Math HW8

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05-12-2019

8.3, 8.4, 8.8, 8.9, 8.10a, 8.11 (you should use a matrix language to do this problem, but be sure to show suitable intermediate steps)

8.3

I made the matrices here for checking work (see code in the appendix), but just to show they work as intended using A as an example:

```
print(A)
```

```
-1
 3
 4
```

```
print(A * 3)
```

```
-3
 9
12
```

(a):

$$3A - 2B,$$
$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

Work: If the number of rows and columns are the same, you can use scalar multiplication.

$$3A = \begin{bmatrix} 3(-1) \\ 3(3) \\ 3(4) \end{bmatrix},$$

```
write_matex(3*A)
```

$$= \begin{bmatrix} -3 \\ 9 \\ 12 \end{bmatrix}.$$

$$2B = \begin{bmatrix} 2(3) \\ 2(-2) \\ 2(1) \end{bmatrix},$$

```
write_matex(2*B)
```

$$= \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}.$$

$$3A - 2B,$$

```
write_matex(3*A - 2*B)
```

$$= \begin{bmatrix} -9 \\ 13 \\ 10 \end{bmatrix}.$$

(b): $A \cdot B =$ -5.

Work:
Element-wise multiplication:

$$\begin{aligned} & A \times B, \\ &= \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \\ &= -1(3) + 3(-2) + 4(1), \\ & -5. \end{aligned}$$

(c):

$$\begin{aligned} & A \times B, \\ &= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}. \end{aligned}$$

Work:
Out-product rule:

$$\begin{aligned} & A \times B, \\ &= \begin{bmatrix} -1(3) & -1(-2) & -1(1) \\ 3(3) & 3(-2) & 3(1) \\ 4(3) & 4(-2) & 4(1) \end{bmatrix}, \end{aligned}$$

```
write_matex(A%o%B)
```

$$= \begin{bmatrix} -3 & 2 & -1 \\ 9 & -6 & 3 \\ 12 & -8 & 4 \end{bmatrix}.$$

(d):

$$\begin{aligned} & CA, \\ &= \begin{bmatrix} -13 \\ 32 \end{bmatrix}. \end{aligned}$$

Work:
Matrix multiplication, which is possible because the number of C's columns are equal to the number of A's rows: C_{23} A_{32}.

$$\begin{aligned} & CA, \\ &= \begin{bmatrix} 3(-1) + 2(3) + -4(4) \\ -8(-1) + 0(3) + 6(4) \end{bmatrix}, \end{aligned}$$

```
write_matex(C%*%A)
```

$$= \begin{bmatrix} -13 \\ 32 \end{bmatrix}.$$

(e): Because the dimensions don't work out (B_{31} D_{32}), we can't multiply.

(f):

$$B \otimes D,$$

$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

Work:

$$B \otimes D,$$

$$= \begin{bmatrix} 3 \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ -1 \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \\ 1 \begin{bmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix} \end{bmatrix}$$

```
write_matex(kronecker(B,D, FUN="*"))
```

$$= \begin{bmatrix} 18 & -6 \\ -3 & 9 \\ -9 & 24 \\ -12 & 4 \\ 2 & -6 \\ 6 & -16 \\ 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{bmatrix}.$$

(g):

$$CD,$$

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

Work:
Same situation as (d), because the dimensions work out: C_{23} and D_{32}.

$$CD,$$

$$= \begin{bmatrix} 3(6)+2(1)+(-4)(-3) & 3(-2)+2(3)+(-1)(8) \\ -8(6)+0(-1)+6(-3) & -8(-2)+0(3)+6(8) \end{bmatrix},$$

```
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(h):

$$DC,$$

$$= \begin{bmatrix} 34 & 12 & -36 \\ -27 & -2 & 22 \\ -73 & -6 & 60 \end{bmatrix}.$$

Work:
Yet again the same situation as (d), because the dimensions work out: D_{32} and C_{23}.

$$DC,$$

$$= \begin{bmatrix} 6(3)+(-2)(-8) & 6(2)+(-2)0 & 6(-4)+(-2)6 \\ -1(3)+3(-8) & -1(2)+3(0) & -1(-4)+3(6) \\ -3(3)+8(-8) & -3(2)+8(0) & -3(-4)+8(6) \end{bmatrix},$$

```
write_matex(C%*%D)
```

$$= \begin{bmatrix} 28 & -32 \\ -66 & 64 \end{bmatrix}.$$

(i):

$$C'C,$$

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$

Work:
First, transpose C:

```
write_matex(t(C))
```

$$= \begin{bmatrix} 3 & -8 \\ 2 & 0 \\ -4 & 6 \end{bmatrix}.$$

$$C'C,$$

$$= \begin{bmatrix} 3(3)+(-8)(-8) & 3(2)+(-8)0 & 3(-4)+(-8)6 \\ 2(3)+0(-8) & 2(2)+0(0) & 2(-4)+0(6) \\ -4(3)+6(-8) & -4(2)+6(0) & -4(-4)+6(6) \end{bmatrix},$$

```
write_matex(t(C)%*%C)
```

$$= \begin{bmatrix} 73 & 6 & -60 \\ 6 & 4 & -8 \\ -60 & -8 & 52 \end{bmatrix}.$$