

Design and Implementation of Flight Dynamics Control Strategies for a Smartphone-based Quadrotor

Thesis for obtaining the degree of

MASTER OF SCIENCE IN ENGINEERING
with emphasis in automation

Alejandro Astudillo Vigoya

alejandro.astudillo@correounivalle.edu.co



School of Electrical and Electronic Engineering

UNIVERSIDAD DEL VALLE

Cali, COLOMBIA

October 27, 2017

Supervised by:

Dr.-Ing. Esteban Rosero
Research Group in Industrial Control - GICI
School of Electrical and Electronic Engineering
Universidad del Valle

Bladimir Bacca Ph.D.
Research Group in Perception and Intelligent Systems - PSI
School of Electrical and Electronic Engineering
Universidad del Valle

Abstract

Contents

Abstract	i
1 Introduction	1
1.1 State of Art	1
1.1.1 Quadrotors	1
1.1.2 Smartphone as a Controller	1
1.1.3 Smartphone-based Quadrotors	1
1.2 Outline	1
2 Dynamic Model	3
2.1 Non-linear Model	3
2.2 Linearized Model	5
3 Smartphone-based Quadrotor Prototype	7
3.1 Description of the Components	7
3.1.1 Smartphone	7
3.1.2 Frame	7
3.1.3 Motors and Electronic Speed Controllers (ESC)	7
3.1.4 Smartphone-ESC Gateway	7
3.1.5 Battery	7
3.1.6 Assembled Smartphone-based Quadrotor	7
3.2 Quadrotor Parameters	7
3.2.1 Mass	7
3.2.2 Inertial Momentum	7
3.2.3 Motors Thrust	7
3.2.4 Motors Torque	7
4 Control Strategies and State Estimation	9
4.1 Linear Quadratic Regulator	9
4.2 H_∞ Controller	9
4.3 State Estimation Through Kalman Filter	9
4.3.1 Particle Model	9
4.3.2 Quadrotor Model	9

5	Implementation and Results	11
5.1	Kalman Filter for State Estimation	11
5.2	Linear Quadratic Regulator Results	11
5.2.1	Simple Translational Movements (LQR)	11
5.2.2	Trajectory Tracking (LQR)	11
5.3	H_∞ Regulator Results	11
5.3.1	Simple Translational Movements (H_∞)	11
5.3.2	Trajectory Tracking (H_∞)	11
	Conclusions and Outlook	12
	Bibliography	15
	Publications	17

List of Figures

2.1	Quadrotor squeme with movement axis and thrust forces.	4
-----	--	---

List of Tables

Chapter 1

Introduction

1.1 State of Art

1.1.1 Quadrotors

1.1.2 Smartphone as a Controller

1.1.3 Smartphone-based Quadrotors

1.2 Outline

Chapter 2

Dynamic Model

2.1 Non-linear Model

This section describes the dynamic modeling used to perform the quadrotor control, based on the study carried out in [Castillo and Lozano \[2004\]](#), [Tamami et al. \[2014\]](#), [Voos \[2007\]](#). This model represents the quadrotor as a solid symmetrical object subject to a total thrust and three torques, without considering the dynamics of the actuators.

The general coordinates representing the position and attitude of the quadrotor are defined as

$$q = [\xi \quad \eta]^T, \quad (2.1)$$

where $\xi = [x \quad y \quad z]^T$ is the vector representing the position of the center of mass of the quadrotor relative to the body reference frame shown in Fig. [2.1](#) and $\eta = [\psi \quad \theta \quad \phi]^T$ represent the quadrotor's attitude.

The Lagrangian of the quadrotor is defined by

$$L(q, \dot{q}) = K_{trans} + K_{rot} - U, \quad (2.2)$$

where $K_{trans} = \frac{m}{2} \dot{\xi}^T \dot{\xi}$ is the translational kinetic energy, $K_{rot} = \frac{1}{2} \dot{\eta}^T J \dot{\eta}$ is the rotational kinetic energy, $U = mgz$ is the potential energy, m is the quadrotor's mass, z is the quadrotor's elevation, g is the gravity acceleration magnitude, and J is the inertial matrix. The dynamic model of the quadrotor is derived from the Euler-Lagrange equation

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} F_\xi \\ \tau \end{bmatrix}, \quad (2.3)$$

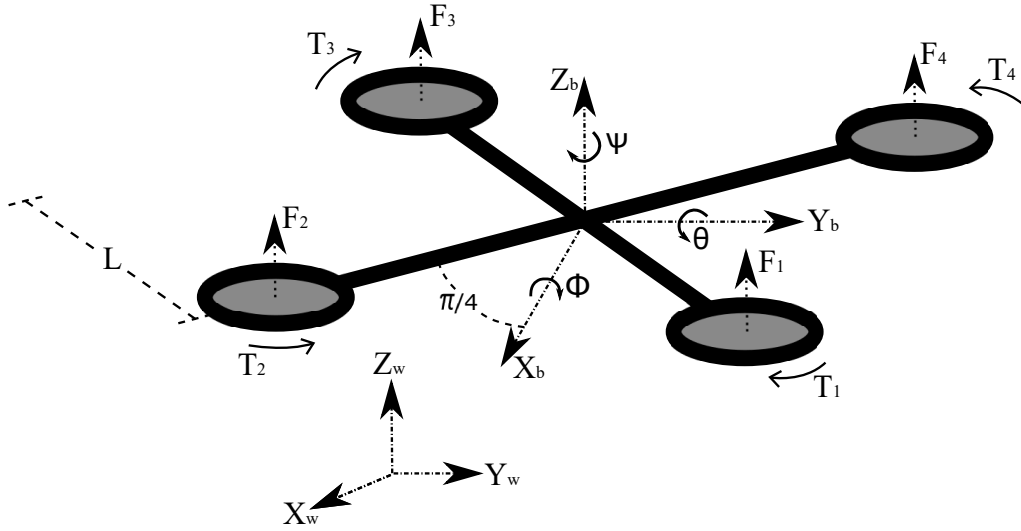


Figure 2.1: Quadrotor squeme with movement axis and thrust forces.

where $F_\xi = R_b^w \hat{F}_b$ is the translational force applied to the quadrotor by the four motors, τ contains the rolling, pitching and yawing torques, and

$$R_b^w = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & s\phi s\psi + c\phi c\psi s\theta \\ c\theta s\psi & s\psi s\theta s\phi + c\phi c\psi & c\phi s\psi s\theta - s\phi c\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (2.4)$$

is the rotation matrix from the body to the Earth frame where $c\theta = \cos \theta$ and $s\theta = \sin \theta$.

In the quadrotor's body frame, the translational force \hat{F}_b is only applied in the z_b axis as shown in Fig. 2.1. This force is represented by

$$\hat{F}_b = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^4 F_i \end{pmatrix}, \quad (2.5)$$

with F_i being the force, in N, exerted by the motor M_i , as shown in Fig. 2.1.

The force F_i has a linear dependency with the square of the motor angular velocity, defined as

$$F_i = k_i w_i^2, \quad (2.6)$$

where w_i is the angular velocity of the motor, and k_i is a proportional constant. However, in practice F_i must be set using the PWM signal input of an ESC. The thrust-PWM relation is found experimentally and is shown in Section ??.

The rolling, pitching and yawing torques contained in vector τ , are generated using the force exerted by each motor as

$$\tau = \begin{bmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\phi \end{bmatrix} = \begin{bmatrix} T_1 + T_3 - T_2 - T_4 \\ L \cos(\pi/4)(F_3 + F_4 - F_2 - F_1) \\ L \cos(\pi/4)(F_2 + F_3 - F_1 - F_4) \end{bmatrix}, \quad (2.7)$$

where T_i is the torque produced by each motor along the z_b axis, L is the distance between each motor's rotor and the quadrotor's center of mass, and $L \cos(\pi/4)$ is the real distance between the point of application of the rolling and pitching torques and the quadrotor's center of mass along the x_b and y_b axes ?.

The Euler-Lagrange equations can be divided in two parts, one for the ξ coordinates and another for the η coordinates, getting

$$\ddot{\xi} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \frac{u_1}{m}(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \frac{u_1}{m}(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ \frac{u_1}{m}(\cos \phi \cos \theta) - g \end{bmatrix}, \quad (2.8)$$

$$\ddot{\eta} = \begin{bmatrix} \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{\phi} \dot{\theta} \frac{J_{xx} - J_{yy}}{J_{zz}} + \frac{u_2}{J_{zz}} \\ \dot{\phi} \dot{\psi} \frac{J_{zz} - J_{xx}}{J_{yy}} + \frac{u_3}{J_{yy}} \\ \dot{\theta} \dot{\psi} \frac{J_{yy} - J_{zz}}{J_{xx}} + \frac{u_4}{J_{xx}} \end{bmatrix}, \quad (2.9)$$

where, $[u_1, u_2, u_3, u_4]^T = [u, \tau_\psi, \tau_\theta, \tau_\phi]^T$, and (J_{xx}, J_{yy}, J_{zz}) are the moments of inertia around the quadrotor's body axes ??.

The Euler-Lagrange equations in (2.8) and (2.9) are linearized using their Jacobian around the hover state where $[\eta, \dot{\eta}, \xi] \rightarrow [0, 0, 0]$, getting

$$\ddot{q} = \begin{bmatrix} g\theta \\ g\phi \\ u_1/m \\ u_2/J_{zz} \\ u_3/J_{yy} \\ u_4/J_{xx} \end{bmatrix}, \quad (2.10)$$

that is a simplified representation of the quadrotor complete model found in ?.

2.2 Linearized Model

The linearised model of the quad-rotor helicopter written as a state space model is given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \\ r(t) &= Cx(t),\end{aligned}$$

where

$$\begin{aligned}A &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T\end{aligned}$$

with the parameters

$$m = 0.64 \text{ kg},$$

$$g = 9.81 \text{ m/s}.$$

The state vector is defined as

$$x(t) = [r_x \quad \dot{r}_x \quad r_y \quad \dot{r}_y \quad r \quad \dot{r}_z]^T,$$

and the control inputs as

$$u(t) = [u_1 \quad u_2 \quad u_3 \quad u_4]^T,$$

and the output vector is defined as

$$r(t) = [r_x \quad r_y \quad r_z]^T.$$

Chapter 3

Smartphone-based Quadrotor Prototype

3.1 Description of the Components

3.1.1 Smartphone

3.1.2 Frame

3.1.3 Motors and Electronic Speed Controllers (ESC)

3.1.4 Smartphone-ESC Gateway

3.1.5 Battery

3.1.6 Assembled Smartphone-based Quadrotor

3.2 Quadrotor Parameters

3.2.1 Mass

3.2.2 Inertial Momentum

3.2.3 Motors Thrust

3.2.4 Motors Torque

Chapter 4

Control Strategies and State Estimation

4.1 Linear Quadratic Regulator

4.2 H_∞ Controller

4.3 State Estimation Through Kalman Filter

4.3.1 Particle Model

4.3.2 Quadrotor Model

Chapter 5

Implementation and Results

5.1 Kalman Filter for State Estimation

5.2 Linear Quadratic Regulator Results

5.2.1 Simple Translational Movements (LQR)

5.2.2 Trajectory Tracking (LQR)

5.3 H_∞ Regulator Results

5.3.1 Simple Translational Movements (H_∞)

5.3.2 Trajectory Tracking (H_∞)

Conclusions and Outlook

In this thesis distributed algorithms

Bibliography

- P. Castillo and R. Lozano. Stabilization of a mini-rotorcraft having four rotors, 2004.
- N. Tamami, E. Pitowarno, and I. Astawa. Proportional derivative active force control for “x” configuration quadcopter, 2014.
- H. Voos. Nonlinear control of a quadrotor micro-uav using feedback-linearization, 2007.

Publications

A. Astudillo, P. Muñoz, F. Alvarez and E. Rosero, “Altitude and attitude cascade controller for a smartphone-based quadcopter,” in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, jun 2017, pp. 1447–1454. [Online]. Available: <http://ieeexplore.ieee.org/document/7991400/>

A. Astudillo, B. Bacca and E. Rosero, “Optimal and robust controllers design for a smartphone-based quadrotor,” in *2017 IEEE 3rd Colombian Conference on Automatic Control (CCAC)*

(Paper Submitted to Journal) A. Astudillo, P. Muñoz and E. Rosero, “Cascade Controller for Autonomous Flight of a Smartphone-based Quadrotor,” in *Journal of Intelligent & Robotic Systems, SI: UAS-2017*.