HW 4

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4.1, 4.2, 4.3, 4.4, 4.5, 4.6

4.1

Answer:
As n approaches infinity, the shape would approach a circle.

4.2

(a)

Answer:
32

Work:

$$\lim_{x \to 5} 2x^2 - 5x + 7,$$

$$2(5^2) - 5(5) + 7,$$

$$32.$$

```
# chck work
2*(5)^2 - (5*5) + 7
```

```
[1] 32
```

(b)

Answer:
Limit is 0. Approaches but never quite gets there from the left nor right.

Work:

$$\lim_{y \to \infty} \frac{1}{y^6}$$

```r
# check work
1 / 2^6
1 / 4^6
1 / 20^6
1 / -2^6
1 / -4^6
1 / -20^6
```

```
[1] 0.015625
[1] 0.0002441406
[1] 0.000000015625
[1] -0.015625
[1] -0.0002441406
[1] -0.000000015625
```

(c)

Answer:
Positive infinity. The larger the absolute value of z, the larger the quotient.

Work:

$$\lim_{z \to 0} \frac{1}{z^6}$$

```r
# check work
1 / 1^6
1 / 0.5^6
1 / 0.01^6
1 / (-1)^6
1 / (-0.5)^6
1 / (-0.01)^6
```

```
[1] 1
[1] 64
[1] 1000000000000
[1] 1
[1] 64
[1] 1000000000000
```

(d)

Answer:
The limit is 0.

Work:

```r
# check work
2*2 / 6 * (2^2)
2*5 / 6 * (5^2)
2*10 / 6 * (10^2)
2*1 / 6 * (1^2)
2*0.1 / 6 * (0.1^2)
2*0.0001 / 6 * (0.001^2)
```

```
[1] 2.666667
[1] 41.66667
[1] 333.3333
[1] 0.3333333
[1] 0.0003333333
[1] 0.00000000003333333
```

Could also solve this way:

$$\lim_{x\to 0} \frac{2x+3}{5x^2},$$

$$\lim_{x\to 0} \frac{2x}{5x^2} + \frac{3}{5x^2},$$

$$\lim_{x\to 0} \frac{2}{5x} + \frac{3}{5x^2},$$

$$\lim_{x\to 0} \frac{2}{5x} + \lim_{x\to 0} \frac{3}{5x^2},$$

$$\frac{2}{5} \lim_{x\to 0} \frac{1}{x} + \frac{3}{5} \lim_{x\to 0} \frac{1}{x^2},$$

$$\frac{2}{5} \lim_{x\to 0} \frac{1}{x} + \frac{3}{5} \lim_{x\to 0} \frac{1}{x^2},$$

The limits to $\frac{1}{x}$ and $\frac{1}{x^2}$ are 0.

(e)

Answer:
$\frac{3}{5}$
Work:
We know that the limit of $\frac{1}{x^y} = 0$, so we can reduce many of the terms of the expression to 0s by making them into the form of $\frac{1}{x^y} = 0$.

$$\lim_{y\to\infty} \frac{3y^7 + 4y^6 - 2y^5 - 8y^3 - 7y + 1}{2y^7 + y^3 - 8} \times \frac{\frac{1}{y^7}}{\frac{1}{y^7}},$$

$$= \lim_{y\to\infty} \frac{3y^7 + \frac{4}{y} - \frac{2}{y^2} - \frac{8}{y^4} - \frac{7}{y^6} + \frac{1}{y^7}}{2 + \frac{1}{4y^4} - \frac{8}{y^7}},$$

$$= \frac{3}{5}.$$

(f)

Answer:
The limit is 1.

Work:

Do some factoring

$$\lim_{z\to 3} \frac{z^2 - 5z + 6}{z - 3},$$

$$= \lim_{z\to 3} \frac{(z-2)(z-3)}{z-3},$$

$$= \lim_{z\to 3} z - 2 = 1$$

```
# check work
import sympy
from sympy import *
x, y, z = symbols('x y z')
init_printing(use_unicode=True)
expr = Limit((z - 2)*(z - 3) / (z - 3), z, 3)
# find the limit
print(expr.doit())
```

```
1
```

(g)

Answer:
∞

Work:
Need limit from the right:

```
1 / (5.1 - 5)
1 / (5.01 - 5)
1 / (5.00001 - 5)
```

```
[1] 10
[1] 100
[1] 100000
```

(h)

Answer:
The limit from the right is ∞ while the limit from the left is $-\infty$, therefore there is no limit.

Work:
Need limit from the right:

```r
1 / (7.1 - 7)
1 / (7.01 - 7)
1 / (7.00001 - 7)
```

```
[1] 10
[1] 100
[1] 100000
```

and left:

```r
1 / (6.9 - 7)
1 / (6.99 - 7)
1 / (6.99999- 7)
```

```
[1] -10
[1] -100
[1] -100000
```

(i)

Answer:

Work:
Aha! This is just Euler's number, which is equal to 2.7182... Therefore:

$$\lim_{z\to\infty}\left(1+\frac{1}{z}\right)^{2z},$$

Becaause the exponenents are mutliplicative, put the z on the inside to get euler's number:

$$\begin{aligned}
&= \lim_{z\to\infty}\left(\left(1+\frac{1}{z}\right)^{z}\right)^{2}, \\
&= \lim_{z\to\infty}(e)^2 \\
&= \lim_{z\to\infty}(2.7182)^2 \\
&\approx 7.388611
\end{aligned}$$

```r
# calculations
2.7182^2
```

```
[1] 7.388611
```