STAT 600 Statistical Computing

HW 4: Monte Carlo

Spring 2024, **Due March 19th**

Homework format: Homework should be submitted as a pdf generated by LaTeX or Rmarkdown. All functions should be coded in `Rcpp/RcppArmadillo`. Please provide explanations of your solutions and appropriate graphics (labeled well).

1. (Book Problem 6.3) Consider finding $\sigma^2 = E[X^2]$, where X has the density proportional to $q(x) = \exp\{-|x|^3/3\}$.

   (a) Estimate σ^2 using importance sampling with standardized weights.

   (b) Repeat the estimation using rejection sampling

   (c) Repeat the estimation using sampling importance resampling.

   (d) Philippe and Robert describe an alternative to importance-weighted averaging that employs a Riemann sum strategy with random nodes [506,507]. When draws $X_1, \ldots, X_n$ originate from $f(\cdot)$, an estimator of $E\{h(X)\}$ is

   $$\sum_{i=1}^{n-1}(X_{[i+1]}) - X_{[i]}h(X_{[i]})f(X_{[i]}),$$

   where $X_{[1]} \leq \cdots \leq X_{[n]}$ is the ordered sample associated with $X_1, \ldots, X_n$. This estimator has faster convergence than the simple Monte Carlo estimator. When $f = cq$ and the normailization constant c in unknown, then

   $$\frac{\sum_{i=1}^{n-1}(X_{[i+1]}) - X_{[i]}h(X_{[i]})q(X_{[i]})}{\sum_{i=1}^{n-1}(X_{[i+1]}) - X_{[i]}q(X_{[i]})}$$

   estimates $E\{h(X)\}$, noting that the denominator estimates $1/c$. Use this strategy to estimate σ^2, applying it post hoc to the output obtained in part (b).

   (e) Carry out an experiment to compare the performance of the 4 estimators (or more generally the 4 estimating techniques). This can take on various forms. Be creative and investigate any aspect of the methods that you find interesting. For example, generate 100 estimates with each method varying the number of iterations taken in each sample (e.g., $n = 100, 1000, 5000$), compare the methods in different model settings (e.g., $q(x) = \exp\{-|x|^\alpha/\beta\}$, varying α and β), computation times with and without Rcpp in various settings, and/or investigate other estimands and compare performance (e.g., $E[X^{-1/2}]$ and others). Discuss your results.

2. Work with a partner (if you want) on this problem: Find an article in one of the journals listed below. The article should be relatively recent (published in 2018 or later). Read the article and meet with your partner to discuss it. You can help one another understand the article and answer the questions below.

   Each of you should write a 1-1.5 page summary of the article in your own words. Your summary should include:

   - A description of the contribution of the work.
   - Discussion of potential applications of the methodology to a problem not described in the paper.
   - Links to topics in our textbook.

- A discussion about things the authors did well and what they could have improved.

Write a coherent essay, not a list of the above topics. When you turn in the assignment, submit the article along with your write-up.

Journals: *Journal of Computational and Graphical Statistics, Statistical Computing, Statistical Computing and Data Analysis, Journal of Statistical Computing and Simulation, Communications in Statistics, Part B–Simulation and Computation, Statistics and Computing, Bayesian Analysis*