Supplementary Appendix A: Algorithm Supp.A.1

Algorithm Supp.A.1 Causality discover for potential factors affecting the doctor consultation

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Input: The doctor set with corresponding profiles and historical interactions.

Output: The DAG consisting of observed doctor attributes.

1: #Pairwise relationship discovery

2: Determine the relevant attributes of doctors regarding to the consultation number by analyzing the collected data.

3: Leverage the determined doctor attributes to construct the objective function of GLasso for estimating the inverse covariance matrix $\hat{\Theta}$ as Eq. (B.1).

4: Transform Eq. (B.1) into the dual optimization function as Eq. (B.2).

5: **while** not reach convergence **do**

6:     Iteratively update $\hat{\beta}_j$ as Eq. (B.4).

7: Obtain the estimated inverse covariance by $\hat{\Theta} = W^{-1}$.

8: Generate the data dependency graph G based on $\hat{\Theta}$ to reflect the dependency relationship between multiple features of doctors.

9: #Causal-effect direction determination

10: **for** each variable x_i **do**

11:     **for** other variables x_j **do**

12:         Determine the causal relationship between x_i and x_j by Eq. (1).

13: Add the causal directions into G and generate DAG.

14: **return** The DAG consisting of observed doctor features.

Supplementary Appendix B: Proof of unbiased stratified estimation

Proof.   As the theorem discussed in Rubin (1974), when both SUTVA and conditionally random treatment assignment hold, the basic stratified estimation with covariates X_i that take values in a discrete space $X_i \in \mathcal{X}$ is unbiased, and this is manifested as follows:

$$\sqrt{n}(\hat{\tau}_{STRAT} - \tau) \Rightarrow \mathcal{N}(0, V_{STRAT}),$$
$$V_{STRAT} = \text{Var}[\tau(X_i)] + \mathbb{E}\Big[\frac{\sigma_{(1)}^2(X_i)}{e(X_i)} + \frac{\sigma_{(0)}^2(X_i)}{1 - e(X_i)}\Big],$$

where $e(x) = \mathbb{P}[W_i = 1 | X_i = x]$ and $e(x)$ satisfies $0 < e(x) < 1$ for all x. The complicated variables of competency and service level consist of multiple base variables as discrimination conditions, where the base variables jointly form a larger discrete space.

Then, benefited from its excellent property of the asymptotic variance, the usage of $\hat{\tau}_{STRAT}$ does not depend on the number of the groups, i.e. $|\mathcal{X}| = p < \infty$. As a result, the stratified estimation on two complicated variables in this paper is unbiased as classical stratified estimation. Therefore, the proof of Theorem 2 is completed.   □

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Supplementary Appendix C: The sensitivity analysis of the E-value

The analysis of the constructed DAGs in causality discovery leads to the proposal of Assumption 1, which elucidates a specific treatment-outcome relationship between service quality and online consultation orders, attributing it to the concern of popularity bias. Inevitably, while the causality discovery incorporates a set of measured confounders, the presence of hidden unmeasured confounders that potentially impact the proposed causal effect is imperatively considered. Inspired by the concept of the E-value in VanderWeele and Ding (2017), it employs a quantitative approach for sensitivity analysis to explore the influence of unmeasured confounders on the association. In other word, the E-value refers to the risk ratio that completely eliminates unmeasured confounding effects under the condition of controlling measured confounding factors (Cheng et al. 2024, Lonati and Wulff 2024, VanderWeele and Ding 2017).

In this study, the exposure denoted as E represents the professional competency y_c, the outcome denoted as D signifies the status of order quantity y_o (where $y_o = 1$ if the order quantity exceeds the average doctor's order, and $y_o = 0$ otherwise), and the measured confounding factor denoted as C corresponds to the service quality y_s. As detailed in Table Supp.E.1, the attributes *Gender* and *Waiting*, labeled as U ($U = 0, 1, \cdots, k, \cdots, K-1$), are identified as potential unmeasured confounding factors. To begin the analysis, the risk ratio (RR) between the exposure and the outcome in the observed data $RR_{ED|c}^{obs}$ is computed by controlling the measured confounders as follows:

$$RR_{ED|c}^{obs} = \frac{P(D=1|E=1, C=c)}{P(D=1|E=0, C=c)}. \tag{Supp.C.1}$$

And RR between unmeasured confounding factors and exposure when $U = k$ is determined as the maximal association impact, i.e. $RR_{EU|c} = max_k \, RR_{EU,k|c}$, where $RR_{EU,k|c} = \frac{P(U=k|E=1, C=c)}{P(U=k|E=0, C=c)}$. Then, the association impact between the unmeasured confounders and outcome $RR_{UD|c}$ is necessary to determine as follows:

$$RR_{UD|c} = max(RR_{UD|E=1,c}, RR_{UD|E=0,c}), \tag{Supp.C.2}$$

where $RR_{UD|E=1,c} = \frac{max_k P(D=1|E=0, C=c, U=k)}{min_k P(D=1|E=0, C=c, U=k)}$ and $RR_{UD|E=0,c} = \frac{max_k P(D=1|E=1, C=c, U=k)}{min_k P(D=1|E=1, C=c, U=k)}$, respectively measuring the impact in the exposure group and non-exposure group. Finally, the true causal RR can be inferred as follows:

$$RR_{ED|c}^{true} = \frac{\Sigma_{k=0}^{K-1} P(D=1 \mid E=1, C=c, U=k)P(U=k \mid C=c)}{\Sigma_{k=0}^{K-1} P(D=1 \mid E=0, C=c, U=k)P(U=k \mid C=c)}. \tag{Supp.C.3}$$

Based on the whole data in this study, we can obtain $RR_{ED|c}^{true} = 1.614$, $RR_{ED|c}^{obs} = 1.087$, $RR_{EU|c} = 5.768$ and $RR_{UD|c} = 5.333$. Following to the discrimination formula in VanderWeele and Ding (2017) under the condition of $RR_{ED|c}^{true} > 1$, the association impact of the gender and waiting time insufficiently eliminates the causal effect between competency and orders with the service level controlled when the following inequality holds:

$$RR_{ED|c}^{true} \geq RR_{ED|c}^{obs} / \frac{RR_{EU|c} RR_{UD|c}}{RR_{EU|c} + RR_{UD|c} - 1}. \tag{Supp.C.4}$$

From the computation of the above inequality, it holds in this context and demonstrates the feasibility of Assumption 1. Moreover, the E-value can be computed as follows:

$$E - value = RR_{ED|c}^{obs} + \sqrt{RR_{ED|c}^{obs} \times (RR_{ED|c}^{obs} - 1)}. \tag{Supp.C.5}$$

$E - value = 2.609$ can be derived, which indicates that the observed RR can be completely explained by the unmeasured confounders when both $RR_{EU|c}$ and $RR_{UD|c}$ is more than 2.609 and the measured confounders have been controlled.

Supplementary Appendix D: Algorithm Supp.D.2

Algorithm Supp.D.2 Pre-training process for competency representation of doctors

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Input: The specialty texts of doctors, the nearest neighbors K, the number of negative edges M and relevant hyper-parameter settings.

Output: The competency embedding $\hat{X}$ of doctors.

1: #Graph construction

2: **for** each doctor $i \in \mathcal{D}$ **do**

3:      Segment the specialty text into word level by pkuseg.

4: Adopt word2vec method to train word segmentation for doctor embedding $\mathbb{R}^{\mathcal{D} \times h_d}$.

5: **for** each doctor $i \in \mathcal{D}$ **do**

6:      Calculate the similarity of i with other doctors.

7:      Select the doctors with top-k similarity to link as edges.

8: The competency graph $\mathcal{G} = \{\mathcal{D}, \mathcal{E}\}$ where nodes are doctors is constructed.

9: #Competency pre-training

10: Initialize the initial embedding of doctors $X \in \mathbb{R}^{\mathcal{D} \times h_c}$.

11: Incorporate the information of neighbors and update embedding by GraphTransformer in Eq. (3).

12: **for** each link $e(u, v) \in \mathcal{E}$ **do**

13:      Generate M negative edges for u.

14:      Generate M negative edges for v.

15: Optimize the pre-train loss function of Eq. (4) to determine final competency embedding of doctors.

16: **return** The final competency embedding of doctors $\hat{X} \in \mathbb{R}^{\mathcal{D} \times h_c}$.

Supplementary Appendix E: Features of empirical data

The performance evaluation of CARE method is examined in two genuine datasets corresponding to two diseases, where they are extracted from a large-scale dataset of Haodf online healthcare platform. Both the two datasets encapsulates dual facets: basic profile of doctors, and matching pair instance, whose feature lists are presented in Tables Supp.E.1 and Supp.E.2. In these feature lists, the features with abbreviations, data types, and their definitions are described in accordance to the data detail of Haodf platform [1]. Note that three features, satisfaction, waiting time and stopped outpatient information of the most doctors is missing within dataset in Table Supp.E.1.

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Table Supp.E.1   Feature list of basic doctor profile

Feature (Abbreviation)	Type	Definition
ID	Numerical	The desensitized ID of the doctor.
Gender	Categorical	The gender of the doctor.
Title (DT)	Categorical	The professional title of the doctor.
Hospital	Categorical	The hospital that the doctor works in.
Department	Categorical	The department of the doctor in the hospital.
Hospital_Level (HL)	Categorical	The grade of the hospital under the Chinese system.
Province	Categorical	The province where the hospital is located.
City	Categorical	The city where the hospital is located.
Specialty	Text	Diseases that doctors excel in diagnosing and treating.
Popularity	Numerical	The recommendation popularity calculated by platform.
Satisfaction	Numerical	Satisfaction with online services on the doctor.
Consultation_Num (HC)	Numerical	The number of historical online consultation of the doctor.
View	Numerical	The number of visits to the doctor's homepage.
Article_Num (AN)	Numerical	The number of articles related to diseases published by doctors on the platform.
After_Consultation (PC)	Numerical	The number of patients that re-consults after consultation.
Vote (PV), Thanks (TL) Gift (GF)	Numerical	The number of votes, thanks letter, and gift operated by patients to the doctor.
Waiting	Numerical	The waiting time to receive the consultation of the doctor.
Online, Registration	Time	The last online time and the registration time of the doctor.
Stopped, Outpatient	Time	The stopped outpatient information and the outpatient schedule of the doctor.

Table Supp.E.2   Feature list of matching pair instance

Feature	Type	Definition
Patient	Numerical	The desensitized ID of the patient.
Disease	Categorical	The disease of the patient diagnosed by the doctor.
Pat_Province	Categorical	The province of the patient living in.
Severity	Categorical	The severity level of the patient instance.
Pay_Time	Time	The time to pay the order.
Type	Categorical	The product type of the order.
Doc	Numerical	The desensitized ID of the doctor.

Supplementary Appendix F: Discrimination criteria of doctor features to conduct RCT

In the estimation of ATE, difference-in-means method requires the randomized control on data instances, which sets the treat group and control group as $[0, 1]$ of features. By observing the potential values within each feature, in line with the domain knowledge on medicine, we utilize the discrimination condition to transform important features for conducting RCT as shown in Table Supp.F.3. That is, if the feature of one doctor satisfies the discrimination criteria of treatment group, it is $y_\bullet = 1$ where $\bullet = \{DT, HL, AN\}$.

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Table Supp.F.3   The discrimination criteria in RCT

Feature	Group	Discrimination condition
Title (DT)	Treatment group	Chief nurse, Chief physician, Chief rehabilitation specialist, Chief pharmacist, Chief technician, Chief inspection physician, Psychotherapist
	Control group	Other doctor titles
Hospital_Level (HL)	Treatment group	Grade A tertiary hospital, Tertiary hospital
	Control group	Other hospital levels
Article_Num (AN)	Treatment group	More than the average of article number
	Control group	Less than the average of article number

Supplementary Appendix G: Complexity analysis of CARE

This section discusses the time complexity of our proposed CARE method to enhance its feasibility in practice. In CARE method, the causality analysis is separated from the real time two-sided matching process so that its implementation time is not included into total time. We just need to analyze the time consumption of pre-training part and weak reciprocal matching.

Pre-training part: For graph construction, the embedding of doctor specialty text via word2vec framework is $\mathcal{O}(kv)$ with sliding time window size k and vocabulary size v. And the usage of KNN nearly consumes the time $\mathcal{O}(|\mathcal{D}|^2)$ in similarity calculation and ranking. Given L layers to repeat GraphTransformer message passing, the complexity is $\mathcal{O}(L(|\mathcal{D}| + M \cdot |\mathcal{E}|) + |\mathcal{E}| \cdot h_c \cdot M)$ with the number of negative sampling M and competency embedding dimension h_c.

Weak reciprocal matching: This process is defined as a binary classification task through two layes MLP, respectively resulting in the complexity of $\mathcal{O}(T \cdot |\mathcal{M}_{obs}| \cdot h_{hidden}(h_{f1} + 1))$ and $\mathcal{O}(T \cdot |\mathcal{M}_{obs}| \cdot h_{hidden}(h_{f2} + 1))$ for P2D and D2P direction, where h_{hidden} is the hidden dimension of MLP. As reciprocity considered in two directions, $h_{f1} = h_0 + h_c + h_s$ and $h_{f2} = h_0 + h_c$ correspond to factors used in prediction.

Hence, the overall time complexity is summarized as follows:

$$\mathcal{O}(kv + |\mathcal{D}|^2 + L(|\mathcal{D}| + M \cdot |\mathcal{E}|) + |\mathcal{E}| \cdot h_c \cdot M) + T \cdot |\mathcal{M}_{obs}| \cdot h_{hidden}(h_{f1} + h_{f2} + 2)). \qquad \text{(Supp.G.6)}$$

## Supplementary Appendix H: The ROC results of the CARE method

Broadly, as Fig. Supp.H.1 shows, the ROC area of Skin disease is 0.6503 (P2D direction) and 0.6548 (D2P direction), and that of Dizziness are 0.5753 (P2D direction) and 0.5769 (D2P direction), which demonstrates that CARE is able to respectively predict and match from doctor side and patient side with close capacity.

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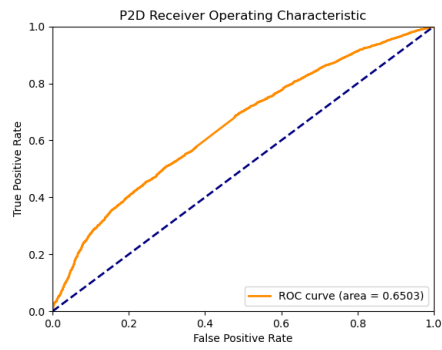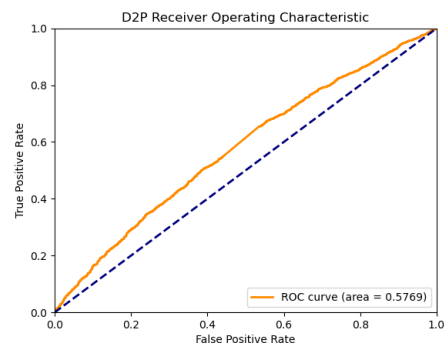
(a) P2D on Skin disease
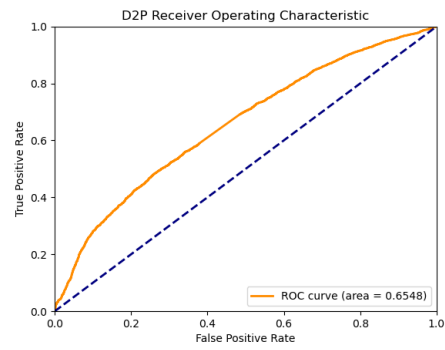
(b) D2P on Skin disease
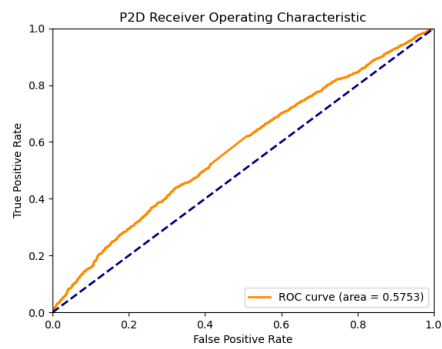
(c) P2D on Dizziness

(d) D2P on Dizziness

Figure Supp.H.1   ROC of CARE method