# Design and Implementation of Flight Dynamics Control Strategies for a Smartphone-based Quadrotor

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# Chapter 2

# Dynamic Model

## 2.1 Non-linear Model

This section describes the dynamic modeling used to perform the quadrotor control, based on the study carried out in Castillo and Lozano [2004], Tamami et al. [2014], Voos [2007]. This model represents the quadrotor as a solid symmetrical object subject to a total thrust and three torques, without considering the dynamics of the actuators.

The general coordinates representing the position and attitude of the quadrotor are defined as

$$q = \begin{bmatrix} \xi & \eta \end{bmatrix}^T, \tag{2.1}$$

where $\xi = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the vector representing the position of the center of mass of the quadrotor relative to the body reference frame shown in Fig. 2.1 and $\eta = \begin{bmatrix} \psi & \theta & \phi \end{bmatrix}^T$ represent the quadrotor's attitude.

The Lagrangian of the quadrotor is defined by

$$L(q, \dot{q}) = K_{trans} + K_{rot} - U, \tag{2.2}$$

where $K_{trans} = \frac{m}{2}\dot{\xi}^T\dot{\xi}$ is the translational kinetic energy, $K_{rot} = \frac{1}{2}\dot{\eta}^T J\dot{\eta}$ is the rotational kinetic energy, $U = mgz$ is the potential energy, $m$ is the quadrotor's mass , $z$ is the quadrotor's elevation, $g$ is the gravity acceleration magnitude, and $J$ is the inertial matrix. The dynamic model of the quadrotor is derived from the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} F_\xi \\ \tau \end{bmatrix}, \tag{2.3}$$

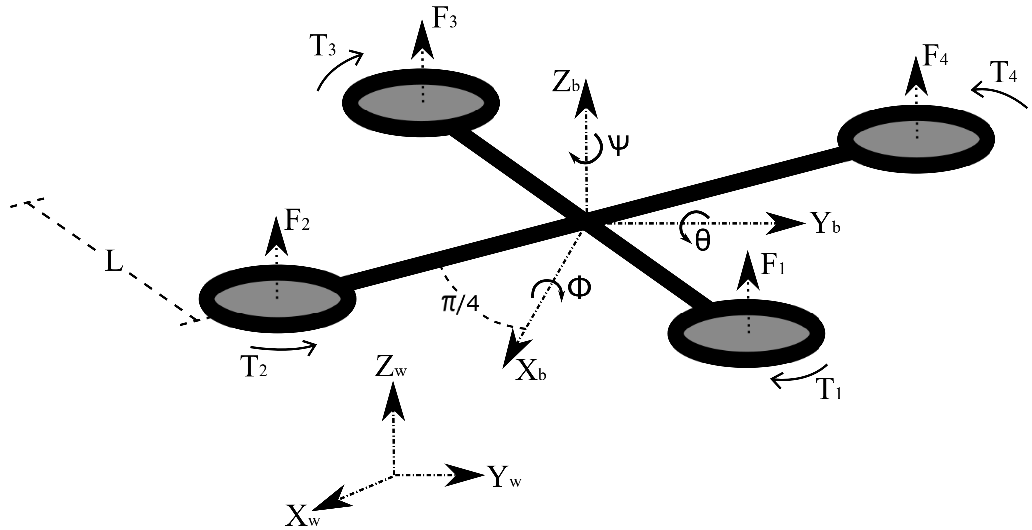


Figure 2.1: Quadrotor squeme with movement axis and thrust forces.

where $F_\xi = R_b^w \hat{F}_b$ is the translational force applied to the quadrotor by the four motors, $\tau$ contains the rolling, pitching and yawing torques, and

$$R_b^w = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & s\phi s\psi + c\phi c\psi s\theta \\ c\theta s\psi & s\psi s\theta s\phi + c\phi c\psi & c\phi s\psi s\theta - s\phi c\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{2.4}$$

is the rotation matrix from the body to the Earth frame where $c\theta = \cos\theta$ and $s\theta = \sin\theta$.

In the quadrotor's body frame, the translational force $\hat{F}_b$ is only applied in the $z_b$ axis as shown in Fig. 2.1. This force is represented by

$$\hat{F}_b = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{pmatrix}, \tag{2.5}$$

with $F_i$ being the force, in N, exerted by the motor $M_i$, as shown in Fig. 2.1.

The force $F_i$ has a linear dependency with the square of the motor angular velocity, defined as

$$F_i = k_i w_i^2, \tag{2.6}$$

where $w_i$ is the angular velocity of the motor, and $k_i$ is a proportional constant. However, in practice $F_i$ must be set using the PWM signal input of an ESC. The thrust-PWM relation is found experimentally and is shown in Section **??**.

The rolling, pitching and yawing torques contained in vector $\tau$, are generated using the force exerted by each motor as

$$\tau = \begin{bmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\phi \end{bmatrix} = \begin{bmatrix} T_1 + T_3 - T_2 - T_4 \\ Lcos(\pi/4)(F_3 + F_4 - F_2 - F_1) \\ Lcos(\pi/4)(F_2 + F_3 - F_1 - F_4) \end{bmatrix}, \tag{2.7}$$

where $T_i$ is the torque produced by each motor along the $z_b$ axis, $L$ is the distance between each motor's rotor and the quadrotor's center of mass, and $Lcos(\pi/4)$ is the real distance between the point of application of the rolling and pitching torques and the quadrotor's center of mass along the $x_b$ and $y_b$ axes **?**.

The Euler-Lagrange equations can be divided in two parts, one for the $\xi$ coordinates and another for the $\eta$ coordinates, getting

$$\ddot{\xi} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \frac{u_1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ \frac{u_1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \\ \frac{u_1}{m}(\cos\phi\cos\theta) - g \end{bmatrix}, \tag{2.8}$$

$$\ddot{\eta} = \begin{bmatrix} \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{\phi}\dot{\theta}\dfrac{J_{xx} - J_{yy}}{J_{zz}} + \dfrac{u_2}{J_{zz}} \\ \dot{\phi}\dot{\psi}\dfrac{J_{zz} - J_{xx}}{J_{yy}} + \dfrac{u_3}{J_{yy}} \\ \dot{\theta}\dot{\psi}\dfrac{J_{yy} - J_{zz}}{J_{xx}} + \dfrac{u_4}{J_{xx}} \end{bmatrix}, \tag{2.9}$$

where, $\begin{bmatrix} u_1, & u_2, & u_3, & u_4 \end{bmatrix}^T = \begin{bmatrix} u, & \tau_\psi, & \tau_\theta, & \tau_\phi \end{bmatrix}^T$, and $(J_{xx}, J_{yy}, J_{zz})$ are the moments of inertia around the quadrotor's body axes **??**.

The Euler-Lagrange equations in (2.8) and (2.9) are linearized using their Jacobian around the hover state where $\begin{bmatrix} \eta, & \dot{\eta}, & \dot{\xi} \end{bmatrix} \rightarrow \begin{bmatrix} 0, & 0, & 0 \end{bmatrix}$, getting

$$\ddot{q} = \begin{bmatrix} g\theta \\ g\phi \\ u_1/m \\ u_2/J_{zz} \\ u_3/J_{yy} \\ u_4/J_{xx} \end{bmatrix}, \tag{2.10}$$

that is a simplified representation of the quadrotor complete model found in **?**.

## 2.2 Linearized Model

The linearised model of the quad-rotor helicopter written as a state space model is given by

$$
\begin{aligned}
\dot{x}(t) =& Ax(t) + Bu(t),\\
r(t) =& Cx(t),
\end{aligned}
$$

where

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}^T
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T
$$

with the parameters

$m = 0.64$ kg,

$g = 9.81$ m/s.

The state vector is defined as

$$
x(t) = \begin{bmatrix} r_x & \dot{r}_x & r_y & \dot{r}_y & r & \dot{r}_z \end{bmatrix}^T,
$$

and the control inputs as

$$
u(t) = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T,
$$

and the output vector is defined as

$$
r(t) = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T.
$$

# Chapter 3

# Smartphone-based Quadrotor Prototype

## 3.1 Description of the Components

### 3.1.1 Smartphone

### 3.1.2 Frame

### 3.1.3 Motors and Electronic Speed Controllers (ESC)

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# Control Strategies and State Estimation

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## 4.3 State Estimation Through Kalman Filter

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## 5.1 Kalman Filter for State Estimation

## 5.2 Linear Quadratic Regulator Results

### 5.2.1 Simple Translational Movements (LQR)

### 5.2.2 Trajectory Tracking (LQR)

## 5.3 H$\infty$ Regulator Results

### 5.3.1 Simple Translational Movements (H$\infty$)

### 5.3.2 Trajectory Tracking (H$\infty$)

# Conclusions and Outlook

In this thesis distributed algorithms

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# Publications

A. Astudillo, P. Muñoz, F. Alvarez and E. Rosero, “Altitude and attitude cascade controller for a smartphone-based quadcopter,” in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, jun 2017, pp. 1447–1454. [Online]. Available: http://ieeexplore.ieee.org/document/7991400/

A. Astudillo, B. Bacca and E. Rosero, “Optimal and robust controllers design for a smartphone-based quadrotor,” in *2017 IEEE 3rd Colombian Conference on Automatic Control (CCAC)*

**(Paper Submitted to Journal)** A. Astudillo, P. Muñoz and E. Rosero, “Cascade Controller for Autonomous Flight of a Smartphone-based Quadrotor,” in *Journal of Intelligent & Robotic Systems, SI: UAS-2017.*