# DISCRETE MATHEMATICS

## ASSIGNMENT 1

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**Q1** Prove $p \rightarrow q = \sim q \rightarrow \sim p$ using truth table.

| $p$ | $q$ | $\sim q$ | $\sim p$ | $p \rightarrow q$ | $\sim q \rightarrow \cancel{p} \sim p$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | ~~F~~ T | F | F | F |
| F | T | F | ~~F~~ T | ~~F~~ T | ~~F~~ T |
| F | F | T | T | T | T |

The output of $p \rightarrow q$ and $\sim q \rightarrow \sim p$ is same. Hence, ~~both~~ PROVED!

**Q2:** Given statements:

$p$ = 12072 is a student

$q$ = 12072 is in CS department.

Convert the symbols into english sentences. Hence, make truth table.

i) $p \wedge q$

12072 is a student AND ~~is in~~ 12072 is in CS department.

ii) $\sim p$

12072 is NOT a student.

iii) $\sim q$

12072 is NOT in CS department.

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(2)

iv) $\cancel{\sim p \vee \sim q}$ $p \wedge \sim q$

12072 is a student 'BUT 12072' is NOT in CS department

v) $\sim p \vee (p \wedge \sim q)$

$$\sim p \vee (p \wedge \sim q)$$

$= (\sim p \vee p) \wedge (\sim p \vee \sim q)$ Distributive law.

$= t \wedge (\sim p \vee \sim q)$ Negation law.

~~$= t \wedge \sim(p \wedge q)$ De Morgan law.~~

$= (t \wedge \sim p) \vee (t \wedge \sim q)$ Distributive law

$= \sim p \vee \sim q$ Identity law $(t \wedge \sim p) = \sim p$, $(t \wedge \sim q) = \sim q$.

~~Either~~ 12072 is NOT a ~~b~~ student OR 12072 is NOT in CS ~~d~~ department

vi) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

In previous question, we have ~~proved~~ simplified:

$$\sim p \vee (p \wedge \sim q) = \sim p \vee \sim q$$

Using this:

$$(p \wedge q) \vee (\sim p \vee \sim q)$$

~~$= (p \wedge q) \vee \sim p$~~

~~$= (p \wedge q) \vee \sim(p \wedge q)$ De Morgan law.~~

~~$= [(p \wedge q) \vee \sim p] \wedge [(p \wedge q)$~~

~~$= [(p \wedge q) \vee \sim p] \wedge [(p \wedge q) \vee (\sim p \vee \sim q)]$~~

$= (p \wedge q) \vee \sim(p \wedge q)$ De Morgan Law.

12072 is a student AND 12072 is in CS department

vi) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

From above eq, we have proved:

$$\sim p \vee (p \wedge \sim q) = \sim p \vee \sim q$$

Substituting:

$(p \wedge q) \vee (\sim p \vee \sim q)$

$= (p \wedge q) \vee \sim(p \wedge q)$ De Morgan law

$= \sim(p \wedge q) \vee (p \wedge q)$ Commutative law

$= [\sim(p \wedge q) \vee p] \wedge [\sim(p \wedge q) \vee q]$ Distributive law

$= [\sim p \vee \sim q \vee p] \wedge [\sim p \vee \sim q \vee q]$ De Morgan law

$= [(\sim p \vee p) \vee \sim q] \wedge [\sim p \vee (\sim q \vee q)]$ Associative law

$= [t \vee \sim q] \wedge [\sim p \vee t]$ Negation law

$= t \wedge t$

$\boxed{= t}$

English sentence:

"It is true that 12072 is NOT a student AND it is true that 12072 is NOT in CS department".

4

Truth Table :-

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $p \wedge \sim q$ | $\sim p \vee (p \wedge \sim q)$ | $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | ~~F~~T | F | F | T |
| T | F | F | T | F | T | T | T |
| F | T | T | F | F | F | T | T |
| F | F | T | T | F | F | T | T |

Since True comes in all conditions; Hence,

$$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)) \equiv t$$