Math HW8

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8.10a, 8.11 (you should use a matrix language to do this problem, but be sure to show suitable intermediate steps)

8.3

I made the matrices here for checking work (see code in the appendix), but just to show they work as intended using A as an example:

```
print(A)
```

```
-1
 3
 4
```

```
print(A * 3)
```

```
-3
 9
12
```

(a):

$$3A - 2B,$$

$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

Work: If the number of rows and columns are the same, you can use scalar multiplication.

$$3A = \begin{bmatrix} 3(-1) \\ 3(3) \\ 3(4) \end{bmatrix},$$

```
write_matex(3*A)
```

$$= \begin{bmatrix} -3 \\ 9 \\ 12 \end{bmatrix}.$$

$$2B = \begin{bmatrix} 2(3) \\ 2(-2) \\ 2(1) \end{bmatrix},$$

```
write_matex(2*B)
```

$$= \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}.$$

$$3A - 2B,$$

```
write_matex(3*A - 2*B)
```

$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

(b): $A \cdot B =$ -5.

Work:
Element-wise multiplication:

$$A \times B,$$
$$= \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$$
$$= -1(3) + 3(-2) + 4(1),$$
$$-5.$$

(c):

$$A \times B,$$
$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

Work:
Out-product rule:

$$A \times B,$$
$$= \begin{bmatrix} -1(3) & -1(-2) & -1(1) \\ 3(3) & 3(-2) & 3(1) \\ 4(3) & 4(-2) & 4(1) \end{bmatrix},$$

```
write_matex(A%o%B)
```

$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

(d):

$$CA,$$
$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

Work:
Matrix multiplication, which is possible because the number of C's columns are equal to the number of A's rows: $C_{23}\ A_{32}$.

$$CA,$$
$$= \begin{bmatrix} 3(-1) + 2(3) + -4(4) \\ -8(-1) + 0(3) + 6(4) \end{bmatrix},$$

```r
write_matex(C%*%A)
```

$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

(e): Because the dimensions don't work out (B_{31} D_{32}), we can't multiply.

(f):

$$B \otimes D,$$

$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

Work:

$$B \otimes D,$$

$$= \begin{bmatrix} 3 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ -1 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ 1 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \end{bmatrix}$$

```r
write_matex(kronecker(B,D, FUN="*"))
```

$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

(g):

$$CD,$$

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

Work:
Same situation as (d), because the dimensions work out: C_{23} and D_{32}.

$$CD,$$

$$= \begin{bmatrix} 3(6) + 2(1) + (-4)(-3) & 3(-2) + 2(3) + (-1)(8) \\ -8(6) + 0(-1) + 6(-3) & -8(-2) + 0(3) + 6(8) \end{bmatrix},$$

```r
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(h):

$$DC,$$

$$= \begin{bmatrix} 34 & 12 & -36 \\ -27 & -2 & 22 \\ -73 & -6 & 60 \end{bmatrix}.$$

Work:
Yet again the same situation as (d), because the dimensions work out: D_{32} and C_{23}.

$$DC,$$

$$= \begin{bmatrix} 6(3) + (-2)(-8) & 6(2) + (-2)0 & 6(-4) + (-2)6 \\ -1(3) + 3(-8) & -1(2) + 3(0) & -1(-4) + 3(6) \\ -3(3) + 8(-8) & -3(2) + 8(0) & -3(-4) + 8(6) \end{bmatrix},$$

```r
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(i):

$$C'C,$$

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

Work:
First, transpose C:

```r
write_matex(t(C))
```

$$= \begin{bmatrix} 3 & -8 \\ 2 & 0 \\ -4 & 6 \end{bmatrix}.$$

$$C'C,$$

$$= \begin{bmatrix} 3(3) + (-8)(-8) & 3(2) + (-8)0 & 3(-4) + (-8)6 \\ 2(3) + 0(-8) & 2(2) + 0(0) & 2(-4) + 0(6) \\ -4(3) + 6(-8) & -4(2) + 6(0) & -4(-4) + 6(6) \end{bmatrix},$$

```r
write_matex(t(C)%*%C)
```

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

8.4

It wil be symmetric because the transposed matrix contains the same information as the original matrix, just in a different order. When multiplying, e.g. 3 and -4, and -8 and 6, will always be grouped together, just in a different order, which means those cells will always come out to be the same number because of the associative property of multiplication. For example, the inner product of (1,3) would be:

$\begin{bmatrix} 3 & -8 \end{bmatrix} \times \begin{bmatrix} -4 \\ 6 \end{bmatrix},$

which ends up being $3(-4) + (-8)6 = -60$.

(3,1) would be:

$\begin{bmatrix} -4 & 6 \end{bmatrix} \times \begin{bmatrix} 3 \\ -8 \end{bmatrix},$

which ends up being $-4(3) + 6(-8) = -60$.

8.8

In the matrix $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, 1 represents latitude and -1 represents longitude. To rotate 90 degrees, just shift the arrow to a new quandrant in the clockwise direction by changing the latitude: $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$. Step 2 is to find the transformation matrix, which is a matrix that when multiplied by $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ will yield $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$. This would work:

$$\begin{bmatrix} 5 & 6 \\ -2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

8.9

Multiply latitude and longitude by the some fraction and the magnitude will change while the direction will remain the same. E.g.:

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix},$$
$$\begin{bmatrix} 2 \times 0.5 \\ 4 \times 0.5 \end{bmatrix},$$
$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

To confirm the direction is the same, the cosine should be 1, meaning the angle is 0:

$$\frac{A \cdot B}{||A||\ ||B||},$$
$$\frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\sqrt{2^2 + 4^2}\ \sqrt{1^2 + 2^2}},$$
$$\frac{10}{10},$$
$$= 1.$$

```r
# check work
do <- matrix(c(2,4), nrow=2, ncol=1)
re <- matrix(c(1,2), nrow=2, ncol=1)
cat(matrixcalc::frobenius.prod(do,re) / (sqrt(2^2+4^2)*sqrt(1^2+2^2)))
```

```
1
```

8.10a