	COLLEGE OF COMPUTING AND INFORMATION SCIENCES		
	Mid-Term Assessment Fall 2020 Semester		
Class Id	105126 / 105128	**Course Title**	Numerical Computing
Program	BSCS	**Campus / Shift**	North Nazimabad Campus / Morning
Date	20th – October 2020	**Total Marks**	30
Duration	02 hours	**Faculty Name**	Saad Akbar
Student Id	63961	**Student Name**	Ali afzal

Instructions:

- Filling out Student-ID and Student-Name on exam header is mandatory.
- Do not remove or change any part of exam header or question paper.
- Write down your answers in given space or at the end of exam paper with proper title "Answer for Question# _ _".
- Answers should be formatted correctly (font size, alignment and etc.)
- Handwritten text or image should be on A4 size page with clear visibility of contents.
- Only PDF format is accepted (Student are advise to install necessary software)
- In case of CHEATING, COPIED material or any unfair means would result in negative marking or ZERO.
- A mandatory recorded viva session will be conducted to ascertain the quality of answer scripts where deemed necessary.

- **Caution:** Duration to perform Mid-Term Assessment is **02 hours only**. Extra 01 hours are given to cater all kinds of odds in submission of Answer-sheet. **Therefore, if you failed to upload answer sheet on LMS (in PDF format) within 03 hours limit, you would be considered as ABSENT/FAILED.**

QUESTIONS	TOTAL MARKS	MARKS OBTAINED
Question # 01	05 + 05 = 10	
Question # 02	05 + 05 = 10	
Question # 03	02 + 02 + 06 = 10	

Question # 01 (a)

Two bus stops are at a distance of 96 km from each other. One bus covers this distance in 40 minutes less than the other. The speed of the first bus is 12 km/h faster than that of the second bus. Determine the speeds of both buses. First create the equation of this problem and then apply Regula Falsi Method to determine their root values. Perform only 2 – iterations, accuracy up to 4 decimal places.

Question # 01 (b)

We have two curves, (USE RADIANS)

$$f(x) = \cos x \; and \; y = x^3 - 1$$

At which point given functions intersect each other? Perform only 2 – iterations, accuracy up to 4 decimal places. Use Newton Raphson Method to justify your answer.

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Q.1 (b) $f(x) = \cos x$

$y = x^3 - 1$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f(0) = \cos(0 \times 57.3) = 1$

$f(1) = \cos(1 \times 57.3) = 0.5402$

$f(2) = \cos(2 \times 57.3) = -0.4162$

$\boxed{x_0 = 1.6}$

Using $n = 0$

$$x_1 = x_0 - \frac{\cos x_0}{-\sin x_0}$$

$$= 1.6 - \frac{\cos(1.6 \times 57.3)}{-\sin(1.6 \times 57.3)}$$

$x_1 = 1.5706$

Using $n = 1$

$$x_2 = x_1 - \frac{\cos x_1}{\sin x_1}$$

$$x_2 = 1.5706 - \frac{\cos(1.5706 \times 57.3)}{-\sin(1.5706 \times 57.3)}$$

$x_2 = 1.5706.$

For $y = x^3 - 1$

$f(x) = x^3 - 1$

$f'(x) = 3x^2$

$f(0) = (0)^3 - 1 = -1$

$f(1) = (1)^3 - 1 = 0$

$f(2) = (2)^3 - 1 = 7$

Using $x_0 = 1.6$

$n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$1.6 - \frac{3.096}{7.68}$$

$x_1 = 1.1968$

$n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.1968 - \frac{0.319}{3.5904}$$

$x_2 = 1.1079$

Question # 02 (a)

Use Muller's Method to find the root of the equation,

$$f(x) = x^3 - 7x^2 + 14x - 6$$

Take, $x_0 = 0.5, x_1 = 1, x_2 = 0.$ Perform only 2 – iterations, accuracy up to 4 decimal places.

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Q.2(a): Use Muller's Method to find the root of the root of the equation.

$$f(u) = u^3 - 7u^2 + 14u - 6$$

Take, $u_0 = 0.5$, $u_1 = 1$, $u_2 = 0$. Perform only 2 iter, accuracy upto 4 decimal places.

First iteration:

For y_0:

$f(u_0) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6$

$y_0 = -0.625$

For y_1:

$f(u_1) = (1)^3 - 7(1)^2 + 14(1) - 6$

$y_1 = 2.$

For y_2:

$f(u_2) = (0)^3 - 7(0)^2 + 14(0) - 6$

$y_2 = -6$

$h_1 = u_1 - u_0 \Rightarrow 1 - 0.5$

$h_1 = 0.5$

$h_2 = u_0 - u_2 \Rightarrow 0.5 - 0$

$h_2 = 0.5$

$h = h_2 / h_1 = 0.5 / 0.5$

$h = 1$

$$a = \frac{hy_1 - y_0(1+h) + y_2}{hh_1^2(1+h)} \Rightarrow \frac{(1)(2) + 0.625(1+1) + (-6)}{1(0.5)^2(1+1)}$$

$a = -5.5$

$$b = \frac{y_1 - y_0 - ah_1^2}{h_1} \Rightarrow \frac{2 + 0.625 - (-5.5)(0.5)^2}{0.5}$$

$b = 8$

$c = y_0 = -0.625$

$$x_r = x_3 = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 0.5 - \frac{2(-0.625)}{8 + \sqrt{(8)^2 - 4(-5.5)(-0.625)}}$$

$$= 0.5 - (-0.0828)$$

$$\boxed{x_r = x_3 = 0.5828}$$

Now $x_r > x_0$

$x_0 = x_r\ (0.5828)$

$x_1 = x_1\ (1)$

$x_2 = x_0\ (0.5)$

$\Rightarrow x_0 = 0.5828, \quad x_1 = 1, \quad x_2 = 0.5$

Second Iteration:

For y_0:

$f(x_0) = (0.5828)^3 - 7(0.5828)^2 + 14(0.5828) - 6$

$y_0 = -0.0204$

For y_1:

$f(x_1) = (1)^3 - 7(1)^2 + 14(1) - 6$

$y_1 = 2$

For y_2:

$f(y_2) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6$

$y_2 = -0.625$

$h_1 = x_1 - x_0 \Rightarrow 1 - 0.5828$

$h_1 = 0.4172$

$h_2 = x_0 - x_2 \Rightarrow 0.5828 - 0.5$

$h_2 = 0.0828$

$h = h_2/h_1 \Rightarrow 0.0828/0.4172$

$h = 0.1984$

$$a = \frac{hy_1 - y_0(1+h) + y_2}{hh_1^2(1+h)}$$

$$\Rightarrow \frac{(0.1984)(2) + 0.0204(1+0.1984) - 0.625}{(0.1984)(0.4172)^2(1+0.1984)}$$

$a = -4.9234$

$$b = \frac{y_1 - y_0 - ah_1^2}{h_1}$$

$$= \frac{2 + 0.0204 - (-4.9234)(0.4172)^2}{0.4172}$$

$b = 6.8968$

$c = y_0 = -0.0204$

$$x_r = x_3 = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 0.5828 - \frac{2(-0.0204)}{6.8968 + \sqrt{(6.8968)^2 - 4(-4.9234)(-0.0204)}}$$

$$x_r = 0.5857$$

Now $x_r > x_0$

$x_0 = x_r$ (0.5857)

$x_1 = x_1$ (1)

$x_2 = x_0$ (0.5828)

$\Rightarrow x_0 = 0.5857$, $x_1 = 1$; $x_2 = 0.5828$.

Question # 02 (b)

The product of a certain negative number less one and three times this number less two is 14. Determine the number by applying Bisection Method. First create the equation and then perform only 2 iterations. (Accuracy up to 4 decimal places).

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Q.2(b)

The equation is:

$$3x^2 - 5x - 12 = 0$$

supposing, $a = 2 \quad b = 4$

$$f(a) = f(2) = 3(2)^2 - 5(2) - 12$$
$$= -10$$

$$f(b) = f(4) = 3(4)^2 - 5(4) - 12$$
$$= 16$$

$$c_2 \quad \frac{a+b}{2} = \frac{2+4}{2}$$

$$c = 3$$

$$f(a) \times f(c) = -10 \times 1 = -10 < 0$$

now $b = c$.

First iteration:

$a = 2 \quad b = 3.$

$$c_2 \quad \frac{a+b}{2} = \frac{2+3}{2}$$

$$c_2 = 2.5.$$

$$f(a) \times f(c) = -10 * -5.75 > 0.$$

now $a = c.$

Second iteration

$$c_2 \quad \frac{a+b}{2} = \frac{2.5+3}{2}$$

$$c_2 = 2.75.$$

Iteration	a	b	c
0	2	4	3
1	2	3	2.5
2	2.5	3	2.75

Question # 03

Use Gaussian Elimination method to solve the following system of linear equation,

$$\begin{aligned} 2x_2 + x_4 &= 0 \\ 2x_1 + 2x_2 + 3x_3 + 2x_4 &= -2 \\ 4x_1 - 3x_2 + x_4 &= -7 \\ 6x_1 + x_2 - 6x_3 - 5x_4 &= 6 \end{aligned}$$

a. Is any row interchange needed?

b. What happens if we add another equation in this system with four variables?

c. Confirms the solution by doing naïve Gaussian Elimination using exact arithmetic (use fractions throughout).

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Q.3 (a) Yes, row interchange will be needed

(b) By doing so our number of equations will get greater than number of unknown variables and we will get infinite number of solutions.

(c) Interchanging R1 with R2 and R3 with R4.

$$2u_1 + 2u_2 + 3u_3 + 2u_4 = -2 \quad -(1)$$
$$2u_2 + u_4 = 0 \quad -(2)$$
$$6u_1 + u_2 - 6u_3 - 5u_4 = 6. \quad -(3)$$
$$4u_1 - 3u_2 + 4u_4 = -7. \quad -(4)$$

÷ing (1) by 2.

$$u_1 + u_2 + 3/2\,u_3 + u_4 = -1 \quad -(1)$$
$$2u_2 + u_4 = 0 \quad -(2)$$
$$6u_1 + u_2 - 6u_3 - 5u_4 = 6 \quad -(3)$$
$$4u_1 - 3u_2 + 4u_4 = -7. \quad -(4)$$

x ing (1) by 6 and (1) by 4 and - by (3) and (4) resp.

$$u_1 + u_2 + 3/2\,u_3 + u_4 = -1$$
$$2u_2 + u_4 = 0$$
$$\otimes \quad \oplus \quad -5u_2$$