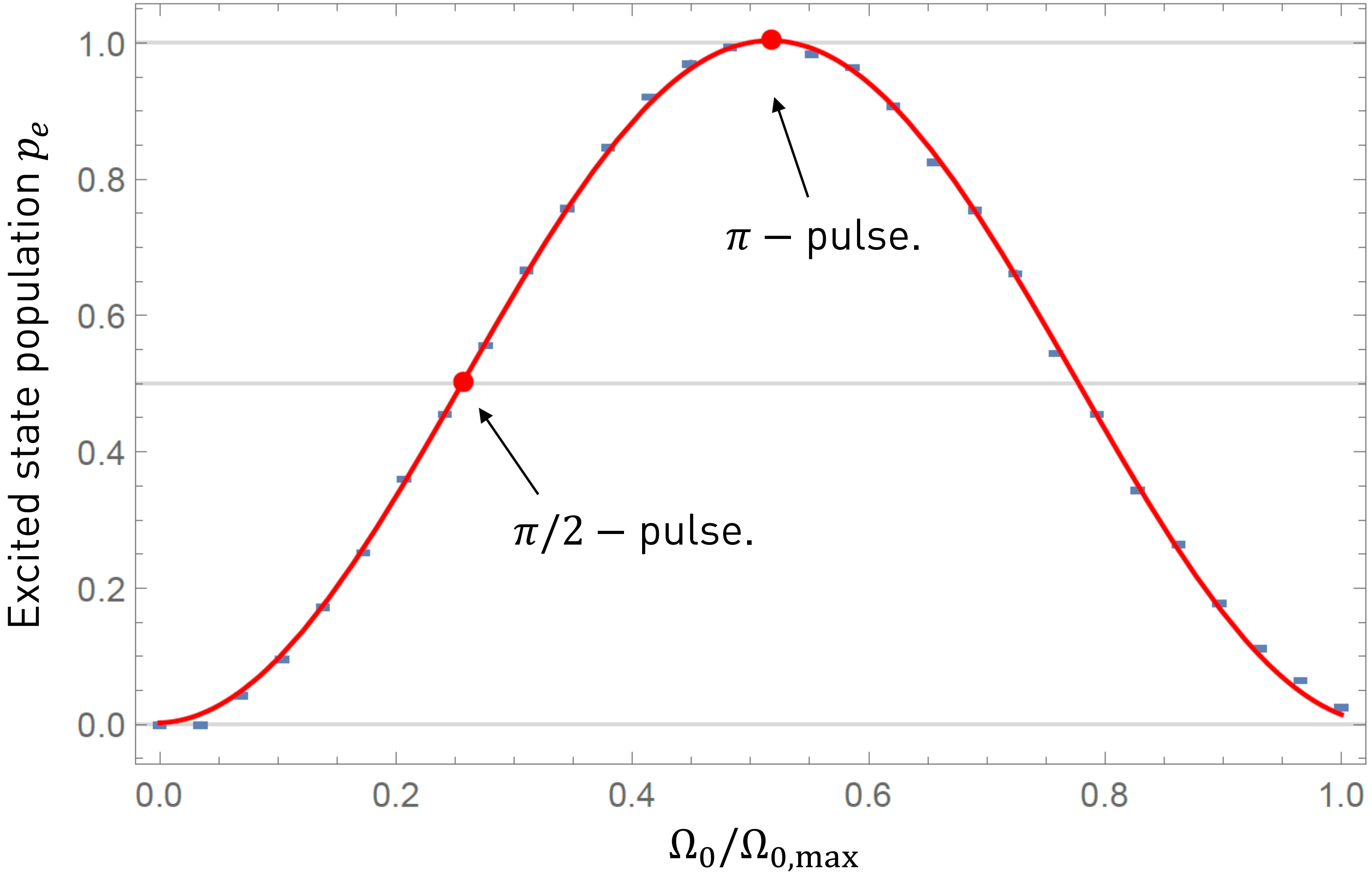


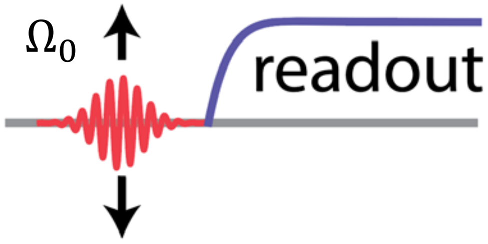






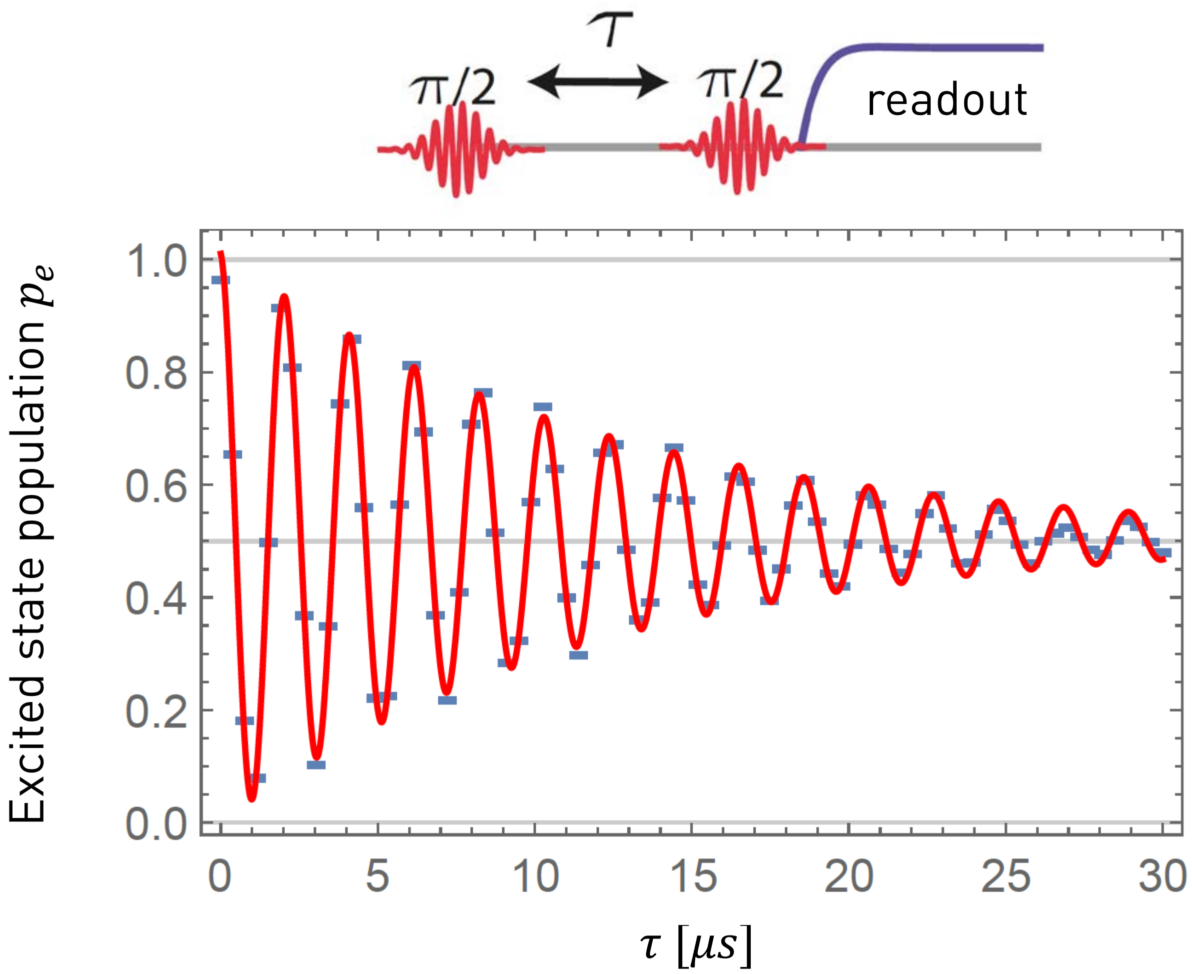
2.2 Measurement of Rabi oscillations





- Qubit frequency $\omega_{ge}/2\pi = 5.758\ \mathrm{GHz}$ determined spectroscopically.
- Initialize qubit in ground state.
- Apply pulse at ω_{ge} with variable amplitude Ω_0.
- Gaussian pulse envelop with characteristic $\sigma \sim 5 - 10ns$.
- Readout qubit state and average over ~10^3 repetitions.
- Sinusoidal fit to extract π- and $\pi/2$-pulse amplitude.

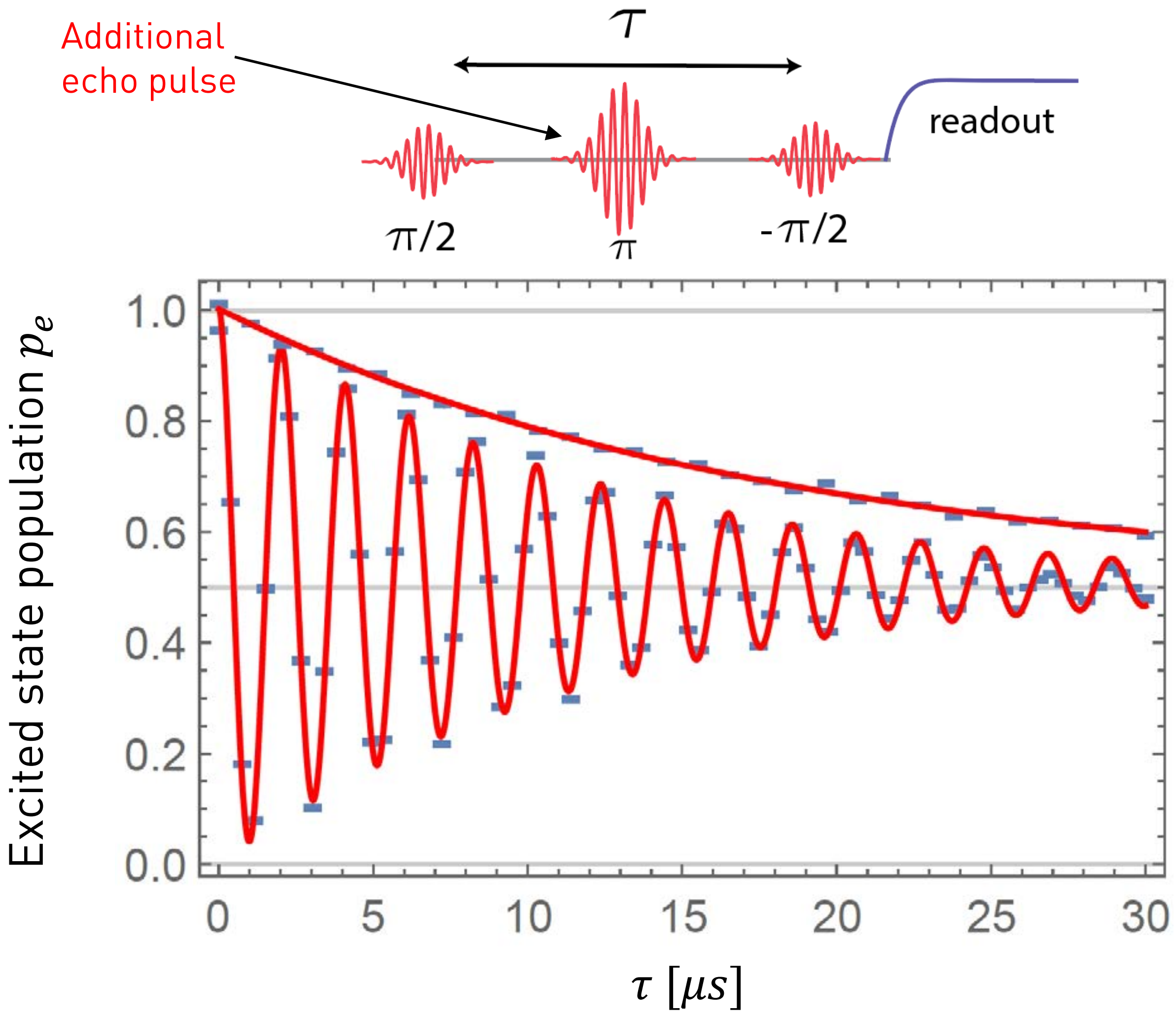
2.3 Measurement of dephasing time



- Initial $\frac{\pi}{2}$-pulse prepares $|g\rangle + |e\rangle$.
- Map remaining coherence after time τ to excited state using a second $\frac{\pi}{2}$- pulse, and measure.
- Detune pulse by $\mathrm{f_{IF}} = 0.5\ \mathrm{MHz}$ from qubit frequency to obtain oscillating pattern. → Higher accuracy in estimating the qubit frequency.
- Fit Characteristic decay time $T_2^* = 13\ \mu s$
- In this case, decay reasonably well described by exponential function $e^{-\tau/T_2^*}$
- Depending on spectral properties of the dominant noise source, decay better described by different functional form, e.g. Gaussian decay for 1/f – noise.
- If relaxation is only source of decoherence: $T_2 = 2\,T_1$ ("T1 limit of dephasing time").

Ithier, G. et al. PRB 72, 134519 (2005).

2.3 Measurement of dephasing time

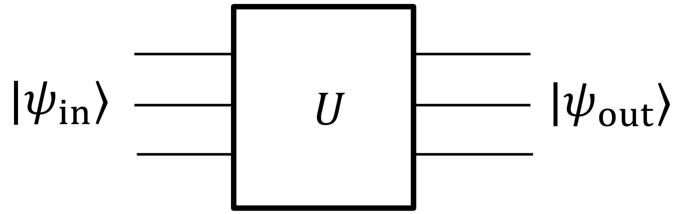


- Low frequency noise can be partly compensated for by applying an echo π-pulse after $\tau/2$ to reverse the direction of the Lamor precision.
- The resulting decay time $T_2^{echo} = 18\ \mu s$ is longer than T_2^*.
- Explanation: Low frequency noise which causes the qubit frequency to change on timescales longer than τ_{max} will cancel out.
- Variants of such dynamical decoupling sequences can be used to do noise spectroscopy → See e.g. *Bylander et al., Nat. Phys. (2011)*

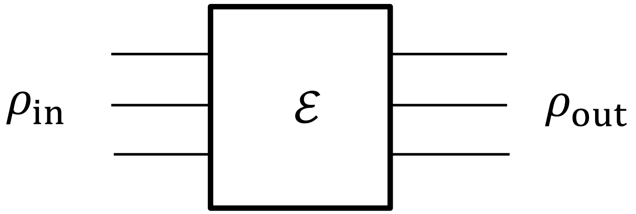
Ithier, G. et al. PRB 72, 134519 (2005).

3.1 Characterization & Benchmarking of Quantum Processes

Ideally



Realistically

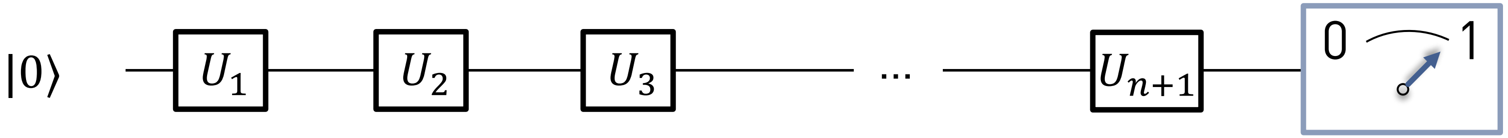


Questions:

- General properties of the map $\mathcal{E}$?
- How to measure the map $\mathcal{E}$?
  - State and process tomography
- Measure of distance between quantum states and processes: Fidelity
- How to benchmark quantum gates with fidelities close to one?
  - Randomized Benchmarking

3.2 Randomized benchmarking

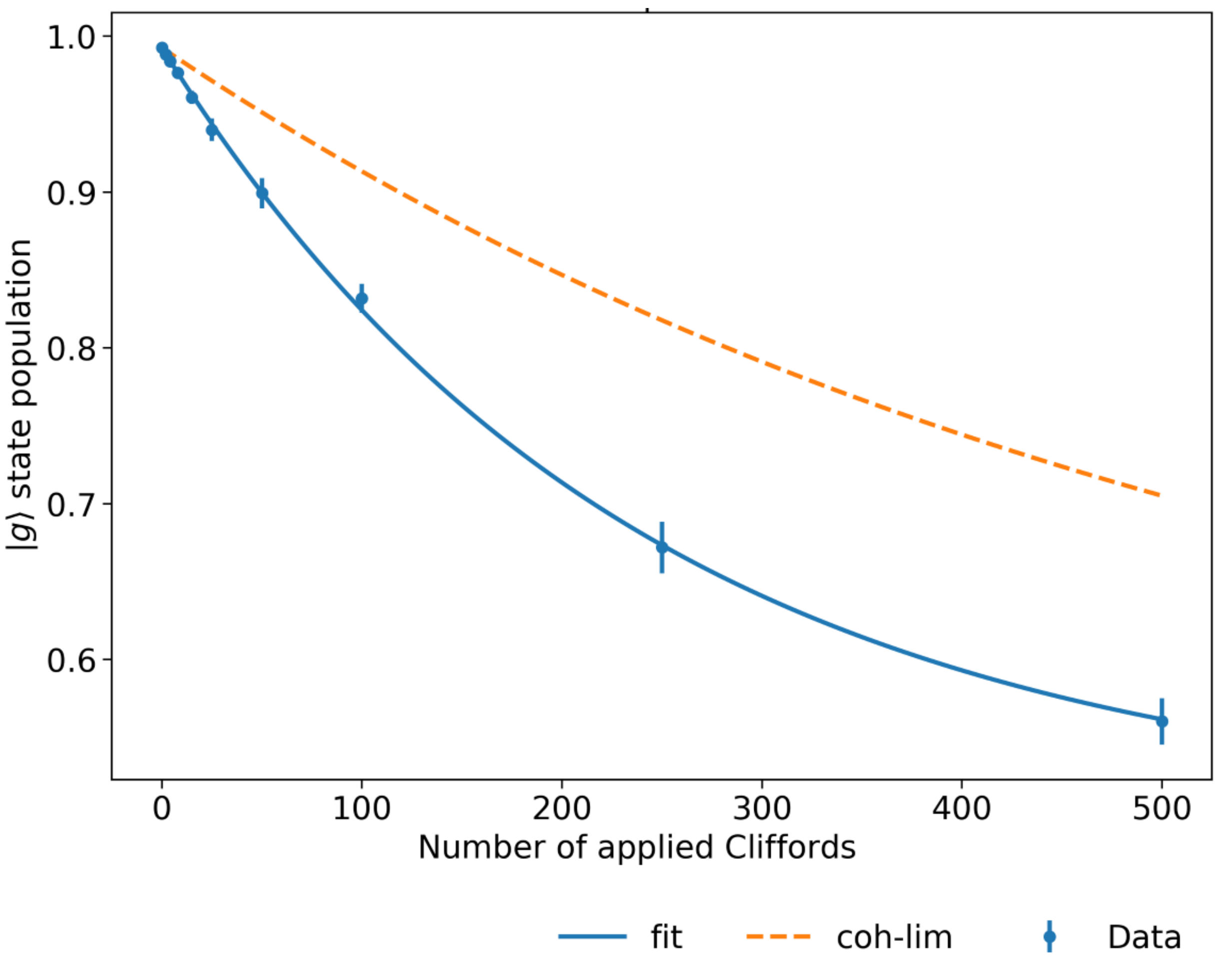
- For simplicity, consider single-qubit case.
- Apply sequence of n gate operations U_i before measuring.



- Gates U_i are chosen randomly from the Clifford group, mapping an element of the Pauli group to an element of the Pauli group. For a single qubit there are 24 Clifford gates.
- Last gate U_{n+1} is chosen such that in the absence of errors state is brought back to initial state, i.e. $U_{n+1} \dots U_2 U_1 = I$.
- Average over m different such sequences.
- Success probability p_0 to recover the initial state decays exponentially with # of gates $p_0 \propto \alpha^n$, with depolarization parameter α.
- The error per gate is given by $\epsilon_{RB} = (1-\alpha)\frac{(d-1)}{d}$ where d is the dimension of Hilbert space (d=2 for a single qubit).

Knill et al., PRA **77**, 012307 2008

3.3 Randomized benchmarking: Example single qubit gates



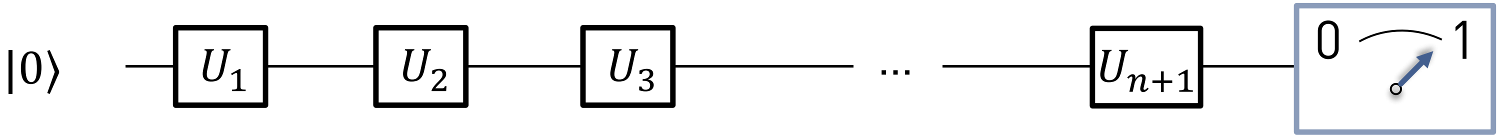
- Pulse duration typically between 15 to 50 ns. Leakage into second excited state avoided by using (DRAG*) pulse parametrization.
- Use Clifford decomposition in terms of X rotations and virtual Z gates (see McKay et al., PRA (2017)).
- Population of ground state decays exponentially.
- Fitted depolarization parameter $\alpha \approx$99.6% and gate error $\epsilon \approx$ 0.2% in this example.
- Orange dashed line indicates the limit expected when only considering qubit decoherence.
- Deviation from coherence limit hints at finite control errors, e.g. resulting in leakage to the f-level.

*Motzoi et al., PRL 103, 110501 (2009)

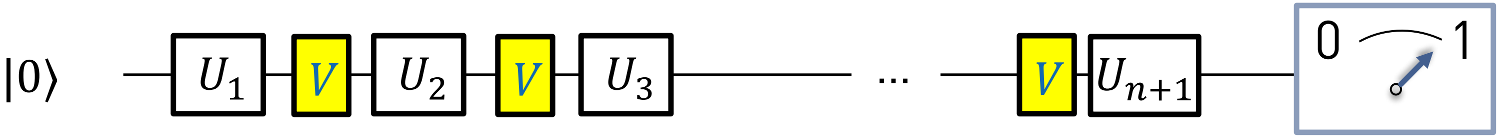
3.4 Interleaved Randomized benchmarking (IRB)

Question how to characterize the fidelity of one particular Clifford gate V?

- Compare decay of standard RB sequence …



- …with result of a 2nd experiment, in which gate V gets interleaved with random Clifford gates.



- Difference between the depolarization parameters α_{RB} and α_{IRB} results in an estimate for the error $\epsilon_V = \frac{d}{d+1}(1 - \frac{\alpha_{IRB}}{\alpha_{RB}})$ per gate V.

- Randomized Benchmarking can be generalized to multi-qubit gates.

Magesan et al., PRL 109, 080505 (2012)