MUHAMMAD UMAR KHAN 10619
NC QUIZ#01

NOTE: QUESTIONS 1-5 ARE DONE IN THIS QUIZ SIR. I HAD AN EYE INFECTION DUE TO WHICH I COULDN'T WRITE PRPERLY BUT I NEVER WANTED TO LOOSE MARKS TO I ATTEMPTED THE QUIZ THANKS.

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1

Q.3 $x^2=9$ $\boxed{A=9}$

$x_{n+1}=\frac{1}{2}\left(x_n+\frac{9}{x_n}\right)$

$f(x)=x^2-9$

$f'(x)=2x$

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

$x_{n+1}=x_n-\frac{x_n^2-9}{2x_n}$

$x_n-\frac{x_n^2}{2x_n}+\frac{9}{2x_n}$

$=x_n-\frac{x_n}{2}-\frac{9}{2x_n}$

$=\frac{x_n}{2}-\frac{9}{2}x_n$

$\boxed{x_{n+1}=\frac{1}{2}\left(x_n+\frac{9}{x_n}\right)}$ where $\boxed{A=9}$

Proved

Q.4

When Newton Raphson method fails.

Sol: The formula of Newton's Raphson Method is:

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

when $f'(x_n)\to 0$, Then Newton's Raphson method fails.

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Question Number #01:

The secant method can also be derived from geometry. Taking two initial guesses, $x_{i-1}$ and $x$, one draws a straight line between $f(x_i)$ and $f(x_{i-1})$ passing through the $x$-axis at $x_{i+1}$. ABE and DCE are similar triangles

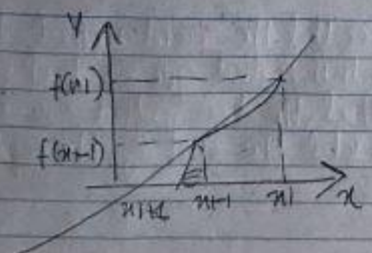
Hence,

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given by

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$



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The newton-raphson method of solving a non linear equation $f(x)=0$ is given by the iterative formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

One of the drawbacks of newton raphson method is that you have evaluate the derivative of the function. To overcome the drawback, derivative of the function $f(x)$ is approximated as:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

Substituting eq-② in eq-① gives

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

This eq is called the secant method. This method now requires two initial guesses, but unlike the bisection method, the two initial guesses do not need to bracket the root of the equation.

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```
p = x0 - f(x0) / df(x0)
if (abs (f(p) TOP)) %.. checks if tolerance is
sroulded.
x = [i, pi f(p), abs (f(p)];
% iteration cont, apronate root, f(apx.root), %dev
disp (single (x)) % single pree-sion of digitals
n = 1;
else
i = i+1
x0 = p
end
end % of whole loop
end
```

letting $z = 0.892$ and creating an equation $f(y)=0$ yields.

$$0.892 = \frac{1+y+y^2-y^3}{(1-y)^3}$$

$$(1-y)^3 (0.892) = 1+y+y^2 = y^3$$

$$\boxed{1+y+y^2+y^3-0.892(1-y)^3 = 0}$$

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solution :- ①

when using numerical approximation techniques, it is useful to have. proframs written to aid in numerical calculations to avoid calculations by hand.

using matlab, creation

m-file named "newton.m" which will contain matlab code that implements the bisection method. Inside this file, write the code to Implements the method. one example of a correctly executable routine based on the algorithm (tole rance 2) in the text is as folows

function rtn = newton (f, df, x0, Tol).

•/. newton's method reaches a ~~cord~~ contain tolerance to approximate root of fx

•/. method takes function f, derivative sal, initial point tolerance if (f(x0) ~=0) and (df(x0)=0))

i=1 ; n=0 (white n==0)

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(4)

looking at the plot, it appears that a positive root (search for a positive root since the equation is associated with a physical problem) occurs near $x=2$, so choose the starting point $x_0=2.5$

the following sequence of commands in the matlab command window to apply the "Newton's method" to the function

$$f(y) = 1+y+y^2-y^3-0.892\ (1-y)^3$$ and its derivate

$f'(y) = 1+2y-3y^2+2.676\ (1-y)^2$, using the points $x=2.5$ to initialize the method to approximate the root for this problem, let the tolerance be $0.00001$. The root approximation will be in the second column of the row output.

INput:-

```
>> format longg
>> f =@(y) 1+y+y^2-y^3-0892 (1-y)^3;
```

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out put :-

f = @ (y) $1+y+y^2-y^3 = 0.892\,(1-y)^3$

INput :-

\>> df = @ (y) $1+2y+3y^2+2.676\,(1-y)^2$

Input :-

\>> newton (f, df, 2.5, 0.00001)

output :-

41.974679 -4.22745.35e-10 4.227435e-10.

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To approximate the root of $f(y) = 1+y+y^2 - y^3 - 0.892(1-y)^3 = 0.$

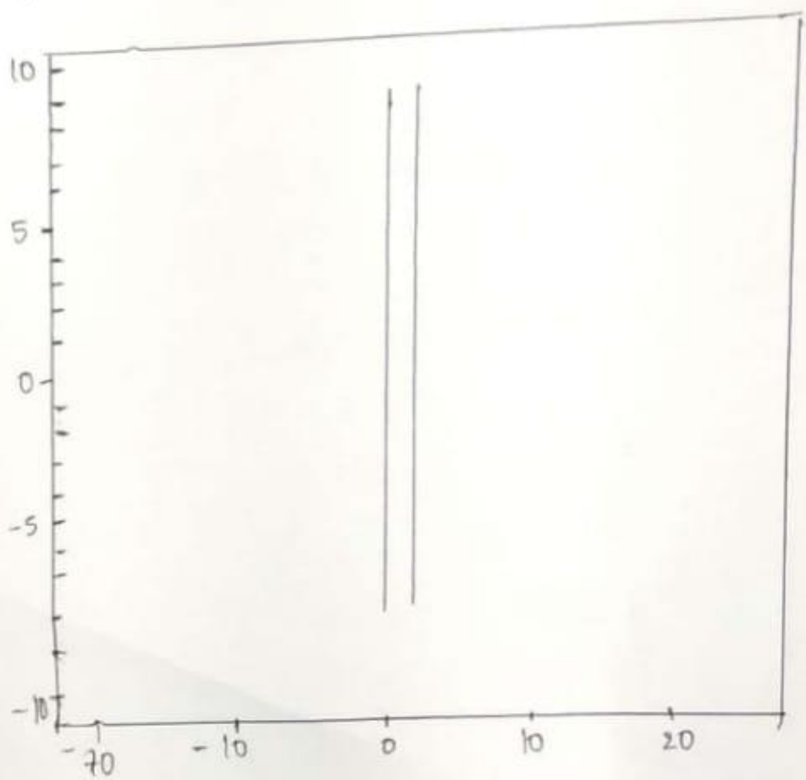
make use of Newtons method.

To fit a good starting point, plt f(y) by using the mathmatical commands.

Input:-

contourplot $[1+y+y^2 - y^3 - 0.892(1-y)^3$ so, $\{y, -20, 20\}, \{fy, -10, 10\}]$

out put:-



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→ 9th iteration:-
Here $f(2.043) = -0.018 < 0$ and $f(2.04) = 0.03 > 0$
∴ Now, Root lies b/w 2.043 and 2.04
$x_8 = \frac{2.043 + 2.04}{2} = 2.0449$
$f(x_8) = f(2.0449) = 2.0449 - \cos(2.0449) - 9$
$= 0.0078 > 0$

→ 10th iteration:-
Here $f(2.043) = 0.0184 < 0$ and $f(2.0449) = 0.0078 > 0$
∴ Now, Root lies b/w 2.043 and 2.0449
$x_9 = \frac{2.043 + 2.0449}{2} = 2.0439$
$f(x_9) = f(2.0439) = 2.04 - \cos(2.043) - 9 = -0.0053 < 0$

→ 11th iteration:
Here $f(2.0439) = 0.0053 < 0$ and
$f(2.0449) = 0.0078 > 0$
∴ Now, Root lies b/w 2.04 and 2.04
$x_{10} = \frac{2.04 + 2.04}{2} = 2.09$

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$f(x_{10}) = f(2.0444) = 2.0444^3 - \cos(2.0444) - 9 = 0.0013 > 0$

12th iteration:

Here $f(2.0439) = -0.0063 < 0$ and
$f(2.044) = 0.0013 > 0$

$\therefore$ Now, Root lies b/w $2.04$ and $2.044$

$x_{11} = \frac{2.04 + 2.0444}{2} = 2.0442$

$f(x_{11}) = f(2.0442) = 2.0442^3 - \cos(2.0442) - 9 = 0.002 < 0$

13th iteration:

Here $f(2.0442) = -0.002 < 0$ and
$f(2.044) = 0.0013 > 0$

$\therefore$ Now, Root lies b/w $2.0442$ and $2.044$

$x_{12} = \frac{2.04 + 2.0444}{2} = 2.0443$

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$f(x_2) = f(2.125) = 2.125^3 - \cos(2.125) - 9$
$= 1.12 > 0$

→ 4th iteration:
Here $f(2) = -0.5839 < 0$ and $f(2.12) = 1.22 > 0$

∴ Now, Root lies b/w 2 and 2.12

$x_3 = \frac{2 + 2.125}{2} = 2.0625$

$f(x_3) = f(2.06) = 2.06^3 - \cos(2.06) - 9 = 0.24 > 0$

→ 5th iteration
Here $f(2) = -0.5839 < 0$ and $f(2.06) = 0.24 > 0$

∴ Now, Root lies b/w 2 and 2.062

$x_4 = \frac{2 + 2.062}{2} = 2.0312$

$f(x_4) = f(2.0312) = 2.0312^3 - \cos(2.0312) - 9 = -0.174$

→ 6th iteration:
Here $f(2.0312) = -0.1748 < 0$ and
$f(2.0625) = 0.2458 > 0$

∴ Now, Root lies b/w 2.03 and 2.06

$x_5 = \frac{2.03 + 2.06}{2} = 2.04$

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Q.2:-

Sol:-

Here $x3 - \cos(x) - 9 = 0$

Let $f(x)$ $x^3 - \cos(x) - 9$

1st iteration:

Here $f(2) = -0.5839 < 0$ and $f(3) = 18.9 > 0$

∴ Now, Root lies b/w 2 and 3

$x_0 = 2+32 = 2.5$

$f(x_0) = f(2.5) = 2.53 - \cos(2.5) - 9 = 7.4 > 0$

2nd iteration:

Here $f(2) = 0.5839 < 0$ and $f(2.5) = 7.4 > 0$

: Now, Root lies b/w 2 and 2.5

$x_1 = 2+2.5 = 2.2$

$f(x_1) = f(2.25) = 2.253 - \cos(2.25) - 9 = 3.01 > 0$

3rd iteration:

Here $f(2) = 0.5839 < 0$ and $f(2.25) = 3.01 > 0$,

∴ Now, Root lies b/w 2 and 2.25

$x_2 = 2+2.2 = 2.152$

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$f(x_5) = f(2.04969) = 2.0469\,3 - \cos(2.04) - 9$
$= 0.03 > 0$

→ 7th iteration:

Here $f(2.0312) = -0.1748 < 0$ and
$f(2.0469) = 0.0341 > 0$

∴ Now, Root lis b/w 2.0312 and 2.0469

$x_6 = 2.0312 + 2.04 = 2.0391$

$f(x_6) = f(2.0391) = 2.03 - \cos(2.0391) - 9 =$
$0.07 < 0$

→ 8th iteration:-

Here $f(2.0391) = 0.07 < 0$ and $f(2.04)$
$= 0.0341 > 0$

∴ Now, Root lies b/w 2.0391 and 2.0469

$x_7 = 2.03 + 2.04 = 2.043$

$f(x_7) = f(2.043) = 2.043 - \cos(24) - 9 =$
$-0.0184 < 0$