Math HW7

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05-10-2019

7.6 (sketch its level sets as well), 7.7, 7.10

7.1a

Answer: $f(a, b, c, d, e, f) = a \times b \times c \times d \times e \times f$

7.3a

Answer: 5

Work:

Plug in 3 and 1:

```
(3^2 - 3 * 1 + (6 * 1)^2) / (3 - 3 * 1)
```

```
[1] Inf
```

So that doesn't work. Need to find some way to cancel out the denominator.

$$x^2 - xy + 6y^2,$$

Set y equal to 1:

$$= x^2 - (x \cdot 1) + (6 \cdot 1)^2,$$
$$= x^2 - x + 6,$$
$$= (x - 3)(x + 2).$$

Plug y back in:

$$= (x - 3y)(x + 2y).$$

Check work:

```
from __future__ import division
from sympy import *
x, y = symbols('x y')
print(expand((x - 3*y) * (x + 2*y)))
```

```
x**2 - x*y - 6*y**2
```

plug back into original expression:

$$\frac{(x - 3y)(x + 2y)}{x - 3y},$$
$$= x + 2y.$$

Plugging in x and y values:

```r
3 + 2 * 1
```

```
[1] 5
```

7.6 (sketch its level sets as well)

(a)

Answer: $\nabla f(xy) = \begin{bmatrix} y - 2x + 2 \\ x - 2y + 1 \end{bmatrix}$.

Work:
Gradient = first partial derivative. Need to find one for x and one for y, and construct a matrix where row 1 is the first partial derivative with respect to x and row 2 is the first partial derivative with respect to y.
With respect to x:

$$\frac{\partial}{\partial x}(-x^2 + xy - y^2 + 2x + y),$$
$$= \frac{\partial}{\partial x}(-x^2) + \frac{\partial}{\partial x}(x) \cdot \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial x}(2) \cdot \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial x}(y),$$
$$y - 2x + 2.$$

With respect to y:

$$\frac{\partial}{\partial y}(-x^2 + xy - y^2 + 2x + y),$$
$$= \frac{\partial}{\partial y}(-x^2) + \frac{\partial}{\partial y}(x) \cdot \frac{\partial}{\partial y}(y) - \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial y}(2) \cdot \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial y}(y),$$
$$= x - 2y + 1.$$

Therefore, the gradient is:

$$\nabla f(xy) = \begin{bmatrix} y - 2x + 2 \\ x - 2y + 1 \end{bmatrix}.$$

(b)

Answer: critical point = $(\frac{5}{3}, \frac{4}{3})$.

Work:

start by solving for x:

$$-2x + y + 2 = 0,$$
$$y = 2x - 2.$$

Plug into the bottom element:

$$x - 2y + 1 = 0,$$
$$x - 2(2x - 2) + 1 = 0,$$
$$x - 4x - 3 = 0,$$
$$3x = 5,$$

$$x = \frac{5}{3}.$$

Back to the top element:

$$\begin{aligned} y - 2x + 2 &= 0, \\ -y &= 2x + 2, \\ -y &= -2(\frac{5}{3}) + 2, \\ y &= \frac{4}{3} \end{aligned}$$

(c)

Answer: $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$.

Work:
The gradient is:

$$\nabla f(xy) = \begin{bmatrix} y - 2x + 2 \\ x - 2y + 1 \end{bmatrix}.$$

Hessian:

$$\begin{aligned} H(f(x,y)) &= \begin{bmatrix} \frac{\partial}{\partial x}(y - 2x + 2) & \frac{\partial}{\partial y}(y - 2x + 2) \\ \frac{\partial}{\partial x}(x - 2y + 1) & \frac{\partial}{\partial y}(x - 2y + 1) \end{bmatrix}, \\ &= \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}. \end{aligned}$$

(d)

$$\begin{aligned} f_{xx}(\frac{5}{3}, \frac{4}{3}) \cdot f_{yy}(\frac{5}{3}, \frac{4}{3}) - f_{xy}(\frac{5}{3}, \frac{4}{3})^2 > 0, \\ -2(-2) - 1^2 > 0, \\ 3. \end{aligned}$$

We therefore know the matrix is either a positive-definite or negative-definite matrix.

$$\begin{aligned} f_{xx}(\frac{5}{3}, \frac{4}{3}) + f_{yy}(\frac{5}{3}, \frac{4}{3}), \\ = -2 + (-2), \\ = -4. \end{aligned}$$

At $(\frac{5}{3}, \frac{4}{3})$, the matrix is negative-definite, and at a local maximum.