Math HW6

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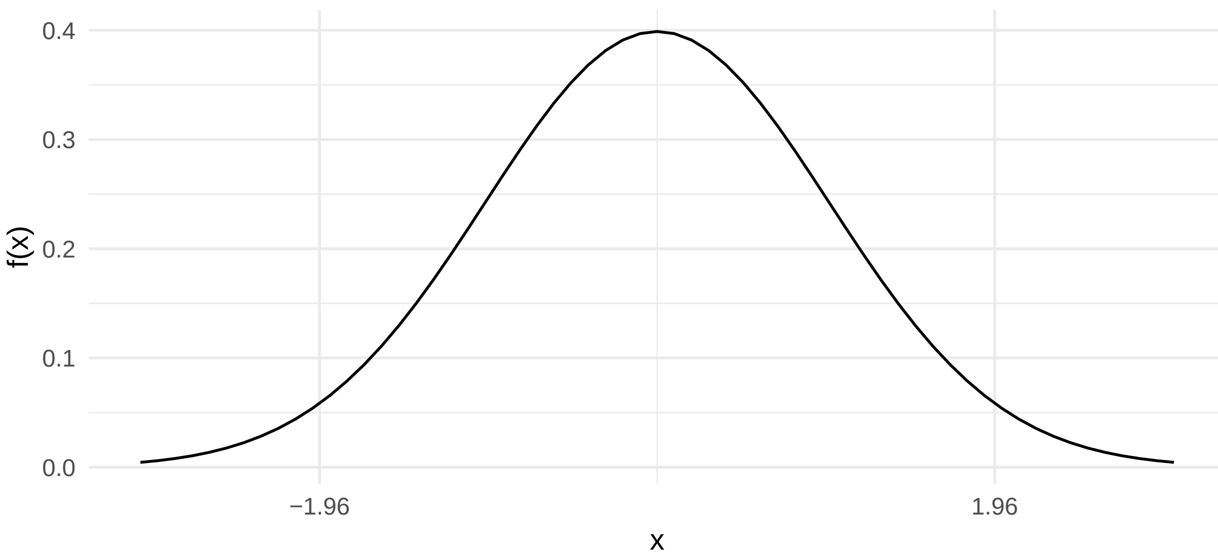
6.1, 6.2, 6.3a, 6.4a, 6.5a, 6.6

6.1

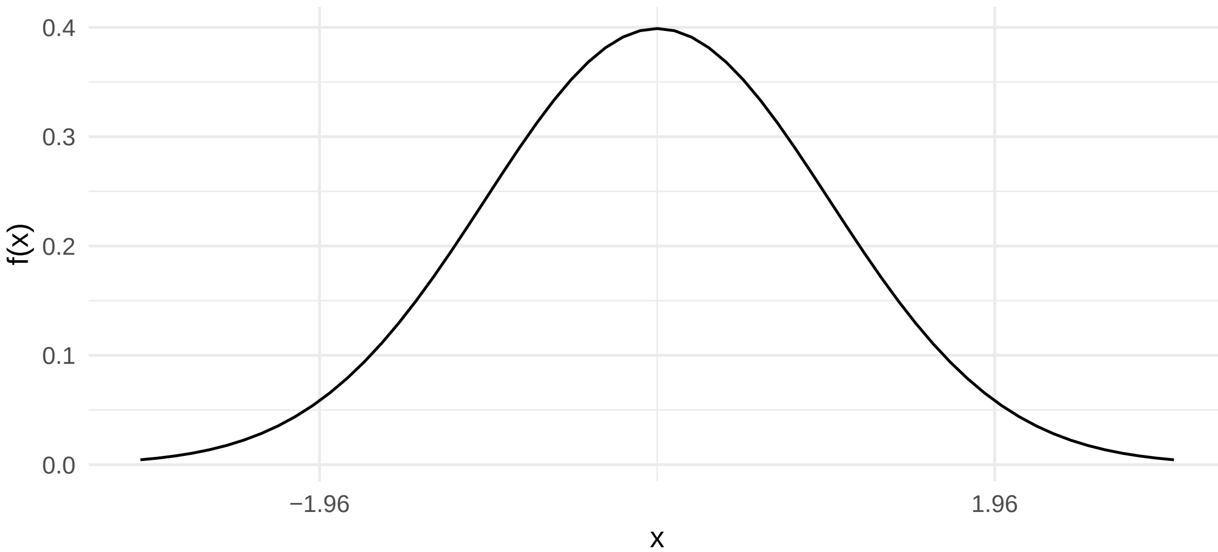
(a)

$$f(x) = \int_{-1.96}^{1.96} \frac{1}{\sqrt{2\pi}} e^{-0.5^2} dx$$

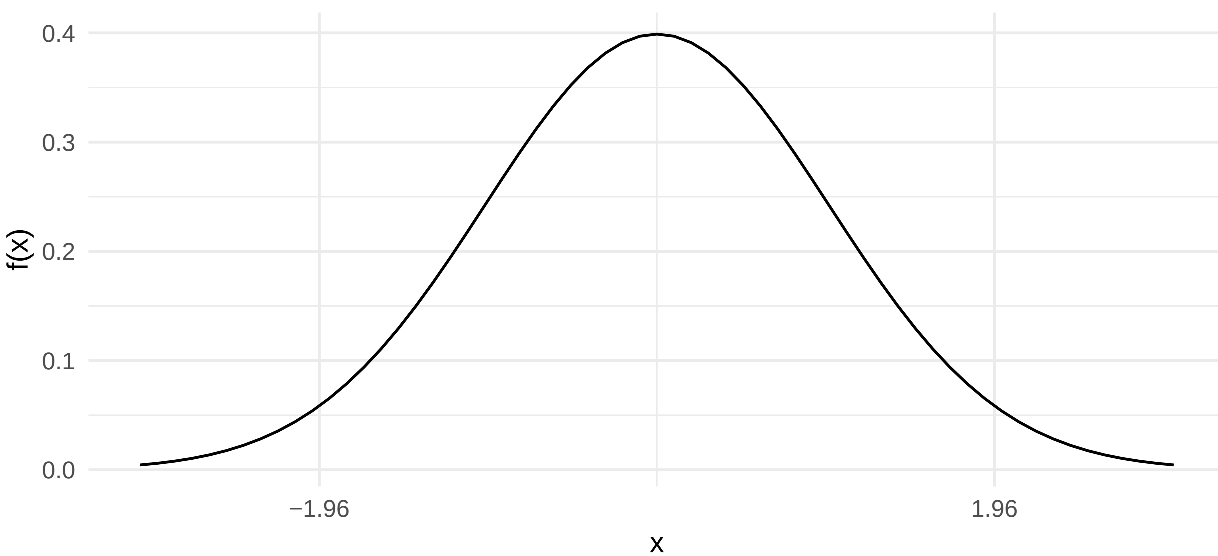
Left



Right



Midpoint



Trapezoidal

(b)

Left

Answer:
$A \approx \frac{0.392}{\sqrt{2\pi}} \left(\sum_{i=0}^{9} e^{-0.5^2\left(0.392i-1.96\right)^2} \right).$

Work:
The left Riemann Sum formula is:

$$A \approx \sum_{i=0}^{n-1} f\left(a + \frac{(b-a)i}{n}\right).$$

We want to find the sums for 10 partitions between -1.96 and 1.96. Which means that n = 10, a = -1.96, and = 1.96. We also plug in the pdf for f. The Reimann Sum formula gets substituted in for x in f(x):

$$A \approx \sum_{i=0}^{n-1} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(a+\frac{(b-a)i}{n}\left(\frac{b-a}{n}\right)\right)^2},$$

$$\approx \sum_{i=0}^{10-1} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-1.96+\frac{(1.96-(-1.96))i}{10}\left(\frac{1.96-(-1.96)}{10}\right)\right)^2},$$

$$\approx \sum_{i=0}^{9} \frac{1}{\sqrt{2\pi}} e^{-0.5\left((0.392i-1.96)0.392\right)^2},$$

0.392 does not depend on index, and neither does the constant $\frac{1}{\sqrt{(2\pi)}}$:

$$\approx \frac{1}{\sqrt{2\pi}}(0.392)\left(\sum_{i=0}^{9} e^{-0.5\left((0.392i-1.96)0.392\right)^2}\right),$$

$$\approx \frac{0.392}{\sqrt{2\pi}}\left(\sum_{i=0}^{9} e^{-0.5^2\left(0.392i-1.96\right)^2}\right).$$

```r
# calculation:
(1.96 + 1.96) / 10
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[1] 0.392
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Right:

Answer:

$A \approx \frac{0.392}{\sqrt{2\pi}}\left(\sum_{i=1}^{10} e^{-0.5^2\left(0.392i-1.96\right)^2}\right).$

Work:

Do the same thing here except the index is from 1 to n instead of from 0 to $n-1$.

$$A \approx \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-1.96+\frac{1.96-(-1.96)}{10}i\left(\frac{1.96-(-1.96)}{10}\right)\right)^2}\right),$$

$$\approx \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left((0.392i-1.96)0.392\right)^2},$$

$$\approx \frac{0.392}{\sqrt{2\pi}}\left(\sum_{i=1}^{10} e^{-0.5^2\left(0.392i-1.96\right)^2}\right).$$

Midpoint:

Answer:

$A \approx 0.392\left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-2.16+0.392i\right)^2}\right).$

Work:

$$A \approx \sum_{i=0}^{n-1} f\left(a + \frac{(b-a)(i-0.5)}{n}\right).$$

We want to find the sums for 10 partitions between -1.96 and 1.96. Which means that n = 10, a = -1.96, and = 1.96. We also plug in the pdf for f. The Reimann Sum formula gets substituted in for x in f(x):

$$A \approx \left(\sum_{i=0}^{n-1} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(a+\frac{(b-a)(i-0.5)}{n}\left(\frac{b-a}{n}\right)\right)^2}\right),$$

$$\approx \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-1.96+\frac{1.96-(-1.96)}{10}\left(\frac{(i-0.5)}{1}\right)\left(\frac{1.96-(-1.96)}{10}\right)\right)^2}\right),$$

$$\approx \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-1.96+0.392(i-0.5)(0.392)\right)^2} \right),$$

$$\approx 0.392 \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-1.96+0.392i-0.196\right)^2} \right),$$

$$\approx 0.392 \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-2.16+0.392i\right)^2} \right),$$

$$\approx 0.392 \left(\sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5\left(-2.16+0.392i\right)^2} \right).$$

```r
# Calculations
0.392 * 0.5
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[1] 0.196
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```r
-1.96 - 0.196
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[1] -2.16
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Trapezoidal

Answer:
$A \approx \left(\frac{0.196}{\sqrt{2\pi}}\right) \sum_{i=0}^{9} \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}.$

Work:

$$A \approx \sum_{i=0}^{n-1} \frac{b-a}{n} \frac{\frac{1}{\sqrt{2\pi}} e^{\left(a+\frac{(b-a)i}{n}\right)^2} + \frac{1}{\sqrt{2\pi}} e^{\left(a+\frac{(b-a)i+1}{n}\right)^2}}{2} \frac{b-a}{n},$$

$$\approx \sum_{i=0}^{10-1} \frac{1.96-(-1.96)}{10} \frac{\frac{1}{\sqrt{2\pi}} e^{\left((-1.96)+\frac{(1.96-(-1.96))i}{10}\right)^2} + \frac{1}{\sqrt{2\pi}} e^{\left((-1.96)+\frac{(1.96-(-1.96))i+1}{10}\right)^2}}{2},$$

$$\approx \sum_{i=0}^{9} (0.392) \frac{\frac{1}{\sqrt{2\pi}} e^{-0.5(0.392i-1.96)^2} + \frac{1}{\sqrt{2\pi}} e^{-0.5(0.392i-0.564)^2}}{2},$$

$$\approx \frac{0.392}{2} \left(\frac{1}{\sqrt{2\pi}}\right) \sum_{i=0}^{9} \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2},$$

$$\approx \left(\frac{0.196}{\sqrt{2\pi}}\right) \sum_{i=0}^{9} \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}.$$

```r
0.392 /2
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[1] 0.196
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