PAF KIET	**COLLEGE OF COMPUTING AND INFORMATION SCIENCES**		
	Final Assessment Fall 2020 Semester		
Class Id	105126 / 105128	**Course Title**	NUMERICAL COMPUTING
Program	BS (CS)	**Campus / Shift**	NORTH CAMPUS
Date	7th – December 2020	**Total Marks**	40
Duration	03 hours	**Faculty Name**	SAAD AKBAR
Student Id	63961	**Student Name**	Ali Afzal

- Filling out Student-ID and Student-Name on exam header is mandatory.
- Do not remove or change any part of exam header or question paper.
- Write down your answers in given space or at the end of exam paper with proper title "Answer for Question# _ _".
- Answers should be formatted correctly (font size, alignment and etc.)
- Handwritten text or image should be on A4 size page with clear visibility of contents.
- Only PDF format is accepted (Student are advised to install necessary software)
- In case of CHEATING, COPIED material or any unfair means would result in negative marking or ZERO.
- A mandatory recorded viva session will be conducted to ascertain the quality of answer scripts where deemed necessary.

- **Caution:** Duration to perform Final-Term Assessment is **03 hours only**. Extra 01 hour is given to cater all kinds of odds in submission of Answer-sheet. **Therefore, if you failed to upload answer sheet on LMS (in PDF format) within 04 hours limit, you would be considered as ABSENT/FAILED.**
- **Timing of the assessment is from 01:00 pm to 04:00 pm extended till 05:00 pm.**

"I promise that all the answers provided in this assessment are provided solely by me. I haven't discussed anything related to this assessment with anyone else"

Name:Ali Afzal

QUESTION	TOTAL MARKS	MARKS OBTAINED
QUESTION # 01	05 + 05 = 10	
QUESTION # 02	05 + 05 = 10	
QUESTION # 03	10	
QUESTION # 04	10	

ALi Afzal 63961

QUESTION # 01 (a):
Find the interpolating polynomial for the following data by,
Divided Difference Formula

X	0	1	2	3
y	1	3	7	13

Ali Afzal (63961)

Q.1(a)

x: $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.

y: $y_0 = 1$, $y_1 = 3$, $y_2 = 7$, $y_3 = 13$

Making Difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		2		
1	3		1	
		4		0.
2	7		1	
		6		
3	13			

$$Y = y_0 + (x-x_0)\,y(x_0,x_1) + (x-x_0)(x-x_1)\,y(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)\,y(x_0,x_1,x_2,x_3).$$

$$Y = 1 + (x-0)2 + (x-0)(x-1)\times 1 + (x-0)(x-1)(x-2)\times 0$$

$$Y = 1 + 2x - 0 + x^2 - x + x^2 - x\,(x-2)\times 0.$$

$$Y = 1 + 2x + x^2 - x + 0.$$

$$Y = x^2 + x + 1.$$

Or

$$f(x) = x^2 + x + 1.$$

QUESTION # 01 (b):
Using Lagrange's Formula, find the value of F(1.5) = ?

X	0	1	2	3
y	1	3	7	13

Q.1 (b)

x	0	1	2	3
y	1	3	7	13

$x = 1.5$

$$F(x) = \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)\left(\frac{x-x_3}{x_0-x_3}\right)y_0 + \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right)y_1 + \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right)y_2 + \left(\frac{x-x_0}{x_3-x_0}\right)\left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right)y_3$$

$$f(1.5) = \left(\frac{1.5-1}{0-1}\right)\left(\frac{1.5-2}{0-2}\right)\left(\frac{1.5-3}{0-3}\right)\times 1 + \left(\frac{1.5-0}{1-0}\right)\left(\frac{1.5-2}{1-2}\right)\left(\frac{1.5-3}{1-3}\right)\times 3 + \left(\frac{1.5-0}{2-0}\right)\left(\frac{1.5-1}{2-1}\right)\left(\frac{1.5-3}{2-3}\right)\times 7 + \left(\frac{1.5-0}{3-0}\right)\left(\frac{1.5-1}{3-1}\right)\left(\frac{1.5-2}{3-2}\right)\times 13.$$

$$f(1.5) = \left(-\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\times 1 + \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\times 3 + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\times 7 + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(-\frac{1}{2}\right) 13.$$

$$f(1.5) = \left(-\frac{1}{16}\right) + \left(\frac{27}{16}\right) + \left(\frac{63}{16}\right) + \left(-\frac{13}{16}\right)$$

$$f(1.5) = \frac{-1+27+63-13}{16} \Rightarrow \frac{76}{16}$$

$$f(1.5) = 4.75.$$

QUESTION # 02 (a):

Apply the Gauss Jacobi method to solve

$$
\begin{aligned}
5x_1 - 2x_2 + 3x_3 &= -1 \\
-3x_1 + 9x_2 + x_3 &= 2 \\
2x_1 - x_2 - 7x_3 &= 3
\end{aligned}
$$

Choose the initial guess as (0.5,0.5,0.5).

Q-2 (a).

initial values:

$x = 0.5$, $y = 0.5$, $z = 0.5$

From $5x_1 - 2x_2 + 3x_3 = -1$

we can derive:

$5x_1 = -1 + 2x_2 + 3x_3.$

$x_1 = \frac{-1 + 2x_2 - 3x_3}{5}.$

From $-3x_1 + 9x_2 + x_3 = 2.$

we can derive.

$9x_2 = 2 + 3x_1 - x_3.$

$x_2 = \frac{2 + 3x_1 - x_3}{9}.$

From $2x_1 - x_2 - 7x_3 = 3$

we can derive,

$-7x_3 = 3 - 2x_1 + x_2$

$x_3 = \frac{3 - 2x_1 + x_2}{-7}$

First iteration:

$x_1 = \frac{-1 + 2(0.5) - 3(0.5)}{5}$

$x_1 = -1.5/5$

$\boxed{x_1 = -0.3}$

$x_2 = \frac{2 + 3(0.5) - 0.5}{9}.$

$x_2 = 3/9$

$\boxed{x_2 = 0.3333}$

$x_3 = 3 - 2(0.5) + 0.5 / -7.$

$x_3 =$ 2.5/-7.

$\boxed{x_3 = -0.3571}$

Second iteration:

$$x_1 = \frac{-1+2(0.3333)-3(-0.3571)}{5}$$

$x_1 = 0.7379/5.$

$x_1 = 0.1485.$

$$x_2 = \frac{2+3(-0.3)-(-0.3571)}{9.}$$

$x_2 = 1.4571/9.$

$x_2 = 0.1619.$

$$x_3 = \frac{3-2(-0.3)+(0.3333)}{-7}$$

$x_3 = 3.9333/-7.$

$x_3 - 0.5619.$

Third iteration

$$x_1 = \frac{-1+2(0.1619)-3(-0.5619)}{5}$$

$x_1 = 1.0095/5$

$x_1 = 0.2019$

$$x_2 = \frac{2+3(0.1485)-(-0.5619)}{9.}$$

$x_2 = 3.0048/9$

$x_2 = 0.3341$

$$x_3 = \frac{3-2(0.1485)+0.1619}{-7}$$

$x_3 = 2.8647/-7$

$x_3 = -0.4092.$

Fourth iteration:

$$x_1 = \frac{-1+2(0.33418)-3(-0.4092)}{5}$$

$x_1 = 0.8960/5$

$x_1 = 0.1791$

$$x_2 = \frac{2+3(0.2019)-(-0.4092)}{9}$$

$x_2 = 3.0152/9.$

$x_2 = 0.33502.$

$$x_3 = \frac{3-2(0.2019)+(0.33418)}{-7}.$$

$x_3 = 2.9303/-7$

$x_3 = -0.41860.$

Fifth iteration,

$$x_1 = \frac{-1+2(0.3350)-3(-0.4186)}{5}$$

$$x_1 = \frac{0.9258}{5}$$

$x_1 = 0.1852.$

$$x_2 = \frac{2+3(0.1791)-(-0.4186)}{9}$$

$x_2 = 2.9564/9$

$x_2 = 0.3285.$

$$x_3 = \frac{3-2(0.1791)+(0.3350)}{-7}.$$

$x_3 = 2.9765/-7$

$x_3 = -0.4252.$

QUESTION # 02 (b):
If number of iterations was not mentioned in the question, then what is the criteria to stop the solution of the problem when find the solution of Linear System of Equations? Also discuss the difference in Gauss Jacobi and Gauss Seidel method for solving system of linear equations? Should the system be diagonally dominant for gauss elimination method?

Q-2(b)

As the number of iterations was not mentioned in the question so I used tolerance as the stopping criteria. As I was solving on paper it was so hard for me to get too accurate so I used tolerance of 0.1 as the stopping criteria. As in my 4th iteration values were

$u_1 = 0.1791$ $u_2 = 0.3350$ $u_3 = -0.4186$

and in 5th iteration the values are:

$u_1 = 0.1852$ $u_2 = 0.3285$ $u_3 = -0.4282$.

the tol = 0.1 is achieved, thats why I decided to stop at 5th iteration.

In Gauss Jacobi values from previous iterations are used and the new values wait till the next iteration. whereas in Gauss ~~Sieet~~ Seidel the updated values are used as soon as new value of unknown variables are obtained.
They don't converge on every set of equations. In order to the methods to converge. They will only work when the equations make a form of ~~strictly~~ diagonally dominant matrices.

QUESTION # 03 (a):

Approximate the given integral using Simpson's (1/3) Rule for **n = 8**.

$$\int_1^3 e^{x^2}\, dx$$

Q-3 (a)

$h = \frac{a-b}{n}$

$h = \frac{3-1}{8} \Rightarrow 0.25$

x	y
1	2.7182
1.25	4.7707
1.50	9.4877
1.75	21.3809
2.00	54.5981
2.25	157.9849
2.5	518.0128
2.75	1924.6511
3	8103.0839

$I = \frac{h}{3}\big[(2.7182) + (8103.0839) + 4(4.7707 + 21.3809 + 157.9849 + 1924.6511) + 2(9.4877 + 54.5981 + 518.0128)\big]$

$I = 0.25/3\,(8105.8022 + 8435.1511 + 1164.1974)$

$I = \frac{0.25\,(17705.1507)}{3}$

$I = \frac{4426.2876}{3}$

$I = 1475.4292.$

QUESTION # 03 (b):

Approximate the area y = sin x on the interval [o, π] using the Trapezoidal Rule with the following data.

X	0	π/6	π/3	π/2	2π/3	5π/6	0
y	0	1/2	$\sqrt{3}/2$	1	$\sqrt{3}/2$	1/2	0

Q.3 (b)

x	0	π/6	π/3	π/2	2π/3	5π/6	π
y	0	1/2	$\sqrt{3}/2$	1	$\sqrt{3}/2$	1/2	0.

$$h = \frac{\pi - 0}{7-1} \Rightarrow h = \frac{\pi}{6}$$

$$I = h\left[\left(\frac{y_0 + y_n}{2}\right) + \sum_{i=1}^{n-1} y_i\right]$$

$$I = h\left[\frac{0+0}{2} + \left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right)\right]$$

$$I = \pi/6\left[0 + \left(1 + 1 + \frac{2\sqrt{3}}{2}\right)\right]$$

$$I = \pi/6\left[2 + \frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right]$$

$$= \frac{\pi}{6}(2+\sqrt{3})$$

$$I = 1.9540$$

QUESTION # 04:
Solve the differential equation,

$$y = x + y;\ \ y(0) = 1$$

in the interval [0,0.5] using Euler's method by taking h=0.1

Q.4

$y' = x + y$; $y(0) = 1$

interval $[0, 0.5]$; $h = 0.1$

x	y
0	1
0.1	1.1
0.2	1.22
0.3	1.362
0.4	1.5282
0.5	1.72102.

$x_1 = x_0 + h$
$= 0 + 0.1 \Rightarrow 0.1$
$x_2 = x_1 + h$
$= 0.1 + 0.1 \Rightarrow 0.2.$
$x_3 = x_2 + h$
$= 0.2 + 0.1 \Rightarrow 0.3$
$x_4 = x_3 + h$
$= 0.3 + 0.1 \Rightarrow 0.4.$
$x_5 = 0.4 + 0.1 \Rightarrow 0.5.$

For $y_1 = y_0 + hf(x_0, y_0)$
$= 1 + 0.1(1)$
$y_1 = 1.1$

For $y_2 = y_1 + hf(x_1, y_1)$
$y_2 = 1.1 + (0.1)(1.2)$
$y_2 = 1.1 + 0.12 \Rightarrow 1.22.$

For $y_3 = y_2 + hf(x_2, y_2)$
$= 1.22 + (0.1)(1.42)$
$y_3 = 1.22 + 0.142 \Rightarrow 1.362.$

For $y_4 = y_3 + hf(x_3, y_3)$
$= 1.362 + (0.1)(1.662)$
$= 1.362 + 0.1662$
$y_4 = 1.5282$

For $y_5 = y_4 + hf(x_4, y_4)$
$= 1.5282 + (0.1)(1.9282)$
$= 1.5282 + 0.19282.$
$y_5 = 1.72102$