Math HW5

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4.8, 4.9, 5.1, 5.2, 5.3

4.8

(a)

Work

First Derivative:

The only layer here is in the exponent of e; everything else is constant.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}},$$

$$A = -\frac{x^2}{2}, \quad f(x) = \frac{1}{\sqrt{2\pi}} e^{A}$$

$$A' = \frac{d}{dx}\Big(-\frac{x^2}{2}\Big) = -\frac{1}{2}\Big(2x^{2-1}\Big) = -\frac{2x^{2-1}}{2} = -x.$$

$$\frac{1}{\sqrt{2\pi}} e^{A}\Big(-x\Big) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

Second Derivative

Because the first derivative was multiplicative, we need the product to get the second derivative.
Product rule: $f''(x) = g'(x)f(x) + g(x)f'(x)$.
$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.
$f'(x) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.
$g(x) = -x$.
$g'(x) = \frac{d}{dx}(-x) = -1$.

$$\begin{aligned} f''(x) &= -1\Big(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big) - x\Big(\frac{-x^2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big) \\ &= -\Big(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big) + \Big(\frac{x^2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big) \\ &= \Big(\frac{x^2-1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big). \end{aligned}$$

Plug in 0 for each version

$$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{0^2}{2}} = \frac{1}{\sqrt{2\pi}}.$$

$$f'(0) = \frac{0^2}{\sqrt{2\pi}} e^{-\frac{0^2}{2}} = 0.$$

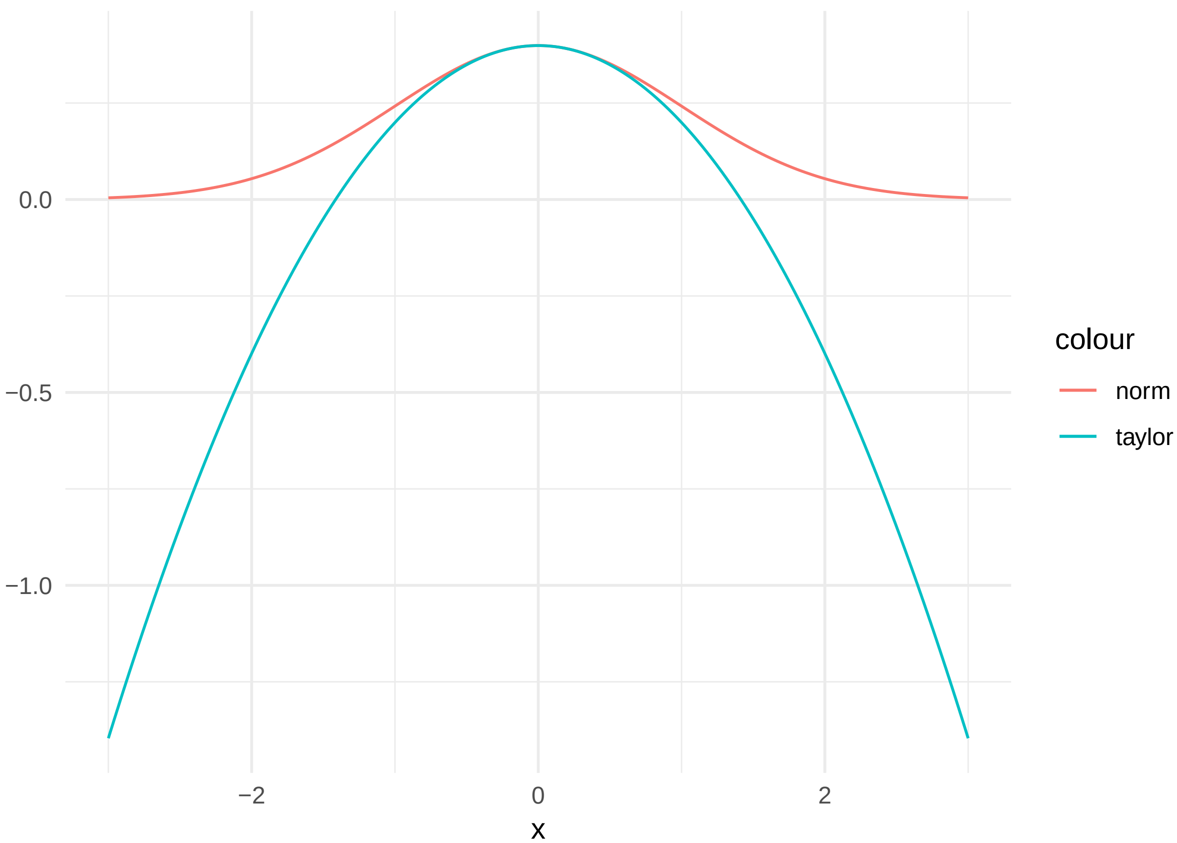
$$f''(0) = \frac{0^2 - 1}{\sqrt{2\pi}} e^{-\frac{0^2}{2}} = \frac{-1}{\sqrt{2\pi}}.$$

Taylor approximation - just plug in what we found:

$$f(x) \approx \frac{1}{\sqrt{2\pi}} + 0x + \frac{-1/\sqrt{2\pi}}{2} x^2.$$

$$f''(x) \approx \frac{-1}{\sqrt{8\pi}} x^2 + \frac{1}{\sqrt{2\pi}}.$$

(b)



Taylor series seems most accurate between about -1 and 1.