3.7 Using an anharmonic oscillator as a qubit

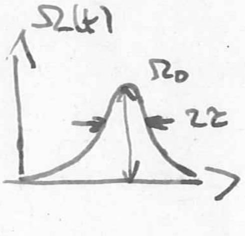
In the previous section we found that the circuit  is described by an effective single particle Hamiltonian H_{CPB}. In the limit $E_C \ll E_J$, called "transmon"-limit, it is well approximated by an anharmonic oscillator model

$$H = \hbar\omega_{01}\, a^\dagger a - \hbar \frac{\alpha}{2}\, a^{\dagger 2} a^2$$

In order to see that this system can be operated as a qubit, we study the effect of a drive field, resonant with the transition frequency $\omega_{01} \equiv \omega_{drive}$. Such a drive field is typically realized as a time-varying voltage $V(t) = V_0(t)\cos(\omega_{01}t + \varphi)$ applied across the transmon. The corresponding drive Hamiltonian reads

$$H_{drive}(t) \approx \hat{Q}\, V(t) \approx \Omega(t)\left(a\, e^{i(\omega_{01}t+\varphi)} + a^\dagger e^{-i(\omega_{01}t+\varphi)}\right)$$

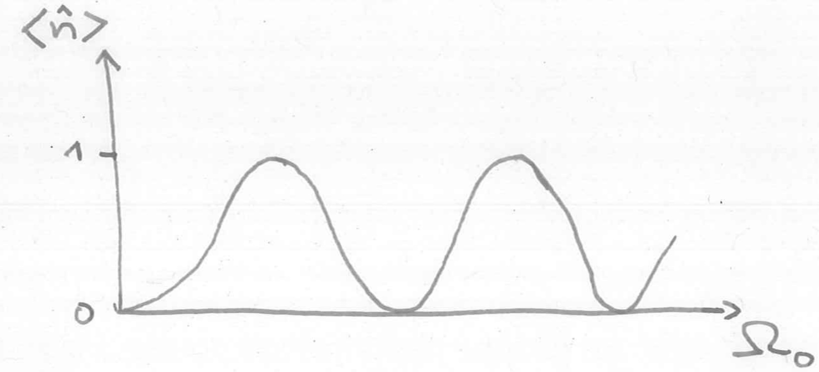
Let's for concreteness assume $\Omega(t)$ follows a Gaussian envelope



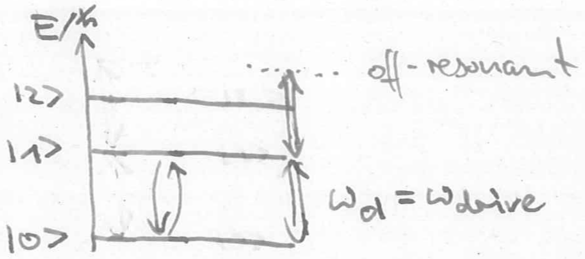
with $2\tau \gg \frac{1}{\alpha}$, small bandwith

Evolving an initial ground state $|0\rangle$ according to this Hamiltonian (see problem set 4) results in so-called Rabi oscillations between $|0\rangle$ and $|1\rangle$.

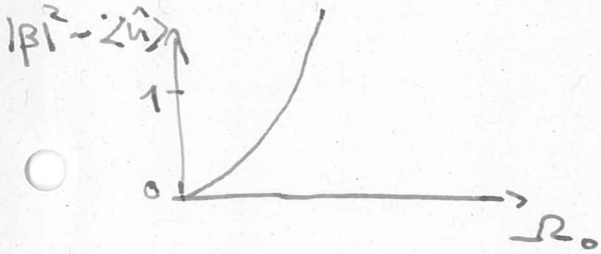
Rabi oscillations



Due to the finite anharmonicity higher excited states do not get populated.



In contrast, for a linear system an initial $|0\rangle \longrightarrow |\beta\rangle$ evolves into coherent state with amplitude $\beta \sim \Omega_0$



for $\alpha = 0$, linear sys

In the presence of large anharmonicity, we may thus describe the system as an effective 2-level sys

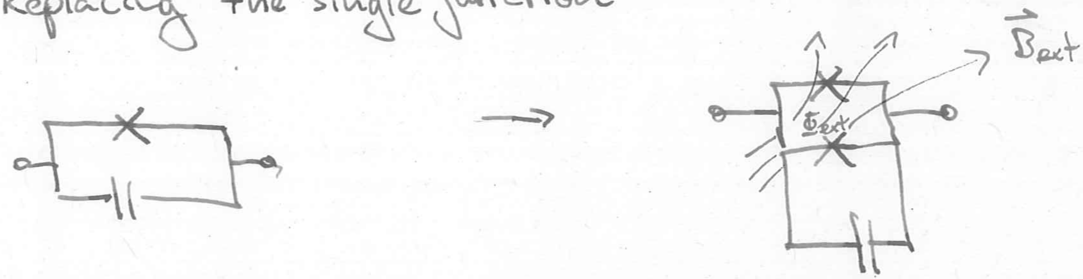
$$\hat{H} = \hbar \omega_{01} \frac{\hat{\sigma}_z}{2}$$

By choosing the drive amplitude Ω_0 and phase φ appropriately, we can perform any rotation of the Bloch vector about an axis in the xy-plane. More about that later.

3.8 Superconducting Quantum Interference Device (SQUID) and flux tunability

The ability to tune the qubit transition frequency ω_{01} by an external control parameter will turn out to be very convenient, e.g. for realizing 2-qubit gates.

Replacing the single junction



by a pair of two Josephson junctions, known as SQUID, allows us to apply an external magnetic flux Φ_{ext} through the loop formed by the two junctions.

Boundary condition imposed by flux quantization results in

$$E_J(\cos\hat{\varphi}_1 + \cos\hat{\varphi}_2) \overset{\uparrow}{=} \dots = \underbrace{2E_J\left|\cos\left(\tfrac{f}{2}\right)\right|}_{E_J(f)} \cos(\hat{\varphi})$$

$$\underbrace{\hat{\varphi}_1 - \hat{\varphi}_2}_{\equiv 2\hat{\varphi}} + \underbrace{\Phi_{ext}/\varphi_0}_{\equiv f} = 2\pi$$

$E_J(f)$: flux tunable Josephson energy

By controlling Φ_{ext} in-situ we can tune the qubit frequency

$$\omega_{01} \sim \sqrt{E_J} \sim \sqrt{|\cos(f/2)|}$$

$\Rightarrow$ SQUID behaves like single junction with tunable Josephson energy $E_J \to E_J(\Phi_{ext})$.

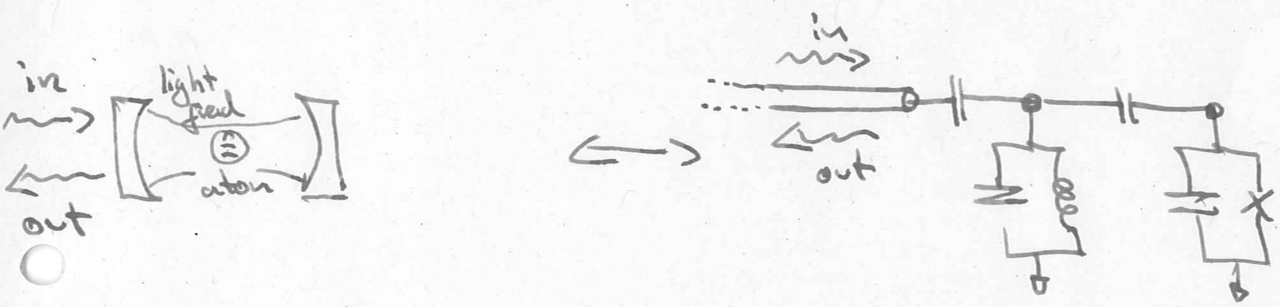
4 Measurement & Control of Superconducting Qubits

4.1 The setting

- Want to isolate qubits from environmental noise to preserve coherence.
- ... and, couple to the environment controllably for readout & control operations.

$\Rightarrow$ Approach: Couple qubits to auxiliary system, and auxiliary system to environment.

$\Rightarrow$ Concept of cavity QED



compare Haroche / Raimond, Exploring the Quantum

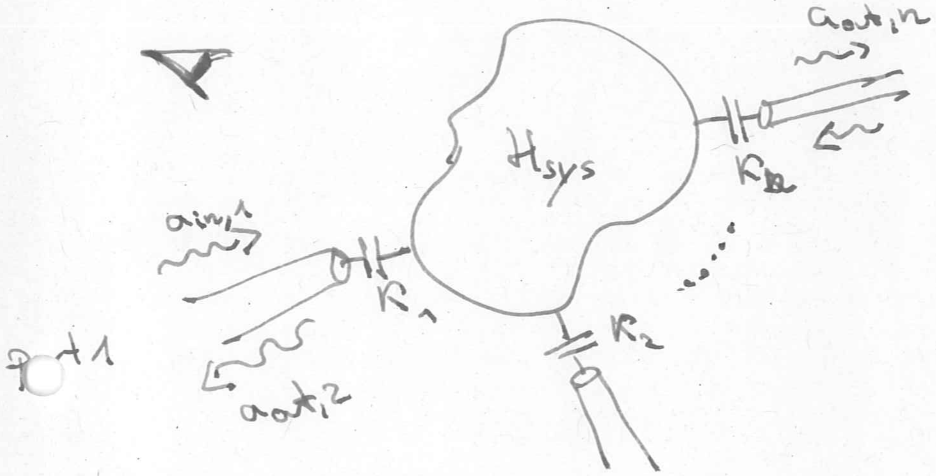
Useful for ...

- studying matter-light interaction
- generation of non-classical states of light
- reading out state of qubits
- coupling qubits
- conversion between stationary & flying qubits
- ...

4.2 Open systems: Two perspectives

Any quantum control experiment requires that classical instruments interact with the quantum system. Apart from that, quantum devices are unavoidably subject to finite coupling to uncontrolled environmental modes. We therefore typically deal with open ~~system~~ rather than closed systems. We will approach this topic from two complementary perspectives

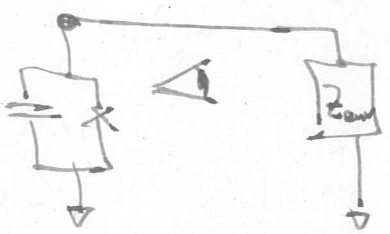
1) The observers' perspective



Scattering matrix

$$S_{ij} = \frac{a_{out,i}}{a_{in,j}}$$

2) The qubit perspective



Decay, decoherence of qubit due to the presence of the environment.