

# Math HW6

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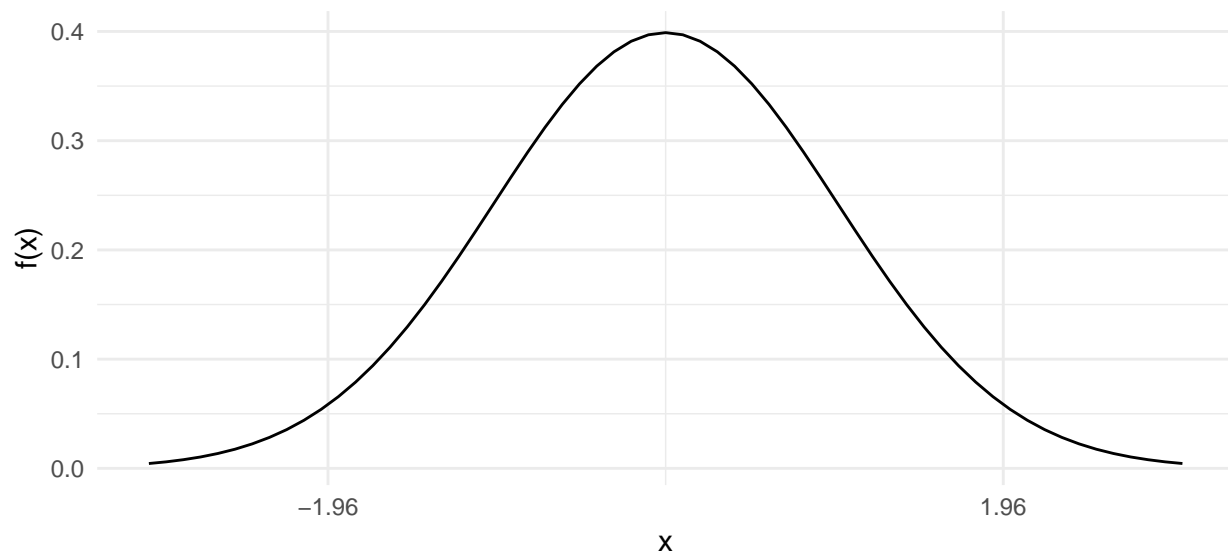
*All work and code are shown in the appendix.*

## 6.1

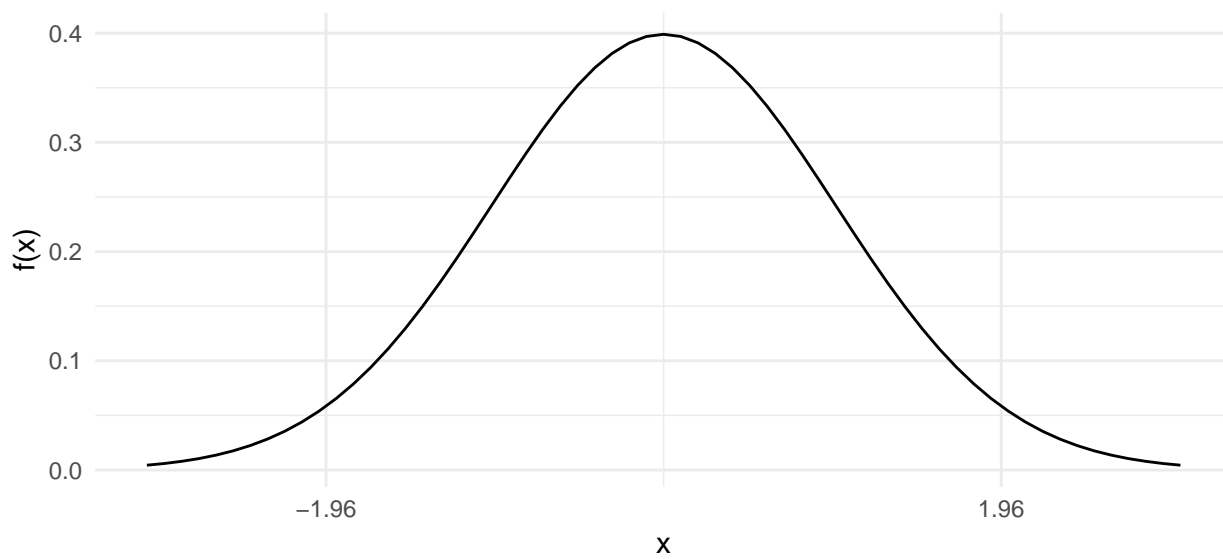
(a)

$$f(x) = \int_{-1.96}^{1.96} \frac{1}{\sqrt{2\pi}} e^{-0.5^2} dx$$

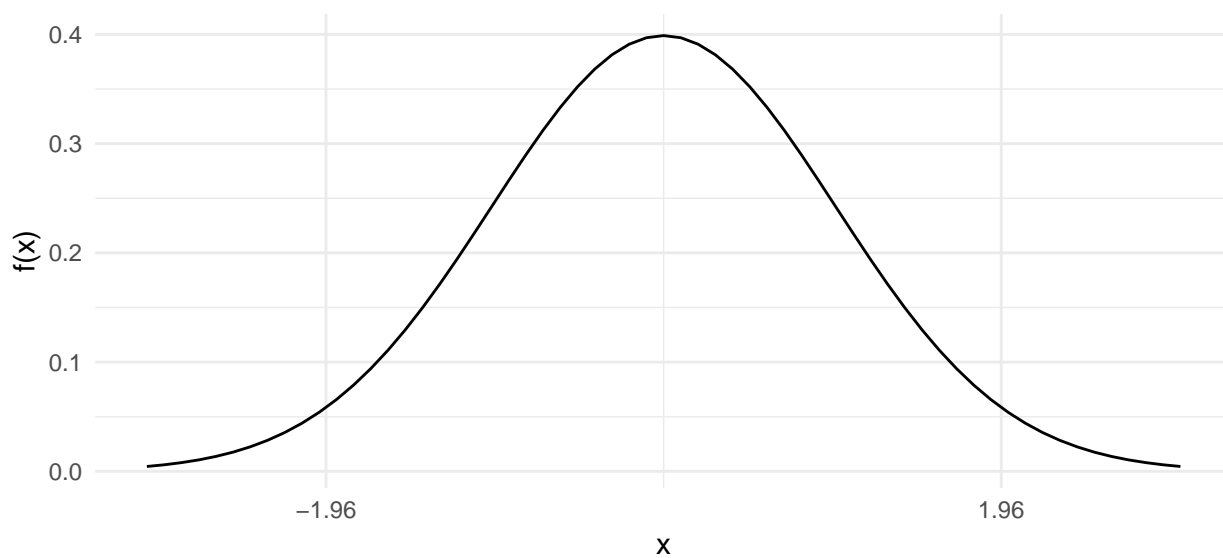
Left



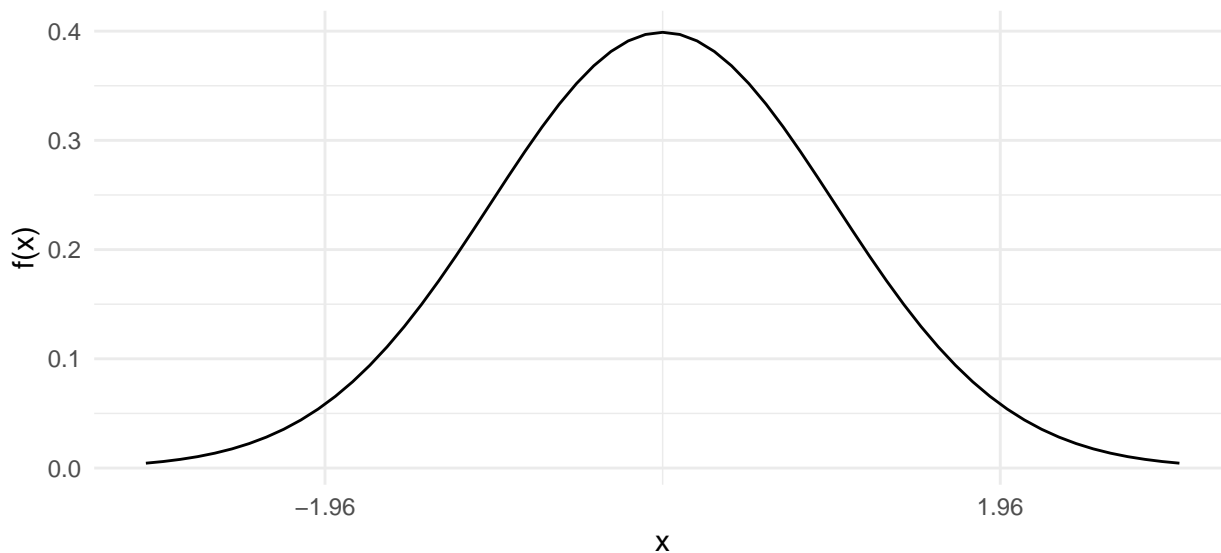
**Right**



**Midpoint**



## Trapezoidal



(b)

Left

**Answer:**

$$A \approx \frac{0.392}{\sqrt{2\pi}} \left( \sum_{i=0}^9 e^{-0.5^2 \left( 0.392i - 1.96 \right)^2} \right).$$

**Work:**

The left Riemann Sum formula is:

$$A \approx \sum_{i=0}^{n-1} f\left(a + \frac{(b-a)i}{n}\right).$$

We want to find the sums for 10 partitions between -1.96 and 1.96. Which means that  $n = 10$ ,  $a = -1.96$ , and  $b = 1.96$ . We also plug in the pdf for  $f$ . The Riemann Sum formula gets substituted in for  $x$  in  $f(x)$ :

$$\begin{aligned} A &\approx \sum_{i=0}^{n-1} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( a + \frac{(b-a)i}{n} \right)^2}, \\ &\approx \sum_{i=0}^{10-1} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -1.96 + \frac{(1.96 - (-1.96))i}{10} \right)^2}, \\ &\approx \sum_{i=0}^9 \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( (0.392i - 1.96)0.392 \right)^2}, \end{aligned}$$

0.392 does not depend on index, and neither does the constant  $\frac{1}{\sqrt{(2\pi)}}$ :

$$\approx \frac{1}{\sqrt{2\pi}} (0.392) \left( \sum_{i=0}^9 e^{-0.5 \left( (0.392i - 1.96)0.392 \right)^2} \right),$$

$$\approx \frac{0.392}{\sqrt{2\pi}} \left( \sum_{i=0}^9 e^{-0.5^2 \left( 0.392i - 1.96 \right)^2} \right).$$

```
# calculation:
(1.96 + 1.96) / 10
```

```
[1] 0.392
```

**Right:**

**Answer:**

$$A \approx \frac{0.392}{\sqrt{2\pi}} \left( \sum_{i=1}^{10} e^{-0.5^2 \left( 0.392i - 1.96 \right)^2} \right).$$

**Work:**

Do the same thing here except the index is from 1 to  $n$  instead of from 0 to  $n - 1$ .

$$\begin{aligned} A &\approx \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -1.96 + \frac{1.96 - (-1.96)}{10} i \left( \frac{1.96 - (-1.96)}{10} \right) \right)^2} \right), \\ &\approx \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( (0.392i - 1.96) 0.392 \right)^2}, \\ &\approx \frac{0.392}{\sqrt{2\pi}} \left( \sum_{i=1}^{10} e^{-0.5^2 \left( 0.392i - 1.96 \right)^2} \right). \end{aligned}$$

**Midpoint:**

**Answer:**

$$A \approx 0.392 \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -2.16 + 0.392i \right)^2} \right).$$

**Work:**

$$A \approx \sum_{i=0}^{n-1} f \left( a + \frac{(b-a)(i+0.5)}{n} \right).$$

We want to find the sums for 10 partitions between -1.96 and 1.96. Which means that  $n = 10$ ,  $a = -1.96$ , and  $b = 1.96$ . We also plug in the pdf for  $f$ . The Riemann Sum formula gets substituted in for  $x$  in  $f(x)$ :

$$\begin{aligned} A &\approx \left( \sum_{i=0}^{n-1} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( a + \frac{(b-a)(i+0.5)}{n} \left( \frac{b-a}{n} \right) \right)^2} \right), \\ &\approx \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -1.96 + \frac{1.96 - (-1.96)}{10} \left( \frac{i+0.5}{1} \right) \left( \frac{1.96 - (-1.96)}{10} \right) \right)^2} \right), \end{aligned}$$

$$\begin{aligned}
&\approx \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -1.96 + 0.392(i-0.5)(0.392) \right)^2} \right), \\
&\approx 0.392 \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -1.96 + 0.392i - 0.196 \right)^2} \right), \\
&\approx 0.392 \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -2.16 + 0.392i \right)^2} \right), \\
&\approx 0.392 \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -2.16 + 0.392i \right)^2} \right).
\end{aligned}$$

# Calculations

0.392 \* 0.5

[1] 0.196

-1.96 - 0.196

[1] -2.16

## Trapezoidal

**Answer:**

$$A \approx \left( \frac{0.196}{\sqrt{2\pi}} \right) \sum_{i=0}^9 \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}.$$

**Work:**

$$\begin{aligned}
A &\approx \sum_{i=0}^{n-1} \frac{b-a}{n} \frac{\frac{1}{\sqrt{2\pi}} e^{\left( a + \frac{(b-a)i}{n} \right)^2}}{2} + \frac{1}{\sqrt{2\pi}} e^{\left( a + \frac{(b-a)i+1}{n} \right)^2} \frac{b-a}{n}, \\
&\approx \sum_{i=0}^{10-1} \frac{1.96 - (-1.96)}{10} \frac{\frac{1}{\sqrt{2\pi}} e^{\left( (-1.96) + \frac{(1.96 - (-1.96))i}{10} \right)^2}}{2} + \frac{1}{\sqrt{2\pi}} e^{\left( (-1.96) + \frac{(1.96 - (-1.96))i+1}{10} \right)^2} \frac{1}{2}, \\
&\approx \sum_{i=0}^9 (0.392) \frac{\frac{1}{\sqrt{2\pi}} e^{-0.5(0.392i-1.96)^2} + \frac{1}{\sqrt{2\pi}} e^{-0.5(0.392i-0.564)^2}}{2}, \\
&\approx \frac{0.392}{2} \left( \frac{1}{\sqrt{2\pi}} \right) \sum_{i=0}^9 \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}, \\
&\approx \left( \frac{0.196}{\sqrt{2\pi}} \right) \sum_{i=0}^9 \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}.
\end{aligned}$$

0.392 /2

[1] 0.196

(c)

Left

Answer:

0.947

Work:

Just plug in to x each level of n from 0 - 9:

$$A \approx \frac{0.392}{\sqrt{2\pi}} \left( \sum_{i=0}^9 e^{-0.5^2 \left( 0.392i - 1.96 \right)^2} \right).$$

```
e <- exp(1) # get euler's number
(0.392 / (sqrt(2 * pi))) * (e^(-0.5 * ((0.392 * 0) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 2) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 3) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 4) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 5) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 6) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 7) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 8) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 9) - 1.96)^2)
)
```

[1] 0.947

Right

Answer:

0.947

\*\* Work:\*\*

Same thing, except runs from 1-10 instead of 0-9. Not surprising that it gives you the same answer:

```
e <- exp(1) # get euler's number
(0.392 / (sqrt(2 * pi))) * (e^(-0.5 * ((0.392 * 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 2) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 3) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 4) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 5) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 6) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 7) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 8) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 9) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 10) - 1.96)^2)
)
```

[1] 0.947

## Midpoint

**Answer:**

0.951

**Work:**

$$A \approx 0.392 \left( \sum_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-0.5 \left( -2.16 + 0.392i \right)^2} \right).$$

```
e <- exp(1) # get euler's number
(0.392 / (sqrt(2 * pi))) * (e^(-0.5 * ((0.392 * 1) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 2) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 3) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 4) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 5) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 6) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 7) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 8) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 9) - 2.16)^2) +
  e^(-0.5 * ((0.392 * 10) - 2.16)^2)
)
```

[1] 0.951

## Trapezoidal

**Answer:**

0.947

**Work:**

$$A \approx \left( \frac{0.196}{\sqrt{2\pi}} \right) \sum_{i=0}^9 \frac{e^{-0.5(0.392i-1.96)^2} + e^{-0.5(0.392i-0.564)^2}}{2}.$$

```
e <- exp(1) # get euler's number
(0.196 / (sqrt(2 * pi))) * (e^(-0.5 * ((0.392 * 0) - 1.96)^2) +
  e^(-0.5 * (0.392 * (0 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 1) - 1.96)^2) +
  e^(-0.5 * (0.392 * (1 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 2) - 1.96)^2) +
  e^(-0.5 * (0.392 * (2 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 3) - 1.96)^2) +
  e^(-0.5 * (0.392 * (3 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 4) - 1.96)^2) +
  e^(-0.5 * (0.392 * (4 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 5) - 1.96)^2) +
  e^(-0.5 * (0.392 * (5 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 6) - 1.96)^2) +
  e^(-0.5 * (0.392 * (6 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 7) - 1.96)^2) +
  e^(-0.5 * (0.392 * (7 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 8) - 1.96)^2) +
  e^(-0.5 * (0.392 * (8 + 1) - 1.96)^2) +
  e^(-0.5 * ((0.392 * 9) - 1.96)^2) +
  e^(-0.5 * (0.392 * (9 + 1) - 1.96)^2)
)
```

```

e^(-0.5 * (0.392 * (6 + 1) - 1.96)^2) +
e^(-0.5 * ((0.392 * 7) - 1.96)^2) +
e^(-0.5 * (0.392 * (7 + 1) - 1.96)^2) +
e^(-0.5 * ((0.392 * 8) - 1.96)^2) +
e^(-0.5 * (0.392 * (8 + 1) - 1.96)^2) +
e^(-0.5 * ((0.392 * 9) - 1.96)^2) +
e^(-0.5 * (0.392 * (9 + 1) - 1.96)^2)
)

```

[1] 0.947

## 6.2

(a)

**Answer:**

$$= \frac{x^{101}}{101} + 3e^x - 7 \frac{4^x}{1.39} + c.$$

**Work:**

$$\begin{aligned}
 F(x) &= \int x^{100} + 3e^x - 7(4^x)dx, \\
 &= \int x^{100}dx + \int 3e^x dx - \int 7(4^x)dx, \\
 &= \frac{x^{100+1}}{100+1} + \int 3e^x dx - \int 7(4^x)dx, \\
 &= \frac{x^{101}}{101} + 3 \int e^x dx - 7 \int (4^x)dx,
 \end{aligned}$$

The constant will cancel out, so leave it out.

$$\begin{aligned}
 &= \frac{x^{101}}{101} + 3e^x - 7 \frac{4^x}{\ln(4)} + c, \\
 &= \frac{x^{101}}{101} + 3e^x - 7 \frac{4^x}{1.39} + c.
 \end{aligned}$$

(b)

**Answer:**

87.7

**Work:**

$$\begin{aligned}
 F(x) &= \int_1^9 5\sqrt{x} dx + \frac{3}{x^4}dx, \\
 &= \int_1^9 5\sqrt{x} dx + \int \frac{3}{x^4}dx,
 \end{aligned}$$



$$\begin{aligned}
&= 5 \int_1^9 \sqrt{x} \, dx + 3 \int \frac{1}{x^4} dx, \\
&= 5 \int_1^9 x^{1/2} \, dx + 3 \int x^{-4} dx, \\
&= 5 \frac{x^{1/2+1}}{\frac{1}{2}+1} + 3 \frac{x^{-4+1}}{-4+1}, \\
&= 5 \frac{x^{3/2}}{\frac{3}{2}} + 3 \frac{x^{-3}}{-3}, \\
&= 5 \left( x^{3/2} \frac{2}{3} \right) + 3 \left( \frac{-1}{3} x^{-3} \right), \\
&= (x^{3/2}) \frac{10}{3} + (3) \frac{1}{x^3}.
\end{aligned}$$

From here, plug in the bounds and take the difference:

$$(9^{3/2} * 10/3 - 1/9^3) - (1^{3/2} * 10/3 - 1/1^3)$$

[1] 87.7

(c)

**Answer:**

6

**Work:**

$$\begin{aligned}
F(x) &= \int_2^\infty \frac{12}{x^2} dx, \\
&= \lim_{k \rightarrow \infty} \int_2^k \frac{12}{x^2} dx, \\
&= 12 \lim_{k \rightarrow \infty} \int_2^k x^{-2} dx, \\
&= 12 \lim_{k \rightarrow \infty} \int_2^k \frac{x^{-2+1}}{-2+1} dx, \\
&= 12 \lim_{k \rightarrow \infty} \frac{-1}{x}.
\end{aligned}$$

Plug in the bounds (plugging for the  $-1/x$  since it approaches zero the higher x gets):

$$12 * (0 + 1/2)$$

[1] 6

(d)

**Answer:**

$2y^2$ .

**Work:**

$$F(x) = \frac{d}{dy} \int_{-3}^{y^2} \sqrt{x} \, dx,$$

$$= \sqrt{y^2} \frac{d}{dy} y^2,$$

Power rule...

$$\begin{aligned} &= \sqrt{y^2} \, 2y, \\ &= 2y^2. \end{aligned}$$

(e)

**Answer:**

$$-e^{\sqrt{z}+\ln(z)} \left( \frac{1}{2\sqrt{z}} + \frac{1}{z} \right).$$

**Work:**

$$\begin{aligned} F(x) &= \frac{d}{dz} \int_{\sqrt{z}+\ln(z)}^{10} e^x dx, \\ &= -1 \times \frac{d}{dz} \int_{\sqrt{z}+\ln(z)}^{10} e^x dx, \\ &= -\frac{d}{dz} \int_{10}^{\sqrt{z}+\ln(z)} e^x dx, \\ &= -e^{\sqrt{z}+\ln(z)} \frac{d}{dy} \sqrt{z} + \ln(z), \\ &= -e^{\sqrt{z}+\ln(z)} \left( \frac{1}{2\sqrt{z}} + \frac{1}{z} \right). \end{aligned}$$

**6.3a**

**Answer:**

$$\frac{(5x^{10} - 25x^4 + 15x)^8}{40} + c.$$

**Work:**

$$\begin{aligned} &\int (10x^9 - 20x^3 + 3)(5x^{10} - 25x^4 + 15x)^7 dx, \\ &\int (10x^9 - 20x^3 + 3)(5x^{10} - 25x^4 + 15x)^7 dx. \end{aligned}$$

$$\begin{aligned} u &= 5x^{10} - 25x^4 + 15x, \\ \frac{du}{dx} &= \frac{d}{dx}(5x^{10} - 25x^4 + 15x) \\ \frac{du}{dx} &= \frac{d}{dx}(5x^{10}) - \frac{d}{dx}(25x^4) + \frac{d}{dx}(15x), \\ \frac{du}{dx} &= 5(10x^9) - 25(4x^3) + 15, \end{aligned}$$

$$\begin{aligned}\frac{du}{dx} &= 50x^9 - 100x^3 + 15, \\ \frac{1}{5} \frac{du}{dx} &= (50x^9 - 100x^3 + 15) \frac{1}{5}, \\ \frac{1}{5} \frac{du}{dx} &= 10x^{10} - 20x^4 + 5x, \\ \frac{1}{5} du &= (10x^{10} - 20x^4 + 5x) dx.\end{aligned}$$

Substitute in  $u$  and  $1/5 \, du$ :

$$\begin{aligned}\int (10x^9 - 20x^3 + 3)(5x^{10} - 25x^4 + 15x)^7 dx, \\ \int (u)^7 \left(\frac{1}{5} du\right), \\ \frac{1}{5} \int (u)^7 (du), \\ \frac{1}{5} \int \frac{u^8}{8} + c, \\ \frac{u^8}{40} + c, \\ \frac{(5x^{10} - 25x^4 + 15x)^8}{40} + c.\end{aligned}$$

## 6.4a

**Answer:**

20.9

$$\int_1^4 x \sqrt{x+5} \, dx,$$

We want to get to:

$$\int u \, dv = uv - \int v \, du.$$

Use some u-substitution:

$$\begin{aligned}u &= x, \quad dv = \sqrt{x+5} \, dx. \\ \frac{du}{dx} &= 1, \text{ therefore } du = dx. \\ w &= x + 5, \\ \frac{dw}{dx}(x+5) &= 1 + 0 = 1, \\ dw &= dx. \\ \int \sqrt{w} \, dw, \\ \int w^{\frac{1}{2}} (1), \\ \frac{w^{\frac{1}{2}+1}}{\frac{1}{2}+1},\end{aligned}$$

$$\frac{w^{\frac{3}{2}}}{\frac{3}{2}},$$

$$\frac{2}{3}w^{\frac{3}{2}},$$

$$\frac{2}{3}(x+5)^{\frac{3}{2}} = v.$$

Plug back into:

$$\int u \, dv = uv - \int v \, du,$$

with

$$u = x.$$

$$dv = \sqrt{x+5} \, dx.$$

$$du = dx = dw.$$

$$w = x + 5.$$

$$v = \frac{2}{3}(x+5)^{\frac{3}{2}}.$$

$$\int_1^4 x \sqrt{x+5} \, dx = x \left( \frac{2}{3}(x+5)^{\frac{3}{2}} \right) - \int_1^4 \frac{2}{3}(x+5)^{\frac{3}{2}} \, dx,$$

$$\int_1^4 x \sqrt{x+5} \, dx = x \left( \frac{2}{3}(x+5)^{\frac{3}{2}} \right) - \frac{2}{3} \int_1^4 (x+5)^{\frac{3}{2}} \, dx.$$

Before we can start plugging in numbers, gotta take care of the remaining integral:

$$\int (x+5)^{\frac{3}{2}} \, dx,$$

$$\int (x+5)^{\frac{3}{2}} \, dx,$$

$$u = x + 5,$$

$$\int (u)^{\frac{3}{2}} \, dx,$$

$$\int \frac{(u)^{\frac{3}{2}+1}}{\frac{3}{2}+1} \, dx,$$

$$\frac{(u)^{\frac{5}{2}}}{\frac{5}{2}},$$

$$\frac{2}{5}(u)^{\frac{5}{2}},$$

$$\frac{2}{5}(x+5)^{\frac{5}{2}}.$$

So in the end you have:

$$\int_1^4 x \sqrt{x+5} \, dx = x \left( \frac{2}{3}(x+5)^{\frac{3}{2}} \right) - \frac{2}{3} \left( \frac{2}{5}(x+5)^{\frac{5}{2}} \right).$$

Plug in the bounds:

$$((2/3 * 4 * ((4+5)^(3/2))) - 2/3 * (2/5 * (4 + 5)^(5/2))) -$$

$$((2/3 * 1 * ((1+5)^(3/2))) - 2/3 * (2/5 * (1 + 5)^(5/2)))$$

[1] 20.9

## 6.5a

**Answer:**

291

**Work:** Second theorem:

$$\int_a^b f(x) \, dx = F(x) \Big|_a^b = F(b) - F(a).$$

$$\begin{aligned} F(x) &= \int_{\ln(8)}^{\sqrt[3]{167}} x^2 + e^x \, dx, \\ &= \int_{\ln(8)}^{\sqrt[3]{167}} x^2 + \int_{\ln(8)}^{\sqrt[3]{167}} e^x \, dx, \\ &= \frac{x^3}{3} + e^x + c, \end{aligned}$$

$$= \left( \frac{x^3}{3} - \frac{x^3}{3} \right) + \left( (e^x + c) - (e^x + c) \right),$$

the constant cancels out

$$\begin{aligned} &= \left( \frac{(\sqrt[3]{167})^3}{3} - \frac{\ln(8)^3}{3} \right) + \left( e^{\sqrt[3]{167}} - e^{\ln(8)} \right), \\ &= \left( \frac{(\sqrt[3]{167})^3}{3} - \frac{\ln(8)^3}{3} \right) + \left( e^{\sqrt[3]{167}} - e^{\ln(8)} \right), \\ &= 291. \end{aligned}$$

```
e <- exp(1)
167 / 3 - (log(8))^3 / 3 + e^(167^(1/3)) - e^(log(8))
```

[1] 291

## 6.6

(a)

**Answer:**

**Work:**