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CSE321-HW1

1) $T_4 < T_6 < T_1 < T_2 < T_5 < T_3$

$$\lim_{n\to\infty} \frac{\ln^2 n}{\sqrt[3]{n}} = \frac{2\ln(n)}{\frac{1}{3}\cdot n^{1/3}} = \frac{2}{\frac{1}{9}\cdot n^{1/3}} = 0 \qquad \boxed{T_3 = O(T_5)}$$

2) $$\lim_{n\to\infty} \frac{\sqrt[3]{n}}{3n^4+3n^3+1} = \frac{n^{1/3}\cdot 1}{n^{1/3}(3\cdot n^{1/3} + 3\cdot n^{1/3} + n^3)} = 0 \qquad \boxed{T_5 = O(T_2)}$$

3) $$\lim_{n\to\infty} \frac{3n^4+3n^3+1}{3^n} = \frac{12n^3+9n^2}{3^n\cdot \ln 3} = \frac{36n^2+18n}{3^n\cdot \ln^2 3} = \frac{72n+18}{3^n\cdot \ln^3 3}$$

$$= \frac{72}{3^n\cdot \ln^4 3} = 0 \qquad T_2 = O(T_1)$$

4) $$\frac{3^n}{2^{2n}} = \frac{3^n}{4^n} = \frac{1}{\left(\frac{4}{3}\right)^n} = 0 \qquad T_1 = O(T_6)$$

5) $$\frac{2^{2n}}{(n-2)!} = \frac{4^n}{\sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n} = \frac{1}{\sqrt{2\pi n}\cdot\left(\frac{n}{4e}\right)^n} = 0$$

$$T_4 = O(T_6)$$

2) This function finds the minimum and the maximum values of given array and divides their summation by 2. Then the function returns the value which is the closest value to divided summation.

Fruits: Represents array

Plum: Refers to the smallest value after computation

Watermelon: Refers to the biggest value after computation

Orange: Refers to the nearest value to divided summation (Return value)

Orange Time: function uses it for loop.

Worst Case:

first loop: $O(n)$

second loop: $O(n)$

If the given array is shifted n times, (n = size) then it is the worst situation for finding minimum value of array.

So $= \Theta(n)$

Best Case: first loop: $O(n)$

Second loop: $O(n)$

If the given array is not shifted, it will be easy for finding minimum value but this time it is difficult to find the largest number. So, loop should rotate n times.

$\Theta(n)$

Average case $= \Theta(n) \leq f(n) \leq \Theta(n)$

$= \Theta(n)$

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3) a) $\sum_{i=0}^{n-1} (i^2+1)^2$

$$\int_{i=0}^{n-2} \underbrace{(x^2+1)^2}_{x^4+2x^2+1} \leq f(x) \leq \int_{i=1}^{n} (x^2+1)^2$$

$$\Big|_{0}^{n-2} \frac{x^5}{5} + \frac{2x^3}{3} + x \leq f(x) \leq \Big|_{1}^{n} \frac{x^5}{5} + \frac{2x^3}{3} + x$$

$$\frac{(n-2)^5}{5} + \frac{2(n-2)^3}{3} + n-2 \leq f(x) \leq \frac{n^5}{5} + \frac{2n^3}{3} + n - \left(\frac{1}{5} + \frac{2}{3} + 1\right)$$

So $f(n) \in \Theta(n^5)$

b) $\sum_{i=2}^{n-1} \log i^2$

$$\int_{i=2}^{n-2} \log x^2 \, dx \leq f(x) \leq \int_{i=3}^{n} \log x^2 \, dx$$

$\log x^2 = u$ $\quad dx = dv$

$\frac{2\log x}{x} dx$ $\quad x = V$

$$\Big\} \; x \log x^2 - 2 \underbrace{\int \frac{\log x}{x} dx}_{\log^2 x}$$

$= x \log x^2 - \log^2 x$

$$\Big|_{i=2}^{n-2} \log x (2\underbrace{x - \log x}_{O(n)}) \leq f(x) \leq \Big|_{i=3}^{n} \log x (2x - \log x)$$

$$f(n) \in \Theta(n \log n)$$

c) $\sum_{i=1}^{n} (i+1)\, 2^{i-1}$

$x+1 = u \qquad 2^{x-1}\, dx = dv$

$dx = du \qquad \dfrac{2^x}{\log 2} = v$

$$\int_{i=1}^{n} (x+1)\cdot 2^{i-1} = (x+1)\cdot\frac{2^x}{\log 2} - \int \frac{2^x}{\log 2}\, dx = (x+1)\cdot\frac{2^x}{\log 2} - \frac{2^x}{\log^2 2}$$

$$\Big|_{i=1}^{n-1} (x+1)\cdot\frac{2^x}{\log 2} - \frac{2^x}{\log^2 2} \leq f(x) \leq \Big|_{i=1}^{n+1} (x+1)\cdot\frac{2^x}{\log 2} - \frac{2^x}{\log^2 2}$$

$$f(n) \in \Theta(n \cdot 2^n)$$

d) $\displaystyle\sum_{i=0}^{n-1}\sum_{j=0}^{i=1} (i+j) \longrightarrow \sum_{j=0}^{i=1} (i+j) = i^2 + \frac{(i-1)\cdot(i-2)}{2}$

$$\sum_{i=0}^{n-1} i^2 + \frac{i^2-3i+2}{2}$$

$$\int_{i=0}^{n-2} x^2 + \frac{x^2-3x+2}{2} \leq f(x) \leq \int_{i=0}^{n} x^2 + \frac{x^2-3x+2}{2}$$

$$\Big|_{i=0}^{n-2} \frac{x^3}{3} + \frac{x^3}{6} - \frac{3x^2}{4} + x \leq f(x) \leq \Big|_{i=0}^{n} \underbrace{\frac{x^3}{3} + \frac{x^3}{6}}_{\frac{x^3}{2} + \frac{3x^2}{4} + x} + \frac{3x^2}{4} + x$$

$$f(n) \in \Theta(n^3)$$

3 - Code implementations (a-d)

a)

```
int funcA(n){
    int sum=0;
    for(i=0; i<n; i++){
        sum+= pow((pow(i,2)+1),2) //sum+=(i^2+1)^2
    }
    return sum;
}
```

b)

```
int funcD(int n){
    int sum2=0;
    for(i=0; i<n; i++){
        for(j=0; j<i; j++){
            sum2+=(i+j)
        }
    }
    return sum2;
}
```

4)

```
int fun(int n){
    int count=0;
    for(int i=n; i>0; i/=2)
        for(int j=0; j<i; j++)
            count += 1;
    return count;
}
```

Lets give a value for variable n.

for $n = 8$

$\xrightarrow{\text{inner loop}}$ $\underbrace{8 + 4 + 2 + 1}_{\text{count values}}$

for $n = 16$

$\xrightarrow{\text{inner loop}}$ $\underbrace{16 + 8 + 4 + 2 + 1}_{\text{count values}}$

So if we assign n to first number of count it will calculate like that;

$\underbrace{n + n/2 + n/4 + n/8 + \cdots + 1}$

$O(n + n/2 + n/4 + \cdots 1) = \boxed{O(n)}$

$= n(1 + 1/2 + 1/4 + 1/8 + \cdots 1/n)$

$= n(1/2^0 + 1/2^1 + 1/2^2 + \cdots 1/2^{n-1})$ so

$$\boxed{\sum_{i=0}^{n-1} n\left(\frac{1}{2}\right)^i}$$

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5) $\lim_{n\to\infty} \frac{n^3}{3^{2n}} = \frac{n^3}{6^n} = \frac{3n^2}{\ln 6 . 6^n} = \frac{6n}{\ln^2 6 . 6^n} = \frac{6}{\ln^3 6 . 6^n} = 0$ so

$\searrow 6^n$

$n^3 \in o(3^{2n})$

b) $\lim_{n\to\infty} \frac{n}{\log\log n} = \frac{1}{\frac{1}{n.\log n} . \log^2 e} = \frac{n \log n}{\log^2 e} = \infty$

$n \notin o(\log\log n)$

c) $\lim_{n\to\infty} \frac{n^2 . \log^2 n}{n!} = \frac{n^2 . \log^2 n}{\sqrt{2\pi n} . \left(\frac{n}{e}\right)^n} = 0$ $\quad n^2 \log^2 n \in O(n!)$

$(n)^n$ always grows faster than $\log n$ so

d) $\lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n} = \frac{\sqrt{n^2\left(10+\frac{7}{n}+\frac{3}{n^2}\right)}}{n} = \frac{\cancel{n} . \sqrt{10}}{\cancel{n}} = \text{constant}$

so $\sqrt{10n^2+7n+3} \in \Theta(n)$