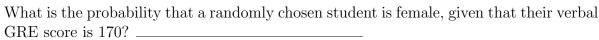
Bootcamp Assessment 2019

Name:	
Please show your work for all problems, even if you do not arrive at a solution	n!
1. For which values of $n > 0$ does $\sum_{n=1}^{\infty} \frac{1}{n}$ converge?	

2. If
$$X = \begin{bmatrix} -2 & 2 \\ -1 & -1 \end{bmatrix}$$
 and $Y = \begin{bmatrix} 3 & -8 \\ 4 & -1 \end{bmatrix}$, does $XY = YX$?

3.	Male verbal GRE scores are normally distributed, with a mean of 149 and a standard
	deviation of 9. Female verbal GRE scores are also normally distributed, with a mean of
	149 and a standard deviation of 8. 55% of the students who take the GRE are female.



4. Let $U \sim Unif(0,1)$ and define $X = -\ln U$. What is the distribution of X? 5. Let X_1, \ldots, X_n be iid samples from $Pois(\lambda)$. Show that both $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ are unbiased estimators for λ .

6. Skewness is a measure of the asymmetry of a probability distribution of a random variable about its mean. We define the skewness of a r.v. X as $\mathsf{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$. A random variable that is right-skewed will have positive skewness, and a variable that is left-skewed will have negative skewness.

If $X \sim Exp(\lambda)$, what is the skewness of X?

7. Let X_1, \ldots, X_n be iid $Beta(\theta, 1)$ random variables, where $\theta > 0$. Find the MLE of $\frac{1}{\theta}$.

8. The joint pdf of X and Y is

$$f(x,y) = \frac{e^{-yx^2/2}}{\sqrt{2\pi/y}} \cdot ye^{-y}, \quad x \in \mathbb{R}, y > 0$$

(a) Find the conditional density $f_{X|Y}(x|y)$ of X given Y = y.

Hint: consider decomposing the joint into the product of conditional and marginal densities.

(b) What is E[X|Y]?

(c) What is Var(X|Y)?

(d) What is Var(X)?

9. What is your favorite distribution?

10. Optional: if you are considering STA 721, you should attempt this problem. Let $\mathbf{Z} = (Z_1, \dots, Z_m)'$ be a vector of m iid N(0,1) random variables. Then $\mathsf{E}[\mathbf{Z}] = \mathbf{0}$ and $Cov(\mathbf{Z}) = \mathbf{I}_m$. Recall that we say a random n-dimensional vector \mathbf{Y} is distributed multivariate normal if it has the same distribution as $\mathbf{AZ} + \mathbf{b}$ where \mathbf{A} is some fixed $n \times m$ matrix and \mathbf{b} is some fixed $n \times 1$ vector. That is, $\mathbf{Y} \sim N_n(\boldsymbol{\mu}, \mathbf{V})$, where $\boldsymbol{\mu} = \mathbf{b}$ and $\mathbf{V} = \mathbf{AA'}$.

However, it is not clear that if we know b and AA' then Y is uniquely or well defined. Notice that the covariance matrix of Y is V = AA'. But because matrix square roots are not unique, we could have some matrix B such that V = BB' but $B \neq A$.

With this information, prove that if $Y \sim N_n(\boldsymbol{\mu}, \boldsymbol{V})$ and $\boldsymbol{W} \sim N_n(\boldsymbol{\mu}, \boldsymbol{V})$, then \boldsymbol{Y} and \boldsymbol{W} follow the same distribution.