Math HW7

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7.6 (sketch its level sets as well), 7.7, 7.10

7.1a

Answer: $f(a,b,c,d,e,f) = a \times b \times c \times d \times e \times f$

7.3a

Answer: 5

Work:

Plug in 3 and 1:

```
(3^2 - 3 * 1 + (6 * 1)^2) / (3 - 3 * 1)
```

```
[1] Inf
```

So that doesn't work. Need to find some way to cancel out the denominator.

$$x^2 - xy + 6y^2,$$

Set y equal to 1:

$$= x^2 - (x \cdot 1) + (6 \cdot 1)^2,$$
$$= x^2 - x + 6,$$
$$= (x-3)(x+2).$$

Plug y back in:

$$= (x - 3y)(x + 2y).$$

Check work:

```
from __future__ import division
from sympy import *
x, y = symbols('x y')
print(expand((x - 3*y) * (x + 2*y)))
```

```
x**2 - x*y - 6*y**2
```

plug back into original expression:

$$\frac{(x-3y)(x+2y)}{x-3y},$$
$$= x + 2y.$$

Plugging in x and y values:

```
3 + 2 * 1
```

```
[1] 5
```

7.6 (sketch its level sets as well)

(a)

Answer: $\nabla f(xy) = \begin{bmatrix} y - 2x + 2 \\ x - 2y + 1 \end{bmatrix}$.

Work:
Gradient = first partial derivative. Need to find one for x and one for y, and construct a matrix where row 1 is the first partial derivative with respect to x and row 2 is the first partial derivative with respect to y.
With respect to x:

$$\frac{\partial}{\partial x}(-x^2 + xy - y^2 + 2x + y),$$
$$= \frac{\partial}{\partial x}(-x^2) + \frac{\partial}{\partial x}(x) \cdot \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial x}(2) \cdot \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial x}(y),$$
$$y - 2x + 2.$$

With respect to y:

$$\frac{\partial}{\partial y}(-x^2 + xy - y^2 + 2x + y),$$
$$= \frac{\partial}{\partial y}(-x^2) + \frac{\partial}{\partial y}(x) \cdot \frac{\partial}{\partial y}(y) - \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial y}(2) \cdot \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial y}(y),$$
$$= x - 2y + 1.$$

Therefore, the gradient is:

$$\nabla f(xy) = \begin{bmatrix} y - 2x + 2 \\ x - 2y + 1 \end{bmatrix}.$$