Design and Implementation of Flight Dynamics Control Strategies for a Quadrotor Based On a Smartphone

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Abstract

The field of autonomous systems control is young, but operational experience is rapidly growing, making research on collaborative systems of great importance. Improving aerial robots in particular could be key in facing future environmental challenges.....

In this work, two main problems are addressed: the cooperative source seeking problem and the cooperative level curve tracking problem by a group of agents under undirected constrained communications. ......

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Chapter 2

Quadrotor Helicopter Model

This section describes the dynamic modeling used to perform the quadrotor control, based on the study carried out in Castillo and Lozano [2004], Tamami et al. [2014], Voos [2007]. This model represents the quadrotor as a solid symmetrical object subject to a total thrust and three torques, without considering the dynamics of the actuators.

The general coordinates representing the position and attitude of the quadrotor are defined as

$$q = \begin{bmatrix} \xi & \eta \end{bmatrix}^T, \tag{2.1}$$

where $\xi = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the vector representing the position of the center of mass of the quadrotor relative to the body reference frame shown in Fig. 2.1 and $\eta = \begin{bmatrix} \psi & \theta & \phi \end{bmatrix}^T$ represent the quadrotor's attitude.

The Lagrangian of the quadrotor is defined by

$$L(q, \dot{q}) = K_{trans} + K_{rot} - U, \tag{2.2}$$

where $K_{trans} = \frac{m}{2}\dot{\xi}^T\dot{\xi}$ is the translational kinetic energy, $K_{rot} = \frac{1}{2}\dot{\eta}^T J\dot{\eta}$ is the rotational kinetic energy, $U = mgz$ is the potential energy, m is the quadrotor's mass , z is the quadrotor's elevation, g is the gravity acceleration magnitude, and J is the inertial matrix. The dynamic model of the quadrotor is derived from the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} F_\xi \\ \tau \end{bmatrix}, \tag{2.3}$$

where $F_\xi = R_b^w \hat{F}_b$ is the translational force applied to the quadrotor by the four motors, τ contains the rolling, pitching and yawing torques, and

$$R_b^w = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & s\phi s\psi + c\phi c\psi s\theta \\ c\theta s\psi & s\psi s\theta s\phi + c\phi c\psi & c\phi s\psi s\theta - s\phi c\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{2.4}$$

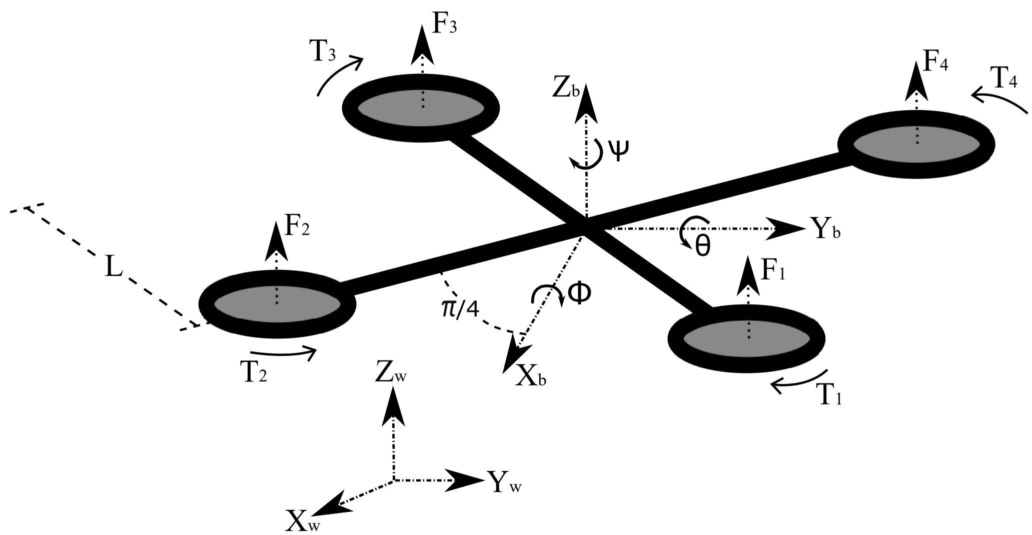

Figure 2.1: Quadrotor squeme with movement axis and thrust forces.

is the rotation matrix from the body to the Earth frame where $c\theta = \cos\theta$ and $s\theta = \sin\theta$.

In the quadrotor's body frame, the translational force $\hat{F}_b$ is only applied in the z_b axis as shown in Fig. 2.1. This force is represented by

$$\hat{F}_b = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{pmatrix}, \tag{2.5}$$

with F_i being the force, in N, exerted by the motor M_i, as shown in Fig. 2.1.

The force F_i has a linear dependency with the square of the motor angular velocity, defined as

$$F_i = k_i w_i^2, \tag{2.6}$$

where w_i is the angular velocity of the motor, and k_i is a proportional constant. However, in practice F_i must be set using the PWM signal input of an ESC. The thrust-PWM relation is found experimentally and is shown in Section **??**.

The rolling, pitching and yawing torques contained in vector τ, are generated using the force exerted by each motor as

$$\tau = \begin{bmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\phi \end{bmatrix} = \begin{bmatrix} T_1 + T_3 - T_2 - T_4 \\ Lcos(\pi/4)(F_3 + F_4 - F_2 - F_1) \\ Lcos(\pi/4)(F_2 + F_3 - F_1 - F_4) \end{bmatrix}, \tag{2.7}$$

where T_i is the torque produced by each motor along the z_b axis, L is the distance between each motor's rotor and the quadrotor's center of mass, and $Lcos(\pi/4)$ is the real distance between the point of application of the rolling and pitching torques and the quadrotor's center of mass along the x_b and y_b axes **?**.

The Euler-Lagrange equations can be divided in two parts, one for the ξ coordinates and another for the η coordinates, getting

$$\ddot{\xi} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \frac{u_1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ \frac{u_1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \\ \frac{u_1}{m}(\cos\phi\cos\theta) - g \end{bmatrix}, \tag{2.8}$$

$$\ddot{\eta} = \begin{bmatrix} \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{\phi}\dot{\theta}\dfrac{J_{xx} - J_{yy}}{J_{zz}} + \dfrac{u_2}{J_{zz}} \\ \dot{\phi}\dot{\psi}\dfrac{J_{zz} - J_{xx}}{J_{yy}} + \dfrac{u_3}{J_{yy}} \\ \dot{\theta}\dot{\psi}\dfrac{J_{yy} - J_{zz}}{J_{xx}} + \dfrac{u_4}{J_{xx}} \end{bmatrix}, \tag{2.9}$$

where, $\begin{bmatrix} u_1, & u_2, & u_3, & u_4 \end{bmatrix}^T = \begin{bmatrix} u, & \tau_\psi, & \tau_\theta, & \tau_\phi \end{bmatrix}^T$, and (J_{xx}, J_{yy}, J_{zz}) are the moments of inertia around the quadrotor's body axes **??**.

The Euler-Lagrange equations in (2.8) and (2.9) are linearized using their Jacobian around the hover state where $\begin{bmatrix} \eta, & \dot{\eta}, & \dot{\xi} \end{bmatrix} \to \begin{bmatrix} 0, & 0, & 0 \end{bmatrix}$, getting

$$\ddot{q} = \begin{bmatrix} g\theta \\ g\phi \\ u_1/m \\ u_2/J_{zz} \\ u_3/J_{yy} \\ u_4/J_{xx} \end{bmatrix}, \tag{2.10}$$

that is a simplified representation of the quadrotor complete model found in **?**.

2.1 Nonlinear Model

2.2 Linearized Model

The linearised model of the quad-rotor helicopter written as a state space model is given by

$$\begin{aligned} \dot{x}(t) =& Ax(t) + Bu(t), \\ r(t) =& Cx(t), \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

with the parameters

$m = 0.64$ kg,

$g = 9.81$ m/s.

The state vector is defined as

$$x(t) = \begin{bmatrix} r_x & \dot{r}_x & r_y & \dot{r}_y & r & \dot{r}_z \end{bmatrix}^T,$$

and the control inputs as

$$u(t) = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T,$$

and the output vector is defined as

$$r(t) = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T.$$

Chapter 3

Smartphone-based Quadrotor Prototype

3.1 Description of the Components

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5.3 H∞ Regulator Results

5.3.1 Simple Translational Movements (H∞)

5.3.2 Trajectory Tracking (H∞)

Conclusions and Outlook

In this thesis distributed algorithms

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