SNR estimation for CO paper

Mathilde Van Cuyck

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Abstract

On scales k<k_{lim}=1.8×10^{-1}Mpc^{-1}, what is the SNR on the cross-power spectra measurement between a line J_{up} at $z \pm dz$ and a galaxy spectroscopic survey complete down to $\mathrm{M}_{*}^{\mathrm{lim}} = 10^{10}\mathrm{M}_{\odot}$, as observed by CONCERTO in a $\Omega_{\mathrm{field}} = 1.5\mathrm{deg}^2$ field for 600h ?

1 Sensitivity

I load the file CONCERTO-SENSITIVITY/SIDES-sensitivity-1Ghz-res.txt of Matthieu.

The beam Ω_{beam} and the NEFD [mJy/beam] at ν_{obs}=J_{up}*115.27120180GHz/(1+z) are interpolated from the file.

The NEI in the frequency channel of width $\mathrm{D}\nu = \mathrm{dz} * \nu_{\mathrm{obs}}/(1+\mathrm{z})$ is then:

$$\mathrm{NEI}_{\nu_{\mathrm{obs}}}[\mathrm{MJy/sr.s}^{(1/2)}] = 10^{-9} * \mathrm{NEFD}_{\nu_{\mathrm{obs}}}/\Omega_{\mathrm{beam}}/\mathrm{D}\nu \tag{1}$$

2 Power spectra

I measure the cross-power spectrum $\mathrm{P}_{\mathrm{J_{up}}\times\mathrm{G}}$ in (the 117deg^2) SIDES-Uchuu map of 50$arcsecs$ resolution, centered on ν_{obs} and of spectral width $\mathrm{D}\nu$ = dz * $\nu_{\mathrm{obs}}/(1+\mathrm{z})$. I compute the cross-power shot noise from the catalogue.

In the same time, I also measure the auto-power spectrum $\mathrm{P}_{\mathrm{J_{up}}}$, the galaxy auto-power spectrum P_{G} and their respective shot noise.

3 Conversion from 2D to 3D

Then I convert my 2D cross-power spectrum to 3D.

$\chi(z)$ is the comoving radial distance:

$$\chi(\mathrm{z}) = \int_0^{\mathrm{z}} \mathrm{c/H(z)dz} \tag{2}$$

The differential comoving distance $\delta\chi$ corresponding to dz is:

$$\delta\chi = \frac{\mathrm{c[km/s](1+z)} \times \mathrm{D}\nu}{\mathrm{H(z)} \times \nu_{\mathrm{obs}}} \tag{3}$$

So the conversion factor from 3D to 2D for **the power spectrum** is: $1/(\chi^2 \times \delta\chi)$.

The conversion factor from 3D to 2D for **the wavenumber is**: $\chi/2\pi$ (to get k in [rad^{-1}])

4 Physical comoving volumes

Moreover, the full comoving volume covered by the CONCERTO 'J_{up}' survey in the bin z±dz/2 is:

$$\mathrm{V_{tot}} = \frac{4}{3}\pi \times (\chi(\mathrm{z}+\mathrm{dz}/2)^3 - \chi(\mathrm{z}-\mathrm{dz}/2)^3). \tag{4}$$

So the comoving volume actually covered by CONCERTO is:

$$\mathrm{V_{survey}} = \frac{\Omega_{\mathrm{field}} \times \mathrm{V_{tot}}}{4\pi} \tag{5}$$

The pixel comoving volume is:

$$\mathrm{V_{pix}} = \mathrm{D_c(z)} \times \Omega_{\mathrm{beam}} * \mathrm{y} * \mathrm{d}\nu \tag{6}$$

where dnu=1.5GHz is the absolute spectral resolution of the instrument and y is the factor to convert the frequency intervals to the comoving distance at the wavelength λ:

$$y = \lambda \frac{(1+z)^2}{H(z)} = \frac{c[km/s](1+z)^2}{H(z) * \nu_{obs} * (1+z)} \tag{7}$$

4.1 Consideration on the bandwidth...

Here I guess that the main assumption is that Eq. 1 which gives a equivalent noise for the entire slice [z-dz/2 - z+dz/2] is well representative of the noise in our data which are actually binned with a spectral resolution dν...

This is important to test, because indeed setting dν = Dν make the SNR drops by orders of magnitudes.

I made this test:
because $\int NEFD(\nu)d\nu$ should be conserved, like $\int S_\nu d\nu$ is conserved, Eq. 1 should give the same result than FluxConservingResampler() from *specutils.manipulation*. This method resamples (NEFD, nu) of CONCERTO-SENSITIVITY/SIDES-sensitivity-1Ghz-res.txt in a NEFD for $\nu_{obs} \pm D\nu/2$. I find that this method gives an NEFD 10 times larger than Eq.1.

However, if measuring the power spectrum on Dν is equivalent to 'summing N=Dν/dν channels', then the measured power spectra should be multiplied by N.

5 Noise auto-power spectrum

Assuming a pixel sees an area of the size of the instrumental beam, the observing time per pixel is given by:

$$\mathrm{t_{obs/pix}} = \mathrm{t_{survey}} \frac{\mathrm{n_{pix}} \Omega_{\mathrm{beam}}}{\Omega_{\mathrm{field}}} \tag{8}$$

Assuming a **spherically-averaged** (i.e, 'a 3D power spectrum measurement') power spectrum, the variance of the power spectrum is [LFO+11]:

$$\mathrm{var}[\bar{\mathrm{P}}_{\mathrm{tot}}(\mathrm{k})] = \frac{[\mathrm{P^{clust}(k)} + \mathrm{P^{shot\ noise}} + \bar{\mathrm{P}}^{\mathrm{noise}}]^2}{\mathrm{N_{modes}(k,z)}} \tag{9}$$

where the number of modes is:

$$\mathrm{N_{modes}(k,z)} = 2\pi \mathrm{k}^2 \Delta \mathrm{k} \frac{\mathrm{V_{survey}}}{(2\pi)^3} \tag{10}$$

with Δk the Fourier bin size. Both k and Δk are in Mpc^{-1}.

The averaged noise power spectrum in Eq. 9 is:

$$\bar{\mathrm{P}}^{\mathrm{noise}}(\mathrm{k})[\mathrm{MJy^2/sr^2.Mpc^3}] = \mathrm{V_{pix}} \frac{\mathrm{NEI}^2}{\mathrm{t_{obs/pix}}} \tag{11}$$

Then the uncertainty in the auto pseudo-power spectrum at each k bin is given simply as [Chu23]:

$$\sigma = \sqrt{\mathrm{var}} = \frac{\mathrm{P(k)} + \mathrm{P_N}}{\sqrt{\mathrm{N_{modes}(k)}}} \tag{12}$$

The signal-to-noise rayio of the auto-power spectrum in clustering regime is:

$$\mathrm{SNR} = \sqrt{\sum_{\mathrm{bins}} (\frac{\mathrm{P^{clust}(k)}}{\sigma})^2} \tag{13}$$

computed only over the bins lower than the k_{lim} from the abstract.

6 LIM x galaxies

The standard deviation in the measurement of the cross-power spectrum $P_{\mathrm{J_{up}\times G}}(k)$ of the intensity map and an overlapping optical spectroscopic redshift survey, in a Fourier bin k of width Δk containing N_{mode} unique Fourier modes, is [WBW17]:

$$\sigma_{\times}(\mathrm{P_{J_{up}\times G}}) = \frac{1}{\sqrt{2\,\mathrm{N_{mode}}}}\sqrt{\mathrm{P^2_{J_{up}\times G}} + (\mathrm{P_J} + \mathrm{P_{noise}})\left(\mathrm{P_G} + \frac{1}{\mathrm{n_g}}\right)} \tag{14}$$

in terms of the surface brightness power spectrum $P_{J_{up}}(k)$ and galaxy power spectrum $P_G(k)$, where $1/n_g$ is the galaxy shot power in terms of the galaxy number density n_g.

At k<k_{lim}, the shot noises are negligible...

The final SNR on the cross-power spectrum is:

$$\mathrm{SNR} = \sqrt{\sum_{\mathrm{bins}}(\frac{\mathrm{P_{J_{up}\times G}(k)}}{\sigma_{\times}})^2} \tag{15}$$

summed over the k bins $< k_{\mathrm{lim}}$.

7 Sensitivity at n σ

The noise power spectrum is defined for the auto-power spectrum, but not for the cross-power spectrum... However we can define a cross-power spectrum such as it is detected at nσ, i.e, $\mathrm{P_{J_{up}\times G}}/\sigma_{\mathrm{X}} = \mathrm{n}$

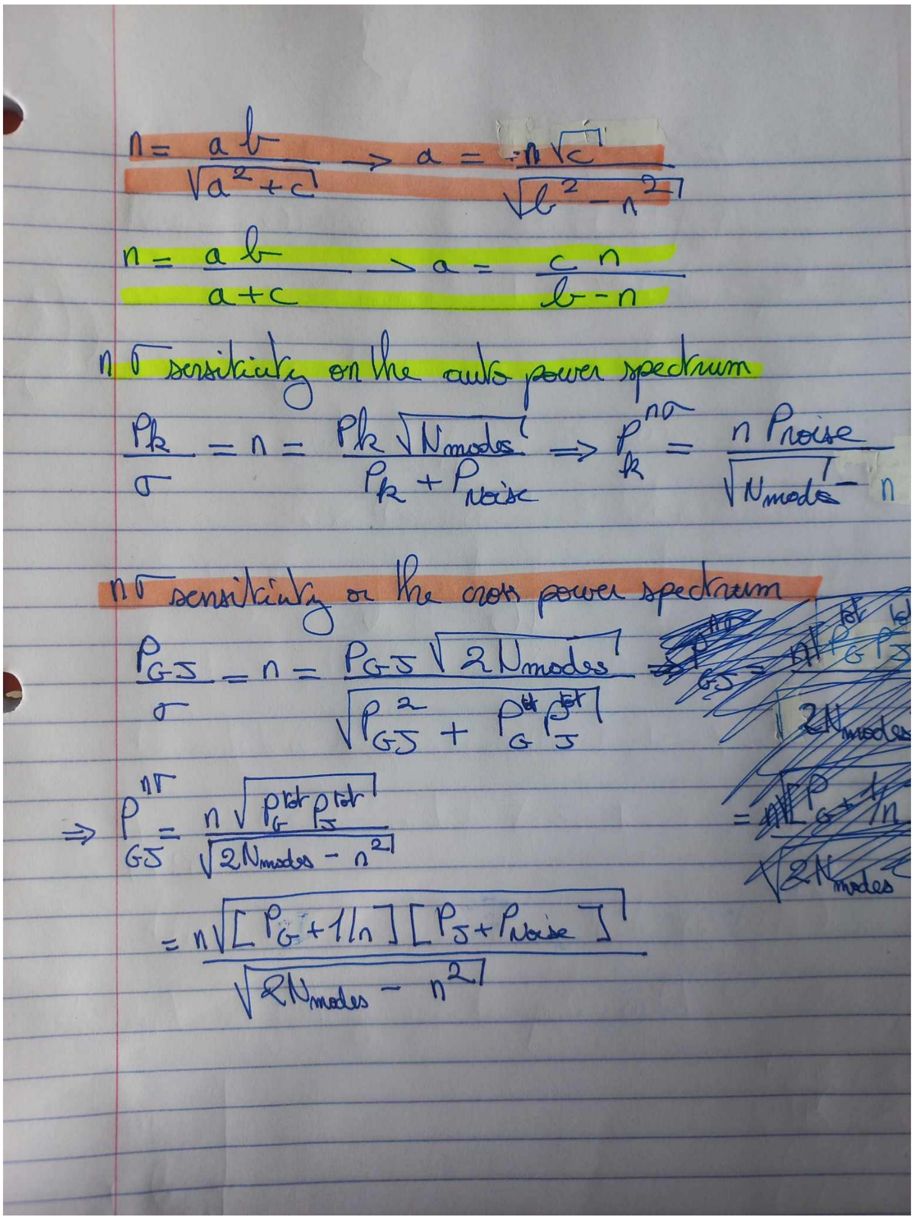
it gives: Fig. 1

8 results

dz=0.1
J=3-Gal, z=1 | SNR = 2.1 See Fig. 2.
J=4-Gal, z=1 | SNR = 1.5
J=5-Gal, z=1 | SNR = 1.2
J=3-Gal, z=1.5 | SNR = 2.1
J=4-Gal, z=1.5 | SNR = 1.7

dz=0.3
J=3-Gal, z=1 | SNR = 5.6
J=4-Gal, z=1 | SNR = 4.2
J=5-Gal, z=1 | SNR = 3.5
J=3-Gal, z=1.5 | SNR = 6.1
J=4-Gal, z=1.5 | SNR = 5.1

dz=0.5 rises the SNR by 0.1 on average...



$$n = \frac{ab}{\sqrt{a^2+c}} \Rightarrow a = \frac{n\sqrt{c}}{\sqrt{b^2-n^2}}$$

$$n = \frac{ab}{a+c} \Rightarrow a = \frac{cn}{b-n}$$

nσ sensitivity on the auto power spectrum

$$\frac{P_k}{\sigma} = n = \frac{P_k\sqrt{N_{modes}}}{P_k+P_{noise}} \Rightarrow P_k^{n\sigma} = \frac{n P_{noise}}{\sqrt{N_{modes}}-n}$$

nσ sensitivity on the cross power spectrum

$$\frac{P_{GJ}}{\sigma} = n = \frac{P_{GJ}\sqrt{2N_{modes}}}{\sqrt{P_{GJ}^2 + P_G^{tot}P_J^{tot}}}$$

$$\Rightarrow P_{GJ}^{n\sigma} = \frac{n\sqrt{P_G^{tot}P_J^{tot}}}{\sqrt{2N_{modes}-n^2}}$$

$$= \frac{n\sqrt{[P_G+1/n][P_J+P_{noise}]}}{\sqrt{2N_{modes}-n^2}}$$

Figure 1: Nσ sensitivity

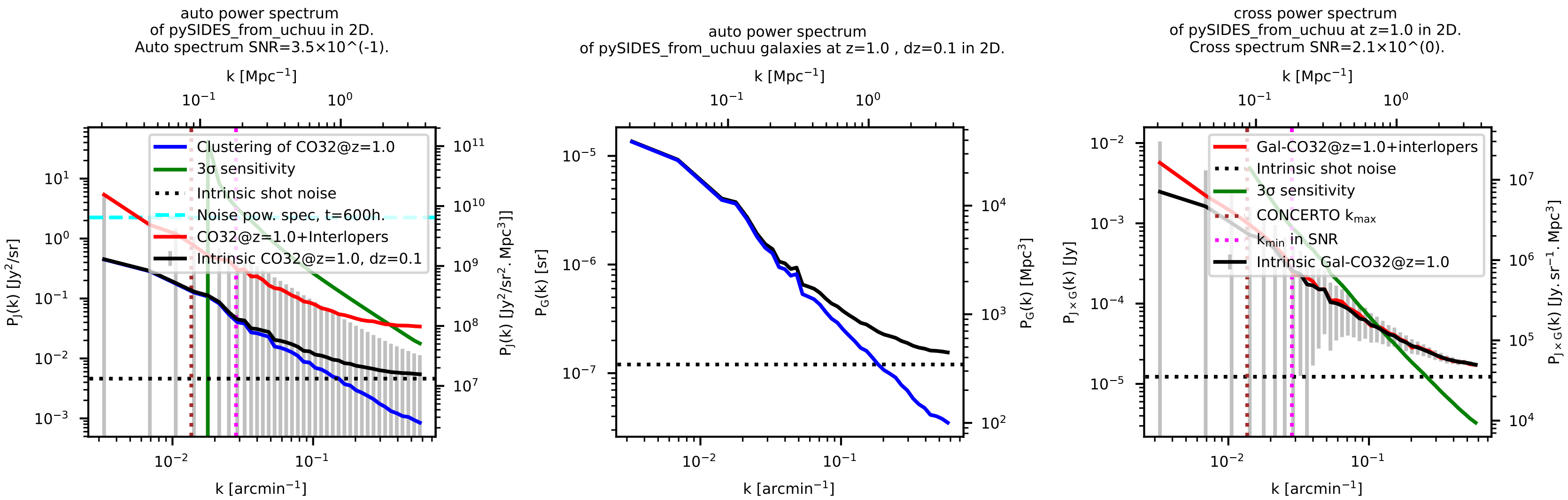


Figure 2: Figure for the results in bold in Sect.8. pink vertical line highlight the k range I use to compute the SNR. The brown vertical line highlight the minimum k accessible with CONCERTO.

References

[Chu23] Dongwoo T. Chung. Constraining the halo-ISM connection through multi-transition carbon monoxide line-intensity mapping. *arXiv e-prints*, page arXiv:2309.03184, September 2023.

[LFO+11] Adam Lidz, Steven R. Furlanetto, S. Peng Oh, James Aguirre, Tzu-Ching Chang, Olivier Doré, and Jonathan R. Pritchard. Intensity Mapping with Carbon Monoxide Emission Lines and the Redshifted 21 cm Line. , 741(2):70, November 2011.

[WBW17] L. Wolz, C. Blake, and J. S. B. Wyithe. Determining the H I content of galaxies via intensity mapping cross-correlations. , 470(3):3220–3226, September 2017.