Math HW8

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8.3

(a):

$$3A - 2B,$$
$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

(b): $A \cdot B =$ -5.

(c):

$$A \times B,$$
$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

(d):

$$CA,$$
$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

(e): Because the dimensions don't work out (B_{31} D_{32}), we can't multiply.

(f):

$$B \otimes D,$$
$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

(g):

$$CD,$$
$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(h):

$$DC,$$

$$= \begin{bmatrix} 34 & 12 & -36 \\ -27 & -2 & 22 \\ -73 & -6 & 60 \end{bmatrix}.$$

(i):

$$C'C,$$

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

8.4

It wil be symmetric because the transposed matrix contains the same information as the original matrix, just in a different order. When multiplying, e.g. 3 and -4, and -8 and 6, will always be grouped together, just in a different order, which means those cells will always come out to be the same number because of the associative property of multiplication. For example, the inner product of (1,3) would be:

$\begin{bmatrix} 3 & -8 \end{bmatrix} \times \begin{bmatrix} -4 \\ 6 \end{bmatrix},$

which ends up being $3(-4) + (-8)6 = -60$.

(3,1) would be:

$\begin{bmatrix} -4 & 6 \end{bmatrix} \times \begin{bmatrix} 3 \\ -8 \end{bmatrix},$

which ends up being $-4(3) + 6(-8) = -60$.

8.8

In the matrix $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, 1 represents latitude and -1 represents longitude. To rotate 90 degrees, just shift the arrow to a new quandrant in the clockwise direction by changing the latitude: $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$. Step 2 is to find the transformation matrix, which is a matrix that when multiplied by $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ will yield $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$. This would work:

$$\begin{bmatrix} 5 & 6 \\ -2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

8.9

Multiply latitude and longitude by the some fraction and the magnitude will change while the direction will remain the same. E.g.:

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

$$\begin{bmatrix} 2 \times 0.5 \\ 4 \times 0.5 \end{bmatrix},$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

To confirm the direction is the same, the cosine should be 1, meaning the angle is 0:

$$\frac{A \cdot B}{||A||\ ||B||},$$

$$\frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\sqrt{2^2+4^2}\ \sqrt{1^2+2^2}},$$

$$\frac{10}{10},$$

$$= 1.$$

```
# check work
do <- matrix(c(2,4), nrow=2, ncol=1)
re <- matrix(c(1,2), nrow=2, ncol=1)
cat(matrixcalc::frobenius.prod(do,re) / (sqrt(2^2+4^2)*sqrt(1^2+2^2)))
```

```
1
```

8.10a

large (a):

$$\begin{bmatrix} 2 & 1 & 6 \\ 8 & 4 & 3 \\ 12 & 1 & 4 \end{bmatrix},$$

Switch the top and second rows

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 6 \\ 12 & 1 & 4 \end{bmatrix},$$

Multiple the first row by -1, then add that to the last row

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 6 \\ 8 & -3 & 1 \end{bmatrix},$$

Switch the first and the last row

$$\begin{bmatrix} 8 & -3 & 1 \\ 2 & 1 & 6 \\ 8 & 4 & 3 \end{bmatrix},$$

Multiply the second row by -4, then add that to the last row

$$\begin{bmatrix} 8 & -3 & 1 \\ 2 & 1 & 6 \\ 0 & 0 & -21 \end{bmatrix},$$

Divide the last row by -21

$$\begin{bmatrix} 8 & -3 & 1 \\ 2 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix},$$

Multiply the second row by 3 and add that to the first row

$$\begin{bmatrix} 2 & 0 & 18 \\ 2 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix},$$

Divide the fist row by 2

$$\begin{bmatrix} 1 & 0 & 9 \\ 2 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix},$$

Multiply the last row by -6 and add that to the second row

$$\begin{bmatrix} 1 & 0 & 9 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Multiply the last row by -9 and add that to the first row

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Multiply the first row by -2 and add that to the second row

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

8.11 (you should use a matrix language to do this problem, but be sure to show suitable intermediate steps) Make the vectors for the dv, errors, and coefficients:

$$Y = \begin{bmatrix} 9 \\ 5 \\ 4 \\ 7 \\ 4 \\ 6 \\ 5 \\ 7 \\ 2 \\ 5 \end{bmatrix}, \ \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \\ \varepsilon_9 \\ \varepsilon_{10} \end{bmatrix}, \ \beta = \begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \end{bmatrix}.$$

Hold all the dv data together. But need to make sure that when the time comes to multiply through, the α stays by itself. One way to make that happen is by making sure is only ever gets multiplied by 1:

$$X = \begin{bmatrix} 1 & 0 & 8 \\ 1 & 0 & 6 \\ 1 & 1 & 7 \\ 1 & 1 & 8 \\ 1 & 1 & 3 \\ 1 & 0 & 4 \\ 1 & 1 & 6 \\ 1 & 0 & 3 \\ 1 & 1 & 5 \\ 1 & 0 & 4 \end{bmatrix}.$$

So now you can multiply through because the dimensions work out to be conformable:

$$Y = {}_{10}X_{3\ 3}\beta_1.$$

The multiplication would look like:

$$\big[(1 \times \alpha) + (0 \times \beta_1) + (8 \times \beta_2)\big]$$

etc.

After you've multiplied through, the resulting matrix will be a 10x1 matrix, which means you'll then be able to add the errors. Each line of the matrix will be the regression equation:

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_i$$

e.g.

$$9 = \alpha + \beta_1 0 + \beta_2 8 + \varepsilon_1.$$

Appendix

8.3

```r
# create some matrices
A <- matrix(c(-1,3,4), nrow=3, ncol=1)
B <- matrix(c(3,-2,1), nrow=3, ncol=1)
C <- matrix(c(3,-8,2,0,-4,6), nrow=2, ncol=3)
D <- matrix(c(6,-1,-3,-2,3,8), nrow=3, ncol=2)
# this function gets rid of the annoying row and  column names of the matrices
print.matrix <- function(m){
write.table(format(m, justify="right"),
            row.names=F, col.names=F, quote=F)
}
# This function writes r matrices in latex
write_matex <- function(x) {
  begin <- "$$= \\begin{bmatrix}"
  end <- "\\end{bmatrix}.$$"
  X <-
    apply(x, 1, function(x) {
      paste(
        paste(x, collapse = "&"),
        "\\\\"
      )
    })
  writeLines(c(begin, X, end))
}
write_matex(A)
```

$$= \begin{bmatrix}-1\\ 3\\ 4\end{bmatrix}.$$

```r
write_matex(A * 3)
```

$$= \begin{bmatrix}-3\\ 9\\ 12\end{bmatrix}.$$

(a):

$$3A - 2B,$$

$$= \begin{bmatrix}-9\\ 13\\ 10\end{bmatrix}.$$

Work: If the number of rows and columns are the same, you can use scalar multiplication.

$$3A = \begin{bmatrix}3(-1)\\ 3(3)\\ 3(4)\end{bmatrix},$$

```
write_matex(3*A)
```

$$= \begin{bmatrix} -3 \\ 9 \\ 12 \end{bmatrix}.$$

$$2B = \begin{bmatrix} 2(3) \\ 2(-2) \\ 2(1) \end{bmatrix},$$

```
write_matex(2*B)
```

$$= \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}.$$

$$3A - 2B,$$

```
write_matex(3*A - 2*B)
```

$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

(b): $A \cdot B =$ -5.

Work:
Element-wise multiplication:

$$A \times B,$$

$$= \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$$

$$= -1(3) + 3(-2) + 4(1),$$

$$-5.$$

(c):

$$A \times B,$$

$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

Work:
Out-product rule:

$$A \times B,$$

$$= \begin{bmatrix} -1(3) & -1(-2) & -1(1) \\ 3(3) & 3(-2) & 3(1) \\ 4(3) & 4(-2) & 4(1) \end{bmatrix},$$

```r
write_matex(A%o%B)
```

$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

(d):

$$CA,$$
$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

Work:
Matrix multiplication, which is possible because the number of C's columns are equal to the number of A's rows: C_{23} A_{32}.

$$CA,$$
$$= \begin{bmatrix} 3(-1) + 2(3) + -4(4) \\ -8(-1) + 0(3) + 6(4) \end{bmatrix},$$

```r
write_matex(C%*%A)
```

$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

(e): Because the dimensions don't work out (B_{31} D_{32}), we can't multiply.

(f):

$$B \otimes D,$$
$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

Work:

$$B \otimes D,$$
$$= \begin{bmatrix} 3 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ -1 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ 1 & \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \end{bmatrix}$$

```r
write_matex(kronecker(B,D, FUN="*"))
```

$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

(g):

$$CD,$$
$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

Work:
Same situation as (d), because the dimensions work out: C_{23} and D_{32}.

$$CD,$$
$$= \begin{bmatrix} 3(6) + 2(1) + (-4)(-3) & 3(-2) + 2(3) + (-1)(8) \\ -8(6) + 0(-1) + 6(-3) & -8(-2) + 0(3) + 6(8) \end{bmatrix},$$

```r
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(h):

$$DC,$$
$$= \begin{bmatrix} 34 & 12 & -36 \\ -27 & -2 & 22 \\ -73 & -6 & 60 \end{bmatrix}.$$

Work:
Yet again the same situation as (d), because the dimensions work out: D_{32} and C_{23}.

$$DC,$$
$$= \begin{bmatrix} 6(3) + (-2)(-8) & 6(2) + (-2)0 & 6(-4) + (-2)6 \\ -1(3) + 3(-8) & -1(2) + 3(0) & -1(-4) + 3(6) \\ -3(3) + 8(-8) & -3(2) + 8(0) & -3(-4) + 8(6) \end{bmatrix},$$

```r
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(i):

$$C'C,$$
$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

Work:
First, transpose C:

```
write_matex(t(C))
```

$$= \begin{bmatrix} 3 & -8 \\ 2 & 0 \\ -4 & 6 \end{bmatrix}.$$

$$C'C,$$

$$= \begin{bmatrix} 3(3) + (-8)(-8) & 3(2) + (-8)0 & 3(-4) + (-8)6 \\ 2(3) + 0(-8) & 2(2) + 0(0) & 2(-4) + 0(6) \\ -4(3) + 6(-8) & -4(2) + 6(0) & -4(-4) + 6(6) \end{bmatrix},$$

```
write_matex(t(C)%*%C)
```

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

8.4 Work shown in answer.

8.8 Work shown in answer.

8.9 Work shown in answer.

8.10a Work shown in answer.

8.11 Work shown in answer.