CatData HW6

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A4.5, A4.15 (3 points)

A4.31 (5 points)

A4.16-A4.17, A5.4 (10 points)

4.3

(a): In the linear probability model, the coefficient is treated the same way as it is in linear regression. That is, it represents a slope such that for every unit increase x (decade, in this case), there is a corresponding unit change in y (the probability of pitching a complete game, in this case, the change of which = -0.0694).

(b):

$$\hat{\pi} = 0.7578 - 0.0694(12),$$
$$= -0.075.$$

Percentages can't be negative! So not possible.

(c):

$$\hat{\pi} = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}.$$
$$\hat{\pi} = \frac{e^{1.148-0.315(12)}}{1+e^{1.148-0.315(12)}},$$
$$= 0.0671071.$$

This is much more plausible.

4.5

(a):

The model comes out to be 15.04 + (-0.23)x. When temperature is 0, the log odds are 15.04. The log odds decrease by 0.23 with every unit increase in temperature.

(b):

$$\hat{\pi} = \frac{e^{15.04-0.23(31)}}{1+e^{15.04-0.23(31)}}.$$
$$= 0.9996331.$$

(c):

You need to solve for x:

$$0.50 = \frac{e^{15.04-0.23(x)}}{1+e^{15.04-0.23(x)}}.$$

The temperature comes out to be 64.8. To get the linear approximation (i.e. rate of change), take the derivative of the logistic function above (with respect to x) to get its probability density function:

$$f'(x) = \frac{\beta e^{\alpha+\beta x}}{(1+e^{\alpha-\beta(x)})^2}$$

$$= \frac{15.04(e^{15.04-0.23(64.8)})}{(1+e^{15.04-0.23(64.8)})^2}$$

$$= -0.0580407.$$

At 64.8 degrees, for every unit increase in temperature, there is a 0.058 decrease in probability.

(d):

Since the coefficient is the log odds ratio, you exponentiate it to get the odds. Exp(-0.2321627) = 0.7928171. This seems kind of difficult to interpret though, so it might be easier to take $1/OR = 1/0.79 = 1.27$ and say that for every unit increase in temperature, the odds of the o-rings *working correctly* increase by 1.27 - that is, every unit increase in temperature means there is 27% greater odds of the orings working the way they're supposed to (or 27% greater probability if we think events are rare enough to warrent using the odds ratio as an approximation of the risk ratio).