MUHAMMAD UMAR KHAN 10619
NC ASSIGNMENT#02

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10619
Assignment #02 NC

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Question Number #01:

Langrangian polynomial:

X	Y
2.0	-11.19
4.0	7.19
7.0	10.19

A = 19

Here $x_0 = 2.0$ $\quad y_0 = -11.19$
$x_1 = 4.0$ $\quad y_1 = 7.19$
$x_2 = 7.0$ $\quad y_2 = 10.19$

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$f(x) = \frac{(x-4)(x-7)}{(2-4)(2-7)} -11.19 + \frac{(x-2)(x-7)}{(4-2)(4-7)} (7.19)$$
$$+ \frac{(x-2)(x-4)}{(7-2)(7-4)}$$

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$$= \frac{(x-4)(x-7)}{-10} \times (11.19) + \frac{(x-2)(x-7)}{-6} (7.19)$$
$$+ \frac{(x-2)(x-4)}{15} (10.19)$$

$$= (x^2 - 11x + 28)(-1.119) + (x^2 - 9x + 14)(-1.19833)$$
$$+ (x^2 - 6x + 8)(0.67933)$$

$$= -1.119x^2 + 12.309x - 31.332 - 1.19833x^2 + 10.7849x$$
$$- 16.7766 + 0.6793x^2 - 4.0758x + 5.4344$$

$$\boxed{f(x) = -1.63803x^2 + 19.0181x - 42.6742}$$

(b) $f(x) = y \quad f'(x) = y'$

$$(y) = -1.63803x^2 + 19.0181x - 42.6742$$

$$\boxed{y' = -3.27606x + 19.0181}$$

(c) $x = 3$

$$f'(x) = -3.27606x + 19.0181$$
$$= -3.27606(3) + 19.0181$$
$$f'(3) = -9.82818 + 19.0181$$
$$\boxed{f'(3) = 9.18992}$$

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(3)

(a)

$$\frac{-11.19-10.19}{2} = -10.69$$

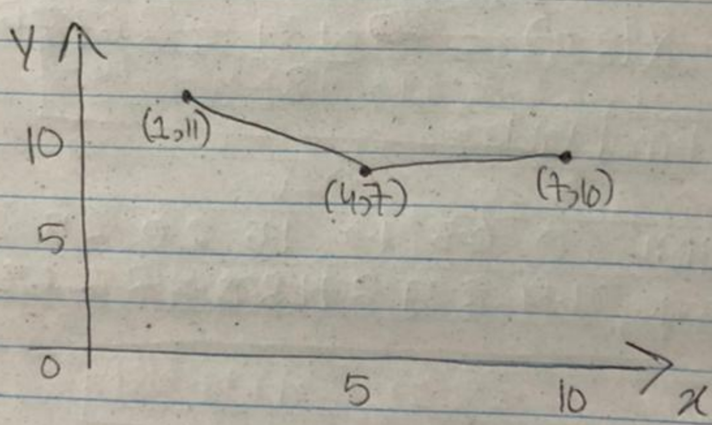
$-11.19+10.69 = -0.5$

$-11.19-(-10.69)x$

$-11.19+10.69(8) = 74.33$

Extropolate $= 74.33$

*Plot :



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Question Numbers #03:

Position	0	0.5	1.0	1.5	2.0	3.0	3.5	4.0
Dosage	1.19	2.39	2.71	2.98	3.2	3.20	2.98	2.74

$$X(2.5) = \frac{1}{n}(1.19+2.39+2.71+2.98+3.2+3.2+2.98+2.74)$$

$$n = 8$$

$$X(2.5) = \frac{21.39}{8} = 2.67375$$

* Final Table:

Position	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Dosage	1.19	2.39	2.71	2.98	3.2	2.6737	3.2	2.98

QUESTION#02:

x	f(x)
1985	19
1986	38
1987	57
1988	356
1989	456
1990	549
1991	659
1992	667
1993	767
1994	836
1995	905

x = 1998

Solution:
The value of table for x and y

x	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
y	19	38	57	356	456	549	659	667	767	836	905

Numerical divided differences method to find solution

Newton's divided difference table is

x	**y**	**1^{st} order**	**2^{nd} order**	**3^{rd} order**	**4^{th} order**	**5^{th} order**	**6^{th} order**	**7^{th} order**	**8^{th} order**	**9^{th} order**	**10^{th} order**
1985	19										
		19									
1986	38		0								
		19		46.6667							

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1987	57		140		-31.625						
		299		-79.8333		11.9167					
1988	356		-99.5		27.9583		-3.1514				
		100		32		-6.9917		0.6216			
1989	456		-3.5		-7		1.2		-0.0884		
		93		4		0.2083		-0.0859		0.0068	
1990	549		8.5		-5.9583		0.5986		-0.0269		0.0006
		110		-19.8333		3.8		-0.301		0.0132	
1991	659		-51		13.0417		-1.5083		0.0919		
		8		32.3333		-5.25		0.4339			
1992	667		46		-13.2083		1.5292				
		100		-20.5		3.925					
1993	767		-15.5		6.4167						
		69		5.1667							
1994	836		0								
		69									
1995	905										

Newton's divided difference interpolation formula is

$$f(x)=y_0+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+(x-x_0)(x-x_1)(x-x_2)f[x_0,x_1,x_2,x_3]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)f[x_0,x_1,x_2,x_3,x_4]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)f[x_0,x_1,x_2,x_3,x_4,x_5]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)f[x_0,x_1,x_2,x_3,x_4,x_5,x_6]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-$$

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$x_6)f[x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)(x-$
$x_7)f[x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)(x-x_7)(x-$
$x_8)f[x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)(x-x_7)(x-x_8)(x-$
$x_9)f[x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}]$

$f(x)=19+(x-1985)\times19+(x-1985)(x-1986)\times0+(x-1985)(x-1986)(x-1987)\times46.6667+(x-1985)(x-1986)(x-1987)(x-$
$1988)\times-31.625+(x-1985)(x-1986)(x-1987)(x-1988)(x-1989)\times11.9167+(x-1985)(x-1986)(x-1987)(x-1988)(x-$
$1989)(x-1990)\times-3.1514+(x-1985)(x-1986)(x-1987)(x-1988)(x-1989)(x-1990)(x-1991)\times0.6216+(x-1985)(x-$
$1986)(x-1987)(x-1988)(x-1989)(x-1990)(x-1991)(x-1992)\times-0.0884+(x-1985)(x-1986)(x-1987)(x-1988)(x-$
$1989)(x-1990)(x-1991)(x-1992)(x-1993)\times0.0068+(x-1985)(x-1986)(x-1987)(x-1988)(x-1989)(x-1990)(x-$
$1991)(x-1992)(x-1993)(x-1994)\times0.0006$

$f(x)=19+(x-1985)\times19+(x^{2}-3971x+3942210)\times0+(x^{3}-5958x^{2}+11832587x-7833171270)\times46.6667+(x^{4}-$
$7946x^{3}+23677091x^{2}-31356354226x+15572344484760)\times-31.625+(x^{5}-9935x^{4}+39481685x^{3}-$
$78450088225x^{2}+77940133040274x-30973393180187600)\times11.9167+(x^{6}-11925x^{5}+59252335x^{4}-$
$157018641375x^{3}+234055808608024x^{2}-186074257930333000x+61637052428573400000)\times-3.1514+(x^{7}-$
$13916x^{6}+82995010x^{5}-274990040360x^{4}+546679923585649x^{3}-$
$652079372868909000x^{2}+432110899967866000000x-122719371385290000000000)\times0.6216+(x^{8}-$
$15908x^{7}+110715682x^{6}-440316100280x^{5}+1094460083982770x^{4}-$
$1741065780651520000x^{3}+1731053010722730000000x^{2}-$
$983484284121279000000000x+244456987799497000000000000)\times-0.0884+(x^{9}-17901x^{8}+142420326x^{7}-$
$660972454506x^{6}+1972010071840810x^{5}-3922324728029180000x^{4}+5200997111561210000000x^{3}-$
$4433472934491680000000000x^{2}+2204541166053210000000000000x-$
$487202776684397000000000000000)\times0.0068+(x^{10}-19895x^{9}+178114920x^{8}-$
$944958584550x^{7}+3289989146125770x^{6}-7854512811279750000x^{5}+13022112619251400000000x^{4}-$
$14804261174944700000000000x^{3}+11044886197429600000000000000x^{2}-$
$4883057861794490000000000000000x+971482336708688000000000000000000)\times0.0006$

$f(x)=19+(19x-37715)+(0)+(46.6667x^{3}-278040x^{2}+552187393.3333x-365547992600)+(-$
$31.625x^{4}+251292.25x^{3}-748788002.875x^{2}+991644702397.25x-492475394330535)+(11.9167x^{5}-$
$118392.0833x^{4}+470490079.5833x^{3}-934863551347.917x^{2}+928786585396599x-369099602063903000)+(-$
$3.1514x^{6}+37580.3125x^{5}-186727150.1597x^{4}+494826801777.604x^{3}-$

$7376008746272 31x^2+586392348949896000x-19424232216726800000\Big)+\Big(0.6216x^7-8650.5611x^6+51591937.7639x^5-170941229453.944x^4+339830992181317x^3-405350133967915000x^2+268611795555422000000x-76285672728196900000000\Big)+\Big(-0.0884x^8+1406.9427x^7-9791967.3118x^6+38942639226.1528x^5-96796742546690.3x^4+153984141215360000x^3-153098587208265000000x^2+86981769771242100000000x-21620377442782900000000000\Big)+\Big(0.0068x^9-122.4379x^8+974116.0966x^7-4520870899.7021x^6+13488009805745.4x^5-26827628899273600x^4+35573398453744800000x^3-30323742899604200000000x^2+15078458923456900000000000x-3332333806577030000000000000\Big)+\Big(-12.6427x^9+113187.0055x^8-600494514.9835x^7+2090695263163.04x^6-4991321247467790x^5+8275185102511510000x^4-9407690219748300000000x^3+7018713506192880000000000x^2-3103045477650490000000000000x+617349611014726000000000000000\Big)$

$f(x)=-12.6359x^9+113064.4791x^8-599518991.3227x^7+2086164591642.31x^6-4977794243393290x^5+8248260505741610000x^4-9371962496827050000000x^3+6988236258617400000000000x^2-3087879767758150000000000000x+613995580350422000000000000000$

Now, differentiate with x

$f'(x)=0.0064x^9-113.7228x^8+904515.8328x^7-4196632939.2587x^6+12516987549853.9x^5-24888971216966500x^4+32993042022966400000x^3-28115887490481100000000x^2+13976472517234800000000000x-3087879767758150000000000000$

$f''(x)=0.0572x^8-909.7827x^7+6331610.8296x^6-25179797635.5521x^5+62584937749269.4x^4-99555884867865800x^3+98979126068899300000x^2-56231774980962300000000x+13976472517234800000000000$

Now, substitute $x=1998$

$f'(1998)=0.0064\times1998^9-113.7228\times1998^8+904515.8328\times1998^7-4196632939.2587\times1998^6+12516987549853.9\times1998^5-24888971216966500\times1998^4+32993042022966400000\times1998^3-28115887490481100000000\times1998^2+13976472517234800000000000\times1998-3087879767758150000000000000=-35184372088832$

$f''(1998)=0.0572\times1998^8-909.7827\times1998^7+6331610.8296\times1998^6-25179797635.5521\times1998^5+62584937749269.4\times1998^4-99555884867865800\times1998^3+98979126068899300000\times1998^2-56231774980962300000000\times1998+13976472517234800000000000=204010946560$