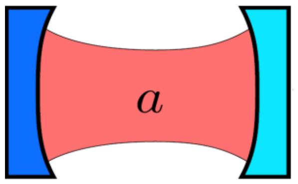
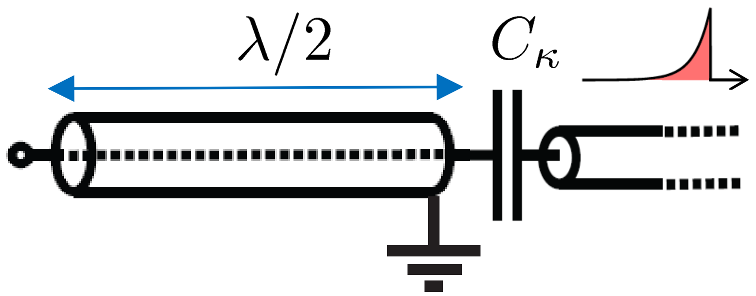
Transmission line resonator

cavity



=

"piece of a coax cable"

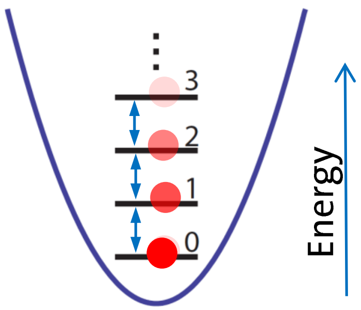


D.M. Pozar, Microwave Engineering, (1993)

Radiation field stored inside:

$$H = \hbar\omega \, a^\dagger a$$

sur
harmonic
oscillators

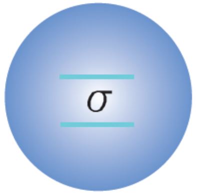


classical
coherent state

Need "artificial atom" for quantum optics experiment!

Artificial atom

atom



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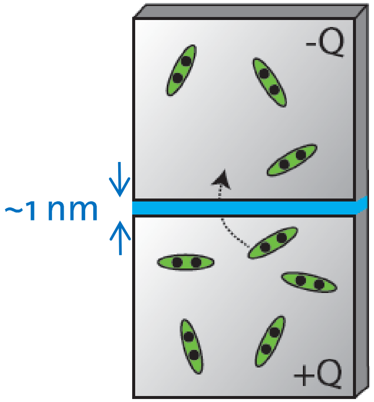
SC

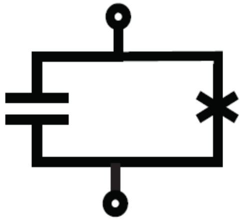


SC

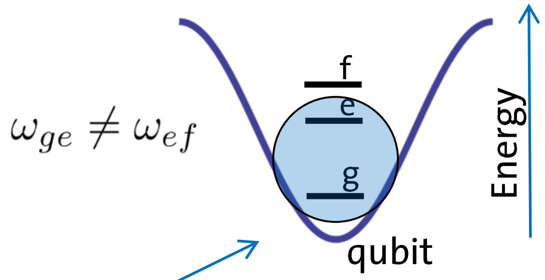
Josephson tunnel barrier

=

electrical oscillator



anharmonic system



Change transition energy on **ns** timescales!

$$\omega_{ge} \sim 5 - 10\,\mathrm{GHz}$$

Can this be quantum coherent?

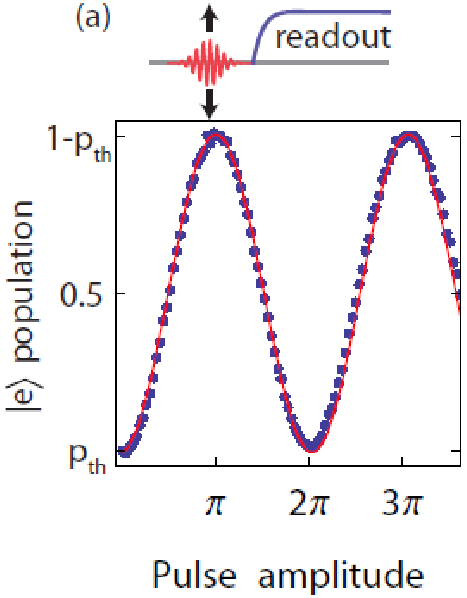
$$|g\rangle + |e\rangle$$

Y. Nakamura *et al., Nature* **398,** *786* (1999)

... since then 4 orders of magnitude improvement in coherence times!

Review: Schoelkopf and Devoret, Science (2013)

Rabi oscillations:



Outline

Last week (lecture 1):

- Quantization of electrical circuits
- Step 1: “Given an electrical circuit composed of inductors and capacitors, find the corresponding system Hamiltonian.”

This week (lecture 2):

- Superconductivity
- Josephson effect
- Superconducting transmon qubit: Hamiltonian and Properties

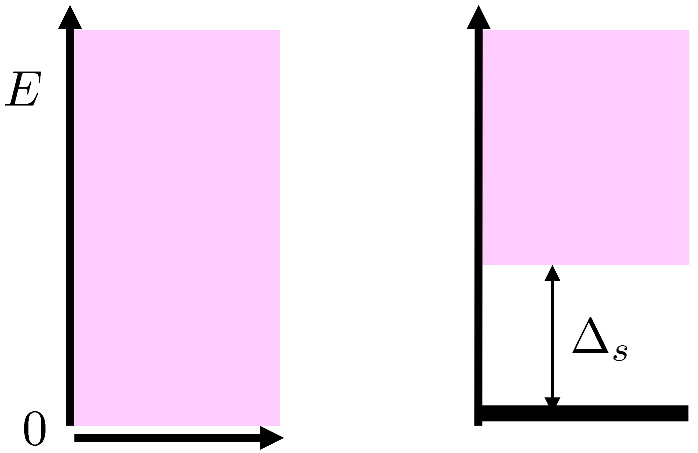
3.1 Basic properties of Superconductors

- Superconductors have zero electrical resistance.
  - First observation: 1911 by H. K. Onnes.
  - Experiments e.g. with persistent current induced in a superconducting ring.

- Superconductors are perfect diamagnets.
- … even more: The Meissner effect.
  - Start with a superconductor above the critical temperature T_c in a finite magnetic field.
  - Cooling down below T_c will result in the built-up of screening currents at the surface causing the B –field to vanish inside the superconductor.

- Microscopic model by Bardeen, Cooper, and Schrieffer (BCS) in 1957
  - Based on the idea that electrons form bound pairs due to long-range attractive interaction mediated by lattice vibrations (phonons).

3.1 Basic properties of Superconductors



density of states: D(E)

normal metal superconductor

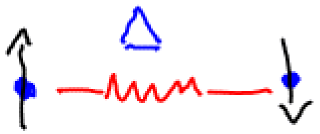
- single non-degenerate macroscopic ground state
- elimination of low-energy excitations
- for $T < T_c$: vanishing internal electrical resistance R_{int}

Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta_S/h = 725$ GHz, $T_c = 9.2$ K
- Aluminum (Al): $2\Delta_S/h = 100$ GHz, $T_c = 1.2$ K

M. Tinkham, Introduction to Superconductivity, McGraw-Hill

Cooper pairs:
bound electron pairs



Bosons (S=0, L=0)

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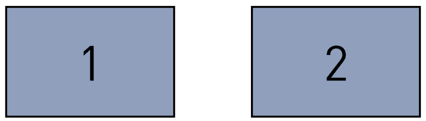
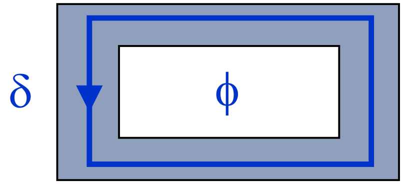
2 chunks of superconductors



macroscopic wave function

$$\psi_i = \sqrt{n_i} \exp(i\delta_i)$$

Cooper pair charge density n_i and global phase δ_i

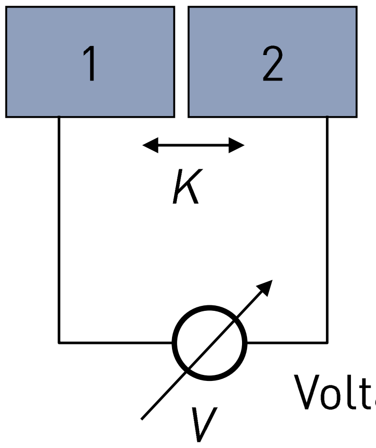


phase quantization: $\delta = n\,2\pi$

flux quantization: $\phi = n\,\phi_0 = n\,h/2e$

3.2 The Josephson effect

2 chunks of superconductors



Voltage source

$$\psi_i = \sqrt{n_i}\exp\left(i\delta_i\right)$$

$$\delta \equiv \delta_2 - \delta_1$$

- K depends on material properties and junction geometry.
- Charge $q = 2e$ is charge of a Cooper pair.
- n1 and n2 are approximately constant and equal.
- Josephson equations relate macroscopic phase variable to current and voltage.

M. Tinkham, Introduction to Superconductivity, McGraw-Hill

Schrödinger equation:

$$i\hbar\frac{\mathrm{d}\psi_1}{\mathrm{d}t} = \frac{qV}{2}\psi_1 + K\psi_2$$

$$i\hbar\frac{\mathrm{d}\psi_2}{\mathrm{d}t} = -\frac{qV}{2}\psi_2 + K\psi_1$$

Potential energy Tunneling energy

Use ansatz and separate variables:

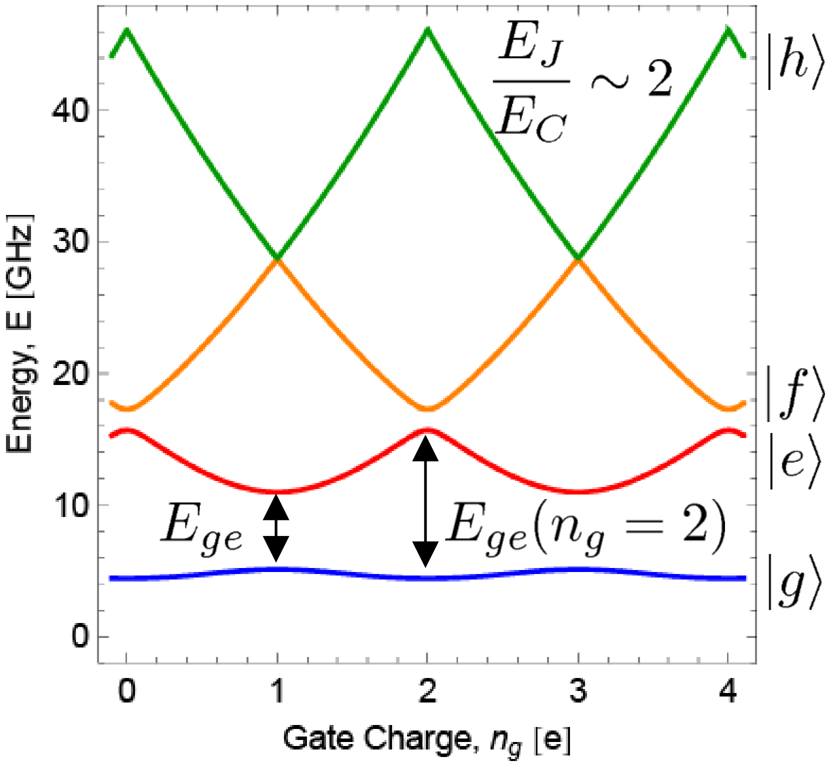
$$I = \frac{\mathrm{d}n_1}{\mathrm{d}t} = \frac{2\sqrt{n_1 n_2}}{\hbar}K\sin\delta \equiv I_0\sin\delta$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{qV}{\hbar}$$

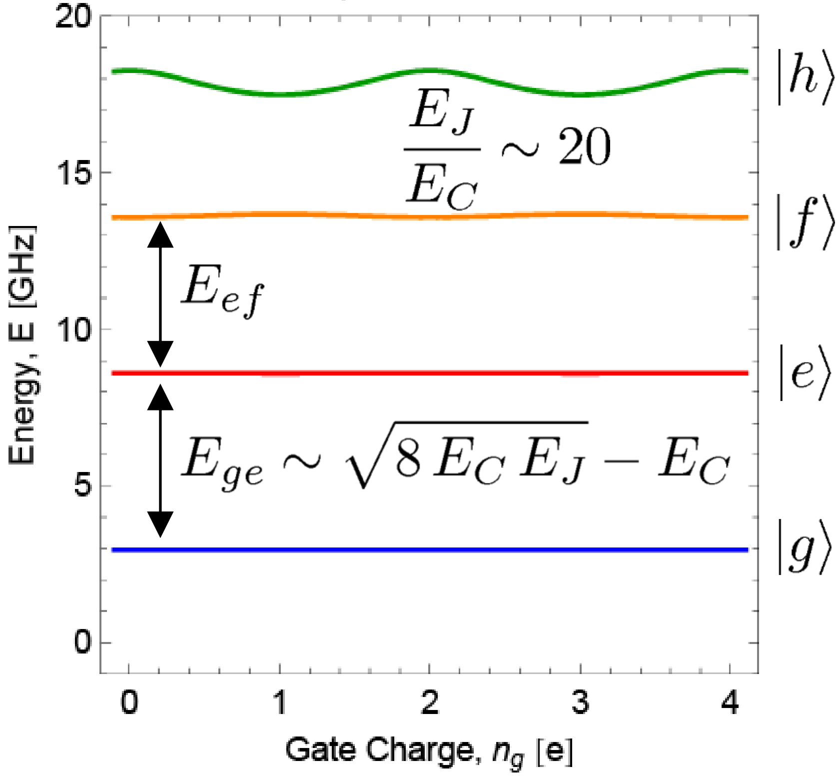
Josephson equations

3.6 Transmon: Sensitivity to charge noise

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

anharmonicity:

$$\hbar\alpha = E_{ef} - E_{ge} \approx E_C$$

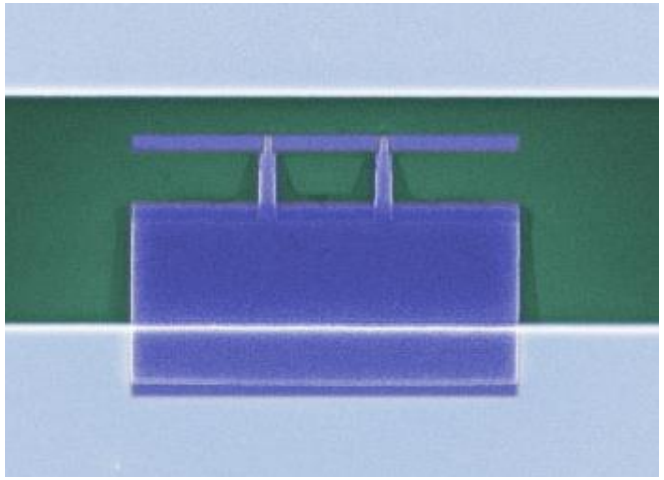
Dispersion for $E_J \gg E_C$:

$$\epsilon \sim e^{-\sqrt{8E_J/E_C}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

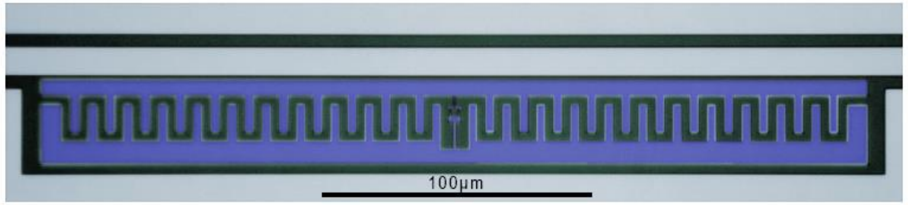
Design variants of the Cooper pair box (transmon)

Cooper pair box:



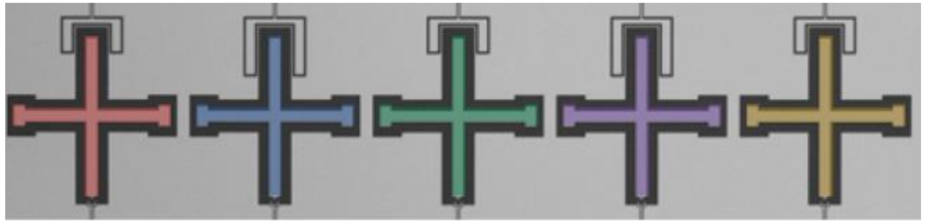
Bouchiat et al., *Physica Scripta* **T76**, 165 (1998).

Transmon:

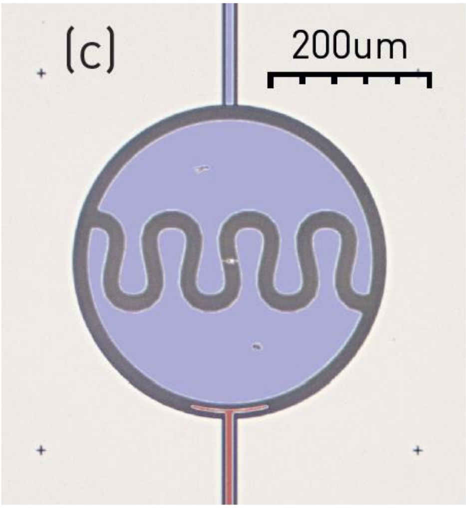


J. Koch *et al.*, PRA **76**, 042319 (2007)

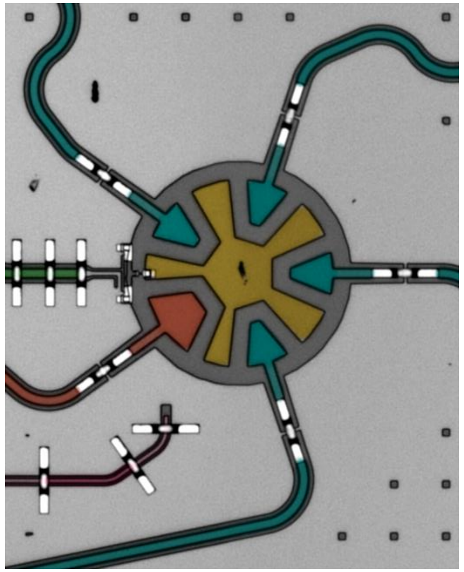
"Xmon" (variant of transmon)



Barends *et al.*, *Phys. Rev. Lett.* **111**, 080502 (2013)



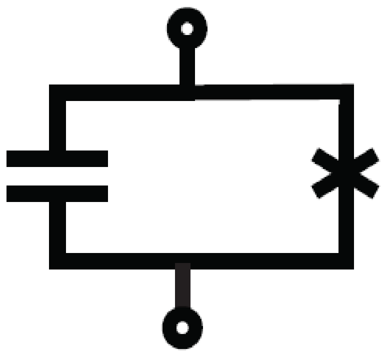
M. Pechal et al., *Phys. Rev. Applied* **6**, 024009 (2016)



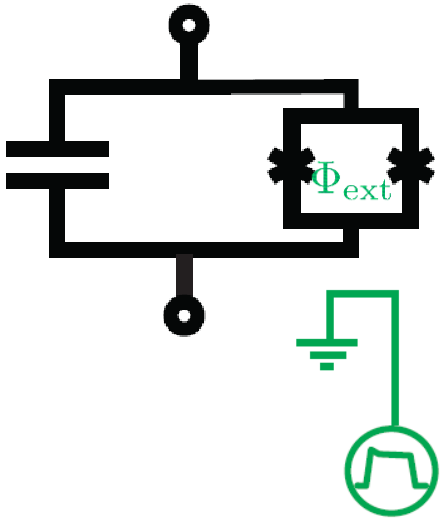
Quantum Device Lab (2019)

Flux tunable Superconducting Qubit

Qubit with constant frequency



Qubit with tunable frequency



Superconducting
Quantum
Interference
Device (SQUID)

External flux applied, using ...

- a coil mounted below the sample
- an on-chip flux line

$$E_{J,0} \rightarrow E_{J,0}(\Phi_{\text{ext}}) = E_{J,\text{max}} |\cos(\pi \Phi_{\text{ext}} / \phi_0)|$$

Effective Josephson energy becomes flux tunable

Outlook

- How to control the state of superconducting qubits?
- How to measure them?
- Coupling to the environment
- Experimental aspects of control and measurement