MEASUREMENT OF THE HIGHER MOMENTS OF TRANSVERSE MOMENTUM OF CHARGED PARTICLES IN PROTON-PROTON COLLISIONS

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ABSTRACT

I. INTRODUCTION

For a given collision centrality, the mean transverse momentum $\langle \mathbf{p_T} \rangle$ of emitted particles in ultra-relativistic proton-proton collisions fluctuates from one event to another. The distribution of $\langle \mathbf{p_T} \rangle$ in event-by-event dynamics reflects the various statistical and dynamical fluctuations. In this project, through various plots, the probability distribution of $\langle \mathbf{p_T} \rangle$ has been shown **not** to be **Gaussian** but to have a **positive skew**, which arises because of the aforementioned fluctuations.

The two dimensionless measures of skewness, namely **standardized skewness** and **intensive skewness**, have been plotted against different multiplicity classes. The former depends on centrality and system size, while the latter has the property of being independent of the system size. Since these are dimensional quantities, both of these are expected to be less sensitive to analysis details, such as those dependent on the detector.

The definitions used below have been given by the **STAR** collaboration. The number of charged particles in an event is denoted by N_{ch}, the transverse momentum of the ith particle is denoted by p_i and the average over events in a centrality class is denoted by angular brackets $\langle\rangle$.

$$\text{Mean Transverse Momentum} = \langle\langle p_T \rangle\rangle = \left\langle \frac{\Sigma_{i=1}^{N_{ch}} p_i}{N_{ch}} \right\rangle \tag{1}$$

The variance of dynamical p_T fluctuations, which is denoted by $\langle \Delta p_i \Delta p_j \rangle$, is defined as

$$\langle \Delta p_i \Delta p_j \rangle = \left\langle \frac{\Sigma_{i;j\neq i}\ (p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)} \right\rangle \tag{2}$$

which can also be written as

$$\langle \Delta p_i \Delta p_j \rangle = \langle (\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2 \rangle \tag{3}$$

The intensive variance of transverse momentum is defined as follows

$$\sigma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \rangle^{1/2}}{\langle\langle p_T \rangle\rangle}. \tag{4}$$

The skewness, which is the third central moment and is denoted by $\langle \Delta p_i \Delta p_j \Delta p_k \rangle$, is defined as follows

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle = \left\langle \frac{\Sigma_{i;j\neq i;k\neq i,j}\ (p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)(p_k - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \right\rangle \tag{5}$$

which can also be written as

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle = \langle (\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^3 \rangle \tag{6}$$

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Standardized skewness and intensive skewness which are denoted by γ_{p_T} and Γ_{p_T} respectively, are defined as follows

$$\gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \tag{7}$$

$$\Gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle \langle p_T \rangle \rangle}{\langle \Delta p_i \Delta p_j \rangle^{2}} \tag{8}$$

The dataset provided has been generated using the **Pythia 8 Monte Carlo Event Generator**.
Number of events : **2 million**
Collisions System : **p + p** at centre of mass energy **13 TeV**

II. EXPERIMENTAL OBSERVATIONS

A. Transverse Momentum and Mean Transverse Momentum for Each Multiplicity Class

In this section, histograms have been plotted for the Transverse Momentum **pT** and the Mean Transverse Momentum ⟨**pT**⟩ of proton-proton collisions corresponding to each multiplicity class. The histogram for **pT** is then approximated using an **Exponential** fit, while that of ⟨**pT**⟩ has been approximated using a **Gaussian** fit. Both the quantities **pT** and ⟨**pT**⟩ have statistical fluctuations arising from the finite number of particles in each event. In each of the subsequent subsections corresponding to each of the 6 multiplicity classes, namely **pytree020**, **pytree2040**, **pytree4060**, **pytree6080**, **pytree80100** and **pytree100**, the histograms and the corresponding fits have been plotted. A logarithmic scale has been used on the *y-axis* to emphasize the skewness of the data.

1. *Multiplicity Class "pytree020"*

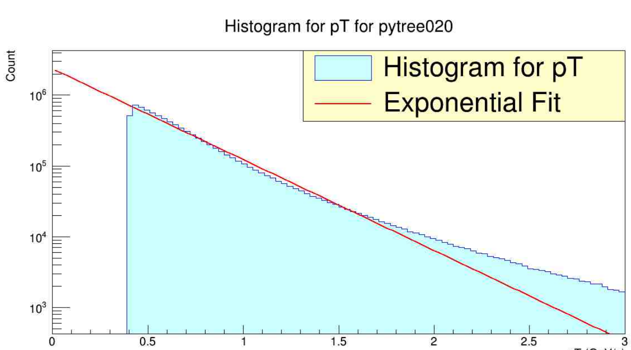

FIG. 1a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree020**. The solid line is an Exponential fit to the data. Owing to the large size of the data, the data has been arranged in a random order and only the first **40%** of the data has been used for analysis.

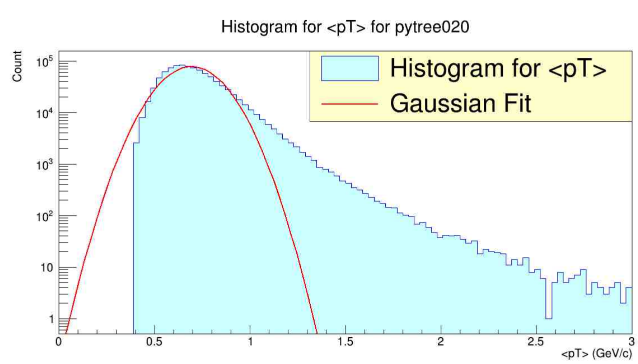

FIG. 1b. (Color Online) Distribution of ⟨**pT**⟩ for proton-proton collision in the multiplicity class **pytree020**. The solid line is a Gaussian fit to the data. Owing to the large size of the data, the data has been arranged in a random order and only the first **40%** of the data has been used for analysis.

2. Multiplicity Class "pytree2040"

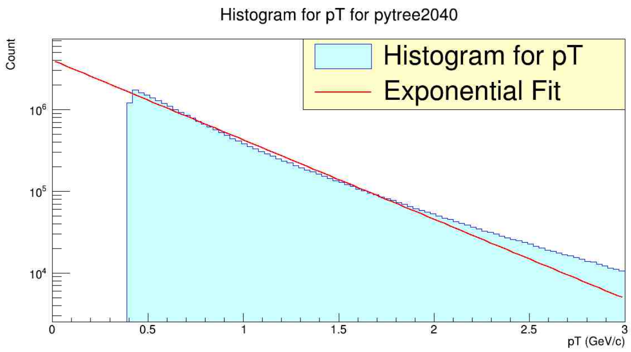

FIG. 2a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree2040**. The solid line is an Exponential fit to the data.

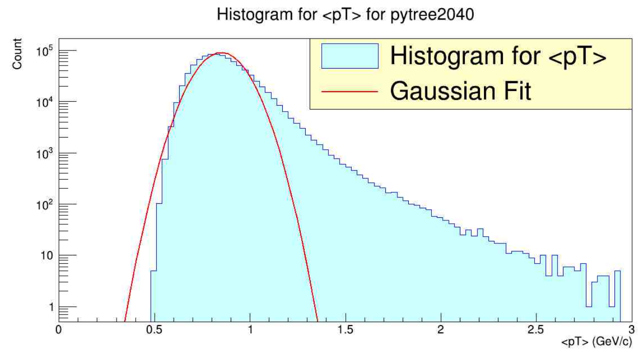

FIG. 2b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree2040**. The solid line is a Gaussian fit to the data.

3. Multiplicity Class "pytree4060"

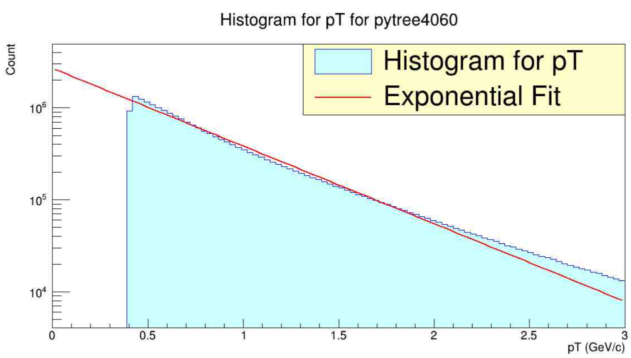

FIG. 3a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree4060**. The solid line is an Exponential fit to the data.

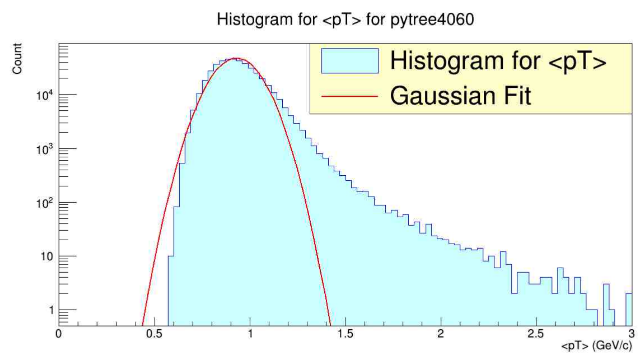

FIG. 3b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree4060**. The solid line is a Gaussian fit to the data.

4. Multiplicity Class "pytree6080"

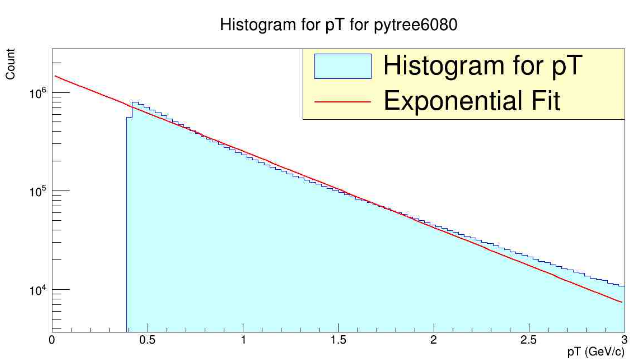

FIG. 4a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree6080**. The solid line is an Exponential fit to the data.

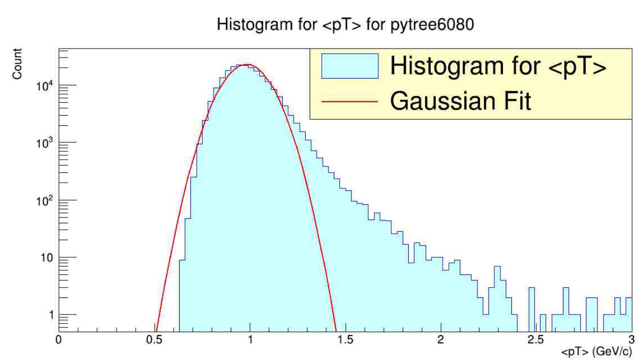

FIG. 4b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree6080**. The solid line is a Gaussian fit to the data.

5. Multiplicity Class "pytree80100"

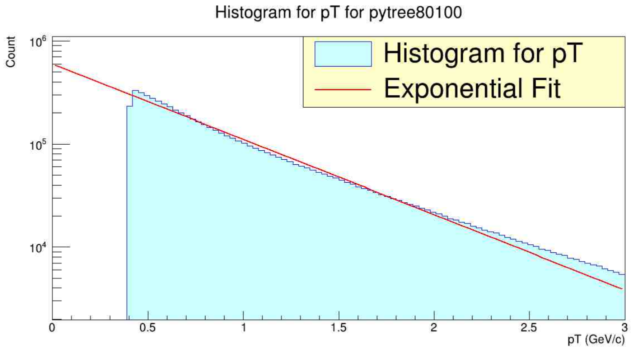

FIG. 5a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree80100**. The solid line is an Exponential fit to the data.

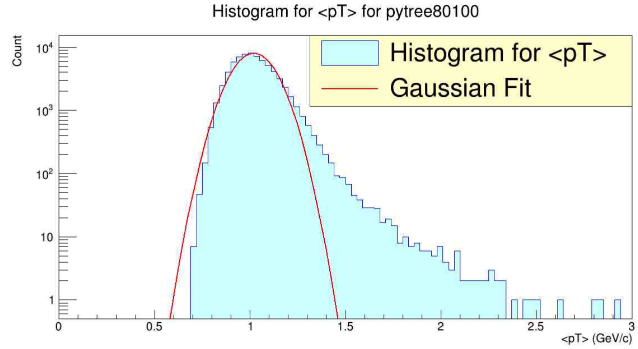

FIG. 5b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree80100**. The solid line is a Gaussian fit to the data.

6. Multiplicity Class "pytree100"

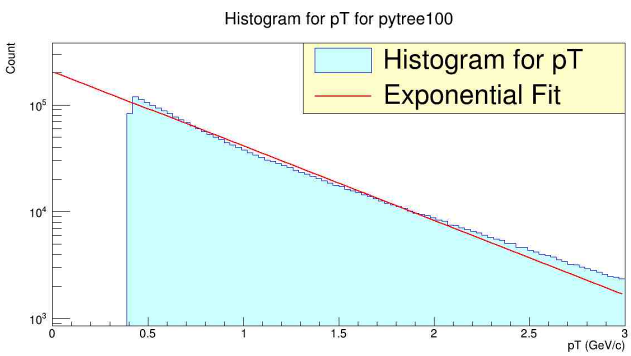

FIG. 6a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree100**. The solid line is an Exponential fit to the data.

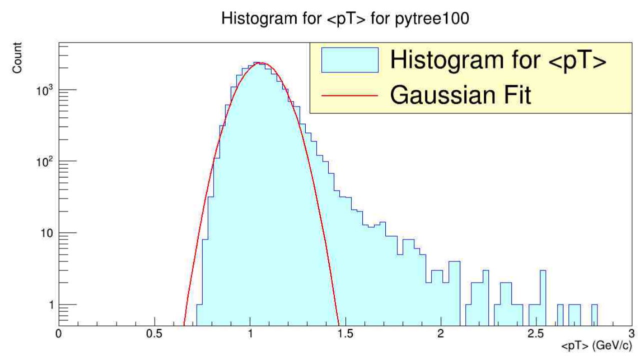

FIG. 6b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree100**. The solid line is a Gaussian fit to the data.

B. Analysis of Mean, Variance and Skewness Versus Multiplicity Class

From the graphs in FIG. 1a., FIG. 2a., FIG. 3a., FIG. 4a., FIG. 5a. and FIG. 6a., it is clear that the Transverse Momenta of the particles produced in a proton-proton collision follows approximately an **exponential distribution**. Graphs in FIG. 1b., FIG. 2b., FIG. 3b., FIG. 4b., FIG. 5b. and FIG. 6b. reveal that there is some **positive skew** in the distribution of the Mean Transverse Momentum.

In this section, the moments of the distribution of Transverse Momentum have been analyzed. The Mean, Variance and Skewness of the Transverse Momenta have been calculated for each multiplicity class and its relation with the multiplicity class has been examined. For each of the multiplicity classes, the Mean Transverse Momentum, the Intensive Variance of the Transverse Momentum, the Standardized Skewness of the Transverse Momentum and the Intensive Skewness of the Transverse Momentum, calculated using formulae 1, 4, 7 and 8 respectively have been summarized in the table and the plots below. It is to be noted that for *pytree2040*, the number of events is too large for a personal computer at today's scale to handle. Hence, the dataset of this multiplicity class has been randomized, and the first **40%** of the data has been used for analysis.

1. *Summary of Data*

The table below summarizes the data.

SUMMARY OF DATA					
Multiplicity Class	**Events**	$\langle \mathbf{p_T} \rangle$ **(GeV/c)**	$\sigma_{\mathbf{p_T}}$	$\gamma_{\mathbf{p_T}}$ **(GeV/c)**	$\mathbf{\Gamma_{p_T}}$
pytree020	952256	0.750912	0.235453	1.22572	5.20581
pytree2040	873322	0.869307	0.174974	1.77742	10.1582
pytree4060	445805	0.940521	0.144591	1.80884	12.51
pytree6080	207990	0.99074	0.130471	3.11451	23.8714
pytree80100	71263	1.03006	0.122603	1.64507	13.4178
pytree100	20981	1.07257	0.132207	4.11805	31.1485

TABLE I. Table Summarizing the Data of Transverse Momenta

2. *Mean Transverse Momentum versus Multiplicity Class*

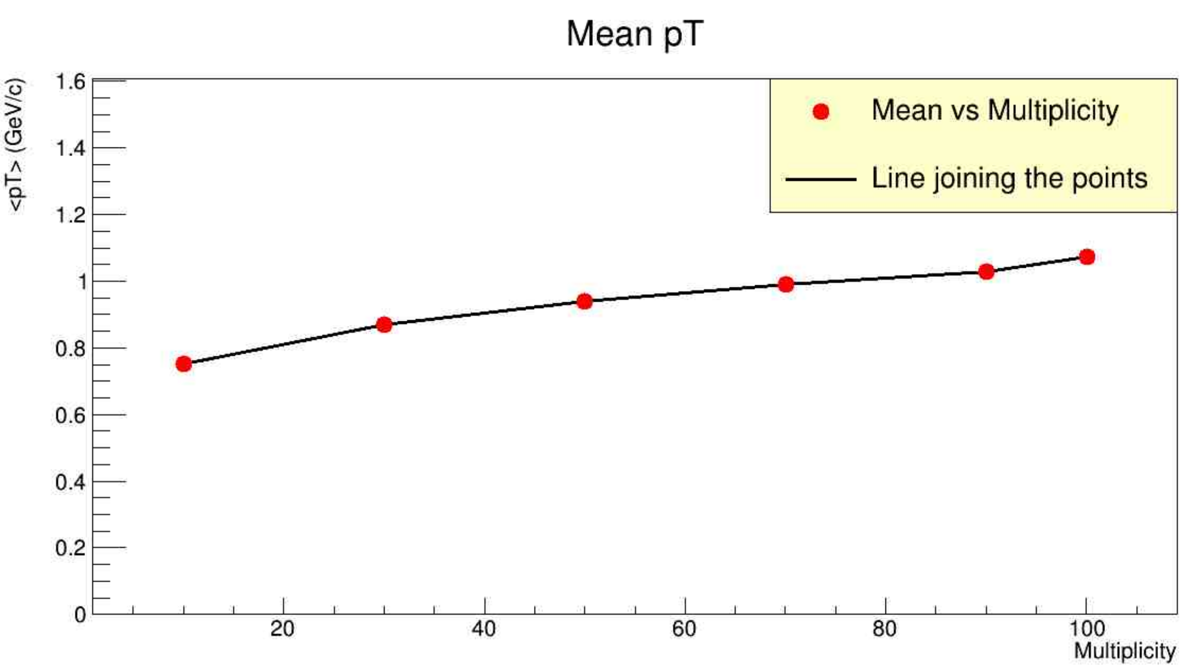

FIG. 6. (Color Online) A plot of **mean** transverse momenta versus multiplicity class. The red dots represent the mean of the transverse momentum. The solid line shows the trend of the mean against the multiplicity class.

3. Intensive Variance of Transverse Momentum versus Multiplicity Class

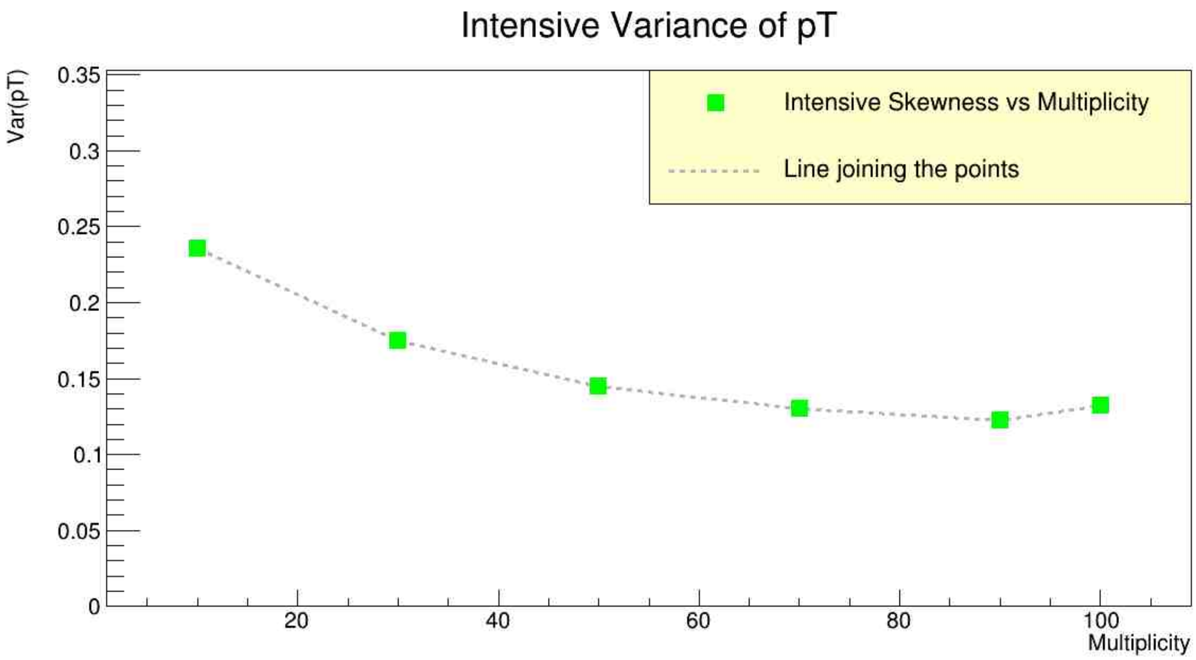

FIG. 7. (Color Online) A plot of the **intensive variance** of transverse momenta versus multiplicity class. The green boxes represent the intensive variance of the transverse momentum. The dashed line shows the trend of the intensive variance against the multiplicity class.

4. Standardized Skewness of Transverse Momentum versus Multiplicity Class

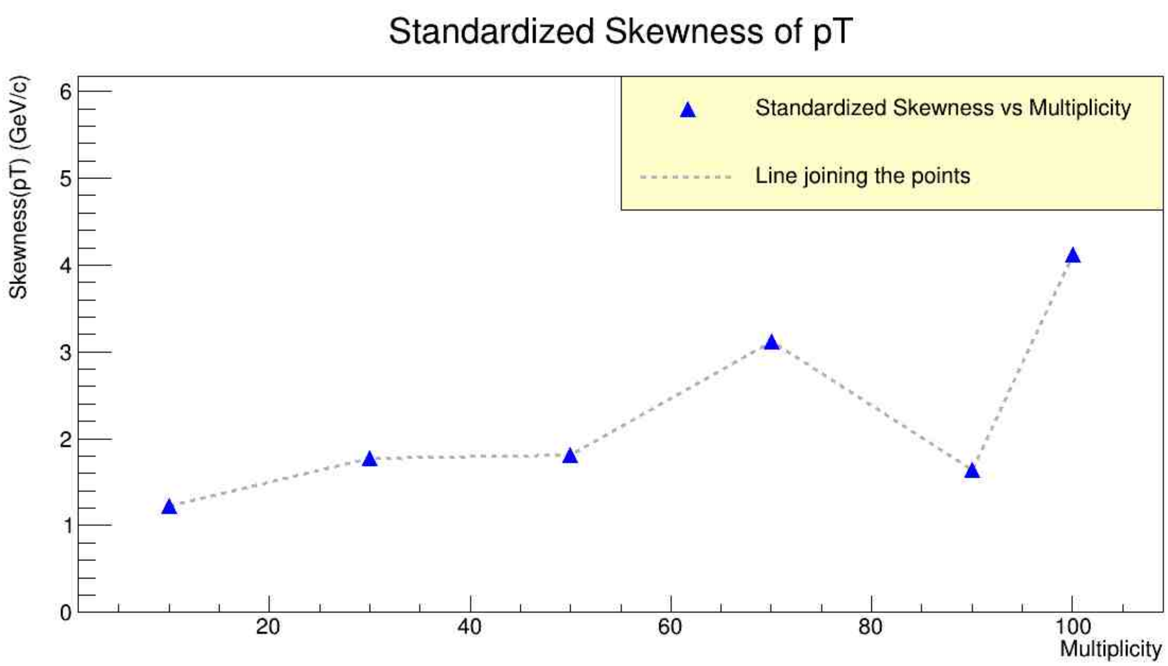

FIG. 8. (Color Online) A plot of the **standardized skewness** of transverse momenta versus multiplicity class. The blue triangles represent the standardized skewness of the transverse momentum. The dashed line shows the trend of the standardized skewness against the multiplicity class.

5. Intensive Skewness of Transverse Momentum versus Multiplicity Class

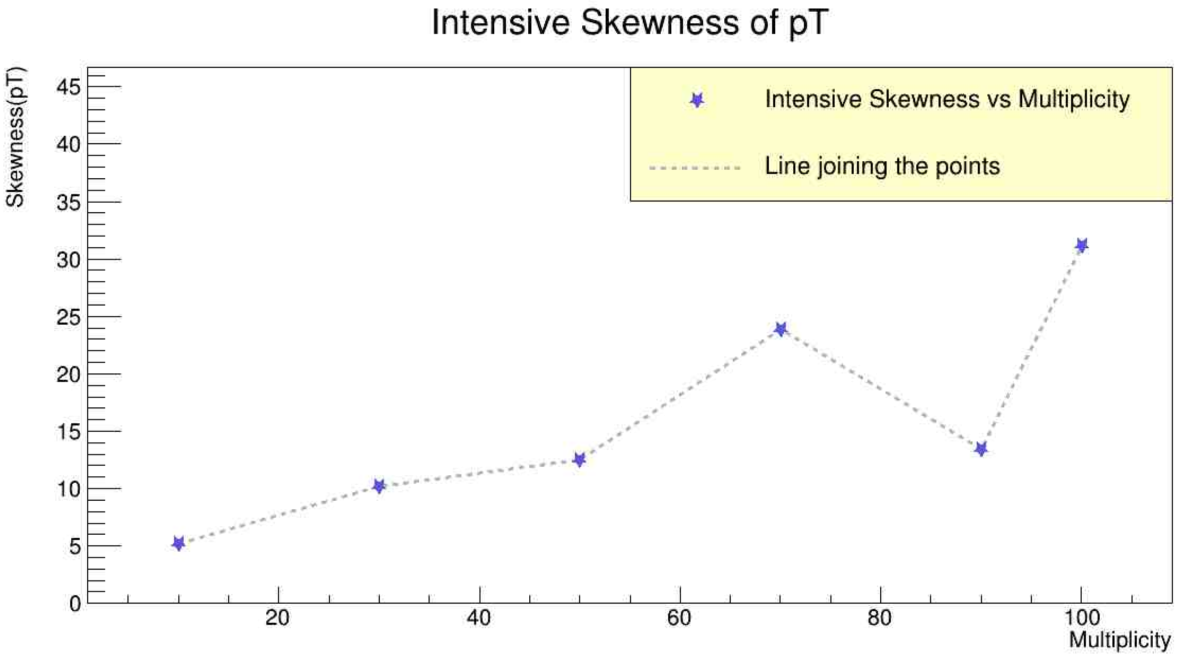

FIG. 9. (Color Online) A plot of the **intensive skewness** of transverse momenta versus multiplicity class. The cyan stars represent the intensive skewness of the transverse momentum. The dashed line shows the trend of the intensive skewness against the multiplicity class.

III. SUMMARY

The fluctuations occurring in the mean transverse momentum $\langle \mathbf{p_T} \rangle$ of emitted particles in various collisions have several underlying causes. It has been shown that the various statistical and dynamical fluctuations are being reflected on the distribution of $\langle \mathbf{p_T} \rangle$. The function of $\langle \mathbf{p_T} \rangle$ versus collision count tends to have a **positive skewness in Gaussian fit**. Hydrodynamics predicts that the event-by-event fluctuations of the mean transverse momentum, $\langle \mathbf{p_T} \rangle$, have a positive skew. The fluctuation can be calculated in terms of standardized skewness and intensive skewness. At the centre of mass energy **13 TeV** for **p+p collision**, the standardized skewness tends to have values between **1.2 to 4.2 GeV/c** and intensive skewness has values between **5.2 to 31.2** for corresponding multiplicity classes. From FIG. 6., it can be concluded that the mean transverse momentum is **monotonically increasing** with respect to the multiplicity class. FIG. 7. shows that the intensive variance of transverse momentum is **monotonically decreasing** with respect to the multiplicity class. From the graphs in FIG. 8. and FIG. 9., it is evident that both the standardized and the intensive skewness of transverse momentum **increases** with multiplicity class, with one exception. For the multiplicity class *pytree80100* there is a dip in the value of skewness. This dip could arise for several reasons. Some of them are mentioned below.

(i) The events are simulated using a Monte Carlo Generator, which in some extreme cases produce data sets that are very close to the mean (having very few outliers).

(ii) The number of events in *pytree80100* is much less than the previous multiplicity classes, as seen from the table.

(iii) As seen from FIG. 5b., the histogram of the distribution of $\langle \mathbf{p_T} \rangle$ has a steep decreasing slope towards the right, which is indicative of higher resemblance to a Gaussian fit. This might occur due to the finite size of the data.

Needless to say, the origin and the behaviour of the skewness is a deep and interesting topic that requires further discussion in the near future.

[1] Giuliano Giacalone *et al.*, *Skewness of mean transverse momentum fluctuations in heavy-ion collisions* (arXiv:2004.09799 [nucl-th])