	COLLEGE OF COMPUTING AND INFORMATION SCIENCES
	Final-Term Assessment Summer 2020 Semester

Class Id	104573-74	**Course Title**	Discrete Mathematics
Program	BSCS	**Campus / Shift**	Main Campus / Morning
Date	11th – August 2020	**Total Marks**	50
Duration	03 hours	**Faculty Name**	Mr. Syed Nabeel Ali / Mr. Adnan Ullah Khan
Student Id	10141	**Student Name**	Shireen Mashal

Instructions:

- Filling out Student-ID and Student-Name on exam header is mandatory.
- Do not remove or change any part of exam header or question paper.
- Write down your answers in given space or at the end of exam paper with proper title "Answer for Question# _ _".
- Answers should be formatted correctly (font size, alignment and etc)
- Handwritten text or image should be on A4 size page with clear visibility of contents.
- Only PDF format is accepted (Student are advise to install necessary software)
- In case of CHEATING, COPIED material or any unfair means would result in negative marking or ZERO.
- A mandatory recorded viva session will be conducted to ascertain the quality of answer scripts where deemed necessary.

- **Caution:** Duration to perform Final-Term Assessment is **03 hours only**. Extra 02 hours are given to cater all kinds of odds in submission of Answer-sheet. **Therefore, if you failed to upload answer sheet on LMS (in PDF format) within 05 hours limit, you would be considered as ABSENT/FAILED.**

[Problem - 1]. [25 Total Points. 5, 5, 2, 3, 10].

a) Find the greatest common divisor of 823 and 2261 using the Euclidean algorithm.
b) Find Bezout identity: Express gcd(823,2261) = 1 as a linear combination of 823 and 2261.
c) Find Bezout Coefficient of above problem.
d) Solve the congruence 823x = 32 mod 2261.
e) Solve the following system of linear congruences by Chinese remainder theorem.(four linear congruential equations)

$x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$,

$x \equiv 3 \pmod 7$ $x \equiv 4 \pmod{11}$.

[Problem - 2]. [15 Total Points. 5, 5, 5].

a) During Eid ul Azha 2020, there was damn shortage of butchers on the first day of Eid as most of the people want to sacrifice their animals on the first day. There was a group of butchers with a very high demand. People continuously asked them to let their animal be sacrificed. So they announced not to worry about, and said the following message **VDE ND NXWHJD** in encrypted form with the following function.

f(p)= (p+3) mod 26

Everyone was confused except DM students who knows what they said.
You are directed to decrypt the message.

b) Books are identified by an International Standard Book Number (ISBN-10), a 10 digit code. The first 9 digits identify the language, the publisher, and the book. The tenth digit is a check digit, which is determined by the following congruence.

$$x_{10} \equiv \sum_{i=1}^{9} i x_i \pmod{11}.$$

i) Suppose that the first 9 digits of the ISBN-10 are 217234505. What is the check digit?
ii) Is 173630143X a valid ISBN10?

c) Find the sequence of pseudorandom numbers generated by the linear congruential method with modulus m = 9, multiplier a = 4, increment c = 2, and seed $x_0 = 5$.

$$x_{n+1} = (ax_n + c) \bmod m.$$

[Problem - 3]. [10 Total Points. 5, 5].

a) Is the sequence {an} a solution of the recurrence relation $a_n = 4a_{n-1} - 5a_{n-2}$ for all integers n, if

$$a_n = n.4^n$$

b) Find the solution to the recurrence relation with the given initial condition. Use an iterative approach.

$$a_n = 2a_{n-1} + 1, \qquad a_0 = 7$$

[Problem - 4]. [15 Total Points. 5, 5, 5]

a) Prove or disprove that 3 divides $n^3 + 2n + 2$ whenever n is a positive integer.
b) The internal telephone numbers in the phone system on a main campus of PAF-KIET consist of seven digits, with the first and second digit not equal to zero and the last digit is even. How many different numbers can be assigned in this system?
c) Suppose that personal identification number (PIN) codes of an ATM card typically consist of four-digit. Find the probability that if you forget your PIN, then you can guess the correct sequence at random in four attempts and you remembered that the last digit is 4.

[Problem - 5]. [10 Total Points. 5, 5]

a) Prove or disprove each of these statements about the floor and ceiling functions.
   a) $\lfloor \lceil x + y \rceil \rfloor = \lceil x + y \rceil$ for all real numbers x.
   b) $\lfloor 2x + 2y \rfloor = 2\lfloor x \rfloor + 2\lfloor y \rfloor$ for all real numbers x and y.
b) Determine whether the given function is a bijection from R to R.

$$f(x) = (x^2 + 3)/(2x^2 + 4)$$

Answer for Question# 01

Question:- 1

(part - a)

a)

GCD = (823, 2261)

$2261 = 823 \times 2 + 615$

$823 = 615 \times 1 + 208$

$615 = 208 \times 2 + 199$

$208 = 1 \times 199 + 9$

$199 = 9 \times 22 + 1$

$9 = 1 \times 9 + 0$

GCD(823, 2261) = 1

Q1 - part b

$GCD(823, 2261) = 1$

$1 \cdot 2261 = 823 \cdot 2 + 615$
$1 \cdot 823 = 615 \cdot 1 + 208$
$1 \cdot 615 = 208 \cdot 2 + 199$
$1 \cdot 208 = 199 \cdot 1 + 9$
$1 \cdot 199 = 9 \cdot 22 + 1$
$1 \cdot 9 = 1 \cdot 9 + 0$

$615 = 1 \cdot 2261 - 2 \cdot 823$
$208 = 1 \cdot 823 - 1 \cdot 615$
$\cancel{199 = 1 \cdot 208 - 1 \cdot 199}$
$199 = 1 \cdot 615 - 2 \cdot 208$
$9 = 1 \cdot 208 - 1 \cdot 199$
$1 = 1 \cdot 199 - 22 \cdot 9$

$1 = 1 \cdot 99 - 22 \cdot 9$
$1 = 1 \cdot 199 - 22(1 \cdot 208 - 1 \cdot 199)$
$1 = 1 \cdot 199 - 22 \cdot 208 - 2 \cdot 199$
$1 = -1 \cdot 199 - 22 \cdot 208$

$1 = -1 \cdot 199 - 22 \cdot 208$
$1 = -1(1 \cdot 615 - 2 \cdot 208) - 22 \cdot 208$
$1 = -1 \cdot 615 + 2 \cdot 208 - 22 \cdot 208$
$1 = -1 \cdot 615 - 20 \cdot 208$

$1 = -1 \cdot 615 - 20 \cdot 208$
$1 = -1 \cdot 615 - 20(1 \cdot 823 - 1 \cdot 615)$
$1 = -1 \cdot 615 - 20 \cdot 823 - 20 \cdot 615$
$1 = -21 \cdot 615 - 20 \cdot 823$

Continue (1 b)

$1 = -21 \cdot 615 - 20 \cdot 823$

$1 = -21(1 \cdot 2261 - 2 \cdot 823) - 20 \cdot 823$

$1 = -21 \cdot 2261 + 42 \cdot 823 - 20 \cdot 823$

$1 = -21 \cdot 2261 + 22 \cdot 823$

Q 1 (part - c)

Find Bezout Coefficient :-

$1 = -21 \cdot 2261 + 22 \cdot 823$

1 inverse 823 mod 2261 = 22

Answer of Q1 (part-d)

Solve the congruence $823x = 32 \mod 2261$

$x = 823 - 32 \mod 2261$

$x = 791 \mod 2261$

$\underline{x = 791}$

Q1 (part-e)

$x \equiv 2 \pmod 3$
$x \equiv 3 \pmod 7$
$x \equiv 3 \pmod 5$
$x \equiv 4 \pmod{11}$

$m = m_1 \times m_2 \times m_3 \times m_4$

$m = 3 \times 7 \times 5 \times 11$

$m = 1155$

$M_1 = \frac{m}{m_1} = \frac{1155}{3} \Rightarrow 385$

$M_2 = \frac{m}{m_2} = \frac{1155}{7} \Rightarrow 165$

$M_3 = \frac{m}{m_3} = \frac{115}{5} \Rightarrow 231$

$M_4 = \frac{m}{m_4} = \frac{115}{11} \Rightarrow 105$

$\underline{Q1\text{ (parte) Remaining}}$

$\underline{y}$:-

$y_1 = 385 \bmod 3 = 1$

$y_2 = 165 \bmod 7 = 4$

$y_3 = 231 \bmod 5 = 1$

$y_4 = 105 \bmod 11 = 6$

$$\left\{ \begin{array}{l} a_1 = 2 \\ a_2 = 3 \\ a_3 = \cancel{4}\,3 \\ a_4 = 4 \end{array} \right\}$$

$x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 y_3 + a_4 M_4 y_4$

$x = 2(385)(1) + 3(165)(4) + 3(231)(1) + 4(105)(6)$

$x = 770 + 1980 + 693 + 2520$

$x = 5963 \bmod 115$

$x = 188$

Answer for Question# 02

Question:-02

(part-a)

ANSWER

	V	D	E	N	D	N	X	W	H	J	D
	21	3	4	13	3	13	23	22	7	9	3
de	18	0	1	10	0	10	20	19	4	6	0
	S	A	B	K	A	K	U	T	E	G	A

Translating this VDE ND NXWHJD, we obtain SAB KA KUTEGA.

(part-b)

ANSWER:-

(i)

$$x_{10} \equiv \sum_{i=1}^{9} i x_i \pmod{11}$$

9 digit of the ISBN-10 are 217234505

$$x_{10} = 1\cdot2+2\cdot1+3\cdot7+4\cdot2+5\cdot3+6\cdot4+7\cdot5+8\cdot0+9\cdot5 \pmod{11}$$

$$x_{10} = 2+2+21+8+15+24+35+0+45 \pmod{11}$$

$$x_{10} = 152 \pmod{11}$$

$$\boxed{x_{10} = 9}$$

Q2 (part-b)

(ii)

Is 1736301 43X a valid ISBN10?

$$\sum_{i=1}^{10} i x_i \equiv 0 \pmod{11}$$

$$1\cdot 1 + 2\cdot 7 + 3\cdot 3 + 4\cdot 6 + 5\cdot 3 + 6\cdot 0 + 7\cdot 1 + 8\cdot 4 + 9\cdot 3 + 10\cdot 10\,)$$

$$\equiv 0 \pmod{11}$$

$$1 + 14 + 9 + 24 + 15 + 0 + 7 + 32 + 27 + 100 \equiv 0 \pmod{11}$$

$$229 \equiv 2 \not\equiv 0 \pmod{11}$$

Since

$$\sum_{i=1}^{10} \not\equiv 0 \pmod{11}$$

It is not a valid ISBN

× ———————— ×

Question:- 02

(part - c)

$$x_{n+1} = (ax_n + c) \bmod m$$

Where $m = 9$, $a = 4$, $c = 2$, and $x_0 = 5$

Where $n = 0$

$x_1 = (4 \times 5 + 2) \bmod 9$

$x_1 = 22 \bmod 9$

$\boxed{x_1 = 4}$

Where $n = 1$

$x_2 = (4 \times 4 + 2) \bmod 9$

$x_2 = 18 \bmod 9$

$\boxed{x_2 = 0}$

Where $n = 2$

$x_3 = (0 \times 4 + 2) \bmod 9$

$x_3 = 2 \bmod 9$

$\boxed{x_3 = 2}$

Where $n = 3$

$x_4 = (4 \times 2 + 2) \bmod 9$

$x_4 = 10 \bmod 9$

$\boxed{x_4 = 1}$

Where $n = 4$

$x_5 = (4 \times 1 + 2) \bmod 9$

$x_5 = 6 \bmod 9$

$\boxed{x_5 = 6}$

Where $n = 5$

$x_6 = (4 \times 6 + 2) \bmod 9$

$x_6 = 26 \bmod 9$

$x_6 = 8$

Sequence 5, 4, 0, 2, 1, 6, 8, ——

Answer for Question# 03

Q3 (b)

$a_n = 2a_{n-1} + 1$ when $a_0 = 7$

$a_1 = 2a_0 + 1 = 2(7) + 1 = 15$

$a_2 = 2a_1 + 1 = 2(15) + 1 = 31$

$a_3 = 2a_2 + 1 = 2(31) + 1 = 63$

$a_4 = 2a_3 + 1 = 2(63) + 1 = 127$

$15, 31, 63, 127 \ldots$

Answer for Question# 04

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Question 4
part a

ANSWER :

$$n^3 + 2n + 2$$

Let $n = 1$

$1 + 2 + 2$

5

5 is not exactly divisible by 3.

Hence,

It is disproved

Question 4
(part b)

ANSWER :

First two digist have nine(9) possibilities each

last digit have 1 possibility

Rest of the digits have 10 possibilities

$= 9 \times 9 \times 1 \times 10 \times 10 \times 10 \times 10$

$= 81 \times 10000$

$= 810,000$ Different numbers can be Assigned

Question 4

(part – c)

ANSWER :-

Probablity of guessing pin $= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot 1$

~~in 4 trys~~

Probability of guessing pin in 4 trys :

$$\frac{1}{1000} \times 4 = \frac{4}{1000} = \frac{1}{250} = 0.004$$

Answer

Answer for Question# 05

Question: 5

part a

(part a) $\lfloor \lceil x+y \rceil \rfloor = \lceil x+y \rceil$ for all real numbers x

ANSWER:

The expression on the R.H.S will first perform ceiling function and then the flooring function will perform ceiling function.

Let $x = 2.8$ & $y = 3$

$[[2.8+3]] = [2.8+3]$

$[[5.8]] = [5.8]$

$[6] = 6$

Proved

part(b) $[2x+2y] = 2[x] + 2[y]$ for all real numbers x & y.

Let $x = 8.5$ and $y = 1.5$

$[2(8.5)+2(1.5)] = 2[8.5] + 2[1.5]$

$[17+3] = 2 \times 8 + 2 \times 1$

$20 = 18$

Disproved

Question 5

part b

$$f(x) = \frac{(x^2+3)}{(2x^2+4)}$$

$$f(x) = \frac{(x^2+3)}{(2x^2+4)} = y$$

$$\frac{x^2+3}{2x^2+4} = y$$

$$x^2+3 = (2x^2+4)y$$

$$x^2+3 = 2x^2y+4y$$

$$3-4y = 2x^2y - x^2$$

$$3-4y = x^2(2y-1)$$

$$\frac{3-4y}{2y-1} = x^2 \quad \text{or} \quad x^2 = \frac{3-4y}{2y-1}$$

S.B.S

$$\sqrt{\cancel{x^2}} = \pm\sqrt{\frac{3-4y}{2y-1}}$$

$$f^{-1}(x) = \pm\sqrt{\frac{3-4y}{2y-1}}$$ It is not a function

There fore it is not a bijeton.