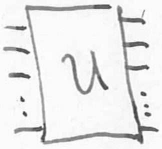
5) Two-qubit coupling and gates

5.1 Universal set of gates

So far: Studied implementation, control & readout of individual qubits.

Important resource still missing: Coupling between multiple qubits to generate entangled states.

Reminder: Need ability to apply arbitrary unitary



to an N-qubit register.

"Any U can be decomposed into single-qubit gates and CNOT gates."
(See 4.5.2 in Nielsen & Chuang)

$$\text{CNOT} = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 0 & 1 \\ 0 & & 1 & 0 \end{pmatrix}$$

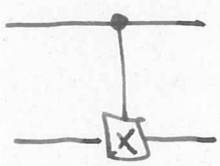
Example of entanglement generation by CNOT

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes |0\rangle \xrightarrow[\text{CNOT}]{} \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

separable state

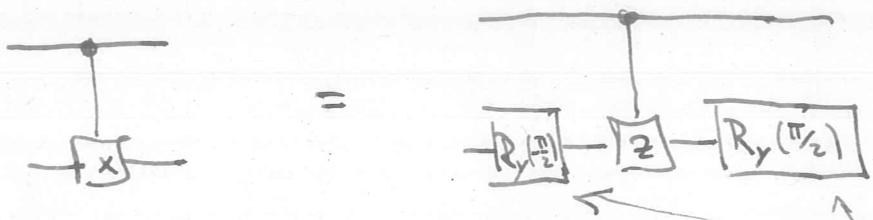
maximally entangled Bell state.

CNOT = CX =

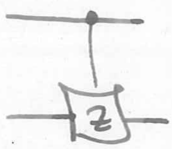


"controlled X"

Can thus be easily transformed into



basis rotation from X into Z basis and back.

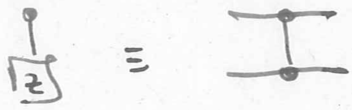


$$= \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & -1 \end{pmatrix}$$

"controlled phase gate"

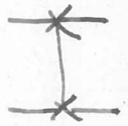
symmetric w.r.t. exchange of qubits

=> Notation :

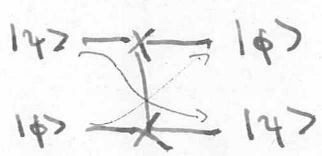


Most frequently implemented in SC circuits.

Another useful 2-qubit gate :

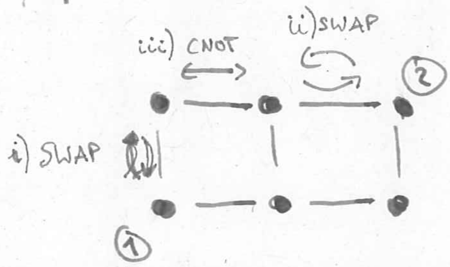


$$= \text{SWAP} = \begin{pmatrix} 1 & & & 0 \\ & 0 & 1 & \\ & 1 & 0 & \\ 0 & & & 1 \end{pmatrix}$$



swaps the state of the 2 qubits

Consider qubit arrangement with finite connectivity (typical for SC qubits)

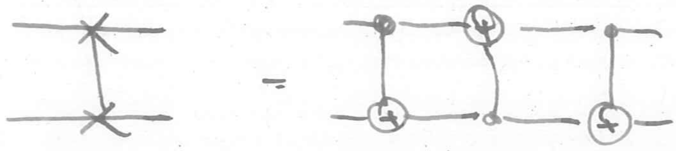


How to perform 2Q gate, e.g. CNOT between 1 and 2 ?

Can result in significant increase in gate count.

Decomposition of U into efficient gate sequence taking finite connectivity into account is non-trivial.

Furthermore



requires 3 CNOTs

While decomposition into CNOTs possible, in principle, reduction of gate count possible by implementing SWAP gate at the hardware level. Same argument applies to other types of gates.

Realization of 2Q-gates requires controlled interactions between qubits.

5.2 Realization of 2-qubit gates in SC circuits

See slides.