CatData HW3

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A2.2, A2.5, A2.13, A2.21, A2.36. (3 points each)

A2.33 (5 points)

All code and work are shown in the appendix.

2.2

(a)

Answer:
Recoding this in a way that makes more sense to me. Will leave X and Y as they are:

- $X(0 = \text{no disease}, 1 = \text{disease})$
- $Y(0 = \text{negative}, 1 = \text{positive})$
- Sensitivity $= \pi_1 = P(Y = 1|X = 1) =$ probability positive diagnosis given disease
- Specificity $= 1 - \pi_2 = 1 - P(Y = 1|X = 0) =$ 1 - probability positive diagnosis given no disease

After subtracting away the probability of positive diagnosis given no disease, you are left with probability of negative diagnosis given no disease:

$1 - P(Y = 1|X = 0) = P(Y = 0|X = 0) =$ probability negative diagnosis given no disease

Likewise, the “noise” of the test is captured by 1 - specificity. A good test would want to show that 1 - specificity yields a large probability as an effective test will have a large probability of correctly screening patients that do not have the disease.

(b)

Answer:
$P(X = 1|Y = 1) = \dfrac{P(Y = 1|X = 1)P(X = 1)}{P(Y = 1|X = 1)P(X = 1) + P(Y = 1|X = 0)P(X = 0)}$

Work:

$$\frac{\pi_1\gamma}{[\pi_1\gamma + \pi_2(1-\gamma)]} = \frac{P(B|A)P(A)}{P(B)} = \text{Bayes' rule}$$

In the above equation, P(B) represents P(X=1), or the probability of having the disease. But this is an unknown value expressed as γ, we have to use a particular version of Bayes' rule that takes into account the conditional probabilities associated with P(B):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\widetilde{A})P(\widetilde{A})}.$$

Just replace with the problem's notation:

$$P(X=1|Y=1) = \frac{P(Y=1|X=1)P(X=1)}{P(Y=1|X=1)P(X=1) + P(Y=1|X=0)P(X=0)}$$

(c)

Answer:
0.07

Work:

We need to solve for $P(X=1|Y=1)$. Lay out the pieces:

- $X(0 = \text{no disease}, 1 = \text{disease}$.
- $P(X=1) = 0.01$.
- $Y(0 = \text{negative}, 1 = \text{positive}$.
- Sensitivity $= \pi_1 = P(Y=1|X=1) = 0.86$.
- Specificity $= 1 - \pi_2 = 1 - P(Y=1|X=0) = 0.88$
- If $1 - P(Y=1|X=0) = 0.88$, then $P(Y=1|X=0) = 0.12$.
- If $P(X=1) = 0.01$, then $P(X=0) = 1 - P(X=1) = 0.99$.

Plug and chug:

$$P(X=1|Y=1) = \frac{P(Y=1|X=1)P(X=1)}{P(Y=1|X=1)P(X=1) + P(Y=1|X=0)P(X=0)},$$

$$= \frac{0.86 \times 0.01}{(0.86 \times 0.01) + (0.12 \times 0.99)},$$

$$= 0.07.$$

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# calculations
(0.86 * 0.01) / ((0.86 * 0.01) + (0.12 * 0.99))
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[1] 0.06750392
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