Enhancing Inflation Forecasting Models through Bayesian Hierarchical Approaches and Stochastic Seasonality

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April 2024

Outline

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Introduction

- Central banks have clearly defined inflation targets
- Inflation affects real income and thus consumer spending decisions
- Inflation is of interest to the financial world
- Literature leaves room for improved estimation methods

Key Literature

- Stock and Watson (2007)
  - Inflation has become less volatile but harder to forecast
  - Introduce the unobserved component stochastic volatility model
- Ciccarelli and Mojon (2010)
  - 70% of inflation in OECD countries can be explained by a common factor
- Lis and Porqueddu (2018)
  - Inflation is seasonal, even when correcting for food and energy prices

Research Questions

1. *Do seasonality features improve the forecasting performance of existing inflation models?*
2. *Does imposing similarity in inflation patterns across countries improve the forecasting performance of inflation models?*

Outline

Inflation dataset

- Global database from the World Bank (Ha et al., 2023)
- Quarterly data
- Headline Consumer Price Index (CPI) inflation
- Core CPI inflation as robustness check
- Subset of 20 countries from Q1 1977 - Q1 2023

Inflation seasonality (1/2)

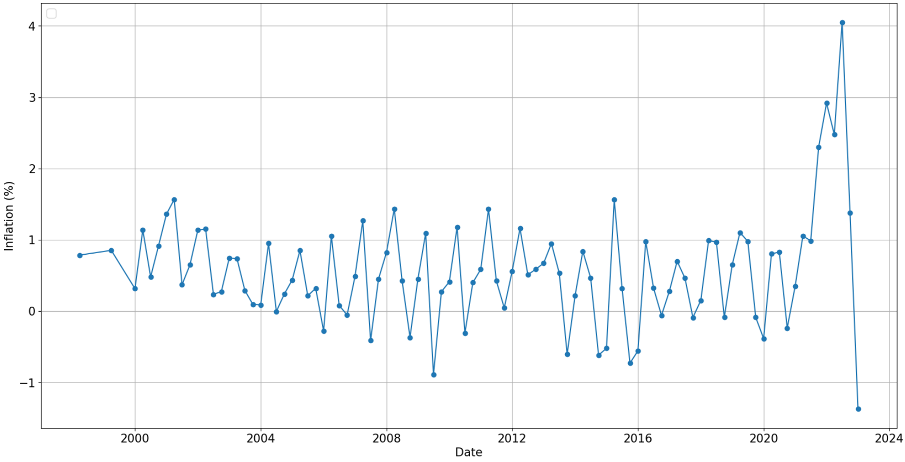


Figure: Quarterly inflation in The Netherlands

Inflation seasonality (2/2)

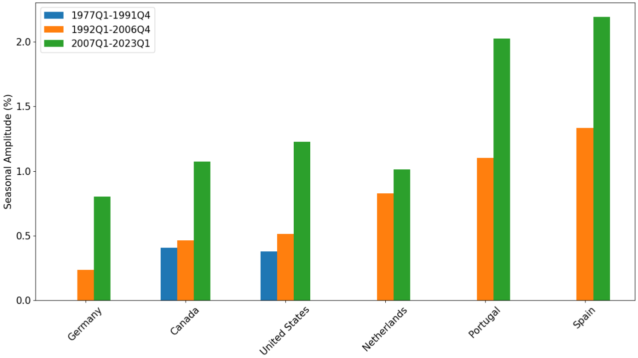


Figure: Range of the seasonal components across countries and sub-samples

- Seasonality stronger for countries with high inflation
- Seasonality increased over time

Inflation correlation and distances

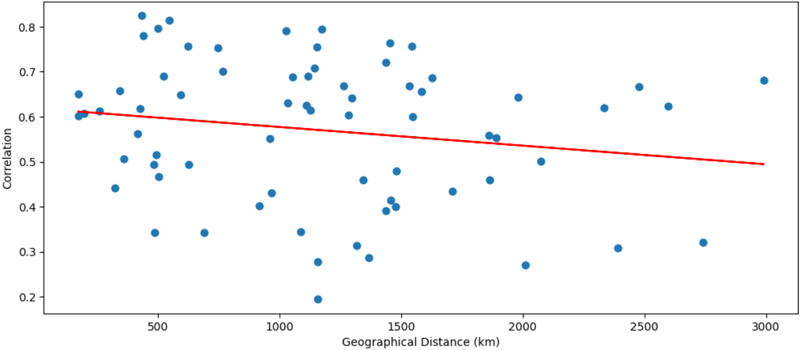


Figure: Correlation of countries and geographical distance

1. Inflation correlation decreasing with distance
2. Distance data from Mayer and Zignago (2011)

Covariates

- Commodity prices (Ha et al., 2023)
- Unemployment rates (ILO, 2024)
- GDP growth (OECD, 2024)
- Interest rates (Bank for International Settlements, 2024)

Outline

Random Effects Bayesian Hierarchical Model

$$y_{t,i} = \alpha_i + \beta_i x_{t-1,i} + \gamma_i w_{t-1} + \epsilon_{t,i}$$

Random Effects Bayesian Hierarchical Model

$$y_{t,i} = \alpha_i + \beta_i x_{t-1,i} + \gamma_i w_{t-1} + \epsilon_{t,i}$$

$$\begin{aligned} \alpha_i &\sim N(\alpha_0, \tau_\alpha^{-1}) & \alpha_0 &\sim N(0, \tau_{\alpha_0}^{-1}) \\ \beta_i &\sim N(\beta_0, \tau_\beta^{-1}) & \beta_0 &\sim N(0, \tau_{\beta_0}^{-1}) \\ \gamma_i &\sim N(\gamma_0, \tau_\gamma^{-1}) & \gamma_0 &\sim N(0, \tau_{\gamma_0}^{-1}) \\ \epsilon_{t,i} &\sim N(0, \tau_\epsilon^{-1}) \end{aligned}$$

$$\tau_{\alpha_0}, \tau_{\beta_0}, \tau_{\gamma_0}, \tau_\alpha, \tau_\beta \tau_\gamma, \tau_\epsilon \sim \Gamma(1, 1)$$

Covariance Priors

$$\boldsymbol{\theta}_{\boldsymbol{s}} \sim \mathcal{N}_n(\mathbf{0}, \Sigma),$$

- Covariance based Bayesian hierarchical model
  - Use sample covariance matrix: $\hat{\Sigma}$
- Distance based Bayesian hierarchical model
  - Matérn kernel to convert distances into covariances: $\Sigma_{\text{Matérn,dist}}$

UC-SV

- Unobserved Component Stochastic Volatility model (Stock and Watson, 2007)
- Split time-series into components
- Variances of the disturbances follow a logarithmic random walk
- Type of Bayesian model

$$
\begin{aligned}
y_{t,i} &= \tau_{t,i} + \eta_{t,i} & \eta_{t,i} &\sim N(0, \sigma^2_{\eta,t,i}) \\
\tau_{t,i} &= \tau_{t-1,i} + \epsilon_{t,i} & \epsilon_{t,i} &\sim N(0, \sigma^2_{\epsilon,t,i}) \\
\ln(\sigma^2_{\eta,t,i}) &= \ln(\sigma^2_{\eta,t-1,i}) + v_{\eta,t,i} & v_{\eta,t,i} &\sim N(0, \gamma_i) \\
\ln(\sigma^2_{\epsilon,t,i}) &= \ln(\sigma^2_{\epsilon,t-1,i}) + v_{\epsilon,t,i} & v_{\epsilon,t,i} &\sim N(0, \gamma_i)
\end{aligned}
$$

UC-SV-SS model

- Add a stochastic seasonality component to the UC-SV model
- seasonal components follow a stochastic process

$$
\begin{aligned}
y_{t,i} &= \tau_{t,i} + \delta_{t,i,1} \cdot seas_1(t) + \ldots + \delta_{t,i,4} \cdot seas_4(t) + \eta_{t,i}, \\
\tau_{t,i} &= \tau_{t-1,i} + \epsilon_{t,i}, \\
\delta_{t,i,j} &= \delta_{t-1,i,j} + seas_j(t) \cdot \xi_{t,i}, \quad j = 1, \ldots, 4, \\
ln(\sigma^2_{\eta,t,i}) &= ln(\sigma^2_{\eta,t-1,i}) + v_{\eta,t,i}, \\
ln(\sigma^2_{\epsilon,t,i}) &= ln(\sigma^2_{\epsilon,t-1,i}) + v_{\epsilon,t,i}.
\end{aligned}
$$

- Estimate using particle filters

Baseline Models

- AO-RW model
  - Introduced by Atkeson and Ohanian (2001), extended by Stock and Watson (2007)
  - Forecast inflation as a four-quarter average
- ARMA(X) model
- PCA-VAR model
  - Apply PCA to the correlation matrix of inflation
  - Apply a VAR(1) to the principal components
  - Transform forecasts back to inflation using eigenvectors

Training and Testing Windows

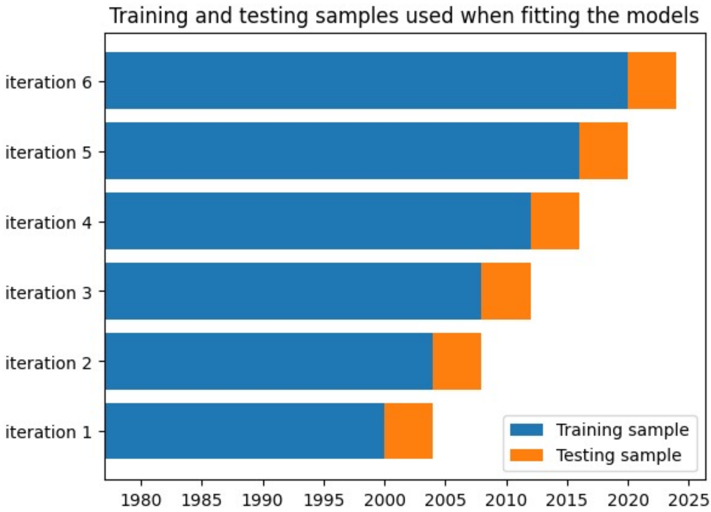
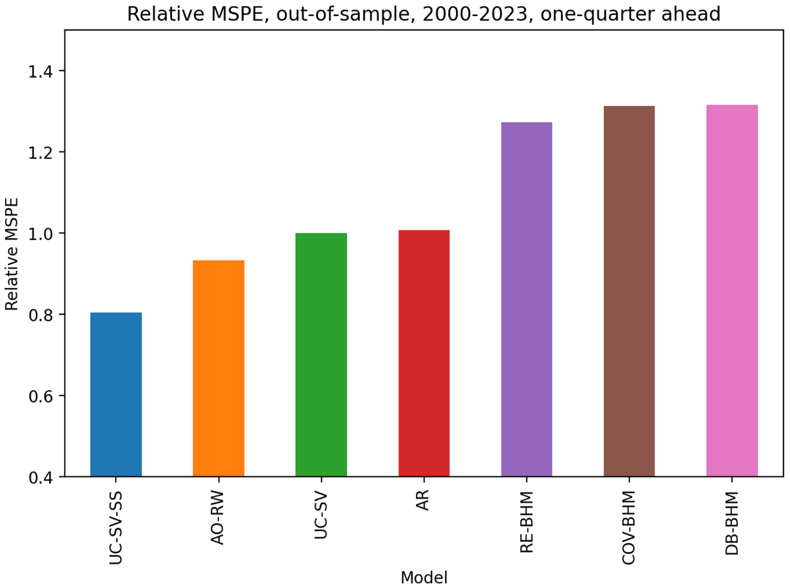


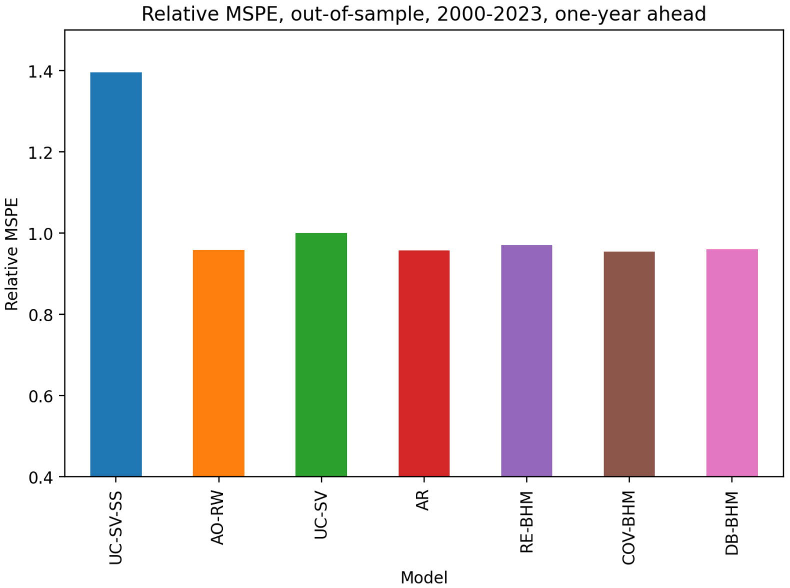
Figure: Training and testing windows used when fitting the models

Outline

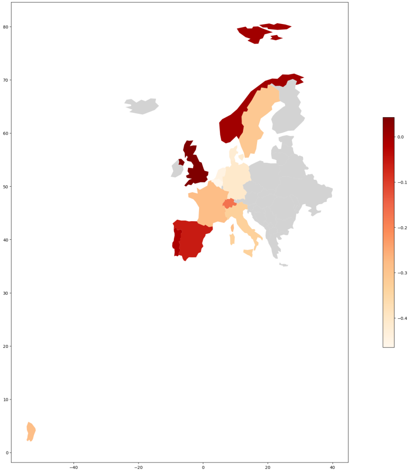
Model Performance



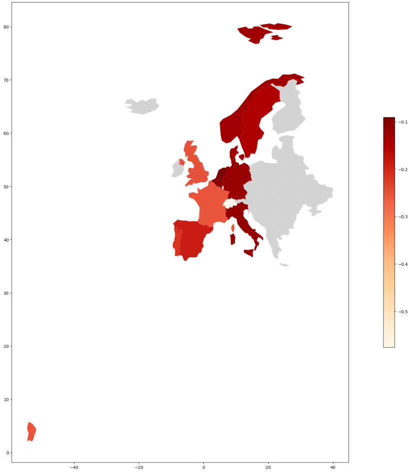
Model Performance



Coefficient Comparison



(a) AR Model



(b) Distance Based Model

Figure: Intercepts of the models per country, 1977-2000

Density Forecasts

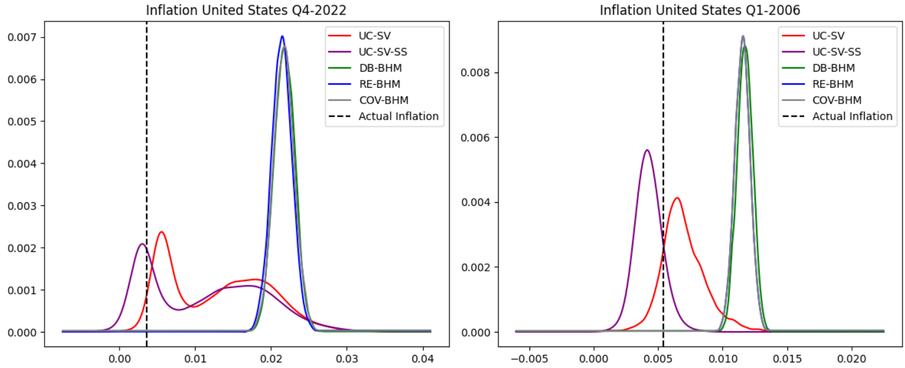


Figure: Density forecasts

Outline

Conclusion

Research Questions:

1. *Do seasonality features improve the forecasting performance of existing inflation models?*

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Further Research:

- Proxies that better measure the political and economic distance
- Weaker priors for the random effects model

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Fitting a BHM

To obtain the posterior distribution of $\theta|x, y$, we can use Bayes' theorem:

$$P(\theta \mid x, y) = \frac{P(x, y \mid \theta) \cdot P(\theta)}{P(x, y)} \propto P(x, y \mid \theta) \cdot P(\theta)$$

However, $P(x, y)$ is difficult to calculate.

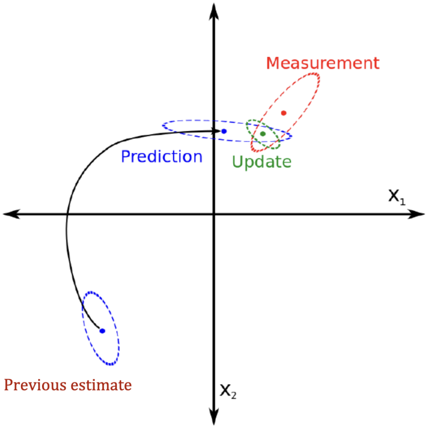
Solution: Markov Chain Monte Carlo (MCMC), e.g. Metropolis-Hastings

Particle filters

- Bayesian method: update priors

Sequential Importance Sampling with Resampling (SISR) algorithm:

1. Predict new hidden state of all points
2. Resample points using measurement likelihood as weights



AO-RW (Baseline model)

- Atkeson–Ohanian Random Walk model
- Forecast inflation as the average of the past four quarters
- Univariate and deterministic model

$$y_{t,i} = \frac{1}{4}(y_{t-1,i} + y_{t-2,i} + y_{t-3,i} + y_{t-4,i}) + \epsilon_{t,i}$$

ARMA(X) (Baseline model)

- Autoregressive Moving Average model with exogenous variables
- Univariate and flexible time-series model

$$y_{t,i} = c + \Sigma_{p=1}^{P} \alpha_p y_{t-p,i} + \Sigma_{q=1}^{Q} \theta_q \epsilon_{t-q,i} + \Sigma_{m=1}^{M} \beta_m x_{m,t-1,i} + \epsilon_{t,i}$$

- Lag length based on AIC

PCA-VAR (Baseline model)

- apply PCA to correlation matrix of inflation
- Vector Autoregression model of order 1

$$f_t = c + A_1 f_{t-1} + \epsilon_t,$$

- transform back to the original data space using the eigenvectors
- lead to parsimonious forecasts

MSPE and MAE

- Evaluate accuracy of point forecasts

$$MSPE_{j,i} = \frac{\sum_{t=1}^{T} (\hat{y}_{j,t,i} - y_{t,i})^2}{T}$$

$$MAE_{j,i} = \frac{\sum_{t=1}^{T} |\hat{y}_{j,t,i} - y_{t,i}|}{T}$$

Mincer-Zarnowitz regression

- test efficiency and bias of forecast
- joint hypothesis of $\alpha_i = 0, \beta_i = 1$

$$y_{t+1,i} = \alpha_i + \beta_i \hat{y}_{t+1,i} + \epsilon_{t+1,i}$$

Country-Specific MSPE

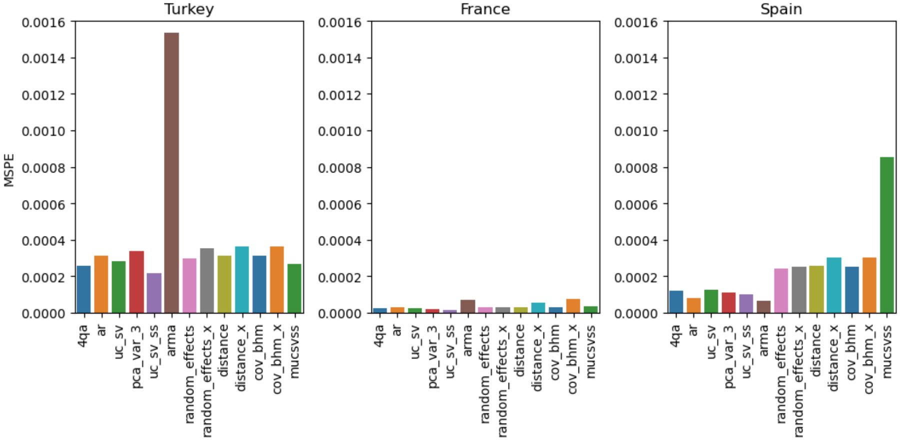


Figure: MSPE of the different models, one-quarter ahead, 2000-2023

Coefficient Comparison

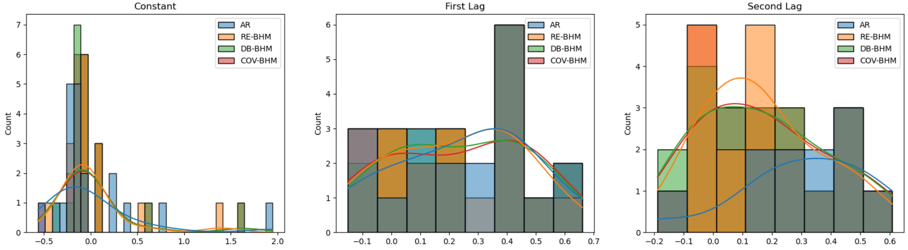


Figure: Coefficients of the AR model, RE-BHM, DB-BHM, and COV-BHM, 1977-2000

Additional Literature

- Introduce a simpel model that forecasts inflation in the next four quarters to be equal to that of the previous four quarters (Atkeson and Ohanian, 2001
- Phillips curve based models outperform simple AR models (Stock and Watson, 1999)
- Bayesian model averaging produces better forecasts than UC-SV model (Medeiros et al., 2021)
- Machine learning techniques can outperform simpler inflation benchmarks (Groen et al., 2013)
- Per sector estimates of inflation can be used to improve the estimate of trend inflation (Stock and Watson, 2016)