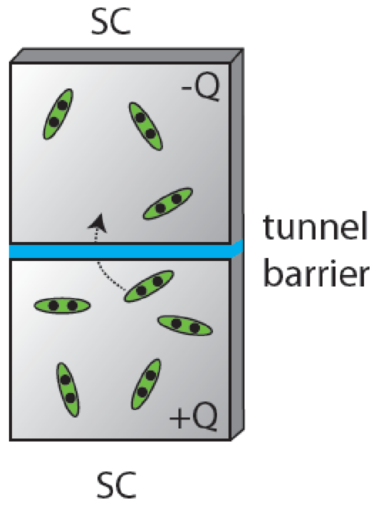
# Outline of today's lecture

- Finish chapter 3 on Josephson junctions based circuits
  - The transmon (limit) and its use as a quantum bit
  - Frequency tunability: The Superconducting Quantum Interference Device
  - Design and fabrication of superconducting qubits

- Chapter 4: Measurement and Control of Superconducting qubits
  - Controlled coupling to the environment
  - Cavity QED and its electrical circuit equivalent

# Summary of last weeks derivation



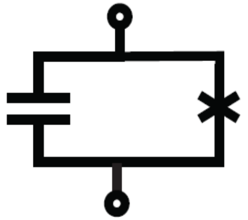

1) Starting point Josephson equations:

$$\boxed{I = I_0 \sin\delta} \qquad \boxed{V = \varphi_0 \dot{\delta} \equiv \dot{\Phi}} \qquad \varphi_0 = \frac{\hbar}{2e}$$

... reduced magnetic flux quantum

2) Josephson junction has properties of a nonlinear, dissipationless inductor with corresponding flux variable $\Phi$.

3) Motivate and derive Hamiltonian, which accounts for tunneling and electrostatic energy



$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

... Cooper pair box Hamiltonian

4) First step today: Show that in so-called transmon-limit $E_C \ll E_J$

$$\hat{H} \approx \hbar\omega_{01} a^\dagger a - \frac{E_c}{2} a^\dagger a^\dagger a a$$

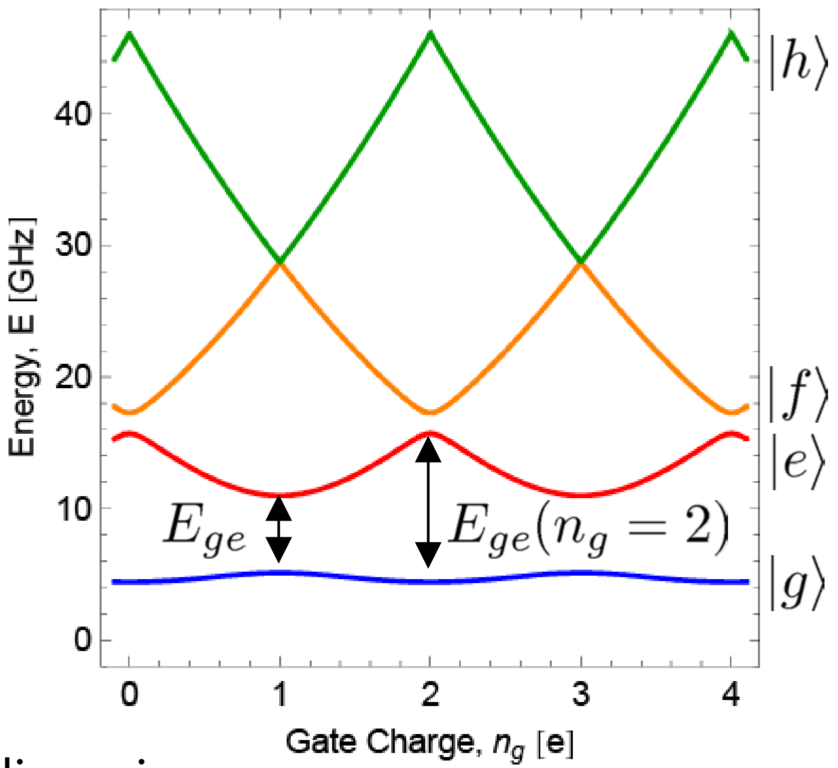
...anharmonic oscillator

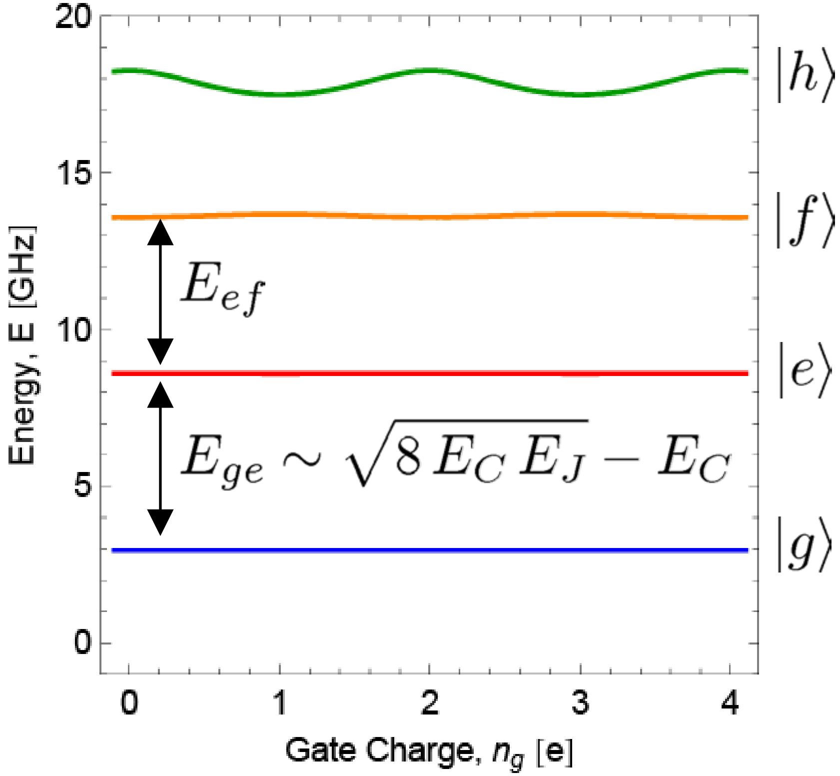
Serves as a qubit, as discussed in today's exercise class and 3.7 of notes!

# 3.6 Transmon: Sensitivity to charge noise

Cooper pair box energy levels: $\frac{E_J}{E_C} \sim 2$

Transmon regime: $\frac{E_J}{E_C} \sim 20$




dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

anharmonicity:

$$\hbar\alpha = E_{ef} - E_{ge} \approx E_C$$

Dispersion for $E_J \gg E_C$:

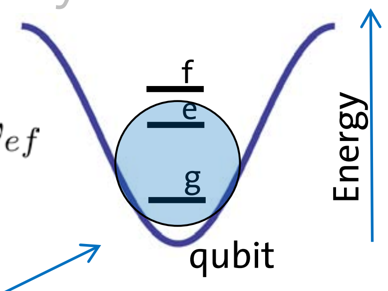
$$\epsilon \sim e^{-\sqrt{8E_J/E_C}}$$

J. Koch *et al.*, Phys. Rev. A 76, 042319 (2007)

# 3.8 Flux tunability

anharmonic system

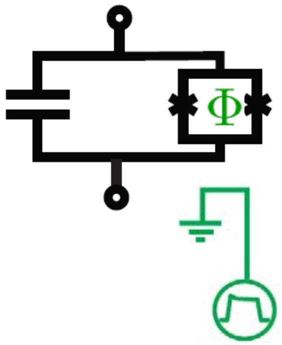
$\omega_{ge} \neq \omega_{ef}$




Change transition energy on ns timescales!

$\omega_{ge} \sim 5 - 10\,\mathrm{GHz}$

electrical oscillator

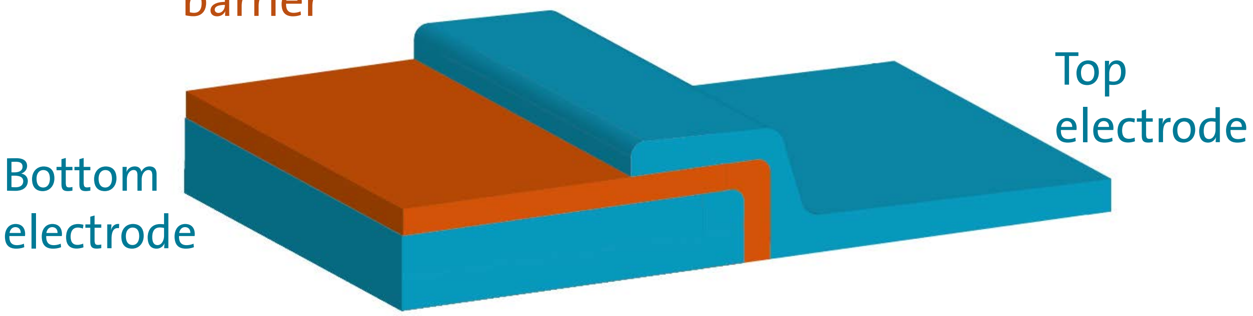



Superconducting Quantum Interference Device (SQUID)

$$E_{J,0} \rightarrow E_{J,0}(\Phi) = E_{J,\max}|\cos(\pi\Phi_{\text{ext}}/\phi_0)|$$

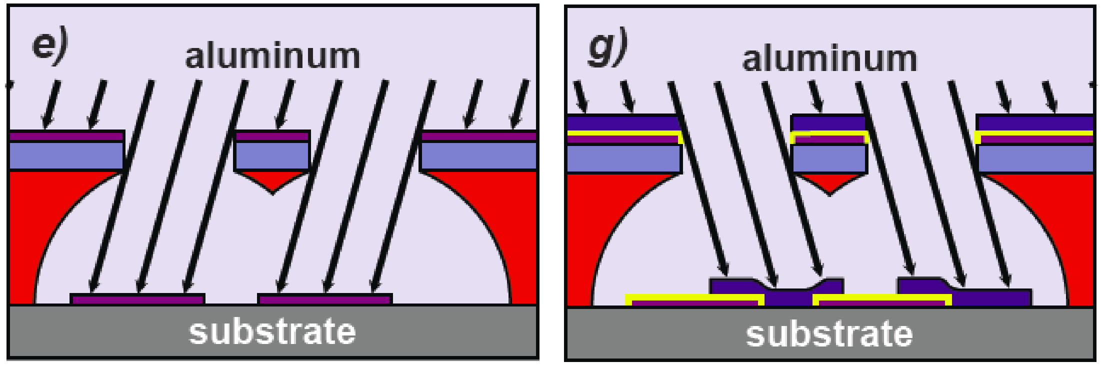
Josephson energy becomes flux tunable

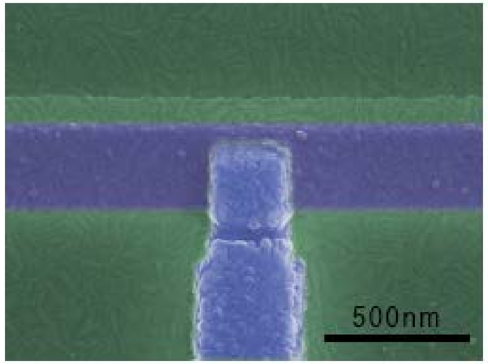
# Fabrication of Josephson Tunnel Junctions




superconductors: Nb, Al
tunnel barrier: $AlO_x$

Josephson junction fabricated by shadow evaporation:




M. Tinkham, Introduction to Superconductivity, McGraw-Hill
Images adopted from M. Goeppl, PhD thesis ETHZ (2009)

# Fabrication of Josephson Tunnel Junctions

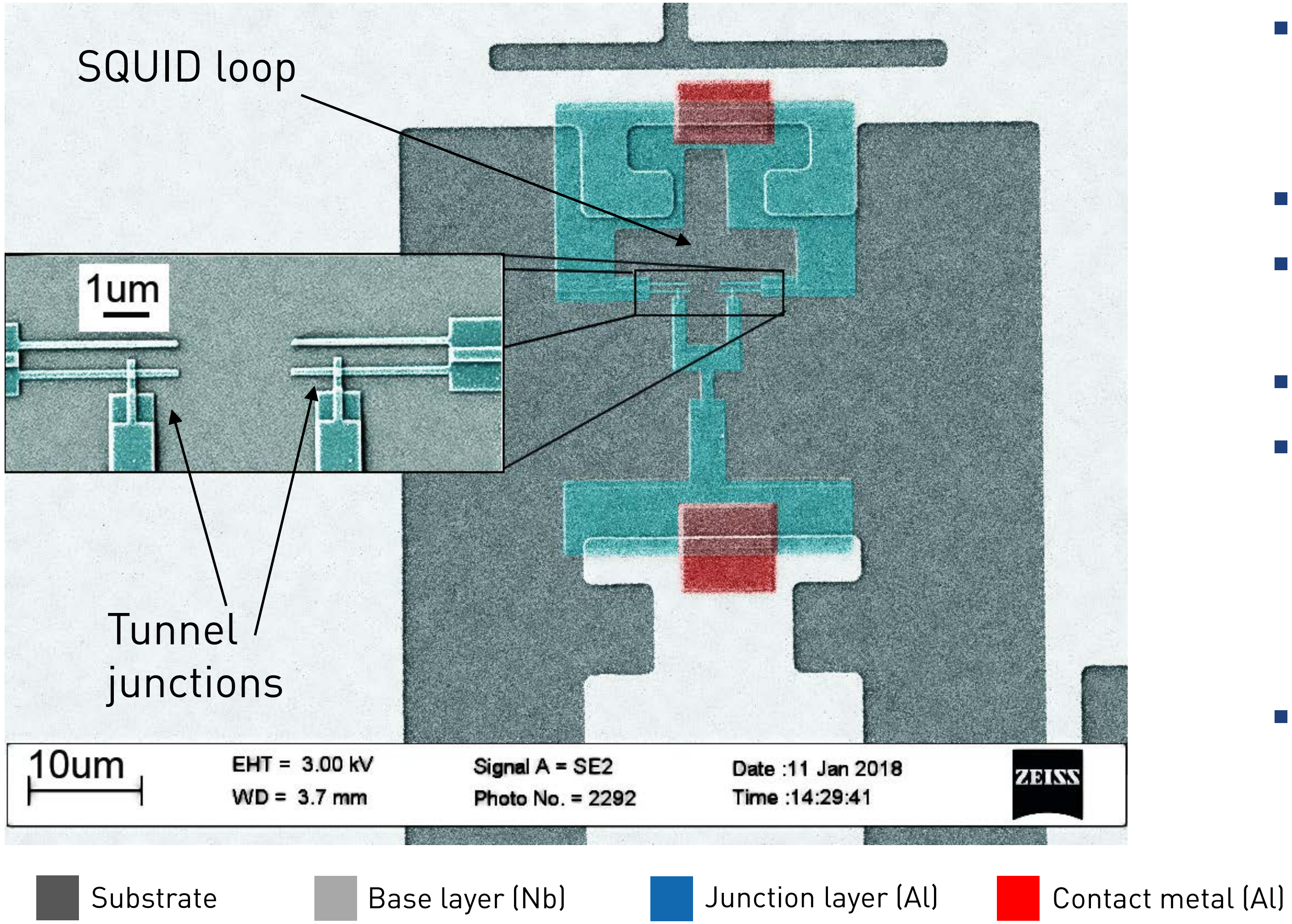



Image: J.C. Besse, Quantum Device Lab (2018)

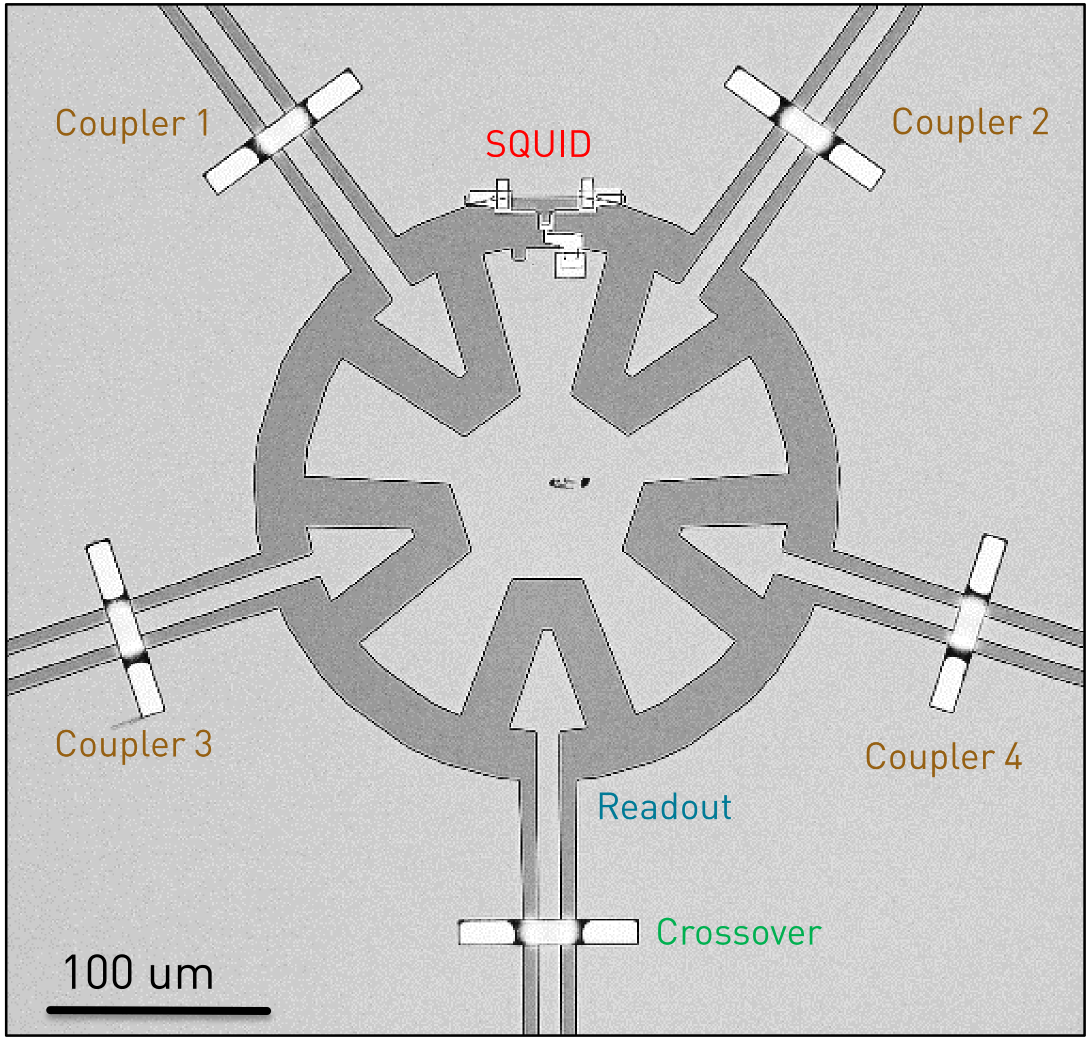
Properties

- The critical current $I_0$ is proportional to the overlap area of the junction $A$ and decreases with the thickness $d$ of the barrier.
- Junction size $A$ controlled lithographically.
- Thickness $d$ of barrier controlled by oxidation parameters (pressure, time, ...)
- Typical values: $A \sim 100 \times 100\,\mathrm{nm}^2$, $d \sim 1\,\mathrm{nm}$
- Critical current related to normal state resistance $R$, which can be probed at room temperature, by Ambegaokar relation (1963)

$$I_0 = \frac{\Delta\,\pi}{2\,R}$$

- Coupling to external field determined by geometry and size of the SQUID loop.

# Design and fabrication of Transmons




Substrate (Si) Superconductor (Nb)

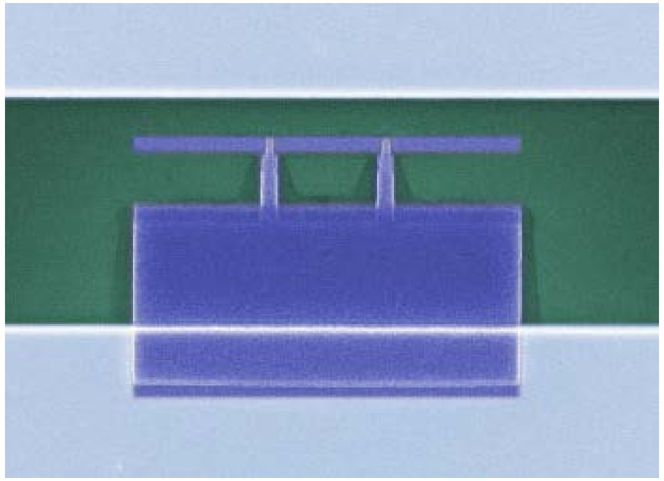
Properties

- SQUID connects transmon island to ground plane.
- Total capacitance between islands and all other elements $C_\Sigma$ sets set charging energy $E_C = e^2/2C_\Sigma$. Typical values $\frac{E_C}{\hbar} \approx 300\ \mathrm{MHz}$.
- Capacitance to coupling elements used to mediate coupling to neighboring qubits, control lines, and the readout circuit.
- Crossovers establish connection between different parts of the ground plane.

Quantum Device Lab , ETHZ (2019)

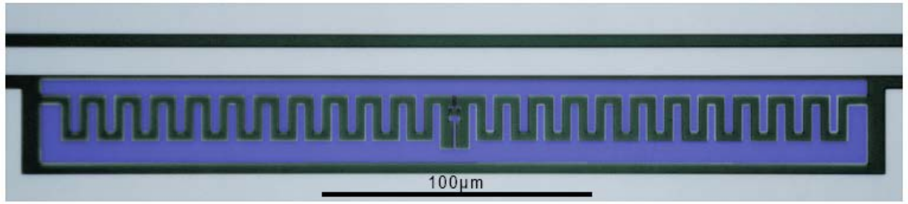
# Many different design variants…

Cooper pair box:



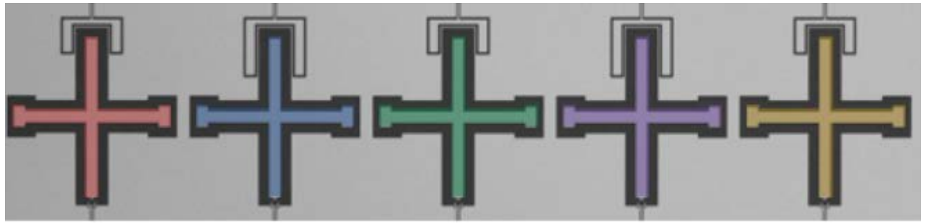
Bouchiat et al., *Physica Scripta* T76, 165 (1998).

Transmon:

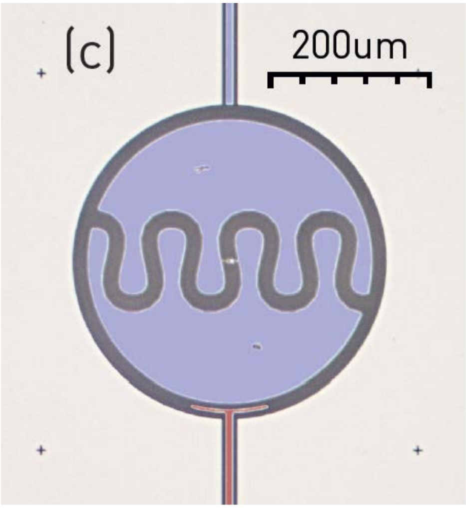



J. Koch *et al.*, PRA 76, 042319 (2007)

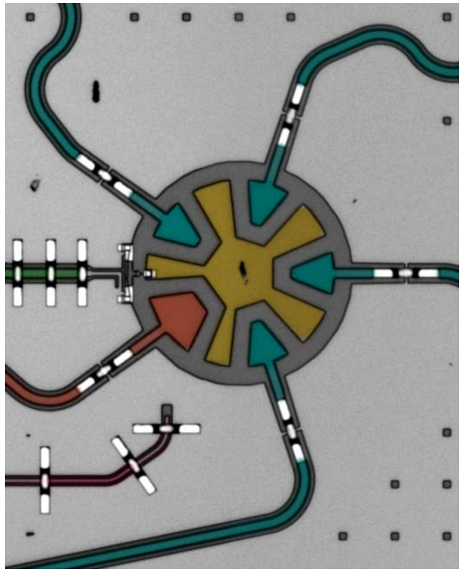
“Xmon” (variant of transmon)



Barends *et al.*, *Phys. Rev. Lett.* 111, 080502 (2013)




M. Pechal et al., *Phys. Rev. Applied* 6, 024009 (2016)



Quantum Device Lab (2019)

# 3.9 Remarks on Superconducting Qubits

- The transmon is the most basic and today's most commonly used type of superconducting qubits. However, a number of variants exist and are under development, e.g. flux qubits, phase qubits, fluxonium qubits, ..., which exhibit interesting properties in terms of the energy levels, coupling rates, etc... In this lecture we will mostly focus on the transmon.

- Recipe to derive Hamiltonian for general circuits (applicable in the transmon limit):
  - Treat Josephson junction as an inductor.
  - Replace $\frac{\Phi^2}{2L}$ term by $-\mathrm{E_J}\cos(\Phi/\varphi_0)$
  - Use boundary condition imposed by flux quantization to eliminate variables.
  - Express in terms of annihilation and creation operators.

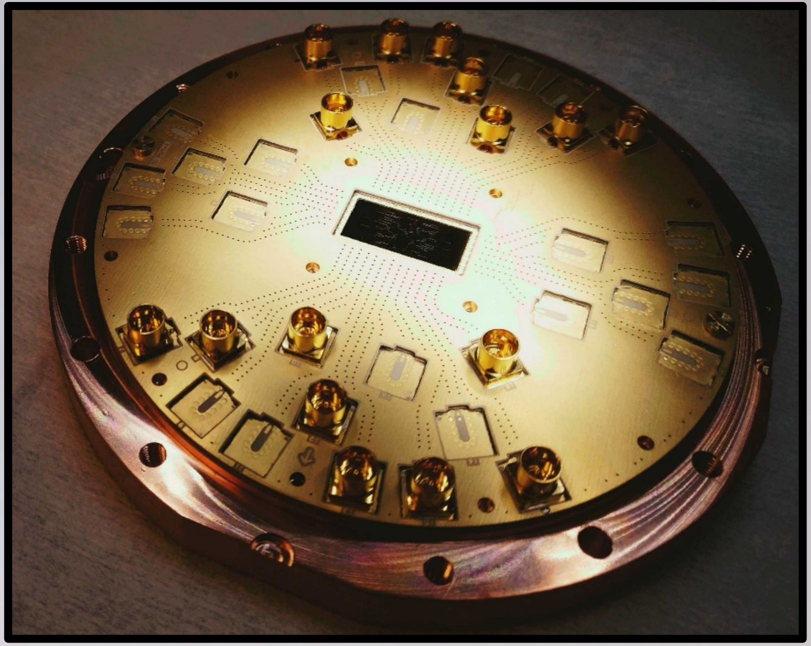
# Reminder: Requirements for physical implementation

DiVincenzo's criteria (2000):

1) Scalable, physical realization of a qubit
2) Ability to initialize qubits in a fiducial state, e.g. the ground state
3) Coherence time needs to be much greater than the gate time
4) Need universal set of gates
5) Need high-fidelity measurement of qubits

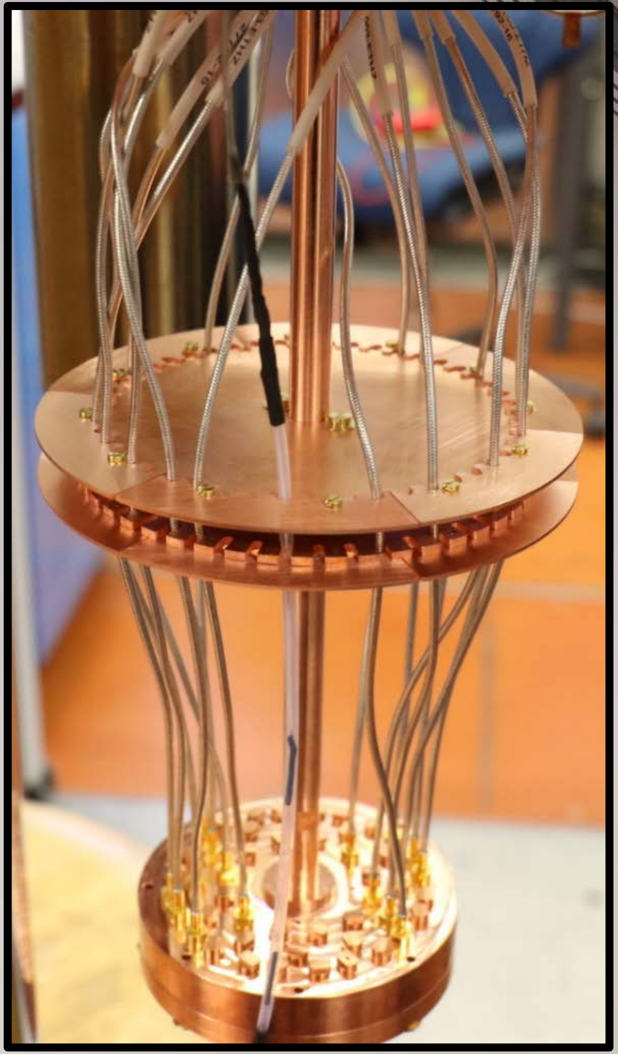
David P. DiVincenzo, The Physical Implementation of Quantum Computation, arXiv:quant-ph/0002077 (2000)

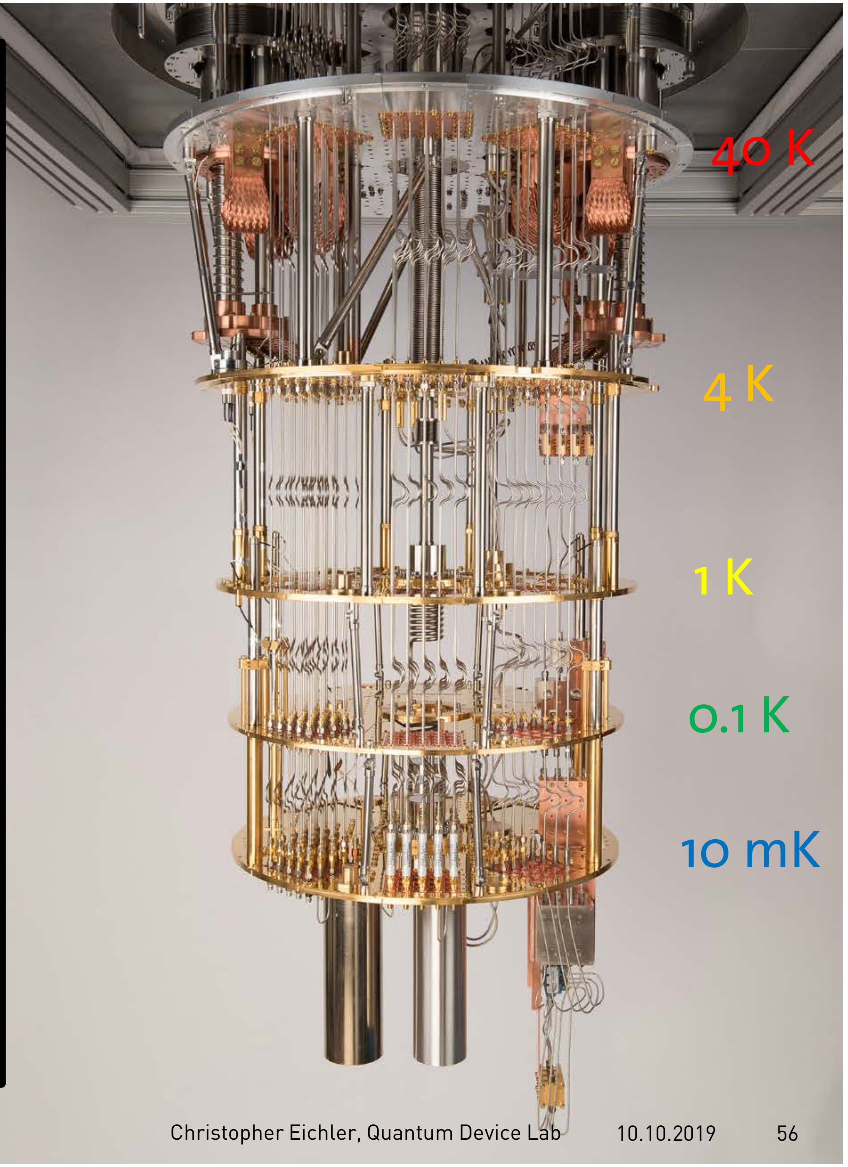
# Cryogenic Measurement setup



Initializing circuit in its ground state requires low temperatures:

$$k_B T \ll \hbar\omega_{ge} \quad (6\,\mathrm{GHz} \mathrel{\hat{=}} 300\,\mathrm{mK})$$






S. Krinner *et al., EPJ Quantum Technology* **6**, 2 (2019)

# Reminder: Requirements for physical implementation

**DiVincenzo's criteria** (2000):

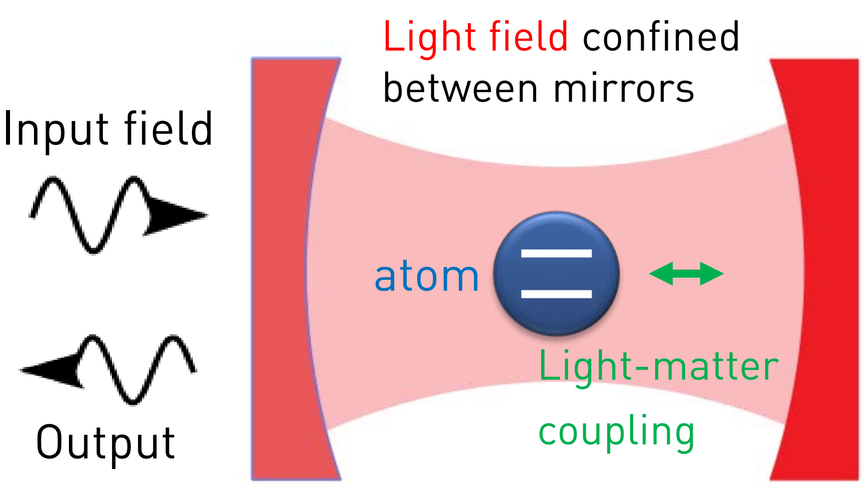
1) Scalable, physical realization of a qubit
2) Ability to initialize qubits in a fiducial state, e.g. the ground state
3) Coherence time needs to be much greater than the gate time → Isolate qubits from environmental noise.
4) Need universal set of gates
5) Need high-fidelity measurement of qubits → Need controlled coupling to environment.

Approach: Couple qubit to auxiliary system used for readout (... and much more!).

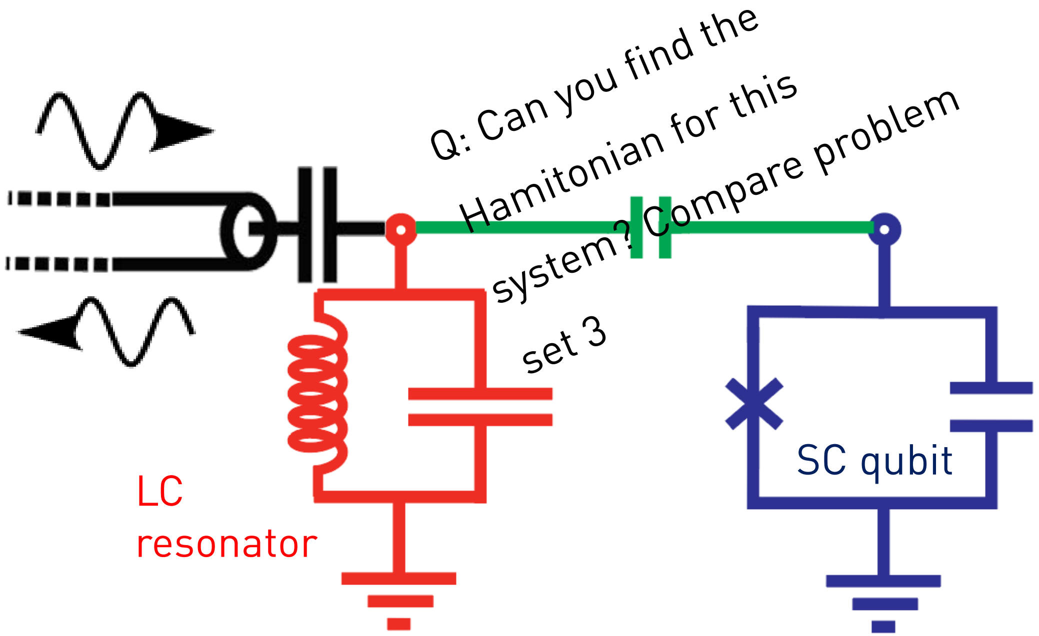
-> Use concepts known from cavity quantum electrodynamics (cavity QED).

# 4) Measurement and Control of Superconducting Circuits

# 4.1 Cavity QED vs. Circuit QED




Circuit
equivalent




Useful for …

- Study light-atom interaction at single particle level
- Generate non-classical states of light (single photon, squeezing, cat states,…)
- Reading out the state of qubits
- Coupling qubits to each other
- Use cavity as long-lived memory

How to model and describe the coupling to the environment?

S. Haroche & J. Raimond, Exploring the Quantum, OUP Oxford (2006)